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Feedback stabilization of drift cyclotron loss cone instability by modulated electron sources

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Abstract. It is shown that the drift cyclotron loss cone instability can be suppressed by modulating electron density within the plasma. With the feedback in $+90°$ phase the critical density gradient needed for the onset of the drift cyclotron loss cone instability increases approximately linearly with the gain. Typically with the gain of $-50\Omega_i$ the critical density gradient can be pushed up by as much as two orders of magnitude and minimum mirror plasma radius can be brought down in the same proportion.

Keywords. Feedback stabilization; drift cyclotron loss cone instability; modulated electron sources; mirror machines; critical density gradient; electron density.

1. Introduction

The drift cyclotron loss cone instability (DCLC) which arises because of the resonance between the positive energy electron drift mode and the negative energy ion Bernstein mode has been conclusively identified in high density mirror machines like PR-6, PR-7 (Kanaev and Yushmanov 1974, 1975), $2\times$II B (Simonen 1976), etc. These modes occur at Ω_i and its harmonics (Ω_i ion gyro frequency) and have growth rates roughly of the same order. They require a critical density gradient (CDG) to become unstable and thus set a minimum limit on the mirror plasma radius (Post and Rosenbluth 1966) ($R=200\ a_i$, R mirror plasma radius, a_i ion gyro radius). In this paper we have examined the feedback stabilization of these dangerous modes by modulated electron sources. This method has been suggested and used before for the stabilization of low frequency drift instabilities and drift temperature instabilities, etc. (Simonen 1969; Kitao 1971; Lakhina and Sen 1974). Here we have shown that when the feedback differs by $+90°$ in phase from the unstable perturbation then the CDG increases approximately linearly with the gain. Typically with a feedback gain of $\sim 50\Omega_i$ the CDG can be pushed by as much as two orders of magnitude thereby considerably improving the stability of mirror plasmas against DCLC instability.

2. Calculations

We consider a high density hot ion plasma *i.e.* $\omega_{pe}\approx\Omega_i$, $T_i \gg T_e$ (ω_{pe} electron plasma frequency, Ω_e electron gyro frequency, T_i ion temperature, T_e electron temperature).

This plasma is embedded in a mirror magnetic field $\vec{B} = B_0\hat{Z}$ and has a density gradient

$$\frac{1}{n}\frac{dn}{dx}\hat{x} = \vec{\varepsilon}.$$

The feedback system consists of an instability amplitude sensing probe whose signals are amplified, phase-shifted and impressed on the suppressor probe which, with appropriate negative DC bias, modulates the local electron flow to the probe in giving rise to modulated electron sources. Although in any real experiment such sources would be present only at finite points in the plasma, nevertheless we assume in this model that sources are distributed uniformly throughout the plasma and they respond linearly to the local density perturbation. Such an assumption yields results in good agreement with the experiment (Furth and Rutherford 1969). Thus we represent the source in the form

$$S = -i\omega_f\, n_{\text{el}}, \tag{1}$$

where the amplitude and the argument of $i\omega_f$ represent the gain and the phase of the feedback, n_{el} is the local electron density perturbation. This source is included in Vlasov's electron equation.

Following the standard procedure outlined in Sen and Sundaram (1976) and using the Vlasov-Poisson system of equations with appropriate source term in electron dynamics the dispersion relation for the electrostatic, flute modes in an inhomogeneous, low β plasma can be written as

$$1 + \chi_e' + \chi_i = 0. \tag{2}$$

χ_i and χ_e' are ion and electron susceptibilities and are given by

$$\chi_i = -\frac{1}{k^2 d_i^2}\left[1 - \sum_{n=-\infty}^{\infty} \int \frac{J_n^2(b_i^2)(\omega + \omega_{Ni}) f_{0i}}{(\omega - n\,\Omega_i)} d\vec{v}\right],$$

