Pramāṇa, Vol. 10, No. 3, March 1978, pp. 247-255, © Printed in India.

Ion acoustic solitary waves in density and temperature gradients

N NAGESHA RAO and RAM K VARMA
Physical Research Laboratory, Ahmedabad 380 009

MS received 22 August 1977

Abstract. The propagation of ion-acoustic K-dV solitary waves in weakly inhomogeneous, collisionless plasmas with gradients both in the density and the temperature of the ions has been considered. The electrons are assumed to be hot and isothermal, and the ions to be warm and adiabatic. The reductive perturbation analysis of the fluid equations is then carried out. The zero order quantities existing in the system due to the presence of the inhomogeneities are taken into account consistently and a set of ' stretched coordinates ' appropriate for the inhomogeneous system is employed. A more general modified K-dV equation has been derived and its soliton solution is obtained explicitly. It is shown that as the soliton propagates along the temperature gradient, its amplitude and the velocity decrease, and the width increases. Further, it is found that when the two gradients are in opposite directions, the amplitude of the soliton remains constant.

Keywords. Solitary wave; soliton; Korteweg-de Vries equation; temperature and density gradients; inhomogeneous plasma; stretched co-ordinates.

1. Introduction

The propagation of certain weakly nonlinear waves in weakly dispersive media is known to be governed by the Korteweg-de Vries equation (Jeffry and Kakutani 1972). In particular, the ion acoustic solitary waves in homogeneous plasmas have been extensively studied, both theoretically and experimentally, by many people over the last several years (Sagdeev 1966; Washimi and Taniuti 1966; Davidson 1972; Ikezi *et al* 1970; Ikezi 1973). These are found to be described by the K-dV equation if one assumes cold ions and isothermal electrons. Sakanaka (1972) and Tappert (1972) have, on the other hand, studied the propagation of ion acoustic waves in inhomogeneous plasmas with warm adiabatic ions.

The propagation of solitary waves in weakly inhomogeneous media has been considered quite generally by Asano (1974) and specialized to the ion acoustic case by Nishikawa and Kaw (1975), and by Gell and Gomberoff (1977). Both these calculations are inadequate and inconsistent since they do not take into account the zero order quantities—the ion-fluid velocity and the electric field which arise due to the presence of inhomogeneity, and further, they have not used the right set of 'stretched coordinates' appropriate for the spatially inhomogeneous system. Recently, Rao and Varma (to be published) have eliminated these shortcomings.

Finally, the case of cold ion plasma with gradients in ion density as well as in electron temperature has been considered by Goswami and Sinha (1976). Since the electron thermal conductivity goes as $T_e^{5/2}$, for high temperature plasmas appreciable temperature gradients cannot be expected. Hence, they assume the electron temperature gradient scale-length to be much larger than that of the ion density gradient. On the other hand, a more realistic problem would be to consider temperature gradient in the ions (not in electrons) coupled with their density gradient, with the scale-lengths of the two gradients to be of the same order.

In the present paper, we extend our previous calculations to include spatial inhomogeneity both in the density and the temperature of the ions. While the electrons are still assumed to be isothermal, the ions are assumed to be warm and adiabatic. The reductive perturbation analysis of fluid equations is then carried out by employing a set of 'stretched co-ordinates' appropriate for spatially inhomogeneous plasmas (see, for instance, Asano 1974). A more general modified K-dV equation has been derived and is integrated analytically to get the soliton solution. The main result of this analysis is that as the soliton propagates in the direction of the temperature gradient, its amplitude and velocity decrease whereas the width increases. Further, it is found that when the two gradients are in opposite directions, the soliton amplitude remains constant whereas the width and the velocity keep changing.

2. Basic equations and stretched coordinates

We consider a collisionless, spatially inhomogeneous plasma having both density and temperature gradients in ions. The scale-lengths of the two gradients are assumed to be of the same order. The electrons are assumed to be isothermal because of their high thermal conductivity, while the ions are assumed to be warm and adiabatic. The basic equations for the problem are then:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nV)=0 \tag{1}$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{\partial \phi}{\partial x}+\frac{1}{n}\frac{\partial P}{\partial x}=0 \tag{2}$$

$$\frac{\partial^2\phi}{\partial x^2}-g(x_0)\,e^{\phi}+n=0 \tag{3}$$

$$\frac{\partial P}{\partial t}+V\frac{\partial P}{\partial x}+3P\frac{\partial V}{\partial x}=0 \tag{4}$$

