arXiv:hep-th/9606184v3 16 Sep 1996

Supersymmetry Breaking and Duality in SU(N) × SU(N − M) Theories

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Abstract

We consider a class of $N = 1$ supersymmetric Yang-Mills theories, with gauge group $SU(N) \times SU(N - M)$ and fundamental matter content. Duality plays an essential role in analyzing the nonperturbative infrared dynamics of these models. We find that Yukawa couplings drive these theories into the confining phase, and show how the nonperturbative superpotentials arise in the dual picture. We show that the odd-N, $M = 2$ models with an appropriate tree-level superpotential break supersymmetry.

1   Introduction.

In order to be relevant to nature, supersymmetry must be spontaneously broken. One attractive idea is that the breaking occurs nonperturbatively. The electroweak scale could then arise from the Planck scale through the logarithmic running of coupling constants and this would help explain its smallness [1].

The past few years have seen remarkable progress in the study of the non-perturbative behavior of SUSY gauge theories [2], [3], [4]. This progress has led in turn to a better understanding of nonperturbative SUSY breaking [5]. In particular, many new examples of theories exhibiting this phenomenon have been found [6]–[13].

In this paper we will study a simple class of $SU(N) \times SU(N-M)$ gauge theories with matter in the fundamental representation, and show that they break supersymmetry dynamically. These models are a generalization of the $SU(N) \times SU(N-1)$ theories discussed in an earlier paper [13]. Various ideas developed in that paper will prove useful for studying the $SU(N) \times SU(N-M)$ theories. In particular, unlike the models considered previously, we find that product-group duality will be essential in understanding the non-perturbative behavior of the theories with $M > 1$[1]. For example, duality will help identify the appropriate degrees of freedom in the low-energy theory, in terms of which the Kähler potential is nonsingular. We will also see, as noted in [12], [13], that Yukawa couplings will sometimes drive these theories into the confining regime, and that this will play a crucial role in the breaking of supersymmetry.

This paper is organized as follows. First we introduce the $SU(N) \times SU(N-M)$ models. We then discuss their $SU(N) \times SU(M)$ duals in Sect. 3. In Sect. 4 we show how the nonperturbative superpotential arises in the dual picture. In Sect. 5, we turn to the question of supersymmetry breaking. In particular, we show that the $M = 2$, odd-N models break supersymmetry, once appropriate Yukawa couplings are added. Finally, in Sect. 6, we discuss the supersymmetry preserving $M = 0$ models.

2   The $\mathbf{SU(N) \times SU(N-M)}$ Models.

The theories we consider in this paper have an $SU(N) \times SU(N-M)$ gauge symmetry with matter content consisting of a single field, $Q_{\alpha\dot{\alpha}}$, that transforms as ($\square$, $\square$) under the gauge groups, $N-M$ fields, $\bar{L}_I^\alpha$, transforming as ($\bar{\square}$, $\mathbf{1}$), and N fields, $\bar{R}_A^{\dot\alpha}$, that transform as ($\mathbf{1}$, $\bar{\square}$). Here, as in the subsequent discussion, we denote the gauge indices of $SU(N)$ and $SU(N-M)$ by α and $\dot\alpha$, respectively, while $I = 1 \ldots N-M$ and $A = 1 \ldots N$ are flavor indices. We note

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[1]Duality in the context of supersymmetry breaking has been discussed in [14].

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that these theories are chiral—no mass terms can be added for any of the matter fields. The models with $M = 1$ were considered in [13]; here we study the $M > 1$ models.

We begin our study by considering the classical moduli space. It is described by the gauge invariant chiral superfields $Y_{IA} = \bar{L}_I \cdot Q \cdot \bar{R}_A$, $\bar{b}^{A_1...A_M} = (\bar{R}^{N-M})^{A_1...A_M}$ and $\bar{\mathcal{B}} = Q^{N-M} \cdot \bar{L}^{N-M}$ (when appropriate, all indices are contracted with ϵ-tensors), subject to the classical constraints $Y_{IA_1}\bar{b}^{A_1...A_M} = 0$ and $\bar{b}^{A_1...A_M}\bar{\mathcal{B}} \sim (Y^{N-M})^{A_1...A_M}$. It is easy to see [15], [13], that the classical superpotential

$$W_{tree} = \lambda^{IA} \, Y_{IA} \tag{2.1}$$

with maximal rank Yukawa-coupling matrix lifts all classical flat directions with the exception of the $SU(N-M)$ baryons $\bar{b}^{A_1...A_M}$. Along the classical $\bar{b}^{A_1...A_M} \neq 0$ flat direction the $SU(N)$ gauge group is completely unbroken and one expects strong quantum effects to be important.

