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Chiral Gauge Theories from D-Branes

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Abstract

We construct brane configurations leading to chiral four dimensional $N = 1$ supersymmetric gauge theories. The brane realizations consist of intersecting Neveu-Schwarz five-branes and Dirichlet four-branes in non-flat spacetime backgrounds. We discuss in some detail the construction in a $\mathbb{C}^2/\mathbb{Z}_M$ orbifold background. The infrared theory on the four-brane worldvolume is a four dimensional $N = 1$ $SU(N)^M$ gauge theory with chiral matter representations. We discuss various consistency checks and show that the spectral curves describing the Coulomb phase of the theory can be obtained once the orbifold brane construction is embedded in M-theory. We also discuss the addition of extra vectorlike matter and other interesting generalizations.

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1. Introduction.

Recently, the study of Dirichlet branes has led to important insights into the behavior of supersymmetric gauge theories. One approach, which has proved especially powerful, is to consider configurations consisting of intersecting Neveu-Schwarz 5-branes and Dirichlet-branes [1]-[20]. It was shown by Witten, [5], that such configurations often correspond to a single 5-brane in M theory. A simple scaling argument shows that the quantum behavior of the resulting gauge theory can then be understood as a classical effect in M theory. So far, in this approach, the background spacetime before adding branes has been taken to be flat (for another important approach which considers branes in Calabi-Yau backgrounds see [21] and references therein), and the resulting gauge theories have been non-chiral (see, however, refs. [22], [11]). The main purpose of this paper is to note that brane configurations in non-trivial backgrounds can often lead to chiral gauge theories. We illustrate this by considering brane configurations consisting of NS 5-branes and intersecting D4-branes in a simple class of orbifold backgrounds. As in the flat space case, the brane construction allows us to deduce various features about the non-perturbative behavior of these theories.

This paper is organized as follows. In Section 2, we describe the $\mathbb{C}^2/\mathbb{Z}_M$ orbifold background and brane configuration consisting of Dirichlet 4-branes placed at the orbifold point and stretched between two Neveu-Schwarz 5-branes. The low-energy dynamics is shown to be described by a 3+1 dimensional $N = 1$ theory with $SU(N)^M$ gauge group and chiral matter content. In fact, the gauge theory turns out to be closely related (apart from some anomalous $U(1)$ factors) to the theories studied in [23], [24]. In Section 3, we study the classical moduli space of this gauge theory and show that it corresponds to the set of allowed motions for the brane configuration; this provides additional evidence that we have identified the correct gauge theory. In Section 4, we turn to the quantum theory and show how by considering the configuration in M theory one can deduce various non-perturbative features of the low-energy dynamics, pertaining to the Seiberg-Witten spectral curves. Finally, some generalizations of the basic brane configuration are discussed in Section 5.

This paper is intended to be a first step in a more complete analysis. Two further generalizations are obvious and will be considered in a subsequent paper. One is to consider orientifold backgrounds. The resulting chiral theories are in many ways more interesting. Another is to blow up the orbifold and consider the brane configuration in the corresponding ALE space. The resulting smooth background allows for a more controlled analysis in M theory. The methods outlined in this paper give rise to theories which are, in a sense, closely related to $N = 2$ theories. As will become clear below, their matter content can be thought of as arising from adjoint fields after a suitable truncation. These methods might consequently have limited use in the study of chiral theories with spinor matter.

2. Brane Configuration and Matter Content.

2.1 The orbifold and brane configuration.

In this paper we will consider $\mathbb{C}^2/\mathbb{Z}_M$ orbifolds. We choose coordinates so that the $\mathbb{C}^2$ involved in the orbifold corresponds to the $X^4 + iX^5$ and $X^8 + iX^9$ directions. The Type IIA brane configuration we consider involves two NS 5-branes and several Dirichlet 4-branes, as shown in Fig. 1. The NS branes stretch along X^1, X^2, X^3, X^4, X^5, are placed at the orbifold point, $X^8 = X^9 = 0$, and have definite positions in X^6, X^7. We take them to be separated by a finite distance in the X^6 direction and to be coincident in the X^7 direction. The D4-branes are taken to lie along X^1, X^2, X^3, and X^6 directions and end on the two NS branes.