where the last equation is the adiabatic law for ions, with $\gamma=3$ (because of one dimensionality), neglecting transport processes such as heat conduction, viscosity, etc. Here, n is the ion density, V the ion-fluid velocity, P the ion pressure, ϕ the electrostatic potential and x and t are the space and time variables. All the quantities are normalized respectively with respect to the standard plasma parameters, viz., plasma density (N), ion acoustic velocity ($\sqrt{KT_e/m_i}$), electron pressure (NKT_e), a characteristic potential (KT_e/e), electron Debye length ($\sqrt{KT_e/4\pi Ne^2}$) and the ion plasma period ($\sqrt{m_i/4\pi Ne^2}$), all these quantities being defined at $x=0$.

Following Asano (1974), we introduce the following stretched coordinates,

$$\left.\begin{aligned} \xi &= \epsilon^{1/2}\left(\int^{x} \frac{dx'}{\lambda_0(x')} - t\right) \\ \eta &= \epsilon^{3/2}\, x \end{aligned}\right\} \tag{5a}$$

where $\lambda_0(x)$ is the phase velocity of the moving frame and will be determined later self-consistently. From eqs (5a) it follows that

$$\left.\begin{aligned} x &= \epsilon^{-3/2}\,\eta \\ t &= \epsilon^{-3/2}\int^{\eta} \frac{d\eta'}{\lambda_0(\eta')} - \epsilon^{-1/2}\,\xi \end{aligned}\right\} \tag{5b}$$

From eqs (5a) and (5b) we easily obtain the following transformations for the space and time derivatives.

$$\left.\begin{aligned} \frac{\partial}{\partial x} &\equiv \frac{\epsilon^{1/2}}{\lambda_0(\eta)}\frac{\partial}{\partial \xi} + \epsilon^{3/2}\frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial t} &\equiv -\,\epsilon^{1/2}\frac{\partial}{\partial \xi} \end{aligned}\right\} \tag{6a}$$

and

$$\left.\begin{aligned} \frac{\partial}{\partial \xi} &\equiv -\,\epsilon^{-1/2}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial \eta} &\equiv \epsilon^{-3/2}\frac{\partial}{\partial x} + \frac{\epsilon^{-3/2}}{\lambda_0(x)}\frac{\partial}{\partial t} \end{aligned}\right\} \tag{6b}$$

Hence, the equilibrium quantities n_0 and P_0 which depend only on x satisfy

$$\left.\begin{aligned} &\frac{\partial n_0}{\partial \xi} = 0 = \frac{\partial P_0}{\partial \xi}, \\ &\frac{\partial \lambda_0}{\partial \xi} = 0. \end{aligned}\right\} \tag{7}$$

Using eqs (5a) and (6a), we write eqs (1)–(4) as

$$-\frac{\partial n}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(nV) + \epsilon\frac{\partial}{\partial \eta}(nV) = 0 \tag{8}$$

$$-n\frac{\partial V}{\partial \xi}+\frac{nV}{\lambda_0}\frac{\partial V}{\partial \xi}+\epsilon nV\frac{\partial V}{\partial \eta}+\frac{n}{\lambda_0}\frac{\partial \phi}{\partial \xi}+\epsilon n\frac{\partial \phi}{\partial \eta}+\frac{1}{\lambda_0}\frac{\partial P}{\partial \xi}+\epsilon\frac{\partial P}{\partial \eta}=0 \tag{9}$$

$$\frac{\epsilon}{\lambda_0^2}\frac{\partial^2 \phi}{\partial \xi^2}+\frac{2\epsilon^2}{\lambda_0}\frac{\partial^2 \phi}{\partial \xi \partial \eta}-\frac{\epsilon^2}{\lambda_0^2}\frac{\partial \lambda_0}{\partial \eta}\frac{\partial \phi}{\delta \xi}+\epsilon^3\frac{\partial^2 \phi}{\partial \eta^2}-g(\eta_0)\,e^{\phi}+n=0 \tag{10}$$

and

$$-\frac{\partial P}{\partial \xi}+\frac{V}{\lambda_0}\frac{\partial P}{\partial \xi}+\epsilon V\frac{\partial P}{\partial \eta}+\frac{3P}{\lambda_0}\frac{\partial V}{\partial \xi}+3\epsilon P\frac{\partial V}{\partial \eta}=0. \tag{11}$$

Next, we carry out the reductive perturbation analysis of the above equations.

