Spin-Reversal Transition in Ising Model under Pulsed Field

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Abstract

In this communication we report the existence of a dynamic "spin-reversal" transition in an Ising system perturbed by a pulsed external magnetic field. The transition is achieved by tuning the strength (h_p) and/or the duration (Δt) of the pulse which is applied in a direction opposite to the existing order. We have studied this transition in the kinetic Ising Model in two dimension using Monte Carlo technique, and solved numerically the mean field equation of motion. The transition is essentially dynamic in nature and it takes the system from one ordered equilibrium phase to another by means of the growth of opposite spin domains (in the kinetic Ising case) induced during the period when the pulsed field is applied.

1 Introduction

The dynamic response of Ising systems under positive pulsed fields has recently been studied extensively employing computer simulation. In particular, some interesting divergent growth of relaxation time and finite time scaling behaviour was observed near the order-disorder transition point of the Ising model for "positive pulses", where the pulsed field in the ordered phase is in the direction of order [1]. Here we report the computer simulation study of the same system where the pulsed field is "negative", i.e in opposition to the existing order having equilibrium magnetization m_0, below the order-disorder transition point. Due to the application of a negative pulse h_p in an ordered equilibrium phase, the down spin domains start growing as long as the pulse is present. Depending on the number and size of these down spin domains at the time of withdrawal of the pulse, either they further grow to reach the other equivalent ordered phase with reversed magnetization $(-m_0)$ or reduce to settle down in the original ordered phase with magnetization (m_0). The relaxation time τ taken by the system to reach either of these equilibrium states depends on the strength (h_p) and duration (Δt) of the pulse, as well as on the temperature T.

It may be mentioned that a number of studies have been made (see for example [2],[3]) on the growth of negative spin domains in a kinetic Ising model when an ordered state is suddenly subjected to a negative field, where the time variation of the field is step function like. The applicability of

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the classical theory of nucleation by Becker and Döring [4] has also been investigated extensively here. However, our problem differs from these studies because the applied field is withdrawn after a finite interval of time. The minimum amplitude of the field required to trigger a spin reversal is non-vanishing if Δt is finite. As we go on to increase the pulse width, in the limit $\Delta t \to \infty$ we recover the results for a field of 'step-function' like nature (in time) where even an infinitesimally small amplitude of the field is sufficient to trigger the spin-reversal.

We have studied the "phase diagram" in the $h_p - \Delta t$ plane for the spin-reversal transition (from $m_0(T)$ to $-m_0(T)$) in a Monte Carlo simulation using Glauber Dynamics [5] and also from the mean field equation of motion. In the kinetic Ising case we have also studied the variation or growth of the relaxation time τ as one approaches this phase boundary. We observe clear divergence of the relaxation time as the phase boundary is approached, indicating the spin-reversal transition to be a clear thermodynamic transition with a divergent correlation length.

2 Model and Simulation

In the Monte Carlo study, we take an Ising system of size 200×200 on a square lattice with nearest neighbour interaction without any disorder. The Hamiltonian of the system is

$$H = -J \sum_{\langle ij \rangle} S_i S_j - h(t) \sum_i S_i \quad, \tag{1}$$

where $S_i = \pm 1$ represents the Ising spins at lattice site i and J denotes the nearest neighbour interaction strength. The time dependent external magnetic field $h(t)$ is applied as a pulse for a short duration Δt :

$$
\begin{aligned}
h(t) \quad &= -h_p, \quad \text{for} \quad t_0 < t < t_0 + \Delta t \\
&= 0, \quad \text{otherwise.}
\end{aligned}
\tag{2}
$$

The pulsed field is applied after the original Ising system comes to an equilibrium ordered phase (corresponding to $T < T_c$, the order-disorder transition temperature). The time t_0 in (2) is thus much larger than the relaxation time of the pure Ising system without any perturbation. The direction of the pulsed field $(-h_p)$ is opposite to the existing order or average equilibrium magnetization $m_0(T)$ existing at times before t_0.

Before the pulsed magnetic field is applied, the system is brought to equilibrium which is characterized by temperature only. The system evolves according to the Glauber single spin flip dynamics. One complete sweep through the entire lattice is defined as one Monte Carlo step (MCS) or one unit of time t. After the system reaches its equilibrium the pulse of strength h_p is applied and is withdrawn after a finite time interval Δt. After that the system is left to itself to come to equilibrium and is attracted or evolves

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towards either of the two equally likely equilibria (determined by the temperature T) having different order parameter values or magnetization. Unless the system is above the critical temperature of the unperturbed system ($T_c \simeq 2.27$), majority of the spins will point to a particular direction. Suppose, before the application of the pulsed field the magnetization of the system is $+m_0(T)$. Now either by tuning the pulse width Δt or the pulse height h_p we can perturb the system in such a manner that after the withdrawal of the pulse the system chooses to go over to the other equilibrium state, characterized by the magnetization $-m_0(T)$. We call it a "spin-reversal" transition when the sign of equilibrium magnetization is flipped by the application of the negative pulse. We have studied in our Monte Carlo calculations the phase diagram for such spin-reversal transition in the $h_p - \Delta t$ plane at a fixed temperature ($T < T_c$). We have also looked at the relaxation behaviour of the dynamics of such systems (in particular the relaxation time τ for the average magnetization) as one approaches the phase boundary. Typical number of samples (Monte Carlo seeds) taken for averaging the data points is 10.