2.2 The gauge group and matter content

As is well known, the low-energy dynamics of this configuration is described by a 3+1 dimensional field theory, which lives in the intersection region of the D4 branes and NS branes. We will show below that NM 4-branes placed at the origin of the $\mathbb{Z}_M$ orbifold give rise to an $N = 1$ $U(N)^M$ gauge theory. The matter content consists of chiral superfields which transform under the gauge groups as:

	$U(N)_1$	$U(N)_2$	$U(N)_3$	$\cdots$	$U(N)_M$
Q_1	$\square$	$\overline{\square}$	1	$\cdots$	1
Q_2	1	$\square$	$\overline{\square}$	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Q_M	$\overline{\square}$	1	1	$\cdots$	$\square$

$$(2.1)$$

Note that the matter content is chiral[2].

We now turn to justifying this claim for the gauge group and matter content. First consider the number of supersymmetries. In the absence of the orbifold this brane configuration preserves 8 supercharges or $N = 2$ supersymmetry in 3+1 dimensions: the IIA theory has 32 supercharges, but the presence of 4-branes and NS branes reduces that by a factor of $2 \cdot 2$. In the $\mathbb{Z}_M$ orbifold we only keep gravitino states for which the vertex operators are invariant under a rotation by $\exp\{(\frac{2\pi i}{M})(J_{45} - J_{89})\}$. This further reduces the supersymmetry by half leading to 4 supercharges or $N = 1$ in 3+1 dimensions.

To arrive at the gauge group and matter content it is useful to consider the final configuration built up in two stages. Let us first look at a configuration without the NS branes where the 4-branes are infinite along X^6 and are placed at the orbifold point. It is well known

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[2] One overall $U(1)$ factor above is "frozen out" while the remaining $U(1)$s are anomalous; we will have more to say on this below.

for compact orbifolds that tadpoles must cancel in the one-loop vacuum amplitude, and that this constraint is often powerful enough to determine the gauge group and matter content [25]. In our case, the one loop amplitude only receives a contribution from the cylinder diagram and is easy to work out. Since the $\mathbb{C}^2$ on which the orbifold group acts is noncompact, we do not expect any constraint on the allowed total number of 4-branes: the corresponding RR flux can always escape to infinity. This is borne out by an explicit calculation. However, there are non-trivial constraints which arise from the tadpole cancellation for twisted RR fields. Let the orbifold group $\mathbb{Z}_M$ act on $v = X^4 + iX^5$ and $w = X^8 + iX^9$ as:

$$(v,\ w) \rightarrow (\alpha\ v,\ \alpha^{-1}\ w)\ ,\quad \alpha \equiv e^{\frac{2\pi i}{M}}\ , \tag{2.2}$$

and the corresponding action of the orbifold group on the Chan Paton factors λ be represented by a matrix γ_α:

$$\lambda \rightarrow \gamma_\alpha\ \lambda\ \gamma_\alpha^{-1}\ . \tag{2.3}$$

The 4-branes are sources of twisted RR scalars that can only propagate in one of the directions transverse to the 4-branes (X^7). As argued in [26], a one-volume is insufficient to allow the Ramond-Ramond flux to escape to infinity, and the tadpole cancellation condition must be satisfied even for infinite volume. The constraints from the twisted RR tadpoles are then given by:

$$\operatorname{tr} \gamma_\alpha^K\ =\ 0,\quad K = 1, \ldots, M - 1\ . \tag{2.4}$$

Note that γ_α must furnish a representation of the orbifold group and thus $\gamma_\alpha^M = 1$. This together with eq. (2.4) allows us to solve for γ_α. We find, first, that the number of 4-branes at the orbifold point must be a multiple of M; we refer to this number hereafter as NM. Second, we find that the matrix γ_α, in a suitable basis, is given by:

$$\gamma_\alpha\ =\ \operatorname{diag}\{1 \times \mathbf{1}_N, \alpha \times \mathbf{1}_N, \ldots, \alpha^{M-1} \times \mathbf{1}_N\}\ , \tag{2.5}$$

with $\mathbf{1}_N$ being the unit $N \times N$ matrix. The gauge and matter content can now be worked out as well. The corresponding gauge group on the 4-brane worldvolume theory turns out to be $U(N)^M$. Fluctuations in the X^7 direction which survive the orbifold projection contribute one adjoint field for each $U(N)$ factor. Together with the gauge bosons these form an $N = 1$ vector multiplet in 4+1 dimensions. Finally, from the X^4, X^5, X^8, X^9 directions we get hypermultiplets transforming under the gauge groups as described in eq. (2.1) (we note that the same orbifold has been considered in [27], [26]).

