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DIFFUSION COEFFICIENTS OF NUCLEONS IN THE INHOMOGENEOUS BIG BANG MODEL *

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ABSTRACT

Recent investigations of the effects of inhomogeneities generated by the quark-hadron phase transition in the early universe on the primordial nucleosynthesis have shown that the difference in the mean free paths of neutrons and protons and the resulting diffusive segregation have a significant influence on the cosmological abundances of the light elements. We calculate the diffusion coefficients of neutrons and protons moving through the background electron and photon gases and their mutual diffusion coefficient, which are important inputs in the nucleosynthesis calculations, in the framework of relativistic kinetic theory in the temperature range $10^8 \leq T \leq 5.\ 10^9\ {}^\circ K$. The coefficients are shown to have explicit dependence on density and temperature.

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1. Introduction

There is a strong possibility that a first order QCD phase transition from the quark–gluon plasma to the confined hadronic matter had occurred in the early the universe, when the temperature was about 100 MeV. Witten [1] pointed out that this phase transition might have produced isothermal baryon number fluctuations because the quark–gluon plasma has a greater baryon concentration than the hadronic phase with which it is in thermal and chemical equilibrium. Applegate and Hogan [2,3] have suggested that the characteristic size of these fluctuations could be such that protons would not be able to diffuse across them before the onset of nucleosynthesis but the neutrons would, as they have no electrical charge and therefore suffer less scattering. This would result in high density regions rich in protons and low density regions rich in neutrons. Consequently the scenario would be quite different from that of the standard big–bang model in which the density of nucleons is assumed to be the same everywhere at the time of nucleosynthesis. Recently several authors [4–9] have calculated the abundance of light elements ^2H,^3He,^4He and ^7Li using this inhomogeneous model of primordial nucleosynthesis.

In this non–standard big–bang model [5] the diffusion of nucleons from regions of density inhomogeneity, especially the diffusion lengths of neutrons and protons moving through the surrounding electron and photon gases, affects the nucleosynthesis in a crucial manner. The calculations of Applegate, Hogan and Scherrer [3] have shown that the difference in the mean free paths of neutrons and protons and the resultant diffusive segregation influences the formation of the light elements very significantly. Therefore it is important to evaluate the diffusion coefficients of neutrons and protons carefully in order to see whether in the inhomogeneous model of the primordial nucleosynthesis, the cosmological abundances of the light nuclei are modified. In ref. 3 the diffusion coefficients were calculated using a mobility formula and the Einstein relation between mobility and the diffusion coefficient [10]. We calculate the diffusion coefficients in the framework of the kinetic theory, assuming all particles to be classical. It turns out that in the temperature range relevant for nucleosynthesis, $10^8 \leq T \leq 5.10^9\ ^0K$, this assumption is not restrictive. The electrons are relativistic in this energy range and so we use the relativistic version of of the kinetic theory [11–13] in our calculations.

In the temperature range under consideration neutrons and protons are no longer in equilibrium with respect to weak interactions and as a result they retain their identity for diffusive segregation to take place. Neutrons are scattered by electrons through the interaction of their magnetic moments and by protons due to nuclear interaction. Protons, on the other hand, undergo Coulomb scattering by electrons and Thomson scattering by photons, and are also scattered by neutrons. With these elementary cross-sections as input we calculate the diffusion coefficients.

2. Diffusion Coefficients

The diffusion coefficient of particle i moving through a gas of particles j is denoted by D_{ij}. The general expression for D_{ij} in the first order Chapman–Enskog approximation [13] may be written as,

$$D_{ij} = \frac{12c}{n\sigma_{ij}(T)} \frac{(1-x_i)}{x_j} \frac{1}{I_{ij}}, \tag{1}$$

where n is the total density of the system of consisting of photons, protons, electrons and neutrons,

$$\begin{aligned} n &= n_\gamma + n_p + n_e + n_n \\ x_i &= \frac{n_i}{n}, \end{aligned} \tag{2}$$

and $\sigma_{ij}(T)$ is the total scattering cross-section at the temperature T under consideration. I_{ij} is the integral,

$$\begin{aligned} I_{ij} = & \frac{\pi z_{ij}^5}{z_i^2 z_j^2 K_2(z_i) K_2(z_j)} \int_1^\infty dt\, t^{-3} \left[(t^2-1)\left\{ t^2 - \left(1 - \frac{4\mu_{ij}}{M_{ij}}\right)\right\}\right]^2 \times \\ & \times \left\{ K_2(z_{ij}t) + (z_{ij}t) K_1(z_{ij}t) \right\} \Sigma_{ij}(t) \end{aligned} \tag{3}$$

where

$$\begin{aligned} z_i &= \frac{m_i c^2}{k_B T}, \\ z_{ij} &= z_i + z_j, \\ \mu_{ij} &= \frac{m_i m_j}{m_i + m_j} \quad \text{and} \quad M_{ij} = m_i + m_j \end{aligned}$$