$$\chi_e = -\frac{\dfrac{1}{k^2 d_e^2}\left[1 - \displaystyle\sum_{n=-\infty}^{\infty} \int \frac{J_n^2(b_e^2)(\omega + \omega_{Ne}) f_{e0}}{(\omega - n\,\Omega_e)} d\vec{v}\right]}{\left[1 - \dfrac{\omega_f}{\omega}\displaystyle\sum_{n=-\infty}^{\infty} I_n(b_e^2)\exp[-b_e^2]\right]},$$

$$b_j^2 = k_\perp^2 v_j^2/2\Omega_j^2, \quad V_j = (2T_j/m_j)^{1/2}, \; d_j = (T_j/4\pi e^2 n_j)^{1/2},$$

$$\Omega_j = \frac{Q_j B_0}{m_j c}, \; \omega_{Nj} = K_\perp \frac{T_j}{m_j}\frac{1}{\Omega_j}\frac{1}{N}\frac{dN}{dX},$$

$$\omega_f = \omega_{fk} + i\,\omega_{fi}, \; i\,\omega_f = R\,e^{i\theta}.$$

Thus, in the dispersion relation (2) the new feature is reflected in X_e' wherein now we have a term in the denominator describing the source characteristics. Since we only consider the electron sources, the ion term is unaffected.

We shall consider the effect of this term on DCLC wave spectrum. For this we make use of the following approximations (Postand Rosenbluth 1966) $k_\perp v_e/\Omega_e \ll 1$, $\omega \ll \Omega_e$ for electrons and $(\epsilon V_\perp^2 k_\perp/\Omega_i\omega) \gg 1$, $(k_\perp v_i/\Omega_i) \gg 1$ for ions. With these approximations and taking a loss cone distribution for ions f_{0i} $(v_1 = 0) = 0$ in equation (2) we arrive at the modified dispersion relation for the DCLC modes as

$$1 + \frac{\omega_{pe}^2}{\Omega_e^2} = \frac{(\omega_{pe}\,\epsilon)/(\Omega_e\,\omega\,|\,k\,|)}{[1-\omega_f/\omega]} - \frac{\omega_{pi}^2\,\Omega_i^2}{k_\perp^3 V_i^3\,\pi} \sum_{n=-\infty}^{\infty} \frac{\omega}{(\omega-n\Omega_i)} \tag{3}$$

Using $\sum_{n=-\infty}^{\infty} 1/(x+n) = \pi \cot \pi x$ and putting $W = \frac{\omega}{\Omega_i} \times \pi$

and $W_f = \frac{\omega_f}{\Omega_i} \times \pi$, equation (3) can be put in the form

$$W^2 \cot W + W\,[\beta - W_f \cot W] - \beta W_f = \beta^{2/3}\,\epsilon\,\langle a_i\rangle\,\pi^{4/3}\left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2}\right)^{-2/3}$$

where $\beta = \pi\,(W\langle a_i\rangle)^3\left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2}\right) > 0$ (4)

For a given density gradient $\epsilon\langle a_i\rangle$ and mode number defined by β, (4) will give the real and the imaginary of the frequency defined by W. But as (4) is transcendental, real and imaginary part of W cannot be evaluated analytically. However, there is yet another method of examining the stability through (4). For W_f=0 equation (4) reduces to the well-known dispersion relation for DCLC modes obtained in Post and Rosenbluth (1966). It is shown there that the left side of (4) admits a saddle point with respect to W and β which gives rise to a critical density gradient $\epsilon_c\,\langle a_i\rangle$. For $\epsilon\,\langle a_i\rangle < \epsilon_c\,\langle a_i\rangle$, all the modes defined by β are stable, while for $\epsilon\,\langle a_i\rangle$ $\epsilon_c\,\langle a_i\rangle$ some of the β are unstable giving rise to the DCLC instability. The idea here is to examine the stability by evaluating CDG in the presence of the feedback *i.e.* for a finite W_f. Such an analysis is possible only when W_f is real *i.e.* for $\theta = \pm 90°$. For other values of θ, W_f becomes complex which makes the evaluation of CDG quite difficult. Accordingly, we proceed to evaluate the CDG for different gains in the phase $\theta = \pm 90°$.

(i) $\theta = +90°\ [\omega_{fk} = -\omega_f,\ \omega_{fi} = 0]$.

