

Quantum theory of dissipation of a harmonic oscillator coupled to a nonequilibrium bath; Wigner-Weisskopf decay and physical spectra

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Abstract

We extend the quantum theory of dissipation in the context of system-reservoir model, where the reservoir in question is kept in a nonequilibrium condition. Based on a systematic separation of time scales involved in the dynamics, appropriate generalizations of the fluctuation- dissipation and Einstein's relations have been pointed out. We show that the Wigner-Weisskopf decay of the system mode results in a rate constant which depending on the relaxation of nonequilibrium bath is dynamically modified. We also calculate the time-dependent spectra of a cavity mode with a suitable gain when the cavity is kept in contact with a nonequilibrium bath.

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I. INTRODUCTION

The problem of dissipative dynamics in quantum system is a key issue in physics and chemistry today. The system-reservoir model describing the evolution of the quantum system coupled to a much larger system regarded as a reservoir has been the standard paradigm for dissipation in classical and quantum systems for many years [1-3]. A popular variant of this model is the spin-boson model, a magnetic dipole coupled to a Boson field; another is a two-level atom in contact with a continuum of radiation field modes. These models describe a large variety of physical situations, such as, spontaneous emission, polaron formations, exiton motion, macroscopic quantum tunneling etc., in atomic physics, solid state physics and quantum optics.

In general, two very distinct situations emerge depending on the strength of the coupling constant between the system and the reservoir. In the weak coupling case, the behavior of the former is only slightly affected by the reservoir which essentially behaves as a free field. In the strong coupling cases (as in polaron theories) the polarization of the reservoir field by the system can not be ignored. The second important approximation that is almost always made is that the correlation time of the reservoir must be very short enough (Markov approximation) for the interaction between the system and the bath to be small (weak coupling approximation). Although several generalizations of the theory [3] which go beyond these approximation schemes are now available which describe the various interesting physical situations in condensed matter and quantum optical physics, one essential step is the assumption of an equilibrium distribution of the reservoir modes. Very little attention has been paid to the problem where the reservoir in contact with the system is itself not in equilibrium. This nonstationarity of the bath is known to affect the kinetics of the system [4-6] leading to nonexponential decay in contrast to exponential decay in equilibrium activation rate theories. Thus the relaxation of the nonequilibrium modes may influence the dissipation of the system in question in a nontrivial way. The present paper addresses a related issue pertaining to quantum optical situations.

We extend the quantum theory of dissipation of a harmonic oscillator coupled to a bath where

the bath in question is in a nonequilibrium condition. The nonequilibrium bath is effectively realized in terms of a semi-infinite dimensional broad-band reservoir which itself kept in contact with a thermal reservoir. We make use of a systematic time scale separation to construct the appropriate Langevin dynamics of the system mode. The fluctuation-dissipation and the Einstein relations have been suitably generalized. A detailed study of two model cases as immediate applications has been carried out. We show that the Wigner-Weisskopf decay rate constant of the system mode is dynamically modified when the system is coupled to a set of relaxing modes. We also calculate the transient noise-spectra of the cavity mode with a positive gain. The result is remarkably different from the steady-state spectra.

The outline of the paper is as follows: In section II we first generalize the quantum theory of dissipation of a harmonic oscillator for a bath which is not in thermal equilibrium, followed by a derivation of fluctuation-dissipation theorem in the next section III. Section IV is devoted to the discussion of the Wigner-Weisskopf decay of the system. In section V we calculate the transient noise spectra of the system mode with a positive gain. The paper is concluded in section VI.

II. THE MODEL AND THE EQUATIONS OF MOTION

To start with we consider a model consisting of a harmonic oscillator (the system) coupled to a set of relaxing modes considered as a semi-infinite dimensional system which effectively constitutes a nonequilibrium reservoir. This in turn is in contact with a thermally equilibrated reservoir. Both the reservoirs are composed of two sets of harmonic oscillators characterized by the frequency sets $\{\omega_j\}$ and $\{\Omega_j\}$ for the equilibrium and nonequilibrium bath, respectively. The system-reservoir combination develops in time under the influence of the total Hamiltonian

$$\begin{aligned}
H = & \hbar\omega_0 a^\dagger a + \hbar \sum_j \omega_j b_j^\dagger b_j + \hbar \sum_\mu \Omega_\mu C_\mu^\dagger C_\mu \\
& + \hbar \sum_\mu g_\mu (C_\mu a^\dagger + C_\mu^\dagger a) + \hbar \sum_\mu \sum_j \alpha_{j\mu} (b_j^\dagger C_\mu + b_j C_\mu^\dagger) .
\end{aligned} \tag{1}$$

The first term on the right-hand side describes the system mode with characteristic frequency ω_0 . The second and the third term represent the thermal and the nonequilibrium linear modes. The next two terms represent the coupling of the nonequilibrium bath with the system mode and the thermal bath where the coupling constants are g_μ and $\alpha_{j\mu}$, respectively. In writing down the Hamiltonian we have made use of the rotating wave approximation.

The Heisenberg equations of motion for the system and the reservoir operators at any given time is given by

$$\dot{a}(t) = -i\omega_0 a(t) - i \sum_\mu g_\mu C_\mu(t) , \tag{2}$$

$$\dot{b}_j(t) = -i\omega_j b_j(t) - i \sum_\mu \alpha_{j\mu} C_\mu(t) , \tag{3}$$

$$\dot{C}_\mu(t) = -i\Omega_\mu C_\mu(t) - ig_\mu a(t) - i \sum_j \alpha_{j\mu} b_j(t) . \tag{4}$$

Making use of the formal integral of the Eq.(3) for $b_j(t)$,

$$b_j(t) = b_j(t_0) e^{-i\omega_j(t-t_0)} - i \sum_\mu \alpha_{j\mu} \int_{t_0}^t dt' C_\mu(t') e^{-i\omega_j(t-t')} \tag{3}$$

in Eq.(4) we obtain

$$\dot{C}_\mu(t) = -i\Omega_\mu C_\mu(t) - ig_\mu a(t) - i \sum_j \alpha_{j\mu} e^{-i\omega_j(t-t_0)} b_j(t_0) - \sum_j \sum_\nu \alpha_{j\mu} \alpha_{j\nu} \int_{t_0}^t dt' C_\nu(t') e^{-i\omega_j(t-t')} . \tag{5}$$

Taking into consideration [1] that the interference time of $\sum_j \alpha_{j\mu} \alpha_{j\nu} e^{-i\omega_j(t-t')}$ is much smaller than the time over which the significant phase and amplitude modulation of the linear modes $C_\mu(t)$ take place, the last term in Eq.(5) can be identified as a relaxation term in the usual way with damping constant

$$\gamma_{\mu\nu}^c = \pi \alpha_{\nu\mu}(\Omega_\nu) \alpha_{\nu\nu}(\Omega_\nu) D(\Omega_\nu) , \tag{6}$$

where $D(\Omega_\nu)$ represents the density of states of the equilibrium modes evaluated at Ω_ν . Thus one can write down the Langevin equation of motion for the nonequilibrium mode C_μ as follows;

$$\dot{C}_\mu(t) = -i\Omega_\mu C_\mu(t) - ig_\mu a(t) - \sum_\nu \gamma_{\mu\nu}^c C_\nu(t) + f_\mu(t). \quad (7)$$

