Astron. Astrophys. 347, 1000–1004 (1999)

ASTRONOMY AND ASTROPHYSICS

Limits on the proton-proton reaction cross-section from helioseismology

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Received 26 April 1999 / Accepted 28 May 1999

Abstract. Primary inversions of solar oscillation frequencies coupled with the equations of thermal equilibrium and other input physics, enable us to infer the temperature and hydrogen abundance profiles inside the Sun. These profiles also help in setting constraints on the input physics that is consistent with the accurately measured oscillation frequencies data. Helioseismic limits on the cross-section of proton-proton nuclear reaction as a function of heavy element abundance in the solar core are derived. We demonstrate that it is not possible to infer the heavy element abundance profile, in addition to temperature and hydrogen abundance profiles, with the helioseismic constraints.

Key words: Sun: abundances – Sun: interior – Sun: oscillations

1. Introduction

The precisely measured frequencies of solar oscillations provide us with a powerful tool to probe the solar interior with sufficient accuracy. These frequencies are primarily determined by the mechanical quantities like sound speed, density or the adiabatic index of the solar material. The primary inversions of the observed frequencies yield only the sound speed and density profiles inside the Sun. On the other hand, in order to infer the temperature and chemical composition profiles, additional assumptions regarding the input physics such as opacities, equation of state and nuclear energy generation rates are required. Gough & Kosovichev (1988) and Kosovichev (1996) have employed the equations of thermal equilibrium to express the changes in primary variables (ρ, Γ_1) in terms of those in secondary variables (Y, Z) and thus obtained equations connecting the frequency differences to variations in abundance profiles. Shibahashi & Takata (1996), Takata & Shibahashi (1998) and Shibahashi et al. (1998) adopt the equations of thermal equilibrium, standard opacities and nuclear reaction rates to deduce the temperature and hydrogen abundance profiles with the use of only the inverted sound speed profile. Antia & Chitre (1995, 1998) followed a similar approach, but they used the inverted density profile, in addition to the sound speed profile, for calculating the temperature and hydrogen abundance profiles, for a prescribed heavy element abundance (Z) profile.

In general, the computed luminosity in a seismically computed solar model is not expected to be in agreement with the observed solar luminosity. By applying the observed luminosity constraint it is possible to estimate the cross-section of proton-proton (pp) nuclear reaction. Antia & Chitre (1998) estimated this cross-section to be $S_{11} = (4.15 \pm 0.25) \times 10^{-25}$ MeV barns. Similar values have been obtained by comparing the computed solar models with helioseismic data (Degl'Innocenti et al. 1998; Schlattl et al. 1998). The main source of error in these estimates is the uncertainties in Z profiles. In this work we try to find the region in the Z–S_{11} plane that is consistent with the constraints imposed by the helioseismic data.

It may even be argued that one can determine the pressure, in addition to the sound speed and density, from primary inversions using the equation of hydrostatic equilibrium. This profile can then be used as an additional constraint for determining the heavy element abundance profile. In this work we explore the possibility of determining the Z profile in addition to the X profile using this additional input. Alternately, we can determine the Z profile (or opacities) instead of the X profile (Tripathy & Christensen-Dalsgaard1998). Roxburgh (1996) has also examined X profiles which are obtained by suitably scaling the hydrogen abundance profiles from a standard solar model in order to generate the observed luminosity. The motivation of this study was to explore the possibility of reducing the neutrino fluxes yielded by the seismic models by allowing for variations in both the composition profiles as well as selected nuclear reaction rates.

2. The technique

The sound speed and density profiles inside the Sun are inferred from the observed frequencies using a Regularized Least Squares technique (Antia 1996). The primary inversions based on the equations of hydrostatic equilibrium along with the adiabatic oscillation equations, however, give only the mechanical variables like pressure, density and sound speed. This provides us with the ratio T/μ, where μ is the mean molecular weight. In order to determine T and μ separately, it becomes necessary to use the equations of thermal equilibrium, i.e.,

$$L_r = -\frac{64\pi r^2 \sigma T^3}{3\kappa\rho} \frac{dT}{dr}, \tag{1}$$

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$$\frac{dL_r}{dr} = 4\pi r^2 \rho\epsilon, \quad (2)$$

where L_r is the total energy generated within a sphere of radius r, σ is the Stefan-Boltzmann constant, κ is the Rosseland mean opacity, ρ is the density and ϵ is the nuclear energy generation rate per unit mass. In addition, the equation of state needs to be adopted to relate the sound speed to chemical composition and temperature: $c = c(T, \rho, X, Z)$. These three equations are sufficient to determine the three unknowns T, L_r, X, provided the Z profile is prescribed (Antia & Chitre 1998).