In this case (4) becomes

$$W^2 \cot W + W\,[\beta + W_f \cot W] + \beta\,W_f = \beta^{2/3}\,\epsilon\,\langle a_i\rangle\,\pi^{2/3}\left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2}\right)^{-2/3}. \tag{5}$$

We shall use a conventional numerical method to estimate CDG in the presence of the feedback source term, following Post and Rosenbluth (1966). In order to determine the saddle points at which critical density gradient occurs, we adopt the procedure described below.

A particular value of W_f is chosen in (5) and a function $f\,[W\,\beta\,W_f]$ is defined as

$$f\,[W\,\beta\,W_f] = \frac{W^2 \cot W + W\,[\beta + W_f \cot W] + \beta\,W_f}{\beta^{2/3} \times \pi^{2/3}}. \qquad (6)$$

We plot in figures 1 and 2, f as a function of W for different values of W_f and β (the values of β are not given in the figure). The plot shows a maximum with

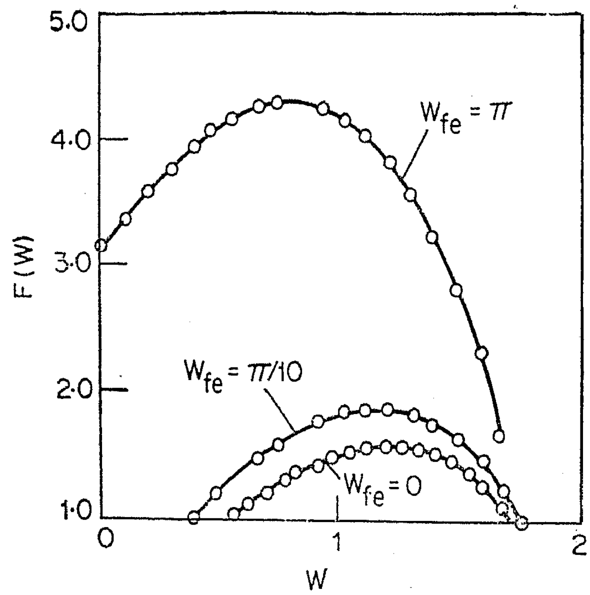

Figure 1. Plot of W vs $F(W)$ for fixed values of W_f

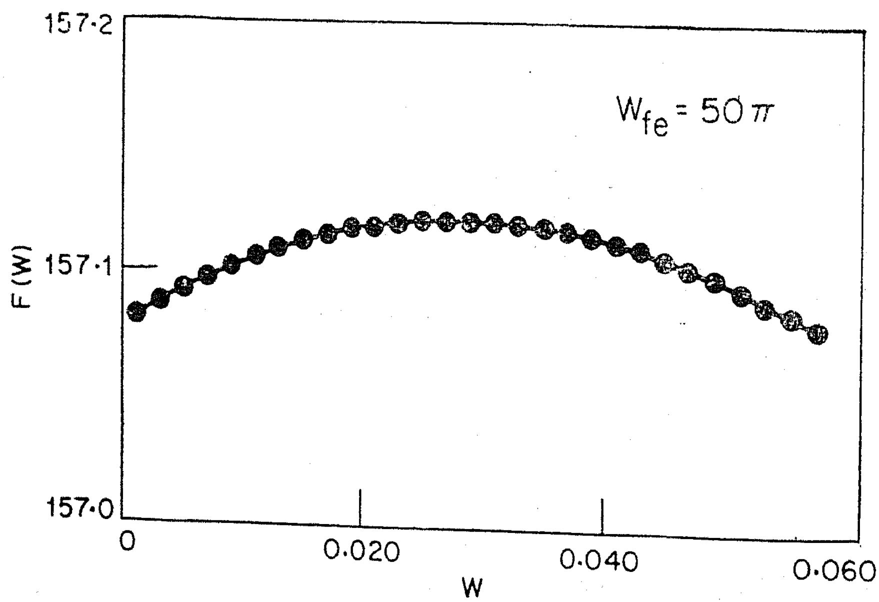

Figure 2. Plot of W vs $F(W)$ for fixed values of W_f

respect to W between 0 and π. The existence of a minimum with respect to β is seen from the following equation.