Here the last term $f_\mu(t)$ represents the usual noise operator arising out of the coupling of the nonequilibrium modes with the thermal bath modes as given by

$$f_\mu(t) = -i \sum_j \alpha_{j\mu} e^{-i\omega_j(t-t_0)} b_j(t_0) , \quad (8)$$

with reservoir average of $f_\mu(t)$ is zero, i.e.,

$$\langle f_\mu(t) \rangle_B = 0 , \quad (9)$$

where by the average $\langle O(t) \rangle_B$ of an operator $O(t)$ we mean $\langle O(t) \rangle_B = \text{Tr}\{O(t)\rho_B\}$. Here ρ_B denotes the initial density operator for the thermal bath $\{b_j\}$ and is a multimode extention of the usual thermal operator. This is given by

$$\rho_B = \prod_j [\exp\{-\frac{(\omega_j b_j^\dagger b_j)}{KT}\}] [1 - \exp(\frac{\omega_j}{KT})] ,$$

where T is the equilibrium temperature. Note that in defining the average we assumed the initial factorization of the total density operator into subsystem densities for the system, thermal bath and the nonequilibrium bath.

Introducing the slowly varying operator

$$\tilde{C}_\mu(t) = C_\mu(t) e^{i\Omega_\mu(t-t_0)} \quad (10)$$

the Eq.(7) reduces to the following form;

$$\dot{\tilde{C}}_\mu(t) = -ig_\mu a(t) e^{i\Omega_\mu(t-t_0)} - \sum_\nu \gamma_{\mu\nu}^c \tilde{C}_\nu(t) e^{i(\Omega_\mu - \Omega_\nu)(t-t_0)} + F_\mu(t) , \quad (11)$$

where

$$F_\mu(t) = f_\mu(t) e^{i\Omega_\mu(t-t_0)} . \quad (12)$$

The relevant properties of the noise operator $F_\mu(t)$ can be summarized as;

$$\langle F_\mu(t) \rangle_B = 0 , \quad (13)$$

and

$$\langle F_\mu^\dagger(t) F_\nu(t') \rangle_B = \gamma_{\mu\nu}^c \bar{N}(\Omega_\mu) \delta(t - t') \delta_{\mu\nu} . \quad (14)$$

The last relation follows from

$$\langle b_m^\dagger(t_0) b_n(t_0) \rangle_B = \bar{N}(\omega_n) \delta_{mn} , \quad (15)$$

where $\bar{N}(\omega_n)$ is the thermal average of the number operator of the equilibrium bath and is given by $\bar{N}(\omega_n) = \frac{1}{\exp(\frac{\omega_n}{kT}) - 1}$. Also note that $\delta_{\mu\nu}$ takes care secular approximation. Eq.(14) also implies a purely Ohmic frequency-independent dissipation of the nonequilibrium modes.

Taking into consideration of the standard fluctuation-dissipation relation for the thermal bath in terms of Eq.(14), we arrive at the following Langevin equation for the nonequilibrium bath modes;

$$\dot{C}_\mu(t) = -i\Omega_\mu C_\mu(t) - ig_\mu a(t) - \gamma_{\mu\mu}^c C_\mu(t) + f_\mu(t) . \quad (16)$$

Eq.(16) constitutes an important result of this section which takes into account of the relaxation of the intermediate oscillator modes due to their coupling to the standard thermal bath whose fluctuations are described by $f_\mu(t)$. It is important to note that the above consideration is based on the rotating wave approximations (RWA) which is frequently used when considering coupling to a heat bath in quantum optics. In the present case as we are to see in the subsequent sections that there are quite subtle interaction effects. The question whether these effects would survive a more complete treatment may naturally arise. We mention two pertinent points at this stage. First, if one retains the nonrotating couplings in the Hamiltonian and carry out the same perturbative procedure one arrives at an equation of motion for $C_\mu(t)$ [instead of Eq.(11)] which additionally contains secular oscillating terms of the form $C_\nu^\dagger e^{i(\Omega_\mu + \Omega_\nu)(t-t_0)}$. RWA amounts to neglecting these contributions which may be important only at very high coupling. Secondly, C_μ modes executes

a slow relaxation dynamics on the time scale $\sim 1/\gamma_{\mu\mu}^c$ as compared to the time scale of correlation of thermal noise. The time scale of secular oscillations being short, those can be safely averaged out from the relevant dynamics.

Another important point regarding RWA in the context of the present linear coupling model Hamiltonian (1) needs to be considered. Ford, O'connell and Lewis [13] have demonstrated that independent oscillator model within RWA (a variant of LC model) is seriously flawed since the Hamiltonian becomes imaginary when the bath is not passive (i.e., there exists an associated spectrum of eigenvalues ranging upto $-\infty$). The problem essentially lies at the specific frequency dependence of the coupling constant in its denominator (for example, as shown in Ref. [13], appropriately transformed g_j in Eq. (1) is proportional to $\frac{1}{\sqrt{\omega_0\omega_j}}$). However, a standard procedure in quantum optics is to replace the summation over modes by an integral over their density $D(\omega_j)$, (in free space equal to $(\frac{V\omega_k^2}{c^3\pi^2})$), i.e., $\sum_j(g_j)^2 \rightarrow \int_{-\infty}^{+\infty} d\omega D(\omega)g^2(\omega)$. One thus gets rid of the unwanted frequency dependence in the denominator of the coupling constant in the calculations (see, for example, calculation of Wigner-Weisskopf decay rate, Lamb et al in Ref.[1]). Since we have followed the same approach, RWA does not pose any problem in the present analysis. The problem however, remains for a strictly discrete spectrum.

We have presented above an extension of the quantum theory of damping from Langevin point of view within a traditional system-reservoir linear coupling scheme. Essentially the model consists of replacing the reservoir by damping terms in the Heisenberg equations of motion for dissipation-free system and adding fluctuating forces as driving terms which add fluctuations to the system. The operator forces are such that (i) the system has the correct statistical properties to agree in the classical limit and (ii) they maintain the commutation relations for Boson operators to ensure that uncertainty principle is not violated. These considerations have been fully taken care of in our analysis with appropriate elaboration in the following sections. The spiritual root of the quantum statistical approach to damping lies in the fluctuation-dissipation relation, which illustrates a dynamical balance of inward flow of energy due to fluctuations from the reservoir into the system and the outward flow of energy from the system to the reservoir due to the dissipation

of the system mode. We address this specific issue in the next section.