Now finally we can add the two NS branes and sandwich the four-branes between them as in Fig. 1. What is the resulting 3+1 dimensional theory? It is useful for this purpose to describe the above matter content in the language of 3+1 dimensions. The component of the gauge field, A_6, can be paired with the adjoint fields coming from the X^7 direction
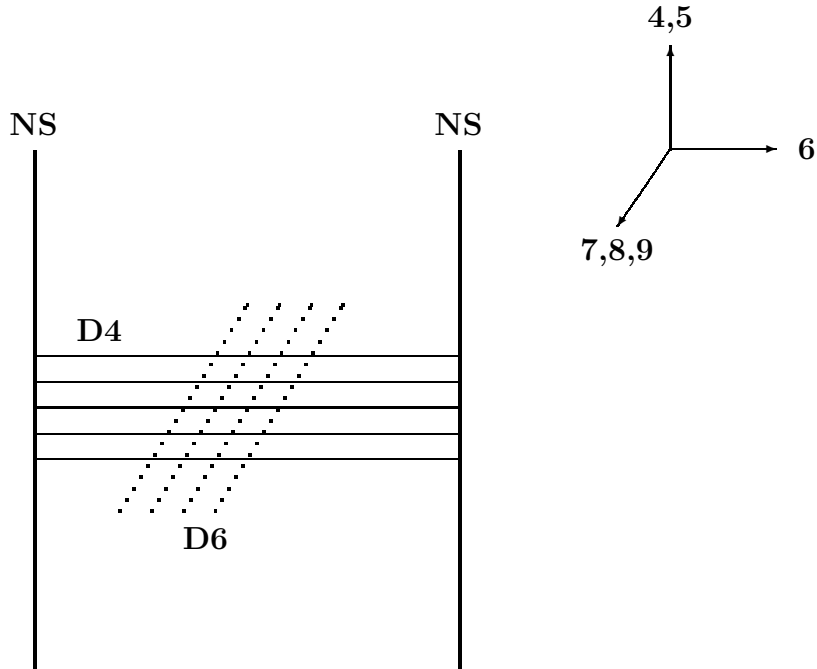
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Figure 1: The Type IIA brane configuration for the $\mathbb{Z}_M$ orbifold models. There are N physical four-branes stretched between the NS five branes, plus their $\mathbb{Z}_M$ images. The six branes, if present, give extra vectorlike matter. The bending of the NS branes is not shown in this figure.

to give a chiral superfield. Each hypermultiplet will transform as two chiral multiplets in $3+1$ dim. language, one of the two chiral multiplets coming from fluctuations in the X^4, X^5 directions, and the other from the X^8, X^9 directions. One expects the boundary conditions coming from the ends of the 4 brane, where it terminates on the 5-brane, to freeze some of these degrees of freedom. Based on the analysis in the absence of the orbifold one expects the gauge field to survive and the chiral mulitiplet coming from the (A_6, X^7), fluctuations to be frozen. Similarly, the matter coming from the fluctuations in the X^4, X^5 directions should survive whereas that from the X^8, X^9 directions will be frozen out. This finally gives rise to the $U(N)^M$ theory with the matter content described in eq. (2.1). We note again that each field in eq. (2.1) represents a chiral multiplet so that the theory is chiral.

Above, we first considered the 4-branes without NS branes in the orbifold background and then introduced the NS branes. It is also illuminating to consider things in the opposite order. Accordingly, let us first consider a configuration of NM 4-branes stretched between the two NS branes in the absence of the orbifold. The resulting field theory is well known to be an $N = 2$ theory, with $SU(NM)$ gauge group. The adjoint scalar field corresponds to fluctuations of the 4 branes along the X^4, X^5 directions. It is natural to expect that the orbifold should correspond to implementing a projection in this theory. In fact, the gauge theory possesses a $U(1)$ global symmetry under which (in $N = 1$ language) the gauge field

and its fermionic partner transform as $(A_\mu, \lambda) \to (A_\mu, \lambda)$, and the adjoint and its fermionic partner as $(\phi, \psi) \to e^{i\alpha}(\phi, \psi)$. In general this symmetry is anomalous, however it has a non-anomalous $\mathbb{Z}_{2NM}$ discrete subgroup. This discrete subgroup in turn has a $\mathbb{Z}_M$ subgroup. In addition, the gauge symmetry has a $\mathbb{Z}_M$ discrete subgroup under which a fundamental representation is multiplied by $\mathrm{diag}\{1 \times \mathbf{1}_N, \alpha \times \mathbf{1}_N, \ldots, \alpha^{M-1} \times \mathbf{1}_N\}$, with $\mathbf{1}_N$ being the unit $N \times N$ matrix. In the $N = 2$ field theory it is natural to identify the orbifold group with the product of these two $\mathbb{Z}_M$ symmetries. On doing so and retaining states invariant under this product discrete symmetry one gets precisely the $U(N)^M$ group and matter content mentioned above.