$K_\nu(z_i)$ is the modified Bessel function of order ν. Also

$$\Sigma_{ij} = \frac{\gamma_{ij}}{2\sigma_{ij}(T)} \int_0^\pi \sin\theta d\theta (1-\cos\theta) d\sigma_{ij}(cM_{ij}t, \theta) \tag{4}$$

where

$$\gamma_{ij} = 1 - \frac{1}{2}\delta_{ij}$$

and $d\sigma_{ij}$ is the differential scattering cross-section in the centre of mass system.

We now list the various diffusion coefficients calculated from Eq. (3) and Eq. (4).

(a) Neutron–Electron

Since $m_e \ll m_n$ we evaluate the integral I_{ne} [Eq. (3)] in the lowest order of z_e/z_n to get,

$$I_{ne} = 32\sqrt{\frac{2}{\pi}}\; z_e^{1/2}\; \frac{K_{\frac{7}{2}}(z_e)}{K_2(z_e)} \tag{5}$$

From Eq. (1) we get for the diffusion coefficient,

$$D_{ne} = \frac{3}{8}\sqrt{\frac{\pi}{2}}\,\frac{c}{n\sigma_{ne}}\,\frac{1-x_n}{x_e}\,\frac{1}{z_e^{1/2}}\,\frac{K_2(z_e)}{K_{\frac{7}{2}}(z_e)}, \tag{6}$$

The neutron–electron interaction is through their magnetic moments and the cross-section σ_{ne} is,

$$\sigma_{ne} = 3\pi\kappa^2\left(\frac{\alpha\hbar c}{m_n c^2}\right)^2 \tag{7}$$

where κ is the anomalous magnetic moment of neutron in nuclear magnetons.

It is interesting to observe that for non-relativistic electrons ($k_B T \ll m_e c^2$) one obtains the same expression for D_{ne} as given by the mobility formula [10]. (This formula has been used in ref. [3] for calculating D_{ne}, albeit for relativistic electrons.) The mobility b of the neutrons moving through the electron gas of density n_e is given by,

$$b^{-1} = \frac{m_e^2}{3k_BT}\int f(v)\sigma_{ne}v^3 d^3p, \tag{8}$$

where $f(v)$ is the Maxwell velocity distribution function of the electrons,

$$f(v) = \frac{n_e}{(2\pi m_e k_B T)^{3/2}}\exp\left(-\frac{p^2}{2m_e k_B T}\right) \tag{9}$$

Carrying out the integral we get for b^{-1},

$$b^{-1} = \frac{8}{3}\sqrt{\frac{2}{\pi}}(m_e c^2 k_B T)^{1/2}\frac{n_e\sigma_{ne}}{c} \tag{10}$$

We now use the Einstein relation between b and the diffusion coefficient,

$$D_{ne} = bk_BT, \tag{11}$$

and obtain

$$D_{ne} = \frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{c}{n_e\sigma_{ne}}\left(\frac{k_BT}{m_ec^2}\right)^{1/2} \tag{12}$$

which is the same as Eq. (6) in the limit of large z_e for a dilute gas of neutrons ($x_n \ll 1$) diffusing through electrons.

(b) Proton–Electron

The differential scattering cross–section in this case is given by the Mott formula

$$\frac{d\sigma_{pe}}{d\Omega}(p_e, \cos\theta) = \frac{(\alpha\hbar c)^2}{4(cp_e)^4\sin^4(\theta/2)}\left[(m_ec^2)^2 + (cp_e)^2\cos^2\frac{\theta}{2}\right], \tag{13}$$

where p_e is the electron momentum. Because of the divergence at small angles, the usual approximation is to cut–off the angular integration at an angle given by the ratio of the Debye shielding length $\lambda_D = (k_BT/e^2n_e)^{1/2}$ and a typical distance of closest approach for which one takes the thermal wave length $\lambda_{th} = (2\pi\hbar^2/m_e k_B T)^{1/2}$ [13]. The cross-section to be used in the diffusion coefficient is then given by,

$$\sigma_{pe} = 4\pi\alpha^2 \left(\frac{\hbar c}{k_B T}\right)^2 \log \delta, \tag{14}$$

where $\delta = \lambda_D/\lambda_{th}$. The diffusion coefficient D_{pe} is then,

$$D_{pe} = \frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{c}{n\sigma_{pe}}\frac{1-x_p}{x_e}\frac{1}{z_e^{1/2}}\frac{K_2(z_e)}{K_{\frac{5}{2}}(z_e)} \tag{15}$$