III. FLUCTUATION-DISSIPATION RELATION FOR NONEQUILIBRIUM BATH

To explore the influence of an initial excitation of the semi-infinite dimensional intermediate reservoir modes and its relaxation, we now consider the evolution of these linear modes C_μ in terms of Eq.(16). The physical situation that has been addressed is the following;

We consider that at $t = t_0$ the excitation is switched on and the bath modes (C_μ, C_μ^\dagger) are thrown into a nonstationary state such that they behave as a nonequilibrium reservoir undergoing relaxation. We follow the stochastic dynamics of the system mode and the relaxation of the nonequilibrium reservoir modes after $t > t_0$. We assume that the effect of back reaction of the system mode on the reservoir modes is small enough to be neglected. Eq.(16) allows a formal solution of the following form

$$C_\mu(t) = C_\mu^s(t) + C_\mu(t_0)e^{(-i\Omega_\mu - \gamma_{\mu\mu}^c)(t-t_0)} - ig_\mu \int_{t_0}^t dt' e^{(-i\Omega_\mu - \gamma_{\mu\mu}^c)(t-t')} a(t') . \quad (17)$$

The first term on the right hand side in the absence of the coupling of the system mode represents the (long time) stationary stochastic solution of the form

$$C_\mu^s(t) = C_\mu^s e^{-i[\Omega_\mu(t-t_0) + \phi_\mu^s]} , \quad (18)$$

where the amplitude C_μ^s (operators) and phases ϕ_μ^s (c-numbers) are assumed to be randomly distributed. The random nature of $C_\mu^s(t)$ may be understood in the following way : Let us first note that in the absence of coupling g_μ Eq.(16) allows the solution

$$C_\mu(t) = C_\mu(t_0)e^{-(i\Omega_\mu + \gamma_{\mu\mu}^c)(t-t_0)} + e^{-(i\Omega_\mu + \gamma_{\mu\mu}^c)(t-t_0)} \int_{t_0}^t dt' f_\mu(t') e^{(i\Omega_\mu + \gamma_{\mu\mu}^c)(t'-t_0)} .$$

In the steady state we neglect the first term which decays rapidly. The second term is a randomly fluctuating term (which is the most important term in almost any Langevin analysis) due to $f_\mu(t)$. Substituting Eq.(8) in the above equation we obtain

$$C_\mu^s(t) = e^{-i\Omega_\mu(t-t_0)} \sum_j (-i\alpha_{j\mu}) e^{-\gamma_{\mu\mu}^c(t-t_0)} b_j(t_0) \int_{t_0}^t dt' e^{(i\omega_j + i\Omega_\mu + \gamma_{\mu\mu}^c)(t'-t_0)} ,$$

where s signifies the steady state. The above solution implies that $C_\mu^s(t)$ is essentially a superposition of unknown (since the initial condition for the infinite number of thermal bath oscillator $b_j(t_0)$ are assumed to be completely uncertain) amplitudes (operators) and phases (c -numbers) and may be written compactly in the form of Eq.(18). When written in the form (18) we obtain an instantaneous realization of the random distribution of C_μ^s and ϕ_μ^s . Thus Eq.(17) represents an instantaneous solution of Eq.(16). To check the consistency of the solution (17) we note the following points; (i) To recover initial deterministic amplitude of $C_\mu(t)$ we need an ensemble average of Eq.(17) at $t = t_0$ so that $\bar{C}_\mu^s(t_0) = 0$. Also we are to see (subsequent to Eq.(24)) that the first term in Eq.(17) is responsible for the usual fluctuation-dissipation relation when the nonequilibrium modes get equilibrated at $t = \infty$. $C_\mu^s(t)$ in $C_\mu(t)$ is thus dictated by the condition of stationarity. (iii) It can be easily seen that the random nature of the operator forces in $C_\mu^s(t)$ is responsible for maintaining the Boson commutation relation for $C_\mu(t)$ which further ensures that the uncertainty principle is not violated.

The second term on the right hand side in Eq.(17) carries the information of relaxation of the C_μ modes due to their coupling to the thermal bath and is a typical memory term. The third term on the other hand represents the effect of coupling of the system mode to the nonequilibrium reservoir.

Substitution of the solution(17) in Eq.(2) yields the equation of motion for the slowly varying system operator $A(t)$;

$$\dot{A}(t) = - \sum_{\mu} g_{\mu}^2 \int_{t_0}^t dt' e^{i(\omega_0 - \Omega_{\mu})(t-t')} e^{-\gamma_{\mu\mu}^c(t-t')} A(t') + Z(t), \quad (19)$$

where

$$A(t) = a(t) e^{-i\omega_0(t-t_0)} \quad (20)$$

and

$$Z(t) = -i \sum_{\mu} g_{\mu} [C_{\mu}^s(t) + C_{\mu}(t_0) e^{(-i\Omega_{\mu} - \gamma_{\mu\mu}^c)(t-t_0)}] e^{i\omega_0(t-t_0)}. \quad (21)$$

Eq.(19) is a non-Markovian equation where the memory effects arise out of the two sources; the first one being the system operator concerned term $A(t')$ which depends on earlier time t' ; the other one $e^{-\gamma_{\mu\mu}^c(t-t')}$ is due to the relaxation of the nonequilibrium bath modes which arises because of their coupling to the thermal bath. $Z(t)$ represents the noise operator for the nonequilibrium bath modes.

In the weak coupling approximation scheme the first term in equation(19) can be simplified to the following form;

$$A(t) \int d\Omega \rho(\Omega) g^2(\Omega) \int_0^{\infty} d\tau e^{i(\omega_0 - \Omega)\tau} e^{-\gamma_{\mu\mu}^c \tau},$$

where the summation over the bath modes is replaced by integration and $\rho(\Omega)$ represents (an a priori known) density of nonequilibrium bath modes. Assuming weak dependence of $\gamma_{\mu\mu}^c$ on the modes one can reduce the last expression (after performing integration over Ω) to obtain the following Langevin equation for the system operator;

$$\dot{A}(t) = -\Gamma A(t) + Z(t), \quad (22)$$

where

$$\Gamma = \pi g^2(\omega_0) \rho(\omega_0) \quad (23)$$

can be identified as a dissipation constant of the system mode due to the fluctuations of the nonequilibrium reservoir in the limit when $\gamma_{\mu\mu}^c$ is vanishingly small. The explicit dependence of the decay constant of the system mode on γ will be revealed clearly (as a more general case) in the next section within the scope of Wigner-Weisskopf approximation.

The nonequilibrium generalization of the fluctuation-dissipation relation is now immediately apparent. From the expression for $Z(t)$ (Eq.(21)) one finds that

$$\begin{aligned} \langle Z^+(t)Z(t') \rangle_{NR} &= \sum_{\mu} g_{\mu}^2 [\langle C_{\mu}^{s\dagger} C_{\mu}^s \rangle_{NR} e^{i(\Omega_{\mu}-\omega_0)(t-t')} \\ &+ \langle C_{\mu}^{\dagger}(t_0) C_{\mu}(t_0) \rangle_{NR} e^{i(\Omega_{\mu}-\omega_0)(t-t')} e^{2\gamma_{\mu\mu}^c t_0} e^{-\gamma_{\mu\mu}^c(t+t')}] . \end{aligned} \quad (24)$$

We denote the average photon number of the nonequilibrium bath by

$$\bar{n}(\Omega_{\mu}, t_0) = \langle C_{\mu}^{\dagger}(t_0) C_{\mu}(t_0) \rangle_{NR} , \quad (25)$$

where t_0 signifies the dependence of average photon number of the nonequilibrium bath on its initial state of preparation. Also the steady state average photon number is given by

$$\bar{n}(\Omega_{\mu}) = \langle C_{\mu}^{s\dagger} C_{\mu}^s \rangle_{NR} .$$

By $\langle O(t) \rangle_{NR}$ we mean $\langle O(t) \rangle_{NR} = Tr\{O(t)\rho_c\}$ where ρ_c indicates the initial thermalized density operator for the intermediate oscillator $\{C\}$ -modes, and is given by

$$\rho_c = \prod_{\mu} \exp\left(-\frac{\Omega_{\mu} C_{\mu}^{\dagger} C_{\mu}}{KT}\right) \left[1 - \exp\left(\frac{\Omega_{\mu}}{KT}\right)\right] .$$

As usual, the initial factorization of densities of $\{b\}$ and $\{C\}$ modes is assumed.