$$[d^2 f/d\beta^2]_{\substack{W_1\\ \beta_1}} = \frac{1}{3}\beta_1^{-5/3}[W_1+W_f] > 0. \tag{7}$$

As the quantities β_1, W_1 and W_f are all positive, there is a minimum with respect to β. This implies the existence of a saddle point $[W_1\,\beta_1]$ which can be located by simultaneously solving the following set of non-linear equations

$$[df/d\beta]_{\substack{W_1\\ \beta_1}} = \beta_1 - 2\,W_1 \cot W_1 = 0, \tag{8}$$

$$[df/dw]_{\substack{W_1\\ \beta_1}} = \sin W_1 \cos W_1\left[1+\frac{W_1+W_f}{3\,W_1}\right] - \frac{W_1+W_f}{3} = 0. \tag{9}$$

From (8) and (9) we obtain the saddle point coordinates W_1 and β_1, as a function of W_f. The CDG for a particular value of W_f is then given by

$$\frac{f\,[\beta_1(W_f),\,W_1(W_f),\,W_f]}{\pi^{2/3}}\left(\frac{m}{M}+\frac{\Omega_i^2}{\omega_{pi}^2}\right)^{+2/3} = \epsilon_c\,\langle a_i\rangle, \tag{10}$$

where $f\,[\beta_1(W_f),\,W_1(W_f),\,W_f]$ is to be evaluated from (6) which can be rewritten as

$$f\,[W_1\,\beta_1\,W_f] = \frac{W_1^2 \cot W_1+\beta_1 W_1}{\beta_1^{2/3}\times\pi^{4/3}} + \left[\frac{W_1 \cot W_1+\beta_1}{\beta_1^{2/3}\times\pi^{4/3}}\right]W_f. \tag{11}$$

Such calculations have been carried out for different values of W_f. The results are tabulated in table 1. The first column gives the value of W_f the second gives the corresponding CDG. In figure 3, CDG is plotted against Gain. The plot shows that

Table 1. Feedback gain vs critical density gradient.

Gain	CDG
0	0·38
3π	3·961
4π	5·231
5π	6·50
6π	7·79
7π	9·07
8π	10·35
9π	11·64
10π	12·76
20π	25·82
30π	38·72
40π	51·52
50π	64·52

CDG increases linearly with W_f in the range 0–50 π. It must be mentioned here that this linearity is only approximate because it so turns out that the value of the first term and the coefficient of W_f in the second term in (11) does not change much for different saddle point coordinates W_1 (ω_f) and β_1 (ω_f), because of which (11) approximately represents a straight line with f and W_f as the two variables. It must also be mentioned that (though it appears) the straight line does not pass through the origin because even in the absence of the feedback there is a CDG. This is also obvious from (11).

From the table it is clear that with a gain of $\sim 50\ \Omega_i$ the CDG can be increased by as much as two orders of magnitude, thereby considerably improving the stability of the mirror plasma against DCLC instability. As shown in Post and Rosenbluth (1966) the minimum plasma radius is given by

$$R_{\min} = \frac{\pi^{4/3}}{f\,[W_1, \beta_1]} (M/m)^{4/3}\, a_i\, \Psi\, (\Omega_e^2/\omega_{pi}^2), \tag{12}$$

where Ψ is a function of Ω_e^2/ω_{pe}^2, and is tabulated in Post and Rosenbluth (1966) for different values of its argument. It is clear from (12) that $R_{\min}$ goes inversely as the gain of the feedback. Typically in the present-day mirror machines where $\Omega_e^2/\omega_{pe}^2 \sim 1$, $a_i \sim 1$ cm, with a gain of $\sim 50\ \Omega_i$, $R_{\min}$ can be brought down from 500 cm to 3 cm, thereby almost removing the constraint imposed by DCLC instability on the radius of the mirror plasma.