(c) Neutron–Proton

In this case both the particles are non-relativistic and the scattering cross-section is given by the triplet and singlet scattering lengths a_t and a_s,

$$\sigma_{np} = 3\pi a_t^2 + \pi a_s^2. \tag{16}$$

We get for the diffusion coefficient in this case,

$$D_{np} = \frac{3\sqrt{\pi}}{4}\frac{c}{n\sigma_{np}}\frac{1-x_n}{x_p}\frac{1}{z_p^{1/2}} \tag{17}$$

(d) Proton–Photon

Since one of the particles has zero mass we have same case as in (a) and the diffusion coefficient is given by,

$$D_{p\gamma} = \frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{c}{n\sigma_{p\gamma}}\frac{1-x_p}{x_\gamma}\frac{1}{z_p^{1/2}}\frac{K_2(z_p)}{K_{\frac{5}{2}}(z_p)}, \tag{18}$$

where the proton Thomson cross-section $\sigma_{p\gamma}$ is

$$\sigma_{p\gamma} = \frac{8\pi}{3}\left(\frac{e^2}{m_p c^2}\right)^2 \tag{19}$$

Again for $k_B T \ll m_p c^2$,

$$D_{p\gamma} = \frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{c}{n\sigma_{p\gamma}}\frac{1-x_p}{x_\gamma}\frac{1}{z_p^{1/2}}. \tag{20}$$

3. Results and Conclusions

We have calculated the numerical values of the diffusion coefficients D_{ne} and D_{np}. The densities n_γ, n_e, n_p and n_n are calculated using the following expressions valid in the radiation era [14,15] :

$$
\begin{aligned}
n_\gamma &= 20.3T^3 \\
n_p = n_{e^-} &= 1.5 \times 10^{-7}\Omega T^3\{1 - 0.17exp(-2.10^{20}T^{-2}/t_n)\} \\
n_n &= 1.5 \times 10^{-7}\Omega T^3(0.17exp(-2.10^{20}T^{-2}/t_n))
\end{aligned}
\tag{21}
$$

where Ω and t_n are respectively the density parameter and neutron life-time. We take $\Omega = 0.2$ and $t_n = 887.6$ sec [16]. The results are shown in Table I. For D_{ne} our results differ widely from those of Applegate et al [3]. This is largely because their diffusion coefficient does not depend on the electron density, whereas it depends very strongly on the temperature—D_{ne} is proportional to $\exp(m_e c^2/k_B T)$ which becomes inordinately large for $k_B T \ll m_e c^2$. For D_{np} our expression Eq. (13) is the same, except for numerical factors, as the elementary kinetic theory expression $\frac{1}{3}v\lambda$, which has been used in ref. [3]. Here v is the velocity of the neutron and $\lambda = 1/n_p\sigma_{np}$ is its mean free path. The agreement of numerical values of the two calculations is to within an order of magnitude.

We would like to emphasize that our expressions for the diffusion coefficients have explicit dependence on the densities of the diffusing particles and we recover the correct expressions in the non–relativistic limit. An important feature that emerges from the calculations is the equivalence between the expression for the diffusion coefficient D_{ne} given by the mobility formula and that derived from the kinetic theory for a dilute neutron gas diffusing through electrons at low temperatures. We have thus shown that the relativistic kinetic theory can be used to calculate the various diffusion coefficients needed for the inhomogeneous nucleosynthesis model as long as the classical approximation is valid. This approximation can be checked by calculating the degeneracy parameter $(h^2/2\pi m k_B T)^{3/2} n$ which is $\ll 1$ throughout the density and temperature range that we have considered. We expect our values for the diffusion coefficients, which differ from the values used so far, to have significant influence on the abundance of light elements in an inhomogeneous cosmological model. However this can only be shown by detailed calculations which are in progress [17].

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TABLE I

$T_9(^\circ K)$	$n(cm^{-3})$	$D_{ne}(cm^2/sec)$	$D_{np}(cm^2/sec)$
5.0	2.5×10^{30}	2.9×10^{18}	1.4×10^{10}
3.0	5.5×10^{29}	1.2×10^{19}	4.9×10^{10}
1.0	2.0×10^{28}	2.3×10^{20}	7.8×10^{11}
0.8	1.0×10^{28}	4.2×10^{20}	1.2×10^{12}
0.6	4.4×10^{27}	8.8×10^{20}	2.5×10^{12}
0.4	1.3×10^{27}	2.3×10^{21}	6.5×10^{12}
0.2	1.6×10^{26}	1.3×10^{22}	3.4×10^{13}
0.1	2.0×10^{25}	7.4×10^{22}	1.9×10^{14}

Table I. Values of D_{ne} and D_{np} as functions of density n and temperature T.