The baryonic flat direction can be lifted, for $M = 2$, by adding the tree-level superpotential

$$W_{tree} = \lambda^{IA} \, Y_{IA} + \alpha_{AB} \, \bar{b}^{AB} \, . \tag{2.2}$$

As above λ_{IA} has to have maximal rank, i.e. in this case rank $N - 2$. The matrix α_{AB} needs to satisy two conditions. First it has to be invertible. Second, the projection of α_{AB} into the cokernel of λ^{IA} needs to be invertible as well. The last condition can be stated more explicitly as follows. By flavor rotations one can go to a basis in which $\lambda_{IA} = \lambda_I \delta_{IA}$ for $A < N - 1$ and $\lambda_{I(N-1)}$ and $\lambda_{IN} = 0$. In this basis, the $N - 2$ dimensional matrix formed from α_{AB} by restricting A and B to be $\leq N - 2$ must be invertible. For even N, there is no nonanomalous R symmetry which is left unbroken by the superpotential (2.2). As we will see below, the even-N, $M = 2$ models do not break supersymmetry, in conformity with the criteria of ref. [16].

For N-odd, $M = 2$, the matrix α_{AB} has to be of maximal rank $(N - 1)$, and its cokernel should contain the cokernel of λ^{IA} (rank $\lambda = N - 2$). As opposed to the even-N case, the superpotential (2.2) that lifts all flat directions preserves a nonanomalous, flavor dependent, R symmetry. To see that, choose for example $\alpha^{AN} = 0$, $\lambda^{IN} = \lambda^{I(N-1)} = 0$ (to lift the classical flat directions). Then one sees that the field $\bar{R}_N$ appears in each of the baryonic terms of the superpotential (2.2), while it does not appear in any of the Yukawa terms. Assigning different R charges to the four types of fields, $\bar{R}_N$, $\bar{R}_{A<N}$, Q, and $\bar{L}_I$, one has to satisfy four conditions: two conditions ensuring that the superpotential (2.2) has R charge 2, and two conditions that the gauge anomalies of this R symmetry vanish. It is easy to see that there is a unique solution to these four conditions (a similar flavor-dependent R symmetry is preserved in the $M = 1$ models, for all N, when all classical flat directions are lifted [13]).

Lifting the baryonic flat directions for $M > 2$ is much more complicated and we have not been able to analyze this fully yet.

Table 1: The field content of the dual $SU(N) \times SU(M)$ theory.

	$SU(N)$	$SU(M)$
q_ν^α	$\bar\square$	$\square$
$\bar r^{A\nu}$	1	$\bar\square$
$\frac{1}{\mu} M_{\alpha A}$	$\square$	1
$\bar L_I^\alpha$	$\bar\square$	1

3   The SU(N) × SU(M) Dual.

We begin our analysis of the quantum theory by noting that it has a dual description in terms of an $SU(N) \times SU(M)$ gauge theory. This dual theory may be constructed as follows: first, turn off the $SU(N)$ coupling. The electric theory is then $SU(N - M)$ with N flavors, whose dual is an $SU(M)$ gauge theory. Turning the $SU(N)$ coupling back on, the $SU(N) \times SU(M)$ dual is obtained. We will find it useful to study the low-energy dynamics by analyzing this dual theory. Duality will help us identify the low-energy degrees of freedom in terms of which the Kähler potential is nonsingular.

Before doing so, however, two comments regarding the various length scales present are in order. First, while the dual theory was constructed in the limit $\Lambda_2 \gg \Lambda_1$, prior experience, based on a study of an $SU(2) \times SU(2)$ theory [13], strongly suggests that the two theories are equivalent in the infra-red for all values of the ratio Λ_2/Λ_1. Therefore, the infra-red behavior of the electric theory can be understood by studying the dual theory.

Second, in our discussion we will assume that the SUSY breaking scale is much lower than the strong coupling scales of both groups. Our analysis will show that SUSY breaking occurs only in the presence of the Yukawa couplings eq. (2.2). Thus, by making the couplings λ^{IA} small enough, one expects that the SUSY breaking scale can be made arbitrarily small too, and in this regime our analysis will be self-consistent.