3. Brane Motion and the Classical Moduli Space.

In this section we compare the set of allowed motions of the brane configuration to the classical moduli space of the gauge theory described above. This will serve two purposes. First, agreement between the two will give additional evidence that we have identified the correct gauge theory. Second, in the process we will understand better the role of the various $U(1)$s in this theory—an issue which we have so far not fully addressed.

It will be convenient in the following discussion to organize the $U(1)$s in the following basis. We will choose the first $U(1)$ to be the sum of the $U(1)$ factors, and the other $U(1)$s to be orthogonal to the first. It is easy to see from eq. (2.1) that none of the matter fields are charged under the first $U(1)$. In fact one can deduce that this $U(1)$ factor is frozen, i.e. its coupling vanishes. There are two arguments in support of this. First, for the case of a flat space time background, it was argued in [5], that in the $N = 2$ theory this overall $U(1)$ must be frozen. We saw above that for the orbifold background the resulting field theory could be understood as a further truncation of the $N = 2$ theory; we thus expect the $U(1)$ to continue to be frozen in it. Second, we will see below that when we interpret this configuration in M theory, the genus of the two dimensional surface spanned by the 5-brane worldvolume will be consistent with the absence of the $U(1)$.

Turning our attention to the remaining $U(1)$s we notice that they are all anomalous[3]. These $U(1)$s are analogous to anomalous $U(1)$ factors which often arise in string compactifications [28]. In the context of D-branes anomalous $U(1)$s were discussed in [27] where they were shown to play an important role in governing the low-energy dynamics. We will discuss these $U(1)$s in some detail below. Here we summarize their essential features which are important in the present discussion of the classical moduli space. The important point is that these anomalous $U(1)$s are broken. The low-energy 3+1 dimensional theory contains axion fields which arise from twisted RR fields, and the anomalies are cancelled by shifting these axions appropriately [29]. In fact the axions can be regarded as the longitudinal com-

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[3]The $\mathbb{Z}_2$ orbifold is an exception: in this case the theory is not chiral.

ponents of the heavy gauge bosons. The only feature that is really important in the present discussion is that each $U(1)$ will give a D-term contribution to the full potential energy, which is important in determining the moduli space of the theory (notice also that the $U(1)$ charges are all traceless, hence no Fayet-Iliopoulos term is generated by loop effects).

We are now ready to study the motion of the 4-branes. We begin with a $\mathbb{Z}_M$ orbifold with NM branes located at the orbifold point. The corresponding gauge group is $SU(N)^M$. The 4-branes can only move along the X^4, X^5 directions, since they end on NS branes which only extend along these directions. Each 4-brane has $M-1$ images under the $\mathbb{Z}_M$ symmetry, so counting images, we can move sets of M branes away from the orbifold point. Moving M branes away breaks $SU(N)^M \to SU(N-1)^M \times U(1)$. If all the 4-branes are moved away from the orbifold point we are left with a $U(1)^{N-1}$ gauge symmetry. Since the motion of each set of M branes is described by one complex number, the moduli space is N dimensional. Finally, we also note that if N_1 physical branes come together away from the orbifold point we get an enhanced $U(N_1)$ gauge symmetry.

Now consider the flat directions in the gauge theory. These are in one-to-one correspondence with gauge invariant chiral superfields made out of the elementary matter fields in eq. (2.1). Ignoring the anomalous $U(1)$s for the moment, these moduli are of two kinds. One class is best described in terms of the operator:

$$\Sigma^i_j = (Q_1 \cdot Q_2 \cdots Q_M)^i_j, \tag{3.1}$$

as:

$$\phi_k = tr(\Sigma)^k, \tag{3.2}$$

for $k = 1, \cdots, (N-1)$. The second class of "baryonic" directions is given by:

$$b_\alpha = (Q_\alpha)^N, \tag{3.3}$$

with $\alpha = 1, \cdots, M$. Altogether, we see that there are $N - 1 + M$ flat directions; these are more than the number of brane degrees of freedom found above. The discrepancy is corrected when we account for the D-term potential generated by the anomalous $U(1)$s. We saw above that there are $M - 1$ of these, thus their D terms get rid of $M - 1$ moduli giving us, finally, a N dimensional moduli space in agreement with what we found for the motion of branes. An analysis of the vacuum expectation values also shows that in the moduli space, generically, a $U(1)^{(N-1)}$ is left unbroken. Finally, one finds subspaces of the moduli space which correspond to partially enhanced gauge symmetry, again in accord with what is found from brane considerations.