The resulting seismic model will not in general have the correct solar luminosity which is an observed quantity. It turns out that we need to adjust the nuclear reaction rates slightly to obtain the correct luminosity and we believe this boundary condition can be profitably used for constraining the nuclear reaction rates. The rate of nuclear energy generation in the Sun is mainly controlled by the cross-section for the pp nuclear reaction, which has not been measured in the laboratory. This nuclear reaction rate is thus calculated theoretically and it would be interesting to test the validity of calculated results using the helioseismic constraints. Since the computed luminosity in seismic models also depends on Z_c, the heavy element abundance in solar core, we attempt to determine the region in the Z_c–S_{11} plane which yields the correct solar luminosity.

Using the density profile along with the equation of hydrostatic equilibrium, it should be possible to determine the pressure profile also from primary inversions. It may even be argued that if we use the additional constraint, $p = p(T, \rho, X, Z)$ it should be possible to determine the Z profile besides other profiles. However, it is not clear if these constraints are independent and in Sect. 3.2 we examine this possibility.

3. Results

We use the observed frequencies from GONG (Global Oscillation Network Group) data for months 4–10 (Hill et al. 1996) which corresponds to the period from 23 August 1995 to 30 April 1996, to calculate the sound speed and density profiles. A Regularized Least Squares (RLS) technique for inversion is adopted for this purpose. With the help of the inverted profiles for sound speed and density, along with the Z profile from Model 5 of Richard et al. (1996), we obtain the temperature and hydrogen abundance profiles by employing the equations of thermal equilibrium. We adopt the OPAL opacities (Iglesias & Rogers 1996), OPAL equation of state (Rogers et al. 1996) and nuclear reaction rates from Adelberger et al. (1998) for obtaining the thermal structure. Recently, Elliot & Kosovichev (1998) have demonstrated that inclusion of relativistic effects in the equation of state improves the agreement with helioseismic data. Since the OPAL equation of state does not include this effect we have applied corrections as outlined by Elliot & Kosovichev (1998) to incorporate the relativistic effects. The inferred mean molecular weight profile is displayed in Fig. 1. The only difference between the present calculations and earlier work of Antia & Chitre (1998) is in the adopted nuclear reaction rates and application of the relativistic correction to the equation of state.

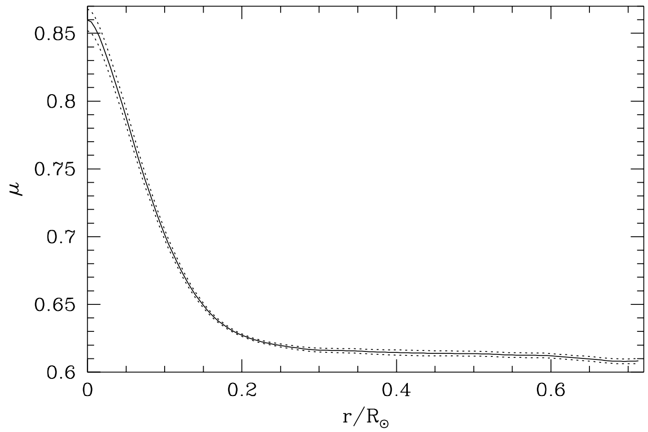

Fig. 1. The mean molecular weight, μ, inferred using the GONG data is shown by the continuous line, while the dotted lines indicate the 1σ error limits.

3.1. Cross-section for pp reaction

With the help of the inverted density, temperature and hydrogen abundance profiles, it is possible to compute the total energy generated by nuclear reactions, and this should be compared with the observed solar luminosity, $L_\odot = 3.846 \times 10^{33}$ ergs/sec. As emphasized by Antia & Chitre (1998) there is an (2σ) uncertainty of about 3% in computing the luminosity of seismic models. This arises from possible errors in primary inversion, solar radius, equation of state, nuclear reaction rates for other reactions. The uncertainty arising from errors in Z profiles is much larger and hence in this work we use seismic models with homogeneous Z profile, covering a wide range of Z values. For each central value of Z we estimate the range of cross-section of pp nuclear reaction, which reproduces the luminosity to within 3% of the observed value. The results are shown in Fig. 2, which delineates the region in Z_c–S_{11} plane that is consistent with helioseismic and luminosity constraints.