3. Derivation of the modified K-dV equation

To carry out the perturbation analysis, we expand the quantities n, V, ϕ and P in terms of the smallness parameter ϵ around their equilibrium values n_0, V_0, ϕ_0 and P_0, respectively, as

$$\begin{aligned} n &= n_0+\epsilon n_1+\epsilon^2 n_2+\epsilon^3 n_3+\dots \\ V &= V_0+\epsilon V_1+\epsilon^2 V_2+\epsilon^3 V_3+\dots \\ \phi &= \phi_0+\epsilon \phi_1+\epsilon^2 \phi_2+\epsilon^3 \phi_3+\dots \\ P &= P_0+\epsilon P_1+\epsilon^2 P_2+\epsilon^3 P_3+\dots \end{aligned} \tag{12}$$

Substituting these expansions into eqs (8)–(11), we get sets of equations to different orders in ϵ. The zero order equations are

$$-\frac{\partial n_0}{\partial \xi}+\frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(n_0 V_0)=0 \tag{13}$$

$$-n_0\frac{\partial V_0}{\partial \xi}+\frac{n_0 V_0}{\lambda_0}\frac{\partial V_0}{\partial \xi}+\frac{n_0}{\lambda_0}\frac{\partial \phi_0}{\partial \xi}+\frac{1}{\lambda_0}\frac{\partial P_0}{\partial \xi}=0 \tag{14}$$

$$-g(\eta_0)\,e^{\phi_0}+n_0=0 \tag{15}$$

$$-\frac{\partial P_0}{\partial \xi}+\frac{V_0}{\lambda_0}\frac{\partial P_0}{\partial \xi}+\frac{3P_0}{\lambda_0}\frac{\partial V_0}{\partial \xi}=0. \tag{16}$$

Combining these equations with eq. (7), we get

$$\frac{\partial V_0}{\partial \xi}=0;\quad \frac{\partial \phi_0}{\partial \xi}=0. \tag{17}$$

The first order equations in ϵ from eqs (8)–(11), respectively, are

$$-\frac{\partial n_1}{\partial \xi}+\frac{n_0}{\lambda_0}\frac{\partial V_1}{\partial \xi}+\frac{V_0}{\lambda_0}\frac{\partial n_1}{\partial \xi}+\frac{\partial}{\partial \eta}(n_0V_0)=0 \tag{18}$$

$$-n_0\frac{\partial V_1}{\partial \xi}+\frac{n_0V_0}{\lambda_0}\frac{\partial V_1}{\partial \xi}+n_0V_0\frac{\partial V_0}{\partial \eta}+\frac{n_0}{\lambda_0}\frac{\partial \phi_1}{\partial \xi}$$

$$+n_0\frac{\partial \phi_0}{\partial \eta}+\frac{1}{\lambda_0}\frac{\partial P_1}{\partial \xi}+\frac{\partial P_0}{\partial \eta}=0 \tag{19}$$

$$-\eta_0\phi_1+n_1=0 \tag{20}$$

$$-\frac{\partial P_1}{\partial \xi}+\frac{V_0}{\lambda_0}\frac{\partial P_1}{\partial \xi}+V_0\frac{\partial P_0}{\partial \eta}+\frac{3P_0}{\lambda_0}\frac{\partial V_1}{\partial \xi}+3P_0\frac{\partial V_0}{\partial \eta}=0 \tag{21}$$

where we have used eqs (7) and (17). We now impose the boundary conditions that the plasma is homogeneous at $|\xi|\to\infty$; that is,

$$\left.\begin{array}{l} n_1=V_1=\phi_1 \\ V_0,\ \phi_0,\ P_0\to 0 \\ n_0,\ \lambda_0\to 1 \end{array}\right\} \text{ as } |\xi|\to\infty.$$

Using these boundary conditions, eqs (18)–(21) can easily be integrated with respect to ξ to give, respectively,

$$V_1=Rn_1-\xi Q \tag{22}$$

$$\phi_1=(\lambda_0-V_0)V_1-\frac{1}{n_0}P_1-\frac{\lambda_0}{n_0}\left(n_0V_0\frac{\partial V_0}{\partial \eta}+n_0\frac{\partial \phi_0}{\partial \eta}+\frac{\partial P_0}{\partial \eta}\right)\xi \tag{23}$$

$$n_1=n_0\phi_1 \tag{24}$$

$$P_1=\frac{1}{(\lambda_0-V_0)}\left[3P_0(Rn_0\phi_1-\xi Q)+\xi\lambda_0\left(V_0\frac{\partial P_0}{\partial \eta}+3P_0\frac{\partial V_0}{\partial \eta}\right)\right] \tag{25}$$

where

$$R=\frac{(\lambda_0-V_0)}{n_0},\quad Q=\frac{\lambda_0}{n_0}\frac{\partial}{\partial \eta}(n_0V_0).$$