After replacing the summation by integration and $\gamma_{\mu\mu}^c$ by the average γ in Eq.(24) we obtain

$$\langle Z^\dagger(t)Z(t')\rangle_{\text{NR}} = \left[\Gamma\bar{n}(\omega_0) + e^{-2\gamma(t-t_0)}\Gamma\bar{n}(\omega_0, t_0) \right] \delta(t - t') .$$

Rewriting $\Gamma\bar{n}(\omega_0, t_0)$ in terms of a deviation from its steady state value $\Gamma\bar{n}(\omega_0)$ as

$$\Gamma\bar{n}(\omega_0, t_0) = D(t_0) - \Gamma\bar{n}(\omega_0) ,$$

we identify a time-dependent diffusion coefficient $D(t)$ in the last equation as

$$D(t) = \Gamma\bar{n}(\omega_0) + [D(t_0) - \Gamma\bar{n}(\omega_0)] e^{-2\gamma(t-t_0)} .$$

We thus obtain

$$\langle Z^\dagger(t)Z(t')\rangle_{\text{NR}} = \left\{ \Gamma\bar{n}(\omega_0) + [D(t_0) - \Gamma\bar{n}(\omega_0)] e^{-2\gamma(t-t_0)} \right\} \delta(t - t') . \quad (26)$$

Eq.(26) is the desired nonequilibrium generalization of the fluctuation-dissipation relationship. This relates instantaneous fluctuations of the nonequilibrium bath (which itself is undergoing relaxation at a rate γ due to its coupling with the thermal bath) to the dissipation of the energy of the system mode through Γ . The nonequilibrium nature of the bath is implicit in the initial preparation which creates an initial diffusion coefficient $D(t_0)$ and also in the exponentially decaying term. In the long time limit one recovers the usual fluctuation-dissipation relation for the thermal bath at equilibrium.

We now express Eq.(26) in terms of energy density of fluctuations of the nonequilibrium modes. The energy density which is proportional to the power spectrum centered around ω_0 is given by [$\hbar = 1$]

$$\begin{aligned} u(\Omega, t) &= \frac{\Omega}{4\Gamma} \int_{-\infty}^{+\infty} d\tau \langle Z^\dagger(t) Z(t + \tau) \rangle e^{i(\Omega - \omega_0)\tau} \\ &= \frac{1}{2} \Omega \bar{n}(\Omega) + e^{-2\gamma(t-t_0)} [u(\Omega, t_0) - \frac{1}{2} \Omega \bar{n}(\Omega)] . \end{aligned}$$

It is important to note that t is the slow time variable which is well separated from the time scale of thermal noise. The fluctuations of the noise operator $Z(t)$ is now explicitly determined by the nonequilibrium state of the intermediate oscillator bath through its energy density $u(\Omega, t)$ at each instant of time t . In other words the instantaneous nonequilibrium energy density distribution of the fluctuating modes is related to the friction coefficient of these modes on the system degree of freedom through a dynamic equilibrium. One can immediately recover the classical version of the above equation in the high temperature limit (where $\bar{n}(\Omega) = \frac{1}{e^{\frac{\Omega}{KT}} - 1} \simeq \frac{KT}{\Omega}$) to obtain

$$u(\Omega, t) = \frac{1}{2} KT + e^{-2\gamma(t-t_0)} \left[u(\Omega, t_0) - \frac{1}{2} KT \right] .$$

The above equation was derived earlier [6] in the context of classical relaxation kinetics of complex nonlinear systems. This reduction to classical version of the nonequilibrium fluctuation-dissipation relation serves as a consistency check for its quantum generalization (26) which is more relevant in quantum optical issues that we address in this paper.

Another point should be emphasized regarding the fluctuation-dissipation relation (26). The very notion of a nonequilibrium bath apparently suggests that frequency distribution function of the modes of this bath should be a function of time and in principle, one should seek for it as a self-consistent solution from a quantum kinetic analysis. We have followed here an alternative route. Since we are working with Heisenberg operator equations of motion, the knowledge of initial total state density which is factorizable in subsystem densities (for the system, thermal bath and

nonequilibrium or intermediate bath) at $t = 0$, is sufficient to describe the complete dynamics in terms of average values and correlation functions. We are concerned here (and also in classical theory [6]) with a priori given frequency distribution functions for equilibrium $\{b_j\}$ bath ($D(\omega)$) and nonequilibrium $\{C_\mu\}$ bath ($\rho(\Omega)$) which are independent of time (nor we have introduced any non-thermal temperature in describing the nonequilibrium bath). The essential content of the nonequilibrium nature of the bath rests on a time-dependent energy density fluctuation distribution function $u(\Omega, t)$ as described above (varying over a slower time scale compared to the time scale of thermal noise) which is a derived quantity rather than a self-consistently obtained function which might be obtainable from a quantum kinetic analysis. The effect of initial excitation is to create an energy density function $u(\Omega, 0)$ which differs from its equilibrium counterpart. This departure sets in a nonequilibrium situation. All these considerations also apply to the classical version of nonequilibrium fluctuation-dissipation relation [6].

Before bringing an end to this section some pertinent points are to be noted. First, we need to stress that in the derivation of the relation (26) we assume that $Z(t)$ is effectively stationary on the fast correlation time scale of the thermal bath. Second, the theory as developed above is based on the consideration of quantum optical situations in mind. It is well known [2,11] that, in general, the noise from the equilibrium bath at low temperatures is very different from a simple white noise and concerns expressions which contain integral from distribution function of the bath over all frequencies. Most often such situations are encountered in condensed matter and in chemical physics of complex systems. However in the problems of quantum optics where the harmonic oscillator bath serves as a standard paradigm for optical fields one routinely uses a broad band white noise spectrum such that $\sum_\mu g_\mu^2 \bar{n}_\mu$ is slowly varying and the summand in Eq.(24) is so strongly peaked at $\Omega_\mu = \omega_0$, that we may convert the sum into an integral and remove the slowly varying factors to obtain the result (26) [p. 422 of Louisell in Ref. 1]. Similar consideration leads us to Einstein's spontaneous emission coefficient or Wigner-Weisskopf decay rate which contains single frequency ω_0 , the characteristic frequency of the system mode. Thus although, in principle, it may be possible to consider a colored noise spectrum and the resulting frequency dependence

of rate constants we restrict ourselves to former situations of broad band reservoir which itself is undergoing a relaxation on the time scale of $1/\gamma$. An important content of the present work is to explore the effect of this secondary relaxation on the primary kinetics of the system mode. The nonequilibrium generalization of the fluctuation dissipation relation as discussed in this section serves as a basis of this exploration in following two quantum optical cases.