We now go on to consider the dual theory in some detail. The field content of the dual theory is summarized in Table 1. We see that the fields q_ν^α and $\bar r^{A\nu}$ are dual to $Q_{\alpha\dot\alpha}$ and $\bar R_A^{\dot\alpha}$ respectively, while the $SU(M)$ singlet, $M_{\alpha A}$, is dual to the $SU(N - M)$ meson, $M_{\alpha A}$. Here $\nu = 1 \ldots M$ is the $SU(M)$ index. The dual theory has a Yukawa superpotential [3]:

$$W = \frac{1}{\mu} \, M_{\alpha A} \, \bar r^A \cdot q^\alpha \; . \tag{3.3}$$

The dimension-one parameter μ and the strong coupling scales Λ_2 and $\bar\Lambda_2$ of the electric and

magnetic theories obey the matching relation[2] [3]:

$$\Lambda_2^{2N-3M} \bar\Lambda_2^{3M-N} \sim \mu^N \ . \tag{3.4}$$

Symmetries also allow us to relate the scale of the $SU(N)$ group in the dual theory, $\bar\Lambda_1$, to the scales of the electric theory:

$$\bar\Lambda_1^{2N} \sim \Lambda_1^{2N+M} \frac{\Lambda_2^{\frac{2N-3M}{2}}}{\mu^{N-\frac{M}{2}}} \ . \tag{3.5}$$

Before proceeding, one comment about the dual theory is worth making. Note that we started with an electric theory that was chiral but in contrast the dual theory is *not* chiral. From Table 1 we see that mass terms can be added for the $\bar L_I^\alpha$ and the $M_{\alpha A}$ fields. As we will see below, these mass terms will correspond to Yukawa couplings in the electric theory. Examples of duality between chiral and non-chiral theories have also been found in [9] and [18].

We now proceed with the discussion of the dual theory. In this theory $SU(N)$ has N flavors (see Table 1) and is therefore confining in the infra-red. At scales below $\bar\Lambda_1$, the light degrees of freedom are the $SU(N)$ mesons:

$$
\begin{aligned}
N_{A\nu} &= \frac{1}{\mu} M_{\alpha A} q_\nu^\alpha \ , \\
K_{AI} &= \frac{1}{\mu} M_{\alpha A} \bar L_I^\alpha = \frac{1}{\mu} Y_{IA} \ ,
\end{aligned}
$$

and the $SU(N)$ baryons:

$$
\begin{aligned}
\mathcal{B} &= \det(M_{\alpha A}/\mu) \ , \\
\bar{\mathcal{B}}' &= q^M \cdot \bar L^{N-M} \\
&\sim \mu^{\frac{M}{2}} \Lambda_2^{\frac{3M-2N}{2}} Q^{N-M} \cdot \bar L^{N-M} = \mu^{\frac{M}{2}} \Lambda_2^{\frac{3M-2N}{2}} \bar{\mathcal{B}} \ .
\end{aligned}
\tag{3.6}
$$

In the last equation we used the baryon operator duality map of SQCD [3], [4]. Note that the field $\mathcal{B}$ in (3.6) vanishes classically in the electric theory. We note also that the second equation in (3.6) relates $\frac{1}{\mu} M_{\alpha A} \bar L_I^\alpha$, which is a mass term in the dual theory, to Y_{IA}, which is a Yukawa coupling in the electric theory.

Adding the confining superpotential of the $SU(N)$ theory to (3.3), the total superpotential, written in terms of the $SU(N)$ mesons and baryons, becomes

$$W = N_{A\nu} \bar r^{A\nu} + \mathcal{A}\left(N^M \cdot K^{N-M} - \mathcal{B}\,\bar{\mathcal{B}}' - \Lambda_{1L}^{2N}\right) \ , \tag{3.7}$$

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[2]Hereafter we leave out the numerical constants appearing in the various scale matching relations. These constants are calculable but are not essential to the present discussion.

with $\mathcal{A}$ a Lagrange multiplier superfield.