We have also solved numerically the mean field equation of motion

$$\frac{dm(t)}{dt} = -m(t) + \tanh\left(\frac{m(t) + h(t)}{T}\right), \tag{3}$$

where $m(t)$ is the magnetization (per site) at time t and $h(t)$ is given by (2).

3 Results

It is quite obvious that all possible combinations of h_p and Δt cannot give rise to the spin-reversal transition at a particular temperature. Fig. 1 shows how the transition can be brought about by increasing Δt for a fixed value of h_p at a constant temperature. Fig. 2 shows similar effect by increasing h_p while keeping Δt and T constant.

At any particular temperature $(T < T_c)$ there exists a combination of h_p and Δt which just manages to induce the spin-reversal transition. This $h_p - \Delta t$ curve is given by the phase diagram shown in Fig. 3. The inner side or the axes side of the curves corresponds to the original phase, whereas one gets "spin-reversed" phase for all the combinations of h_p and Δt outside the phase boundary. The limitation arising out of the discrete time simulations force the value of Δt to start from 1, i.e one MCS. With this kind of technique for estimating the critical value of pulsed field strength $h_p(\Delta t)$, the estimate of the phase boundary for $\Delta t < 1$ is not possible. The phase boundaries tend to touch the abscissa at large values of Δt. This is well anticipated because even with an infinitesimally small strength, a negative field will eventually give rise to spin-reversal if applied for sufficiently long time.

An important observation can be made from the series of figures shown in Fig. 4. In Figs. 4(a)-(c) the time series plots of $m(t)$ are shown when h_p is increased (at fixed Δt and T) to reach the phase boundary from below. In Figs. 4(d)-(f) the phase boundary is approached from above by decreasing

h_p at the same value of Δt and T. In either case, it is clear that as one approaches the phase boundary for a particular temperature and pulse width, the (relaxation) time taken by the system to reach its final equilibrium state increases.

This prompts us to define the relaxation time τ of the system as the time (MCS) taken by it to reach the final equilibrium state from the time of withdrawal of the pulse. In Fig. 5, we look for the variation of the quantity τ with h_p for a particular temperature and fixed pulse width. It is clearly seen that τ seems to diverge as we approach the phase boundary from either side for that particular temperature and pulse width. Similar growth of τ is also observed while approaching the phase boundary by varying Δt at fixed T and h_p. Such divergent growths of relaxation time clearly indicate the thermodynamic nature of the spin-reversal transition (divergent correlation length). It may be mentioned here that due to very large scatter (sometimes by order of magnitude) in the values of τ for different Monte Carlo seeds (otherwise thermodynamically identical samples) , log averaging turned out to be a better choice than ordinary averaging of τ. The data for τ shown in Fig. 5 are obtained using log-averages for τ, thereby keeping the relative error less than $O(10^{-2})$.

The mean-field phase diagram is shown in fig. 6. Since there is no fluctuation, there exists a finite coercive field. The spin reversal does not occur even for infinite pulse width ($\Delta t \to \infty$), if the pulse height h_p does not ex-

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ceed the coercive field. The coercive field $h_p(\infty)$ decreases as $(T_c - T)^{\frac{3}{2}}$ with increasing temperature in the mean field case, where $T_c = 1$ in (3). This is because $m_0 h_p(\infty) \sim$ free energy $F(m_0) \sim (T - T_c)m_o^2 + O(m_0^4) \sim (T - T_c)^2$. The inset of Fig. 6 shows the variation of $h_p(\infty)$ as a function of temperature. Unlike the kinetic Ising case, the mean field phase diagrams can be extended beyond $\Delta t = 1$. However since the tanh function saturates to negative unity (for large h_p), the spin reversal can not occur by further increasing the magnitude of the negative field if it is not applied for sufficient time. From (3) we can write $dm = [-m + \tanh \beta(m - h_p)]dt \approx -(m_0 + 1)dt$. Now $\int_0^{\Delta t} dm \approx -(m_0 + 1)\Delta t$ should be sufficient to make the value of m decrease from m_0 to 0. At low temperatures $(m_0 \simeq 1)$ it requires $\Delta t \simeq 1$ while for higher temperatures spin reversal occurs for $\Delta t \underset{\sim}{<} 1$.