Equations (22)–(25) form a set of nonhomogeneous equations for n_1, V_1, ϕ_1 and P_1. Using eqs (22), (24) and (25), we eliminate V_1 and P_1 in eq. (23) in terms of ϕ_1 alone to get

$$\phi_1=\frac{\xi\left[-n_0Q(\lambda_0-V_0)^2+3P_0Q-\lambda_0\left(V_0\frac{\partial P_0}{\partial \eta}+3P_0\frac{\partial V_0}{\partial \eta}\right)-\lambda_0(\lambda_0-V_0)\left(n_0V_0\frac{\partial V_0}{\partial \eta}+n_0\frac{\partial \phi_0}{\partial \eta}+\frac{\partial P_0}{\partial \eta}\right)\right]}{n_0(\lambda_0-V_0)-Rn_0^2(\lambda_0-V_0)^2+3Rn_0P_0} \tag{26}$$

Here, the right hand side depends only on the zero order quantities whereas ϕ_1 is a first order quantity. Since the first order quantities cannot be determined in terms of zero order quantities alone, we make the above expression for ϕ_1 an indeterminate quantity with respect to the zero order quantities; that is, we put both the numerator as well as the denominator of the above expression equal to zero separately.

Putting the denominator equal to zero, we get

$$\lambda_0 = V_0 + \sqrt{1 + \frac{3P_0}{n_0}} \tag{27}$$

which determines λ_0 self-consistently. Similarly, setting the numerator equal to zero one gets, after using eq. (27),

$$n_0\lambda_0(\lambda_0 - V_0)\frac{\partial V_0}{\partial \eta} + V_0\frac{\partial n_0}{\partial \eta} + \lambda_0\frac{\partial P_0}{\partial \eta} + n_0(\lambda_0 - V_0)\frac{\partial \phi_0}{\partial \eta} = 0. \tag{28}$$

This equation gives a self-consistent relationship between the zero order quantities.

To derive the modified K-dV equation, we now consider the second order quantities coming respectively from eqs (8)–(11):

$$-\frac{\partial n_2}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(n_0V_2 + n_1V_1 + n_2V_0) + \frac{\partial}{\partial \eta}(n_0V_1 + n_1V_0) = 0 \tag{29}$$

$$\left.\begin{aligned} &-\frac{\partial V_2}{\partial \xi}n_0 - n_1\frac{\partial V_1}{\partial \xi} + \frac{n_0V_0}{\lambda_0}\frac{\partial V_2}{\partial \xi} + n_0V_0\frac{\partial V_1}{\partial \eta} \\ &+ \frac{1}{\lambda_0}(n_0V_1 + n_1V_0)\frac{\partial V_1}{\partial \xi} + (n_0V_1 + n_1V_0)\frac{\partial V_0}{\partial \eta} \\ &+ \frac{1}{\lambda_0}\left(n_0\frac{\partial \phi_2}{\partial \xi} + n_1\frac{\partial \phi_1}{\partial \xi}\right) + \left(n_0\frac{\partial \phi_1}{\partial \eta} + n_1\frac{\partial \phi_0}{\partial \eta}\right) + \frac{1}{\lambda_0}\frac{\partial P_2}{\partial \xi} + \frac{\partial P_1}{\partial \eta} \end{aligned}\right\} = 0 \tag{30}$$

$$\frac{1}{\lambda_0^2}\frac{\partial^2 \phi_1}{\partial \xi^2} - n_0\phi_2 - \tfrac{1}{2}n_0\phi_1^2 + n_2 = 0 \tag{31}$$

$$\left.\begin{aligned} &-\frac{\partial P_2}{\partial \xi} + \frac{1}{\lambda_0}\left(V_0\frac{\partial P_2}{\partial \xi} + V_1\frac{\partial P_1}{\partial \xi}\right) + \left(V_0\frac{\partial P_1}{\partial \eta} + V_1\frac{\partial P_0}{\partial \eta}\right) \\ &+ \frac{3}{\lambda_0}\left(P_0\frac{\partial V_2}{\partial \xi} + P_1\frac{\partial V_1}{\partial \xi}\right) + 3\left(P_0\frac{\partial V_1}{\partial \eta} + P_1\frac{\partial V_0}{\partial \eta}\right) \end{aligned}\right\} = 0 \tag{32}$$