4. The Quantum Behavior via M Theory

We will now turn to considering the quantum behavior of the gauge theory described above. It was found in the previous section that generically in moduli space the theory

is in the Coulomb phase with the gauge symmetry being broken to a $U(1)^{(N-1)}$ subgroup. We would like to see if the corresponding spectral curves, [30], can be determined. In this analysis we will closely follow [5] where it was pointed out that in M theory, the brane configuration corresponding to that in Fig. 1 can be thought of as the worldvolume of a single NS 5-brane, and that this insight leads to determining the curves.

In [5] the 5-brane worldvolume had infinite extent along the X^0, X^1, X^2, X^3 coordinates, while spanning a two dimensional surface in the four-manifold parametrized by $v = X^4 + iX^5$ and $t = \exp(-s) = \exp(-(X^6 + iX^{10})/R)$. In our case v and $w = X^8 + iX^9$ are modded by the $\mathbb{Z}_M$ transformation eq. (2.2). A more convenient representation of this $\mathbb{C}^2/\mathbb{Z}_M$ orbifold is obtained by embedding it as a hypersurface in $\mathbb{C}^3$:

$$yz - x^M = 0. \tag{4.1}$$

The coordinate mapping is $y = v^M$, $z = w^M$, $x = vw$; the orbifold singularity is at $y = z = x = 0$. In the M theory limit the 5-brane is described by a Riemann surface Σ embedded in $\mathbb{C}^3 \times R^1 \times S^1$. This surface is smooth except at the orbifold point, and can be parametrized as a rational curve by y and t, with z set equal to zero.

Now consider the configuration shown in Fig. 1, consisting of two NS branes and NM 4-branes (we are counting the branes and their images as distinct) stretching between them. The two dimensional surface Σ can now be described by the curve:

$$t^2 + B(y)\, t + 1 = 0. \tag{4.2}$$

Here B is a polynomial of degree N (in $y = v^M$), i.e.,

$$B(y) = y^N + u_1 y^{N-1} + u_2 y^{N-2} + \cdots + u_N. \tag{4.3}$$

Note this surface corresponds to genus $N-1$ as would be expected for a curve with $N-1$ photons. As discussed in [5] and [30], the periods of this Riemann surface determine the gauge couplings of the $N-1$ $U(1)$ gauge groups.

The asymptotic behavior of t for large y is given by $t \simeq -y^N$, and $t \simeq -y^{-N}$. This tells us how the two NS branes bend for large y and determines the asymptotic form of the beta function which goes like

$$\frac{4\pi}{g^2} \simeq 2N \ln |y| \ . \tag{4.4}$$

This agrees with the expected beta function for each of the $SU(N)$ factors.

The coefficients u_i in eq. (4.2) parametrize the moduli space of the theory. It would be useful to express them in terms of the gauge invariants built out of the elementary fields in eq. (2.1). When the 4-branes are sufficiently far (compared to the strong coupling scale(s)) from the orbifold point the leading order dependence of the u_i can be determined by classical

considerations. To see this, note that eq. (4.2), at fixed t, can be used to solve for y and thereby yield the positions of the 4-branes. Furthermore, at large enough separation these positions can be unambiguously related to the gauge invariants, thereby determining the leading dependence of the u_i.

In Section 2, we had described how the gauge theory corresponding to NM 4-branes placed at a $\mathbb{Z}_M$ orbifold point can be thought of as being obtained by starting from an $SU(NM)$, $N = 2$, theory and only keeping states invariant under a certain $\mathbb{Z}_M$ symmetry. In fact this provides the simplest way of determining the leading dependence of the coefficients u_i. One starts with the $N = 2$ curve,

$$t^2 + B(v)\ t + 1 = 0, \tag{4.5}$$

where $B(v)$ is a polynomial of degree NM given by:

$$B(v) = v^{NM} + a_1 v^{NM-1} + a_2 v^{NM-2} + \cdots + a_{NM}. \tag{4.6}$$

In this case the coefficients are easily determined as (trace of) the appropriate powers of the adjoint field. We now only allow fields invariant under the $\mathbb{Z}_M$ symmetry to have vacuum expectation values. This means that only integer powers of v^M survive in $B(v)$. The resulting curve thus has a $\mathbb{Z}_M$ symmetry, under which $v \to e^{\frac{2\pi i}{M}} v$. To obtain the curve in the orbifold theory it is natural to identify points related by this symmetry. This amounts to parametrizing the curve with a variable $y = v^M$. The curve, eq. (4.5), then turns into the required one, eq. (4.2). As mentioned before, the coefficients in eq. (4.5) can be determined in terms of the adjoint field and can then be easily expressed in terms of the moduli in the orbifold theory.