IV. DECAY OF THE SYSTEM MODE ; WIGNER-WEISSKOPF APPROXIMATION

We now obtain the solution of Heisenberg-Langevin integro-differential equation of motion [Eq.(19)] for the system mode coupled to nonequilibrium bath under Wigner-Weisskopf approximation. If we take the Laplace transformation of Eq.(19), we have after some algebra

$$\bar{A}(s) = \frac{a(0) + \bar{Z}(s)}{s + \sum_{\mu} \frac{g_{\mu}^2}{s + i(\Omega_{\mu} - \omega_0) + \gamma_{\mu\mu}^c}}, \quad (27)$$

where

$$\bar{A}(s) = \int_0^{\infty} dt A(t) e^{-st} \quad (28)$$

and

$$\bar{Z}(s) = \int_0^{\infty} dt Z(t) e^{-st},$$

or,

$$\bar{Z}(s) = -i \sum_{\mu} g_{\mu} C_{\mu}(0) \frac{1}{s + i(\Omega_{\mu} - \omega_0) + \gamma_{\mu\mu}^c}, \quad (29)$$

where we have used the fact that the amplitudes and the phases of $C_{\mu}^s(0)$ are random. Also we have $A(0) = a(0)$. Here we have chosen the initial time $t_0 = 0$ for convenience.

We make Wigner-Weisskopf approximation to solve for the zeros of Δ in Eq.(27), where

$$\frac{1}{\Delta} = (s + \sum_{\mu} \frac{g_{\mu}^2}{s + i(\Omega_{\mu} - \omega_0) + \gamma_{\mu\mu}^c})^{-1}. \quad (30)$$

For weak interaction, zeroth approximation is $\Delta = 0$ if $s = 0$. As a next approximation let $s \rightarrow 0$ in the denominator of the sum in Δ . In other words under Wigner-Weisskopf approximation we calculate the first order shift in the simple pole which is approximately given by

$$\Delta(0) - s \simeq Lt_{s \rightarrow 0} \sum_{\mu} \frac{g_{\mu}^2}{s + i(\Omega_{\mu} - \omega_0) + \gamma_{\mu\mu}^c} = \gamma^W + i\delta\omega \quad (31)$$

where γ^W and $\delta\omega$ are real quantities. Explicit calculation in the usual way yields,

$$\gamma^W = \int d\Omega \rho(\Omega) g^2(\Omega) \frac{\gamma}{(\Omega - \omega_0)^2 + \gamma^2}, \quad (32)$$

and

$$\delta\omega = \int d\Omega \rho(\Omega) g^2(\Omega) \frac{(\Omega - \omega_0)}{(\Omega - \omega_0)^2 + \gamma^2}. \quad (33)$$

The expressions for the line width γ^W and the frequency shift $\delta\omega$ of the system mode thus obtained due to the relaxation of the nonequilibrium bath are markedly different from the usual expressions of Wigner-Weisskopf theory. This is because of the explicit dependence of γ^W and $\delta\omega$ on the γ in Eqs.(32) and (33) which arises due to the relaxation of the nonequilibrium modes due to their coupling with the thermal bath. In the limit $\gamma \rightarrow 0$ one recovers the usual decay rate Γ and the level-shift terms, i.e.,

$$\lim_{\gamma \rightarrow 0} \gamma^W = \Gamma .$$

We thus see that the effect of Wigner-Weisskopf approximation is to replace the more exact

equation(19) by the Langevin equation whose solution obtained after appropriate inverse Laplace transform of equation(27) is given by

$$A(t) = a(0)e^{-(\gamma^W+i\delta\omega)t} - \sum_{\mu} g_{\mu} C_{\mu}(0) \frac{e^{-\gamma t} e^{-i(\Omega_{\mu}-\omega_0)t} [1 - e^{-(\gamma^W-\gamma)t} e^{i(\Omega_{\mu}-\omega_0-\delta\omega)t}]}{(\omega_0 - \Omega_{\mu} + \delta\omega) - i(\gamma^W - \gamma)} . \quad (34)$$

It may be noted that while deriving γ^W and $\delta\omega$, weak coupling and smooth density of states $\rho(\Omega)$ for the intermediate oscillators have been assumed. By the same token, the correction terms in Eq. (34) are small and the result for $A(t)$ as expressed in Eq.(34) could be simplified further to the following form,

$$A(t) = a(0)e^{-(\gamma^W+i\delta\omega)t} + \chi(\omega_0)C_{\omega_0}(0) [e^{-\gamma t} - e^{-(\gamma^W+i\delta\omega)t}] , \quad (35)$$

where $\chi(\omega_0)$ is given by

$$\chi(\omega_0) = \left[\frac{g(\omega_0)\rho(\omega_0)}{\delta\omega - i(\gamma^W - \gamma)} \right] .$$

The primary evolution of the system mode A thus depends on the secondary relaxation of the intermediate oscillators explicitly.

The expression for γ^W (Eq.(32)) illustrates a dynamical modification of the Wigner-Weisskopf decay rate constant since it incorporates the effect of coupling of the nonequilibrium bath to the thermal bath through γ . The modification of atomic spontaneous decay rate both in the form of enhancement and suppression by appropriate tailoring of vacuum modes of the cavity is now well known in cavity QED [7]. Whereas in the cavity QED problems one essentially 77manipulates the boundary conditions in various ways, the present modification is effectively dynamical in nature in the sense that it carries the effect of relaxation of the nonequilibrium modes on the dissipation of

the system mode. It is then also expected that the atomic decay rate might be similarly affected in appropriately modified situations.

Before completing this section we point out that in the present problem of quantum theory of dissipation in the quantum optical context we are concerned with the frequency spectrum of the radiation field modes. In the context of solid state the frequency density is assumed to be of the Debye type with appropriate regularization by cutoff at high frequency. In the cavity QED problems adjustment of boundary conditions may lead to different density of states. Although there is no generalization of the dependence of friction on the frequency spectrum, in general, one encounters a time-retarded friction.