It can be seen that current best estimates for Z_c and S_{11} (Bahcall et al. 1998) are only marginally consistent with helioseismic constraints and probably need to be increased slightly. This figure also shows the limits on the values of Z_c obtained by Fukugita & Hata (1998) as well as the range of S_{11} as inferred from various theoretical calculations so far (Bahcall & Pinsonneault 1995; Turck-Chièze & Lopes 1993). One therefore, expects that the values of Z_c and S_{11} should fall within the region with vertical shading in Fig. 2.

The neutrino fluxes in seismic models with the correct luminosity (for the value of S_{11} corresponding to the central line in Fig. 2) as a function of Z_c are shown in Fig. 3. It can be seen that the neutrino flux in ^{71}Ga detector is never as low as the observed value, while the ^{8}B neutrino flux and the neutrino flux in ^{37}Cl are within observed limits, although for disjoint values of Z_c. Thus, a variation of Z_c values does not yield neutrino fluxes that are simultaneously consistent with any two of the three

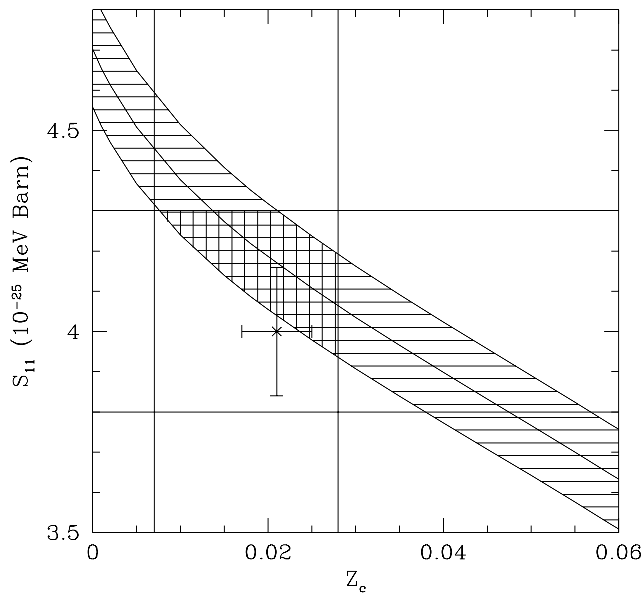

Fig. 2. The region in Z_c–S_{11} plane that is consistent with helioseismic data is marked by horizontal shading. The central line defines the values where the seismic model matches the observed solar luminosity. The point with 2σ error bars shows the current best estimates for Z_c and S_{11}. The vertical lines denote the limits on central Z values obtained by Fukugita & Hata (1998) and the horizontal lines mark the limits on S_{11} as obtained by various calculations so far. The region with vertical shading indicates the area that is consistent with all data.

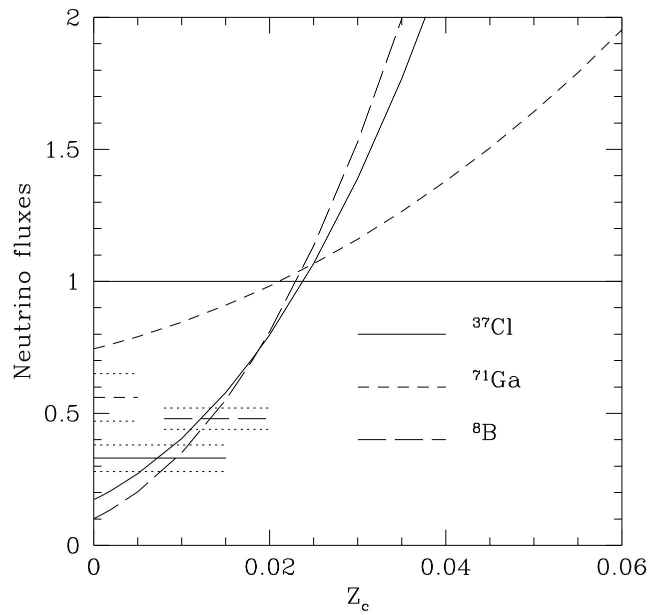

Fig. 3. The neutrino fluxes scaled in terms of those in standard solar model (Bahcall et al. 1998) are displayed as a function of heavy element abundance in the solar core, for the seismic model with the correct observed luminosity. For each neutrino experiment, the horizontal lines mark the observed value with dotted lines denoting the 2σ error limits. The error bars on computed values is not shown for clarity. These error estimates can be found in Table 1 of Antia & Chitre (1998).

solar neutrino experiments. Similar conclusions were reached from more general considerations by Hata et al. (1994), Heeger & Robertson (1996), Bahcall (1996), Castellani et al. (1997), Antia & Chitre (1997).