The leading dependence of the u_i on the gauge invariants can thus be determined. However, there can be subleading terms in these relations, depending on strong coupling scales of the gauge theories involved, which cannot be determined by classical considerations alone[4]. In fact, such terms are present in the theories at hand. We know this because these theories are essentially identical, (apart from the anomalous $U(1)$s discussed above) to the $SU(N)^M$ theories studied in [23], [24], and their curves have been worked out from field theoretic considerations.

For illustrative purposes we consider the example of an $SU(2)^3$ theory, which corresponds to taking six 4-branes (two physical branes and their images) in a $\mathbb{Z}_3$ orbifold. The related theory was discussed in [24] and the curve was obtained to be:

$$\tilde{t}^2 = (x^2 - (\Lambda_1^4\ M_2 + \Lambda_2^4\ M_3 + \Lambda_3^4\ M_1 - M_1 M_2 M_3 + T^2))^2 - 4\Lambda_1^4\Lambda_2^4\Lambda_3^4. \tag{4.7}$$

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[4]Such terms are absent in the $N = 2$ $SU(N_c)$ theory studied in [5], provided $N_c > N_f$. In this case dimensional arguments and the fact that these corrections arise from instanton effects and are therefore proportional to Λ^{b_0} is enough to explain their absence.

Here, $\Lambda_{1,2,3}$ are the three strong coupling scales, while $M_i = Q_i^2$ and $T \sim Q_1 \cdot Q_2 \cdot Q_3$ are the moduli. This curve is related to the one obtained in the brane construction, eq. (4.2) by a shift and rescaling of the variables y and t. On doing so and comparing one finds that the $M_1 M_2 M_3$ and T^2 terms in the first bracket in eq. (4.7) correspond to the leading dependence of the coefficients u_i, while the Λ-dependent terms in the first bracket correspond to the subleading terms we were worried about. Actually, strictly speaking we need to incorporate the effects of the anomalous $U(1)$s in the curve, eq. (4.7), before comparing the two. This is relatively simple to do in the orbifold limit where the Fayet-Iliopoulos terms for the two anomalous $U(1)$s are zero[5].

Let us pause for a moment to sketch this out. In a convenient basis, the $U(1)$ charge assignments of the three elementary fields are, $Q_1 : (2,0)$, $Q_2 : (-1,1)$, $Q_3 : (-1,-1)$. The corresponding D terms then imply:

$$4|M_1|^2 - 2|M_2|^2 - 2|M_3|^2 = 0, \tag{4.8}$$

and

$$2|M_2|^2 - 2|M_3|^2 = 0. \tag{4.9}$$

In this example, the $U(1)$ anomalies cancel due to appropriate shifts in two axion fields. One consequence is that the Λ dependent terms in eq. (4.7) acquire an axion dependence. In describing the resulting curve it is simplest to carry out appropriate $U(1)$ rotations (and shifts in the axion fields) to go to a gauge where the three fields M_1, M_2 and M_3 have the same phase. Eq. (4.8) and (4.9) can now be used to solve for two of the fields, say, M_2 and M_3 in terms of third, M_1. On substituting back in eq. (4.7) the resulting curve in this gauge in terms of the moduli, M_1 and T is given by:

$$\tilde{t}^2 = (x^2 - (\Lambda_1^4 + \Lambda_2^4 + \Lambda_3^4) \, M_1 + M_1^3 - T^2)^2 - 4\Lambda_1^4 \Lambda_2^4 \Lambda_3^4. \tag{4.10}$$

The axion dependence in eq. (4.10) enters through the dependence of the strong coupling scales on these fields and can be easily worked out. The important point is that after going through this procedure, in eq. (4.10) one sees that the subleading terms mentioned above continue to persist, while the leading terms on which there was agreement in the two cases are not changed in an essential way.

5. Generalizations of the Orbifold Brane Construction.

5.1 Additional vectorlike matter.

There are three obvious generalizations of our $\mathbb{Z}_M$ orbifold brane construction which add massless vectorlike matter.

[5]Determining the curve away from the orbifold limit is an interesting problem which we hope to address in a subsequent paper. This will also allow us to see whether the subleading terms arise in part because of the orbifold nature of the background and can be determined by blowing it up.