Below the scale $\bar{\Lambda}_1$, the $SU(M)$ theory has N flavors: $N_A/\bar{\Lambda}_1$ and $\bar{r}^A$. It therefore seems, at first glance, to be in the dual regime. However, the confining dynamics of the $SU(N)$ theory has turned the Yukawa couplings of eq. (3.3) into mass terms for the fields $N_A/\bar{\Lambda}_1$, and $\bar{r}^A$, see eq. (3.7)[3]. Thus, the $SU(M)$ theory too is driven into the confining regime. This will play an important role in our discussion of SUSY breaking below. One feature of this low-energy $SU(M)$ theory is that its scale $\bar{\Lambda}_{2L}$ is field dependent. We determine this scale in the next section and show how it helps recover, in the dual theory, some features of the electric theory.

4 Gaugino Condensation in the Dual Picture.

Below the confining scale of $SU(N)$, we have an $SU(M)$ theory with N flavors. The scale of this theory, which we denote by $\bar{\Lambda}_{2L}$, can be determined by symmetry considerations to be

$$\bar{\Lambda}_{2L}^{3M-N} \sim \bar{\Lambda}_2^{3M-N} \, \frac{\mathcal{B}}{\bar{\Lambda}_1^N} \, f\left(\frac{\bar{\Lambda}_1^{2N}}{\mathcal{B}\,\bar{\mathcal{B}}'}\right) \,, \tag{4.8}$$

where f is an unknown function which will be determined below. The physics behind this expression was explained in ref. [13]: the field dependence of the scale of the low-energy $SU(M)$ theory is required by Green-Schwarz anomaly cancellation. A $U(1)$ symmetry, which is anomaly free in the ultraviolet, above the $SU(N)$ confinement scale, has an $SU(M)$ anomaly in the infrared, below the $SU(N)$ confinement scale. The anomaly is cancelled by the shift of an axion-dilaton superfield, proportional to $\mathcal{B}f$ in eq. (4.8).

The dilaton superfield can be determined using duality. Since an analogous calculation is described in Section 3.2 of ref. [13], we only outline the main idea here. While the dilaton superfield of eq. (4.8) is generated by strong-coupling (confining) dynamics, one can construct a dual theory in which the dilaton arises as a weak-coupling ("higgsing") effect. Specifically, adding one extra flavor of $SU(N)$ to the $SU(N) \times SU(M)$ theory discussed above, one can construct a dual of this theory with gauge group $SU(N) \times SU(N - M + 1)$. In this theory $SU(N)$ again confines. But this time, below its confining scale, one is left with an $SU(N - M + 1)$ gauge theory whose scale is field-*independent*. On giving a mass to the extra $SU(N)$ flavor we added, $SU(N - M + 1)$ is higgsed to $SU(N - M)$. The relevant vevs, and therefore the $SU(N - M)$ scale, depend on the $SU(N)$ baryon $\mathcal{B}$. But this $SU(N - M)$ is precisely the dual of the $SU(M)$ with which we started, so that its scale determines the scale of $SU(M)$, $\bar{\Lambda}_{2L}$. On carrying out this exercise, one finds that $f \equiv 1$ in eq. (4.8), giving the relation:

$$\bar{\Lambda}_{2L}^{3M-N} \sim \bar{\Lambda}_2^{3M-N} \, \frac{\mathcal{B}}{\bar{\Lambda}_1^N} \,. \tag{4.9}$$

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[3]On accounting for the correct normalization of the field $N_A/\bar{\Lambda}_1$, its mass term is seen to be of order $\bar{\Lambda}_1$.

As was mentioned in the previous section, the fields $N_A/\bar{\Lambda}_1$ and $\bar{r}^A$ acquire mass. In order to obtain the superpotential after they are integrated out one can proceed, somewhat heuristically, as follows. Since all N flavors are heavy we expect to be left with a pure $SU(M)$ Yang-Mills theory at low energies. The scale of this theory can be determined by the standard SQCD scale matching relation, at the scale of the heavy quark mass $\bar{\Lambda}_1$, to be:

$$\bar{\Lambda}_{2LL}^{3M} \sim \bar{\Lambda}_{2L}^{3M-N} \bar{\Lambda}_1^N. \tag{4.10}$$

Gaugino condensation in the pure $SU(M)$ theory generates a superpotential [2]:

$$W_{LL} \sim \bar{\Lambda}_{2LL}^3 \sim \left(\bar{\Lambda}_2^{3M-N} \, \mathcal{B} \right)^{\frac{1}{M}}, \tag{4.11}$$

where in the last expression we have substituted for $\bar{\Lambda}_{2LL}$ from eq. (4.9) and eq. (4.10). One then expects the full superpotential to be given by adding the terms that remain in eq. (3.7) to the superpotential induced by gaugino condensation, eq. (4.11).