3.2. Determination of X and Z profiles

It is clear that Z profile is the major source of uncertainty in helioseismic constraint on the pp nuclear reaction cross-section. We, therefore, explore the possibility of determining the Z profile in addition to the T, X profiles using the equations of thermal equilibrium, along with the sound speed, density and pressure profiles. This would require a determination of two of the three unknowns T, X, Z, with the two constraints obtained from primary inversions, namely, $p(T,\rho,X,Z)$ and $c(T,\rho,X,Z)$. We can thus write

$$\frac{\delta c}{c} = \left(\frac{\partial \ln c}{\partial \ln \rho}\right)_{T,X,Z} \frac{\delta\rho}{\rho} + \left(\frac{\partial \ln c}{\partial \ln T}\right)_{\rho,X,Z} \frac{\delta T}{T} + \left(\frac{\partial \ln c}{\partial X}\right)_{\rho,T,Z} \delta X + \left(\frac{\partial \ln c}{\partial Z}\right)_{\rho,T,X} \delta Z, \qquad (3)$$

$$\frac{\delta p}{p} = \left(\frac{\partial \ln p}{\partial \ln \rho}\right)_{T,X,Z} \frac{\delta\rho}{\rho} + \left(\frac{\partial \ln p}{\partial \ln T}\right)_{\rho,X,Z} \frac{\delta T}{T} + \left(\frac{\partial \ln p}{\partial X}\right)_{\rho,T,Z} \delta X + \left(\frac{\partial \ln p}{\partial Z}\right)_{\rho,T,X} \delta Z. \qquad (4)$$

Since ρ is known independently, we ignore the variation in ρ and consider only T, X, Z. Now for a fully ionized nonrelativistic perfect gas, it is well known that

$$2\left(\frac{\partial \ln c}{\partial \ln T}\right)_{\rho,X,Z} = \left(\frac{\partial \ln p}{\partial \ln T}\right)_{\rho,X,Z} = 1, \qquad (5)$$

$$2\left(\frac{\partial \ln c}{\partial X}\right)_{\rho,T,Z} = \left(\frac{\partial \ln p}{\partial X}\right)_{\rho,T,Z} \approx \frac{5}{5X+3-Z}, \qquad (6)$$

$$2\left(\frac{\partial \ln c}{\partial Z}\right)_{\rho,T,X} = \left(\frac{\partial \ln p}{\partial Z}\right)_{\rho,T,X} \approx -\frac{1}{5X+3-Z}, \qquad (7)$$

It is clearly not possible to determine any two of these three quantities T, X, Z, from c and p, since if the ρ variations are ignored, we always have $2\delta c/c = \delta p/p$, and these constraints are not independent. Thus, we need to check if the actual equation of state used in solar model computations allows these quantities to be independent. Another basic problem in trying to determine Z using Eqs. (3–4) is that in general we would expect $|\delta Z| << |\delta X|$, while the derivatives w.r.t. Z are smaller than those w.r.t. X and hence we would expect the δZ term to be much smaller than the δX term, making it difficult to determine Z using these equations. Thus we can only hope to use these equations to determine T and X, while Z can be determined from equations of thermal equilibrium through the opacity, which depends sensitively on Z.

Fig. 4 shows the ratio of partial derivatives for c^2 and p, as a function of r in a solar model and it is clear that these derivatives are almost equal. The wiggles in the curve are probably due to

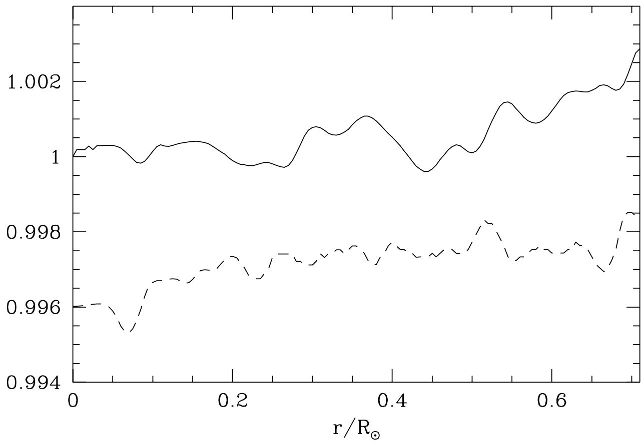

Fig. 4. The continuous line shows the ratio $\frac{\partial \ln c^2}{\partial X} / \frac{\partial \ln p}{\partial X}$ for a solar model, while the dashed line displays the ratio $\frac{\partial \ln c^2}{\partial \ln T} / \frac{\partial \ln p}{\partial \ln T}$.