The first is obtained by adding $M \cdot N_f$ Dirichlet six branes at the origin in the (X^4, X^5) plane. These 6-branes extend in the directions $X^1, X^2, X^3, X^7, X^8, X^9$ and do not break any additional supersymmetries [3]. Once again, the tadpoles in the one-loop vacuum amplitude must cancel in this theory. The only additional constraints arise from the twisted Ramond-Ramond tadpole amplitudes for strings ending on these 6-branes[6]: the 6-branes are sources of twisted RR flux which can only propagate in the X^6 transverse direction, which is insufficient to allow the flux to escape to infinity [26]. Therefore, the total twisted RR charge of the 6-branes has to vanish, and the matrices that represent the $\mathbb{Z}_M$ action on the six brane Chan-Paton factors must also obey the conditions (2.4).

The massless excitations of the $4 - 6$ strings give vectorlike matter with the following transformation law under the $SU(N)^M \times SU(N_f)^M$ symmetry (here we have denoted by $SU(N_f)$ the 6-brane gauge group, which appears as a global symmetry in the 4-brane theory). There are M fields F_i $(i = 1, \cdots, M)$, transforming as $(\square, \bar{\square})$ under $SU(N)_i \times SU(N_f)_i$, which are singlets under the other gauge and flavor groups, and M fields $\bar{F}_i$ that transform as $(\bar{\square}, \square)$ under $SU(N)_i \times SU(N_f)_{(i+1)(\mathrm{mod}\,M)}$ (and, similarly, are singlets under the other gauge and flavor groups). The shift of indices for the $\bar{F}$ fields is due to the fact that the vertex operator for the massless $4 - 6$ string excitations transforms by a factor of $e^{-i\pi/M}$ under the $\mathbb{Z}_M$ symmetry [25] and the Chan-Paton factors for the 6-branes obey $\gamma_\alpha^M = -1$. Finally, as a vestige of $N = 2$ supersymmetry, the following Yukawa couplings that preserve the $SU(N_f)^M$ global symmetry will appear in the superpotential:

$$W = F_1 Q_M \bar{F}_M + F_2 Q_1 \bar{F}_1 + F_3 Q_2 \bar{F}_2 + \cdots + F_M Q_{M-1} \bar{F}_{M-1} . \qquad (5.1)$$

It will become clear in the following that this spectrum (and superpotential) is the only one consistent with field theoretic considerations and nonabelian duality.

Another generalization is obtained by adding more NS branes. The simplest example is illustrated in Fig. 2. This configuration can be obtained by starting with N_f physical four-branes stretched between two NS branes without any six-branes. One then brings a third NS brane in from infinity along the X^7 direction, until it intersects the middle of the four-branes. One can then break the four-branes on this new NS brane; the gauge group at this point is clearly $SU(N_f)^M \times SU(N_f)^M$. Now one can move $N_f - N$ of the left-hand physical four-branes together with their $\mathbb{Z}_M$ images off to infinity in the (X^4, X^5) plane, where they have no effect on the light spectrum of the remaining brane configuration. Thus we deduce that Fig. 2 represents an orbifold model with gauge group $SU(N)^M \times SU(N_f)^M$, with chiral matter content under $SU(N)^M$ and $SU(N_f)^M$ of the form (2.1). In addition there is vectorlike matter corresponding to chiral multiplets F_i $(i = 1, \cdots, M)$, transforming as $(\square, \bar{\square})$

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[6]The twisted RR amplitudes from 4-6 strings can be already seen to vanish since the matrices representing the action of the twist on the Chan-Paton factor of the 4-branes obey (2.4).
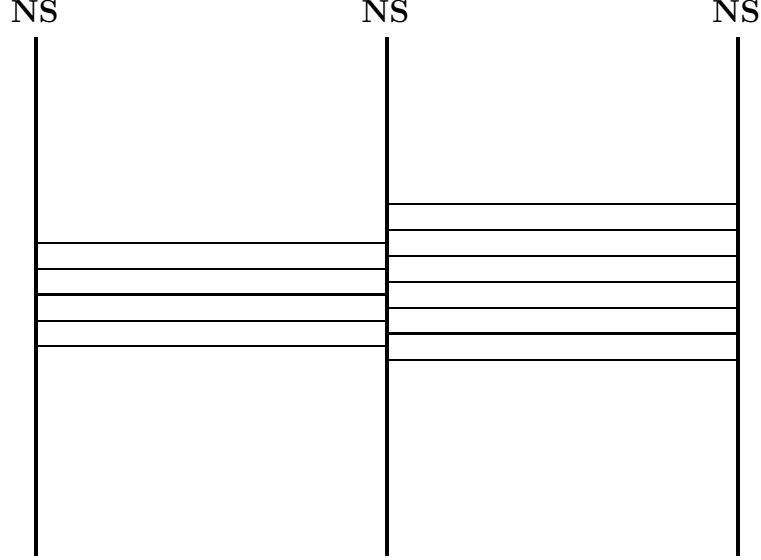
Figure 2: The Type IIA brane configuration for a class of generalized $\mathbb{Z}_M$ orbifold models. There are N physical four-branes stretched between the first pair of NS five branes, plus their $\mathbb{Z}_M$ images. There are N_f physical four-branes stretched between the second pair of NS five branes, plus their $\mathbb{Z}_M$ images.