V. TIME-DEPENDENT SPECTRUM OF A CAVITY MODE WITH GAIN IN CONTACT WITH NONEQUILIBRIUM BATH

An immediate consequence of the nonequilibrium generalization of the fluctuation-dissipation relation is the explicit time dependence of the diffusion constant, as evident in Eq.(26). It is therefore expected that this time-dependence may make its presence felt if one analyses the transient noise spectrum of the system mode. With this end in view we now calculate the time-dependent spectrum of a cavity field mode coupled to a nonequilibrium reservoir which causes the field mode to decay at the rate Γ . Generally we find the spectrum by applying the quantum regression theorem to the Langevin equation for a quantized field mode interacting with a medium described by a gain α . In general, the complex gain $\alpha(t)$ is an operator that is saturated by the number operator $A^\dagger(t)A(t)$ [p. 467 of Ref.12]. The Langevin equation for our problem is given by,

$$\dot{A}(t) = -(\Gamma + i\delta - \alpha)A(t) + Z(t) , \quad (36)$$

where $A(t)$ and $Z(t)$ are slowly varying annihilation and noise operators, respectively. In general, Eq.(36) applies to laser-like situations including those with two-level and semiconductor media

[12]. Here $\delta (= \omega_0 - \nu)$ is the detuning of the mode oscillation frequency ν from the passive cavity resonance frequency ω_0 and α , the gain coefficient is assumed to be a real number ($\Gamma > \alpha$). $Z(t)$ is the noise source operator as given by

$$Z(t) = -i \sum_{\mu} g_{\mu} [C_{\mu}^s(t) + C_{\mu}(t_0) e^{(-i\Omega_{\mu} - i\nu - \gamma_{\mu\mu}^C)(t-t_0)}]. \quad (37)$$

The noise is characterized by the following properties :

$$\langle Z(t) \rangle_{NR} = 0, \quad (38)$$

$$\langle Z^{\dagger}(t)Z(t') \rangle_{NR} = [\Gamma\bar{n} + \{D(t_0) - \Gamma\bar{n}\}e^{-2\gamma(t-t_0)}] \delta(t - t') . \quad (39)$$

It has also to be noted that since we are dealing with a non-stationary situation the standard steady state spectrum is not applicable. We therefore take resort to non-steady state spectrum or the so-called “physical spectrum” of the cavity mode [8] where the attention is focused on the dynamic evolution of the spectrum following an abrupt excitation of a near-resonant cavity mode. The main reason for studying the time-dependent spectrum is that the familiar power spectrum of the Wiener-Khintchine theorem is not applicable to nonstationary processes. Although in quite a number of earlier cases time-dependent spectrum of Page and Lampard were used widely serious objections were raised against this spectrum (e.g., it can be negative). Eberly and Wodkiewicz have shown that the suitably normalized counting rate of a photodetector can be used to define a time-dependent spectrum or physical spectrum. This allows the influence of the spectrum analyzer (basically a Fabry-Perot interferometer, for example) to be exhibited in the spectrum so that the band limit of the measuring device is appropriately incorporated which makes the spectrum free from ambiguities and unphysical characteristics of earlier spectrum. It has also been emphasized [9] that when the instrument width, W is narrow enough such that $W \ll \Gamma$, the spectrum appears to be qualitatively similar to Wiener-Khintchine spectrum. This transient

spectrum has been used earlier in several occasions in connection with resonance fluorescence studies [9], micromaser problem [10] etc. One can define the time-dependent spectrum or the physical spectrum as follows;

$$S(t, \omega, W) = 2W \text{Re} \int_0^t dt_2 e^{-W(t-t_2)} \int_0^{t-t_2} d\tau e^{(\frac{W}{2}-i\Delta)\tau} \langle A^\dagger(t_2 + \tau) A(t_2) \rangle. \quad (40)$$

Here the symbols have the following meaning : t is the elapsed time after the system and the reservoir have been subjected to initial excitation at $t = t_0 (= 0)$, W is the full width of the transmission peak of the interferometer and $\Delta (= \omega - \nu)$ is the detuning, or frequency offset of the Fabry-Perot line center above the frequency of the field ω . It is important to note that the time-dependent spectrum is expressed in terms of two integrals in Eq.(40). The first integral is over the correlation time and is actually the counterpart of the Wiener-Khintchine spectrum band limited by the width of the measuring device, W , while the second one over t_2 takes into account of the nonstationarity which makes the correlation function t_2 dependent. The device width in the second integral also sets the limit over the time scale of this nonstationarity.

Since the time-evolution of the system is governed by Eq.(36), one can make use of the quantum regression hypothesis which yields two-time correlation function

$$\langle A^\dagger(t + \tau) A(t) \rangle = e^{-(\Gamma - i\delta - \alpha)\tau} \langle A^\dagger(t) A(t) \rangle. \quad (41)$$

We emphasize that t in Eq.(41) [or t_2 in Eq.(40)] refers to the non-stationary time. We therefore calculate the explicit time dependence of $\langle A^\dagger(t) A(t) \rangle$ using Einstein's relations (see Appendix A for details);

$$\frac{d}{dt} \langle A^\dagger(t) A(t) \rangle = -2(\Gamma - \alpha) \langle A^\dagger(t) A(t) \rangle + 2\Gamma \bar{n} [1 + r e^{-2\gamma t}] , \text{ where } r = \frac{D(0)}{D(\infty)} - 1. \quad (42)$$

The solution of Eq.(42) is given by

$$\begin{aligned}
\langle A^\dagger(t)A(t) \rangle &= e^{-2(\Gamma-\alpha)t} \langle A^\dagger(0)A(0) \rangle \\
&+ \Gamma \bar{n}(\omega_0) \left[\frac{1}{\Gamma-\alpha} - \frac{(1+r)(\Gamma-\alpha)-\gamma}{(\Gamma-\alpha)(\Gamma-\alpha-\gamma)} e^{-2(\Gamma-\alpha)t} + r \frac{e^{-2\gamma t}}{(\Gamma-\alpha-\gamma)} \right]. \quad (43)
\end{aligned}$$

From Eq.(42) we obtain the steady state condition

$$(\Gamma - \alpha) \langle A^\dagger(\infty)A(\infty) \rangle = \Gamma \bar{n}. \quad (44)$$

We let $\langle A^\dagger(\infty)A(\infty) \rangle = N$. Eqs.(44) and (43) may then be rewritten, respectively, as

$$\bar{n} = \frac{\Gamma - \alpha}{\Gamma} N \quad (45)$$

and

$$\langle A^\dagger(t)A(t) \rangle = N(1 - rke^{-2at} + rke^{-2\gamma t}), \quad (46)$$

where $a = \Gamma - \alpha$, $k = \frac{a}{a-\gamma}$.

Combining Eqs.(41) and (46) we obtain the two-time correlation function

$$\langle A^\dagger(t_2 + \tau)A(t_2) \rangle = e^{-(\Gamma-i\delta-\alpha)\tau} N(1 - rke^{-2at_2} + rke^{-2\gamma t_2}). \quad (47)$$

Making use of this relation and performing the integration over τ and t_2 , we extract the real part [Eq.(40)] which yields

$$\begin{aligned}
S(t, \Delta, W) = & \frac{2NW}{W_-^2 + \Delta^2} \left[\frac{W_-}{W_+} \left\{ (1 + kr)e^{-2Wt} - \frac{1}{2} k r e^{-2(W+a)t} - (1 + \frac{1}{2} kr)e^{-Wt} \right\} \right. \\
& \frac{W_+ W_- + \Delta^2}{W_+^2 + \Delta^2} \left\{ 1 + k r e^{-2\gamma t} \right\} + k r e^{-2at} - \frac{(1 + kr)2a\Delta}{W_+^2 + \Delta^2} e^{W_+ t} \sin \Delta t \\
& \left. + \frac{2akr - (W_+ W_- + \Delta^2)}{W_+^2 + \Delta^2} e^{-W_+ t} \cos \Delta t \right] , \tag{48}
\end{aligned}$$

where, $W_+ = \frac{W}{2} + (\Gamma - \alpha)$ and $W_- = \frac{W}{2} - (\Gamma - \alpha)$.