Using eq. (27), the second order quantities in eqs (29)–(32) are eliminated exactly. In this, we substitute for n_1, V_1 and P_1 in terms of ϕ_1 using eqs (22), (24) and (25). Simplifying the various coefficients, we finally get the following modified K-dV equation

$$\left.\begin{aligned}&\frac{\partial\phi_1}{\partial\eta}+\left(\frac{1}{\mu_0\lambda_0^2}\right)\phi_1\frac{\partial\phi_1}{\partial\xi}+\left(\frac{1}{2n_0\mu_0\lambda_0^4}\right)\frac{\partial^3\phi_1}{\partial\xi^3}\\&\quad-\left(\frac{1}{\lambda_0}\frac{\partial\lambda_0}{\partial\eta}-\frac{1}{\mu_0}\frac{\partial\mu_0}{\partial\eta}\right)\xi\frac{\partial\phi_1}{\partial\xi}+\tfrac{1}{2}\left(\frac{1}{n_0}\frac{\partial n_0}{\partial\eta}+\frac{3}{\mu_0}\frac{\partial\mu_0}{\partial\eta}\right)\phi_1\end{aligned}\right\}=0 \tag{33}$$

where $\mu_0=\sqrt{1+3\,P_0/n_0}$.

It can be noted that for cold ion plasma, $p_0=0$ and hence, $\mu_0=1$. The eq. (33) then reduces to the one obtained earlier by Rao and Varma for the case of density gradients alone, and further, for the approximation $\lambda_0\approx 1$, it reduces to the equation obtained by Nishikawa and Kaw. Hence, eq. (33) is a general equation for ion acoustic K-dV solitons in inhomogeneous plasmas with the zero order quantities taken into account self-consistently.

4. Solution of the modified K-dV equation

To solve the modified K-dV equation (33) we make transformations of dependent and independent variables similar to those given by, for example, Asano (1974), and Nishikawa and Kaw (1975). Defining the new variables

$$\tilde{\phi}_1=(n_0\mu_0^3)^{1/2}\cdot\phi_1 \tag{34a}$$

$$\left.\begin{aligned}\tilde{\xi}&=\left(1+\log\frac{\lambda_0}{\mu_0}\right)\xi\\ \tilde{\eta}&=\eta\end{aligned}\right\} \tag{34b}$$

eq. (33) can easily be reduced to

$$\frac{\partial\tilde{\phi}_1}{\partial\tilde{\eta}}+\frac{1}{\sqrt{N_0}}\tilde{\phi}_1\frac{\partial\tilde{\phi}_1}{\partial\tilde{\xi}}+\frac{1}{2N_0}\frac{\partial^3\tilde{\phi}_1}{\partial\tilde{\xi}^3}=0 \tag{35}$$

where $N_0=n_0\mu_0\lambda_0^4$.

In order to reduce eq. (35) to the usual K-dV equation with constant coefficients, we introduce a set of independent variables (ζ,τ) defined by

$$\left.\begin{aligned}\zeta&=[N_0(\tilde{\eta})]^{1/4}\,\tilde{\xi}\\ \tau&=\int^{\tilde{\eta}}d\tilde{\eta}'\,[N_0(\tilde{\eta}')]^{-1/4}\end{aligned}\right\} \tag{36}$$

Then, eq. (35) can be written as

$$\frac{\partial\tilde{\phi}_1}{\partial\tau}+\tilde{\phi}_1\frac{\partial\tilde{\phi}_1}{\partial\zeta}+\tfrac{1}{2}\frac{\partial^3\tilde{\phi}_1}{\partial\zeta^3}+K\zeta\frac{\partial\tilde{\phi}_1}{\partial\zeta}=0 \tag{37}$$

with $$K = \frac{1}{4N_0^{3/4}}\left(\frac{\partial N_0}{\partial \eta}\right).$$

The last term on the L.H.S. of eq. (37) can be eliminated by defining another set of variables

$$\left.\begin{aligned}\tilde{\zeta} &= (1 - \int K(\tau)\cdot d\tau) \\ \tilde{\tau} &= \tau\end{aligned}\right\} \tag{38}$$

Thus, eq. (37), finally, reduces to the usual K-dV equation for a homogeneous plasma, namely,

$$\frac{\partial \tilde{\phi}_1}{\partial \tilde{\tau}} + \phi_1 \frac{\partial \tilde{\phi}_1}{\partial \tilde{\zeta}} + \frac{1}{2}\frac{\partial^3 \tilde{\phi}_1}{\partial \tilde{\zeta}^3} = 0. \tag{39}$$