under $SU(N)_i \times SU(N_f)_i$, $\bar{F}_i$ that transform as $(\bar{\square}, \square)$ under $SU(N)_i \times SU(N_f)_{(i+1)(\mathrm{mod}\, M)}$ (both F and $\bar{F}$ are singlets under all the other gauge groups). There is also a superpotential, which is the sum of the superpotentials (5.1) for the gauge groups $SU(N)^M$ and $SU(N_f)^M$, respectively.

The third generalization consists of attaching semi-infinite four-branes to the left- or right-hand NS brane. This is equivalent (modulo the discussion in [19]) to taking the configuration of Fig. 2 and moving the left- or right-hand NS brane off to infinity in the X^6 direction. The new vectorlike matter consists of N_f flavors for each $SU(N)$, with superpotential (5.1) (in the limit that the left- or right-hand NS brane is pushed off to infinity, the gauge coupling of the corresponding 4-brane theory goes to zero and the contribution to the superpotential from the $SU(N_f)^M$ gauge group vanishes). It appears therefore that this construction is related to the construction with 6-branes by the Hanany-Witten process: after pushing the 6-branes through one of the NS branes, a set of N_f 4-branes stretched between the NS brane and the 6-branes is created; after moving the 6 branes to infinity we obtain the construction with semi-infinite 4-branes described above.

It is clear that the generalizations discussed above can also be obtained from $N = 2$ theory with matter after eliminating states that are not invariant under an appropriately chosen $\mathbb{Z}_M$ discrete global symmetry, in the same way that was discussed in the end of

Section 2 for the pure Yang-Mills $N = 2$ theory.

5.2 Nonabelian duality.

Here we discuss one more check on our orbifold construction with extra matter fields. The $N = 1$ theory with gauge group $SU(N)^M$, with matter content given by eq. (2.1) plus additional N_f flavors of each $SU(N)$ factor, and a superpotential given by eq. (5.1) was considered in ref. [31]. By an iterative application of the $N = 1$ SQCD dualities it was found that the theory has an equivalent infrared description—along the Higgs branch—in terms of an $SU(N_f - N)^M$ theory with the same matter content and superpotential. The theories along their respective Coulomb branches are clearly different, as follows from the different number of unbroken $U(1)$s at a generic point on the Coulomb branch moduli space.

The $SU(N_c)^M \Leftrightarrow SU(N_f - N_c)^M$ duality is related to the duality of the Higgs branches of $N = 2$ SQCD with gauge groups $SU(MN_c)$ and $SU(M(N_f - N_c))$. This duality is easy to see in the brane construction [1], [3]. Consider the brane configuration of Fig. 1. Pushing the N_f 6-branes (we count only the physical branes here) to the left of the left NS brane, we obtain a configuration with N_f 4-branes stretching between the N_f 6-branes and the left NS brane. Then we enter the Higgs branch of the theory by reconnecting the N_c 4-branes stretching between the two NS branes with N_c of the newly created 4-branes and rearranging them in the most general way consistent with the s-rule [3]. Thus we obtain a configuration where N_c 4-branes stretch between the 6-branes and the right NS brane while $N_f - N_c$ 4-branes stretch between the 6- branes and the left NS brane. Now we can move the two NS branes past each other in the X^6 direction and reconnect once more the 4-branes, obtaining thus a configuration where $N_f - N_c$ 4-branes stretch between the two NS branes, and N_f 4-branes between the 6-branes and the leftmost NS brane. This setup describes the Higgs branch moduli space of the $SU(M(N_f - N_c))$ $N = 2$ theory with N_f flavors. Orbifolding by the $\mathbb{Z}_M$ symmetry does not affect the previous argument in any essential way. We thus obtain a brane realization of the Higgs branch duality between the $SU(N_c)^M$ and $SU(N_f - N_c)^M$ theories.

Acknowledgements

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