The fields that remain in the theory after integrating out the heavy quarks are the $SU(N)$ mesons $K_{IA} \sim Y_{IA}$, the antibaryon $\bar{\mathcal{B}}' \sim \bar{\mathcal{B}}$, and the baryon, $\mathcal{B}$. The resulting superpotential in terms of these fields is then given to be :

$$W = -\mathcal{A} \left(\mathcal{B} \, \bar{\mathcal{B}}' + \bar{\Lambda}_1^{2N} \right) + \mathrm{C} \left(\bar{\Lambda}_2^{3M-N} \mathcal{B} \right)^{\frac{1}{M}}, \tag{4.12}$$

where C above is a constant.

We can now use the constraint of eq. (4.12) to express $\mathcal{B}$ in terms of $\bar{\mathcal{B}}'$. In addition, by expressing $\bar{\mathcal{B}}'$ in terms of $\bar{\mathcal{B}}$, eq. (3.6), and using the scale matching relations, eq. (3.4) and eq. (3.5), we find that the resulting superpotential is given (on appropriately identifying the numerical constant C) by:

$$W_{gaugino} = M \left(\frac{\Lambda_1^{2N+M}}{\bar{\mathcal{B}}} \right)^{\frac{1}{M}}. \tag{4.13}$$

This superpotential has a natural explanation in terms of the electric theory. Since the $SU(N)$ group has $N-M$ flavors, we expect a non-perturbative superpotential to arise due to gaugino condensation in the unbroken electric subgroup $SU(M) \subset SU(N)$. This superpotential is exactly given by eq. (4.13) [2]. Note that due to this superpotential, the quantum theory has no moduli space.

5   Supersymmetry Breaking.

With this understanding of the $SU(N) \times SU(M)$ dual theory at hand, we turn in this section to the question of SUSY breaking. We will mainly focus our attention on the $M = 2$ case

in which, as was discussed above, eq. (2.2), all the flat directions can be raised by adding appropriate terms in the tree level superpotential. Among the $M = 2$ theories we will analyze the odd-N theories first, then consider the even-N theories. Towards the end of this section we will briefly comment on the case of general M as well.

5.1 The $M = 2$ odd - N Theories.

We begin our analysis by returning to the superpotential eq. (3.7). Recall that below the scale $\bar{\Lambda}_1$ the $SU(M = 2)$ theory has N flavors, N_A and $\bar{r}^A$. Furthermore, as is clear from eq. (3.7) all of them have mass and we expect the $SU(2)$ theory to be driven into the confining regime. In the subsequent discussion we will find it sometimes convenient to adopt a common notation U_i for all the quarks of the $SU(2)$ group with $U_i = N_i/\bar{\Lambda}_1$ for $i \leq N$ and $U_i = \bar{r}^{i-N}$ for $N < i \leq 2N$. The mesons of $SU(2)$ will then be referred to as $V_{ij} \equiv U_i \cdot U_j$. Since, as was mentioned above, one expects the $SU(2)$ theory to be driven into the confining regime one can adequately account for the non-perturbative dynamics by working in terms of the $SU(2)$ meson fields and adding a superpotential to the theory of the form:

$$W = \left(\frac{\text{Pf}V \ \bar{\Lambda}_1^N}{\bar{\Lambda}_2^{6-N} \mathcal{B}} \right)^{\frac{1}{N-2}}. \tag{5.14}$$

The $\mathcal{B}$ dependence above arises because the scale of the $SU(2)$ theory is field dependent, eq. (4.8). The full superpotential is then given by a sum of eq. (2.2), eq. (3.7) and eq. (5.14) to be:

$$W = N_A \cdot \bar{r}^A + \mathcal{A} \left(N^2 \cdot K^{N-2} - \mathcal{B} \ \bar{\mathcal{B}}' - \Lambda_{1L}^{2N} \right) \tag{5.15}$$
$$+ \left(\frac{\text{Pf}V \ \bar{\Lambda}_1^N}{\bar{\Lambda}_2^{6-N} \mathcal{B}} \right)^{\frac{1}{N-2}} + \mu \ \lambda^{IA} \ K_{AI} + \alpha_{AB} \ \frac{\Lambda_2^{N-3}}{\mu} \ \bar{r}^A \cdot \bar{r}^B .$$