4 Discussions

Using Monte Carlo simulations for Ising system evolving under Glauber dynamics (with non-conserving order parameter), we have studied a new dynamic "spin-reversal" transition, where the system goes from one stable equilibrium to another due to the application of a pulsed field (of finite duration) opposite to the existing order of the system. We have determined the phase diagram for such a transition in the pulse strength (h_p)-pulse width (Δt) plane for a fixed temperature T less than the order disorder transition temperature T_c. We observe that the typical relaxation time τ tends to diverge as

the phase boundary is approached, indicating a divergent correlation length associated with such dynamic transition.

In the kinetic Ising Model, the transition is actually triggered by the eventual growth of the "negative" spin domains formed during the period when the negative field was "on". According to the classical nucleation theory, number of droplets of size l is given by

$$n_l = N \exp(-\epsilon_l/T)$$

where N is a normalization constant. Here the free energy for formation of a droplet is given by

$$\epsilon_l = 2h_p l + \sigma l^{\frac{d-1}{d}}$$

in d dimensions, where σ is proportional to the surface tension. From the optimality of ϵ_l the estimated nucleation rate from Becker-Döring theory is given by

$$I = I_0 exp(-\epsilon_{l_c}/T); \quad l_c = \left(\frac{\sigma(d-1)}{2dh_p}\right)^d \tag{4}$$

where I_0 is some constant depending on temperature. Equating the rate I with the inverse pulse width Δt one gets approximately $h_p \sim 1/(\ln \Delta t)$ in $d = 2$ for the phase boundary. However, it can be checked from Fig. 3, this is not the case even for high temperature phase boundaries. This is because the spin-reversal does not necessarily have to take place during the presence of the field, it may occur long after the withdrawal of the pulse. In fact, in our case ϵ_{l_c} is expected to have also a Δt dependence . This can be clearly

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seen from Fig. 1, where the relaxation rate, after the withdrawal of the field, is strongly dependent on the pulse width (Δt).

From the linearized limit of the mean field equation (3), one gets

$$m(t) = \left(m_0 - \frac{h_p}{1-T}\right) e^{(1-T)(t-t_0)/T} + \frac{h_p}{1-T}, \qquad (5)$$

for $t > t_0$ and very close to t_0, so that linearization of (3) is possible. Since there is no fluctuation in the mean field limit, there cannot be any spin reversal if the magnetization remains positive at the time of withdrawal of the pulse. Demanding that the magnetization should at least be zero at the time of withdrawal of the field for an eventual spin reversal, we find the relation between h_p and Δt at the phase boundary :

$$\Delta t = \left(\frac{T}{1-T}\right) \ln\left(\frac{h_p}{h_p + (T-1)m_0}\right). \qquad (6)$$

For temperatures close to $T_c = 1$, equation (6) can be approximated as

$$h_p \Delta t \simeq m_0 T, \qquad (7)$$

which indeed compares fairly well with the mean field phase boundaries (cf. Fig. 6) in the region long before saturation.

The detailed study of the nature of the domain growth in this case of pulsed fields and their statistics are in the process, and shall be published elsewhere.

Acknowledgement

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Figure Captions

Figure 1. Time series plots of the pulse $(h(t))$ and the magnetization $(m(t))$ for $T = 1.0$ and $h_p = 1.04$: (a) $\Delta t = 10$ (b) $\Delta t = 15$ (c) $\Delta t = 28$.

Figure 2. Time series plots of the pulse $(h(t))$ and the magnetization $(m(t))$ for $T = 1.0$ and $\Delta t = 10$: (a) $h_p = 1.04$ (b) $h_p = 1.17$ (c) $h_p = 1.34$.

Figure 3. Phase diagram (Monte Carlo) in the $h_p - \Delta t$ plane for $T = 0.1$ ($\bullet$), $0.5(\nabla)$, $1.0(\square)$, $1.5(\triangle)$, $2.0(*)$. The typical errors present in the data points are less than the symbol sizes.

Figure 4. Time series plots of the pulse $(h(t))$ and the magnetization $(m(t))$ for $T = 1.5$ and $\Delta t = 40$: (a) $h_p = 0.40$ (b) $h_p = 0.45$ (c) $h_p = 0.47$ (d) $h_p = 0.55$ (e) $h_p = 0.49$ (f) $h_p = 0.48$.

Figure 5. The behaviour of τ as the phase boundary is approached from either side: (a) $T = 0.50, \Delta t = 10$ (b) $T = 1.00, \Delta t = 2$ (c) $T = 2.00, \Delta t = 10$.

Figure 6. Phase diagram (mean field) in the $h_p - \Delta t$ plane for $T = 0.1$, 0.3, 0.5, 0.8, 0.9. Inset : Variation of $h_p(\infty)$ with T.

Fig. 1

Fig. 2

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Fig. 3

Fig. 4

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Fig. 5

Fig. 6