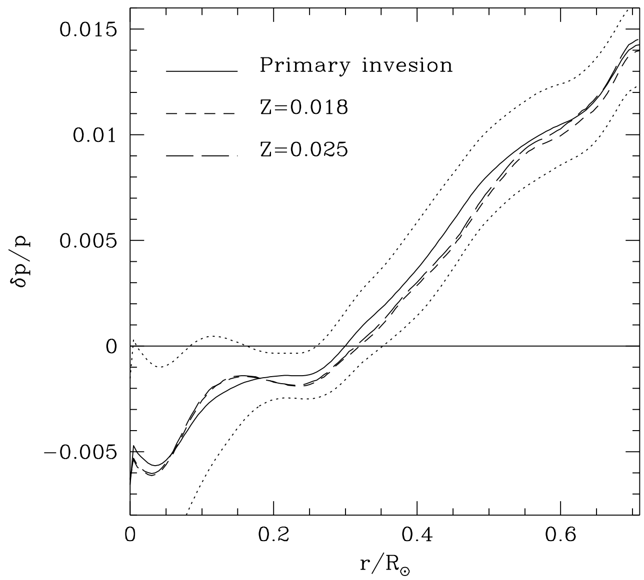

Fig. 5. The relative difference in pressure between the Sun and Model S (Christensen-Dalsgaard et al. 1996) as inferred by primary inversion and by secondary inversion using the OPAL equation of state and labelled value of Z at the surface. The dotted lines are 1σ errors in primary inversion.

errors in estimating these derivatives and it is clear that the departure of the ratio from unity is comparable to these errors, particularly, for the derivatives with respect to X. Thus, for the solar case these two constraints are not independent and it is demonstrably not possible to get any additional information by using the pressure profile. Any attempt to do so will only yield arbitrary results magnifying the errors arising from those in the equation of state and primary inversions.

In order to estimate the extent of error magnification we can try to compute the ratio

$$R_{T,X} = \frac{\left(\frac{\partial \ln c}{\partial \ln T}\right)_{\rho,X,Z} \left(\frac{\partial \ln p}{\partial X}\right)_{\rho,T,Z}}{\left(\frac{\partial \ln c}{\partial \ln T}\right)_{\rho,X,Z} \left(\frac{\partial \ln p}{\partial X}\right)_{\rho,T,Z} - \left(\frac{\partial \ln p}{\partial \ln T}\right)_{\rho,X,Z} \left(\frac{\partial \ln c}{\partial X}\right)_{\rho,T,Z}}, \tag{8}$$

and similar ratios between derivatives with respect to (T, Z) or (X, Z). It turns out that all these quantities are greater than 200 over the entire solar model. Thus all errors will be magnified by a factor of at least 200, if we attempt to determine the Z profile, in addition to T, X profiles.

Even if we do not impose the additional constraint arising from pressure, we can calculate the pressure profile using the OPAL equation of state from the inferred T, ρ, X and assumed Z profiles. As mentioned earlier, we also apply the relativistic corrections (Elliot & Kosovichev 1998) to the equation of state. This p-profile can be compared with that inferred from primary inversions using the equation of hydrostatic equilibrium and Fig. 5 shows the results. It is clear that even without applying the additional constraint from $p(T, \rho, X, Z)$ the resulting profile comes out to be very close to the "independently" inferred profile, well within the 1σ error limits. Moreover, the inferred profile is rather insensitive to Z and hence effecting a change in Z is unlikely to produce the profiles that will match the primary inversion exactly. It is, therefore, evident that the pressure profile does not provide an independent constraint. There are only two independent constraints (e.g., c, ρ) that can be calculated from the primary inversions and it becomes well nigh impossible to determine Z profile in addition to the T, X profiles.