In the equation above we have set $Y_{IA} = \mu K_{AI}$ and used the baryon operator map in SQCD, [3], [4], to write

$$\bar{b}^{AB} = \frac{\Lambda_2^{N-3}}{\mu} \ \bar{r}^A \cdot \bar{r}^B. \tag{5.16}$$

We now restrict our attention to the odd-N case and show that the superpotential in eq. (5.15) implies that SUSY is broken. We work in a basis in flavor space where $\lambda_{IA} = \lambda_I \delta_{IA}$ for $A < N - 1$ and $\lambda_{IA} = 0$ for $A \geq N - 1$. The matrix α_{AB} has rank $N - 1$ with $\alpha^{AN} = 0$. As was mentioned in the discussion following eq. (2.2) this choice of couplings lifts all flat directions in the classical theory. Our strategy will be to start by assuming that all the F term conditions are valid and to solve for some fields using them. But doing so will lead to the conclusion that the constraint enforced by the Lagrange multiplier $\mathcal{A}$, namely $N^2 \cdot K^{N-2} - \mathcal{B} \ \bar{\mathcal{B}}' = \Lambda_{1L}^{2N}$, cannot be met and therefore SUSY must be broken.

We begin by noticing that the F term equation for $K_{A<NI}$ implies that $\mathcal{A} \neq 0$. From the equation for $\bar{\mathcal{B}}'$ we learn then that $\mathcal{B} = 0$ and therefore that the second term in the constraint vanishes. We now prove that the first term vanishes too, thereby showing that the constraint cannot be met. For this purpose notice first that the equations for K_{NI} imply that $(N^2 \cdot K^{N-3})^{IN} = 0$. If in addition we can show that $N_A \cdot N_N$ vanishes then the first term in the constraint will have to vanish. In order to show this, it is in fact convenient to consider the vevs of all the $SU(2)$ mesons together. Notice that the first term in eq. (5.15) is a mass for the $N \cdot \bar{r}$ mesons, the last term a mass for the $\bar{r} \cdot \bar{r}$ mesons, while the second term in eq. (5.15) can be regarded as a mass of order, $\mathcal{A} \, K^{N-2}$ for the $N \cdot N$ mesons. The vevs for the $SU(2)$ mesons can now be expressed in terms of these masses in the standard way. On doing so, one finds that the expectation values of the $N_A \cdot N_N$ mesons do indeed vanish. This completes the proof of SUSY breaking.

A few more points need to be addressed with regards to the above discussion. First, we did not allow for the possibility of a runaway vacuum, with some fields going to infinity, in our discussion. This should be a good assumption since we start with a theory in which classically all the flat directions are lifted. Second, we assumed that the Kähler potential is non-singular. There are in fact some points in moduli space where this assumption is invalid. When $\mathcal{B} \to 0$, eq. (4.9) shows that $\bar{\Lambda}_{2L} \to 0$ as well and one expects a singularity in the Kähler potential since extra fields will enter the low-energy theory. We have analyzed these points in two ways. First, we added an extra flavor (with a mass term) for $SU(N)$. In this case $\bar{\Lambda}_{2L}$ is not field dependent and no singularity arises in the Kähler potential when $\mathcal{B} \to 0$. Second, since the scale of the $SU(2)$ group goes to zero at these points, we worked directly at the point $\mathcal{B} = 0$ in terms of the quarks of the $SU(2)$ theory. Both ways of analyzing the theory show that SUSY cannot be restored when $\mathcal{B} \to 0$.

There is another argument, involving an R symmetry, which shows that SUSY must be broken in these theories. As was pointed out in the discussion following eq. (2.2), there is a flavor dependent R symmetry that is left unbroken in this case. It turns out that all the fields involved in the constraint implemented by the Lagrange multiplier $\mathcal{A}$ in eq. (5.15) are charged under this R symmetry. Thus if this constraint is met the R symmetry must be broken. In the absence of any flat directions one concludes then that SUSY must be broken as well. The only alternative is that the constraint is not met, but then again, SUSY must be broken. The behavior of these models is therefore in accord with the considerations of ref. [16].

Finally, recall that the $SU(N) \times SU(2)$ theory we have analyzed here is dual to the $SU(N) \times SU(N-2)$ electric theory we started with. The low-energy degrees of freedom and the superpotential identified in the $SU(N) \times SU(2)$ theory should also provide a good description of the infra-red dynamics in the electric theory. As was discussed in the beginning

of section 3, for small enough tree-level couplings, eq. (2.2), the SUSY breaking scale is small too. The low-energy theory we considered is therefore a valid framework for studying SUSY breaking, and the electric theory will break SUSY as well.