Here we have set the detuning $\delta = 0$.

In the long limit ($t \rightarrow \infty$) the spectrum reaches the steady state value

$$S(\Delta, W) = \frac{2NW}{\left\{ \frac{W}{2} - (\Gamma - \alpha) \right\}^2 + \Delta^2} \left[\frac{\left(\frac{W}{2} \right)^2 - (\Gamma - \alpha)^2 + \Delta^2}{\left\{ \frac{W}{2} + (\Gamma - \alpha) \right\}^2 + \Delta^2} \right]. \tag{49}$$

It is interesting to note that at short time the effect of nonequilibrium bath is prominent through γ and r . While $r (= \frac{D(0)}{D(\infty)} - 1)$ includes the effect of preparation of the initial nonequilibrium condition by a sudden external excitation at $t = 0$ which makes the initial diffusion coefficient $D(0)$ to be different from its stationary long time value $D(\infty)$, γ carries the information of relaxation. As expected, the steady state spectrum is independent of both γ and r . This is because at large time when the nonequilibrium bath is equilibrated, the system forgets its past and the time-dependence of diffusion coefficient is erased and the spectrum becomes the steady state spectrum.

We now look for the transient characteristics of the spectra of the cavity mode. In Fig 1 we plot the spectra for different scaled time t after the initial excitation, Γ being used as a scaling parameter. For numerical computation we choose the following scaled parameter set : $W = 4, \alpha = 0.1, r = 1$ and $\gamma = 0.1$. One observes that after the initial excitation the spectra grow to a maximum and then the peak height start diminishing and eventually reaches the steady state

value. Thus the effect of relaxation of the nonequilibrium bath becomes prominent in the short time region. The variation of peak intensity with time for the three different γ has been shown in Fig2. It is apparent that the spectrum reaches the steady state more quickly for larger values of γ and also for small γ the maximum peak is larger than that for larger γ .

VI. CONCLUSIONS

We have developed the quantum theory of dissipation of a harmonic oscillator coupled to a nonequilibrium bath in terms of a microscopic model. Making use of appropriate separation of time scales one can construct an effective Langevin dynamics with memory (where the memory functions are not the phenomenological inputs) which is due to the relaxation of the nonequilibrium bath modes and identify the relevant noise sources. An essential offshoot is the nonequilibrium generalizations of the familiar fluctuation-dissipation and Einstein's relations. It is important to note that the Wigner-Weisskopf decay rate constant of the oscillator is dynamically modified. The theory is further applied to calculate the time-dependent spectrum of a cavity mode with suitable gain. One observes that the nonequilibrium nature of the bath modes makes its presence felt in the time-dependence of the diffusion constant leading to the differential behavior of the transient spectra from the steady-state ones. Although in the present problem we are mainly concerned with the dissipation of energy, it may also be worthwhile to investigate the problem of decoherence on similar footing. We hope to address this and related issues in future.

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Appendix A

Generalized Einstein's relations

In this section we outline the derivation of the nonequilibrium generalization of Einstein's relations Eq.(41).

The system operators follow the Langevin equation of motion Eq.(35)

$$\dot{A} = -(\Gamma + i\delta - \alpha)A + Z(t) ; \quad (50)$$

where the first term within the parenthesis of the right hand side is the drift term and $Z(t)$ is the nonequilibrium noise operator [Eq.(37)]. From the identity

$$A^\dagger(t) = A^\dagger(t - \Delta t) + \int_{t-\Delta t}^t dt' \dot{A}^\dagger(t') \quad (51)$$

we first obtain the correlation function of the system and the noise operator ;

$$\langle A^\dagger(t)Z(t) \rangle = \langle A^\dagger(t - \Delta t)Z(t) \rangle + \int_{t-\Delta t}^t dt' \langle [-(\Gamma - i\delta - \alpha)A^\dagger(t') + Z(t')]Z(t) \rangle. \quad (52)$$

Because the operator $A^\dagger(t')$ at time t' is not affected by fluctuation at a later time t , the first term on the right hand side is zero. Similarly the correlation $\langle A^\dagger(t')Z(t) \rangle$ is zero except at the point $t' = t$ but the integration is zero. Thus we have

$$\langle A^\dagger(t)Z(t) \rangle = \int_{t-\Delta t}^t dt' \langle Z^\dagger(t')Z(t) \rangle. \quad (53)$$

Note that we have not assumed the stationary property of the noise.

Next we determine the equation of motion for the average $\langle A^\dagger(t)A(t) \rangle$;

$$\frac{d}{dt} \langle A^\dagger(t) A(t) \rangle = \langle \dot{A}^\dagger(t) A(t) \rangle + \langle A^\dagger(t) \dot{A}(t) \rangle. \quad (54)$$

From Eq.(50) we have after some algebra

$$\frac{d}{dt} \langle A^\dagger(t) A(t) \rangle = -2(\Gamma - \alpha) \langle A^\dagger(t) A(t) \rangle + \langle Z^\dagger(t) A(t) \rangle + \langle A^\dagger(t) Z(t) \rangle. \quad (55)$$

Substituting Eq.(52) and its adjoint in Eq.(55) and performing the integral over t' where we use Eq.(39), we obtain the nonequilibrium generalization of Einstein's relation

$$\frac{d}{dt} \langle A^\dagger(t) A(t) \rangle = -2(\Gamma - \alpha) \langle A^\dagger(t) A(t) \rangle + 2\Gamma \bar{n}(\omega_0) \left[1 + \left\{ \frac{D(t_0)}{D(\infty)} - 1 \right\} e^{-2\gamma(t-t_0)} \right], \quad (56)$$

where we have used the notation $D(\infty) = \Gamma \bar{n}(\omega_0)$.

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Figure Captions

1. Time-dependent spectra of the cavity mode with gain for different dimensionless times
(a) $\Gamma t = 0.05$, (b) $\Gamma t = 0.5$, (c) $\Gamma t = 3.0$, (d) $\Gamma t = 10.0$ with $\alpha = 0.1$, $\delta = 0.0$, $W = 4.0$ and $r = 1.0$ (Scales arbitrary).

2. Variation of intensity of the time-dependent spectra vs. dimensionless time for different dimensionless decay rate constants of the nonequilibrium modes;
(a) $\frac{\gamma}{\Gamma} = 0.1$, (b) $\frac{\gamma}{\Gamma} = 0.2$, (c) $\frac{\gamma}{\Gamma} = 0.3$ with α, δ, W and r same as Fig. 1 (Scales arbitrary).