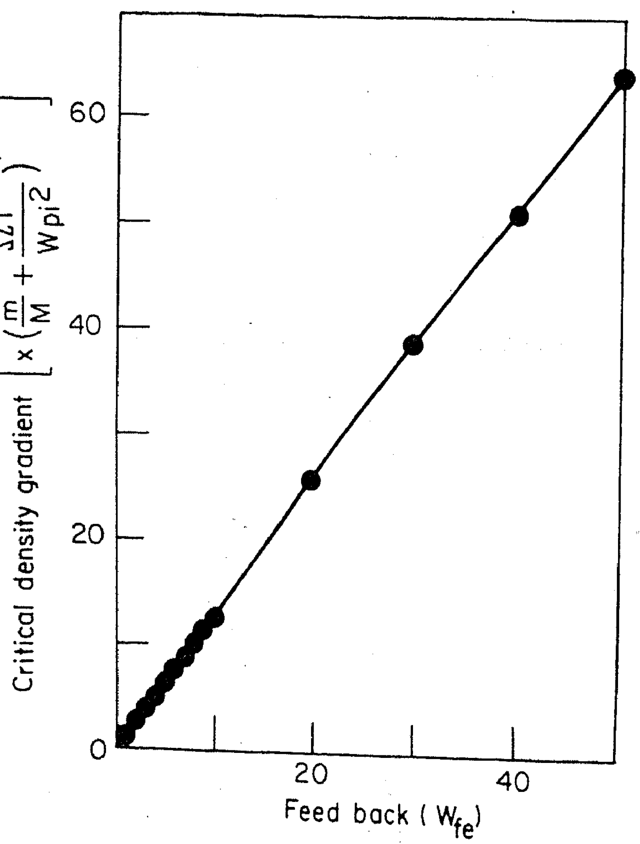

Figure 3. Plot of feedback gain vs critical density gradient.

(ii) $\theta = -90°$ $[W_{fk} = W_f, W_{f1} = 0]$. In this case, as follows from (7) the sign of $d^2f/d\beta^2$ is not fixed. For $W > W_f$, $d^2f/d\beta^2$ is positive while for $W < W_f$, $d^2f/d\beta^2$ is negative. Thus the saddle point and the consequent CDG does not exist, and the effect of the feedback on the overall DCLC spectrum cannot be evaluated. The stability of a particular K in such phase angles should be evaluated numerically from the dispersion relation (4).

3. Discussion

We have shown here that the stability of the mirror plasma against DCLC instability can be considerably improved by modulating electron sources at ion gyro frequencies and at a $+90°$ phase difference from the unstable perturbation. The question of the number of feedback loops and their location has to be decided by the experiment. For instance in Simonen's (1969) experiment on quenching of drift instabilities in Q-machines by modulated electron sources only one feedback loop consisting of two Langmuir probes located in the region of maximum wave amplitude was sufficient to bring about a considerable improvement in the density build-up, confinement time, etc. In our problem, however, more than one loop may be required as DCLC modes are not localized and in fact are spread over a larger plasma cross-section. In the case of mirror plasmas, as the probe would be in actual contact with the hot plasma some complication may arise due to the heating and sputtering of the probe. But we do not expect these effects to be very important as mirror plasma experiments are pulsed (a few msec) and their thermal energy content is very low (a few calories).

Arsenin *et al* (1968 abc) have reported stabilization of $m=1$ flute mode and ion cyclotron instabilities in the low density plasmas ($\sim 10^7/\text{cm}^3$) by placing feedback loops at the radial boundaries to appropriately control them. This method cannot be very effective for the suppression of drift instabilities where the boundary conditions play only a supplementary role. To suppress drift instabilities, the signals have to be injected in the plasma to modulate the particle sources.

Our calculations mainly highlight the importance of feedback systems in stabilizing DCLC mode and provide an upper limiting value of the feedback gain. Considerations such as warm plasma streams (Baldwin *et al* 1976; Gerver 1976) in the loss region of velocity space and the compressional perturbations of magnetic field (Tang 1972) will significantly lower the limit for the gain and thereby will make the stabilizing action of modulated electron sources easier.

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