3.3. *Computation of Z profile*

We have stressed earlier that it is not feasible to determine both X and Z profiles, in addition to the temperature, from equations of thermal equilibrium and primary inversions. However, we can reverse the process and determine the Z profile instead of the X profile, using these equations. We, therefore, prescribe an X profile from some solar model and seek to determine the Z profile using the equations described earlier. In this case the equation of state $c = c(T, \rho, X, Z)$ is used to determine T and then using Eqs. (1–2) we can determine L_r and κ. From the opacity κ we can determine the required value of Z using the OPAL opacity tables. Thus in this process we would also get an estimate of opacity variations required to make the solar model consistent with helioseismic data. This is similar to what has, indeed, been done by Tripathy & Christensen-Dalsgaard(1998) except for the fact that they have used only the inverted sound speed profile, while we constrain, in addition, the density profile.

The resulting Z profiles from our calculations are shown in Fig. 6. This figure displays the results using an X profile from a model without diffusion (Bahcall & Pinsonneault 1992) and some models with diffusion (Bahcall et al. 1998; Richard et al. 1996). From Fig. 6 it is clear that for an X profile from a solar model without diffusion, the required change in Z or opacities is rather large, thus supporting other evidence for diffusion of helium below the solar convection zone. The long-dashed line in Fig. 6 has been obtained using the X profile inferred by Antia & Chitre (1998) with the Z profile from Richard et al.

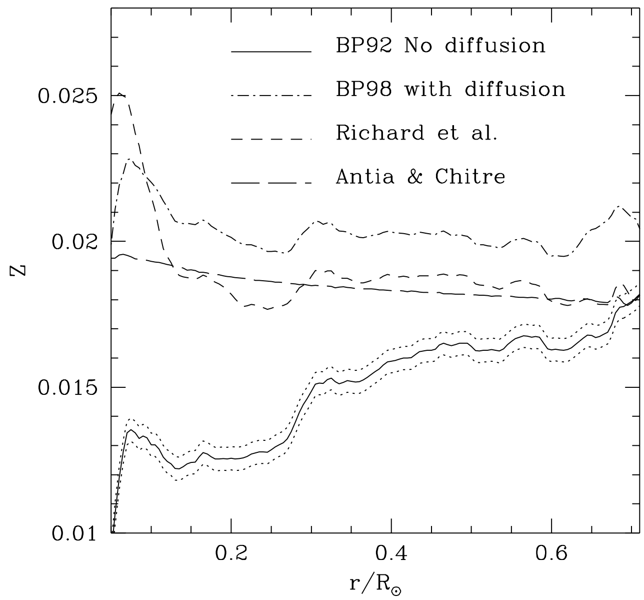

Fig. 6. The Z profiles inferred using a prescribed profile for X from different solar models as labelled in the figure. For clarity, only for one profile the error estimates are shown with dotted lines indicating 1σ error limits.

(1996). The Z profile is evidently reproduced, demonstrating the consistency of the calculations. It may be noted that the error limits displayed in this figure denote the statistical error resulting from uncertainties in observed frequencies and do not include systematic errors arising from other sources. Possible errors in opacity tables may introduce much larger uncertainties in the inferred Z profile. But it is difficult to estimate these errors and hence we have not included them in our analysis. The only purpose of this exercise is to estimate the extent of opacity (or Z) modifications required to get a solar model that is consistent with helioseismic constraints. Of course, this does not give us an estimate of actual error in opacity calculations as there could be other uncertainties in solar models which have not been addressed.

4. Discussion and conclusions

Using the primary inversions for c, ρ, it is possible to infer the T, X profiles in solar interior, provided Z profile is known. The resulting seismic models have the correct solar luminosity only when the heavy element abundance Z_c in the solar core and the cross-section for pp nuclear reaction rate are within the shaded region shown in Fig. 2. It appears that the currently accepted values of Z_c or S_{11} need to be increased marginally to make them consistent with helioseismic constraints.

It is not possible to uniquely determine all three quantities T, X, Z using equations of thermal equilibrium along with results from primary inversions, as there are only two independent constraints that emerge from primary inversions. Incorporation of the pressure profile as an additional input from primary inversions does not yield an independent constraint for determining Z, in addition to T and X. However, it may be possible to determine the Z profile using equations of thermal equilibrium, provided the X profile is independently prescribed. This gives an estimate of variation in opacity required to match the helioseismic data. From these results it is clear that X profile for solar models without diffusion of helium is not consistent with helioseismic data, unless opacity (or Z) is reduced by a large amount.

Acknowledgements. This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, a Division of the National Optical Astronomy Observatories, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofisico de Canarias, and Cerro Tololo Interamerican Observatory.

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