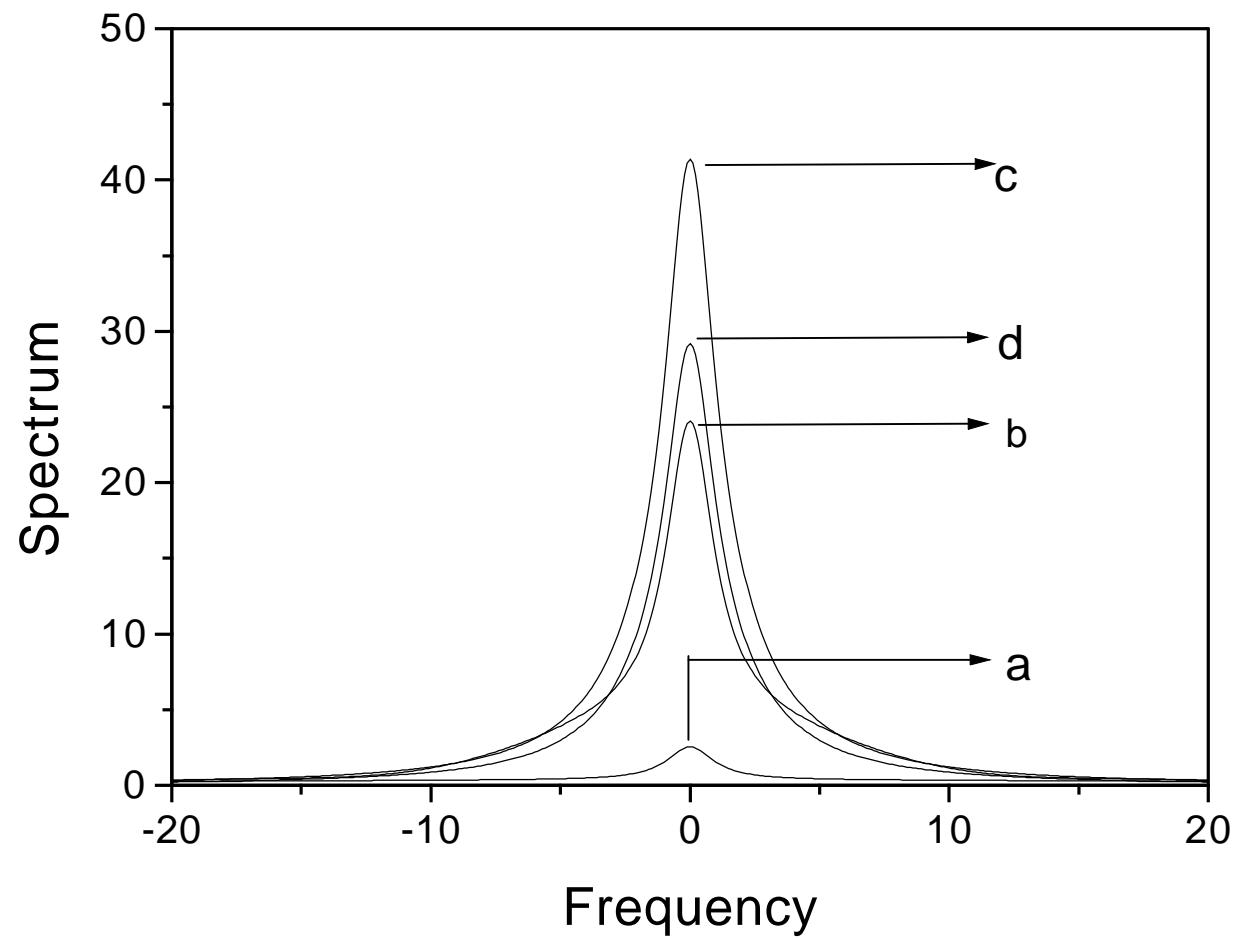


Fig. (1)

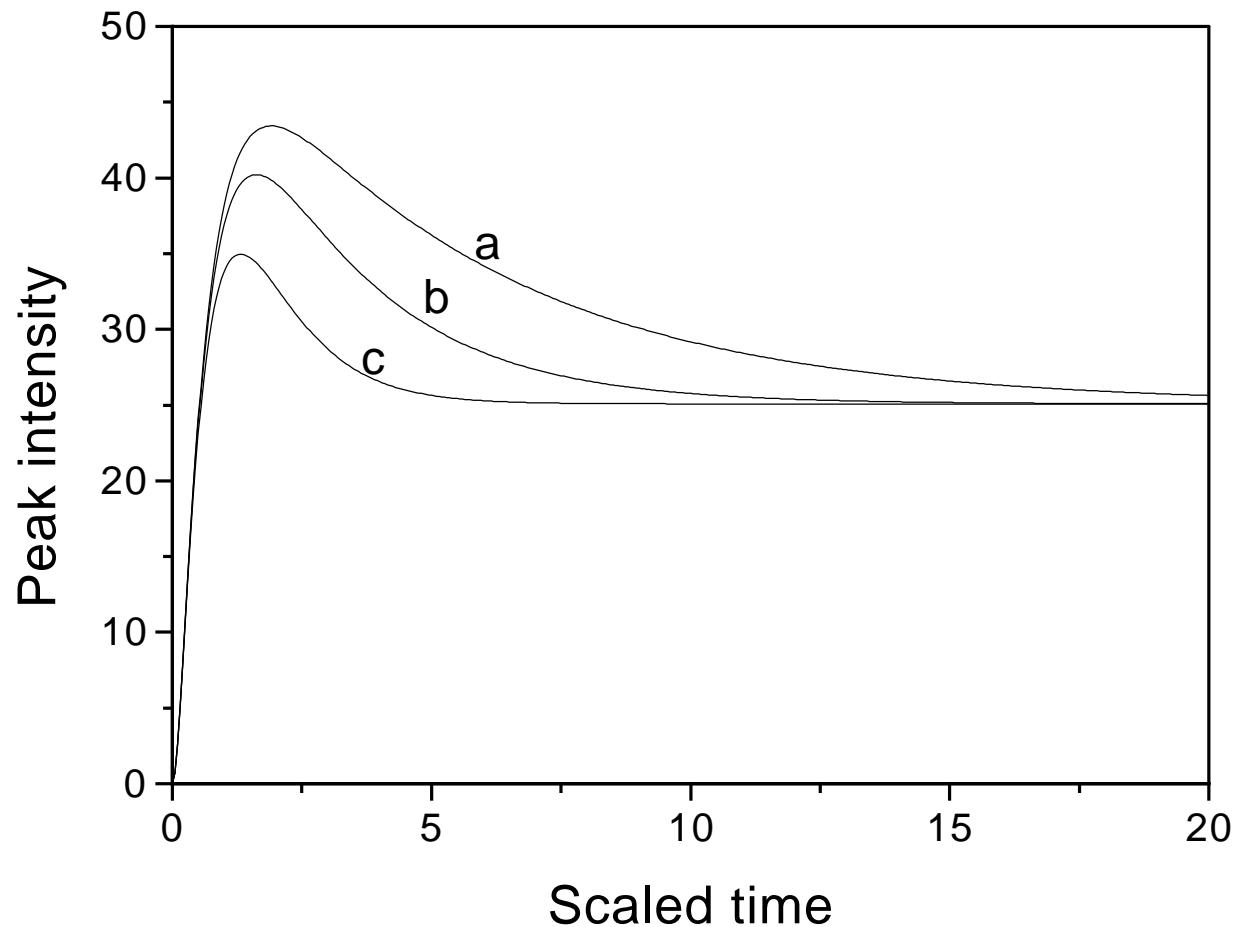


Fig. (2)