The soliton solution of this equation can be written as

$$\tilde{\phi}_1 = 3a\cdot \mathrm{sech}^2\left[\left(\frac{a}{2}\right)^{1/2}(\tilde{\zeta} - a\tilde{\tau})\right] \tag{40}$$

In terms of ξ and η variables, this solution becomes

$$\left.\begin{aligned}\phi_1 = \frac{3a}{\sqrt{n_0\mu_0^3}}\cdot \mathrm{sech}^2\Bigg[\left(\frac{a}{2}\right)^{1/2}\Bigg\{&(N_0^{1/4}(1 - \log N_0^{1/4}) \\ &\cdot\left((1 + \log\frac{\lambda_0}{\mu_0}\right)\xi - a\int \frac{d\eta}{N_0^{1/4}}\Bigg\}\Bigg]\end{aligned}\right\} \tag{41}$$

It is obvious from this solution that the temperature gradients do modify the propagation characteristics of the solitary waves.

5. Results and discussions

If we assume the ideal gas law for the ions, viz., $P = \gamma nT$ with $\gamma = 3$ for one-dimensional adiabatic motion of the ions, then, $P_0 = 3n_0 T_0$ and the expression for μ_0 becomes

$$\mu_0 = \sqrt{1 + 9T_0} \tag{42}$$

where $T_0(x)$ is the given ion-temperature distribution with respect to the space variable x and is normalized with respect to the constant electron temperature (T_e). Combining the expression (42) with the solution (41), we note the following result: As the soliton propagates in the direction of the temperature gradient, its amplitude and velocity decrease, and the width increases. This result is as expected and is easily understood physically as follows. As the temperature of the ions increases, the dispersive effects tend to increase; thus, as the soliton propagates towards higher temperature regions, a given non-linearity in the wave is always overbalanced by the dispersive effects and hence the amplitude keeps on decreasing. Corresponding results follow for the width and the velocity of the soliton.

The fact that the effects of the temperature gradients on the propagation characteristics of the solitons is similar to those of density gradients brings out another interesting result. From the solution (41), we notice that if the quantity $(n_0\, \mu_0^3)$ remains constant with respect to the space variable x, then, the amplitude of the soliton remains constant as it moves in the plasma. However, the width and the velocity of the soliton will change. Hence, for this to happen, the gradients have to be in the opposite directions and satisfy the relation

$$\frac{1}{n_0}\left(\frac{\partial n_0}{\partial x}\right) = -\frac{3}{\mu_0}\left(\frac{\partial \mu_0}{\partial x}\right).$$

This result can, again, be easily explained by the reasons given above.

To conclude, we have derived a general modified K-dV equation for ion-acoustic solitary waves in plasmas with gradients both in the density and the temperature of the ions. We have shown that along the temperature gradient, the amplitude and velocity of the soliton decrease whereas the width increases. Thus, the ion-temperature gradient affects the propagation characteristics of the soliton in a way similar to that of the density gradient. Further, we have pointed out that in a plasma with the two gradients in opposite directions, the soliton amplitude remains constant. These results are easily explained on simple physical grounds.

References

Asano N 1974 *Prog. Theor. Phys.* Suppl. No. **55** 52
Davidson R C 1972 *Methods in Nonlinear Plasma Theory* (New York, London: Academic Press Ch. 2
Gell Y and Gomberoff L 1977 *Phys. Lett.* **A60** 125
Goswami B N and Sinha M 1976 *Pramana* **7** 141
Ikezi H, Taylor R J and Baker D R 1970 *Phys. Rev. Lett.* **25** 11
Ikezi H 1973 *Phys. Fluids* **16** 1668
Jeffry A and Kakutani T 1972 *SIAM (Soc. Ind. Appl. Maths) Rev.* **14** 582
Nishikawa K and Kaw P K 1975 *Phys. Lett.* **A50** 455
Rao N N and Varma R K (to be published)
Sagdeev R Z 1966 *Reviews of Plasma Physics* ed. M A Leontovich (New York: Consultants Bureau) Vol. 4
Sakanaka P H 1972 *Phys. Fluids* **15** 304
Tappert F 1972 *Phys. Fluids* **15** 2446
Washimi H and Taniuti T 1966 *Phys. Rev. Lett.* **17** 996