5.2 The $M = 2$ even-N Theories.

We turn next to the $M = 2$ theories with even N. In this case, when all the flat directions are lifted there is no R symmetry that is left unbroken. This might make one suspect that there is no SUSY breaking. Indeed, by analyzing the theory with the superpotential, eq. (5.15), we can establish this result for a large class of couplings, λ^{IA} and α_{AB}, in eq. (2.2) (for which all the flat directions are lifted). While we do not present the details here, one finds that all the F-term conditions can be met at a point where $\mathcal{B} \to 0^4$. The class of couplings for which we have been able to establish this can be described as follows. As in the discussion following eq. (2.2), let us go to a basis where λ^{IA} is non-zero when $A \leq N - 2$. In this basis as long as α_{AB} is zero when $A \leq N - 2$ and $B > N - 2$ one can show that SUSY is restored. We strongly suspect this to be true in general.

One legitimate concern about this analysis might be that, as was mentioned in our discussion of the odd-N theories, at points where $\mathcal{B} \to 0$, there is a singularity in the Kähler potential and the effective Lagrangian used in the analysis breaks down. As in the odd-N case, in order to address this concern, we study the theory in two ways. First, we add an extra flavor (with a mass term) for $SU(N)$. In this case $\bar{\Lambda}_{2L}$ is not field dependent and no singularity arises in the Kähler potential when $\mathcal{B} \to 0$. Again, we find that all the F-term conditions can be met at a point where $\mathcal{B} \to 0$. In fact, we recover the vevs obtained in the analysis above without the extra $SU(N)$ flavor, after relating the strong-coupling scales in the two cases. Second, since the strong coupling scale of the $SU(2)$ theory tends to zero when $\mathcal{B} \to 0$, we work directly at the point $\mathcal{B} = 0$, in terms of the quarks of $SU(M = 2)$, and verify again that all the F-term constraints can be satisfied.

5.3 The $M > 2$ Theories.

We end this section with some comments on the $M > 2$ case. As was mentioned earlier, finding a tree-level superpotential that lifts all the baryonic flat directions is considerably more complicated in this case, and we have not been able to solve this problem yet. One can of course analyze these theories with just the Yukawa couplings eq. (2.1). However in this case there are baryonic flat directions and a major concern is that these theories have a runaway

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[4] In terms of the description used above the Pfaffian of the mass matrix for the $SU(2)$ mesons goes to zero at this point as well, thereby keeping the vevs of all the moduli fields finite.

SUSY preserving vacuum[5]. The situation here is analogous to that for the $SU(N) \times SU(N-1)$ models which was discussed in some detail in [13]. Consequently, we discuss it only briefly here. Let us return to the electric theory we started with in the presence of the tree level superpotential eq. (2.1). The direction in question corresponds to taking the $\bar{R}$ fields to infinity along a baryonic flat direction. The $SU(N-2)$ group is completely broken along this direction while the $SU(N)$ group is strongly coupled with a scale that diverges asymptotically. As in [13] one can satisfy all the F-term conditions along this direction when the $\bar{R}$ fields go to infinity. Moreover, a preliminary investigation suggests that the corrections to the the classical Kähler potential for $\bar{R}$ are small along this direction leading to the conclusion that SUSY is probably restored when $\bar{R} \to \infty$.

6   The $\mathbf{SU(N) \times SU(N)}$ Models.

For completeness, we also briefly discuss the $SU(N) \times SU(N)$ models. In this case, one finds that SUSY is unbroken. To see this, we note that the $SU(N) \times SU(N)$ models are very similar to some of the $SU(2) \times SU(2)$ models—the "[1, 1]" models—first considered in ref. [20]; see also [13]. The moduli fields are the mesons $Y_{IA} = \bar{L}_I \cdot Q \cdot \bar{R}_A$, and the baryons, $X = \det Q$, $\bar{B}_L = \det \bar{L}$ and $\bar{B}_R = \det \bar{R}$. The superpotential, which can be derived as in [20], [13], is:

$$W = A \left(\det Y - \bar{B}_L \, \bar{B}_R \, X + \bar{B}_L \, \Lambda_R^{2N} + \bar{B}_R \, \Lambda_L^{2N} \right) \, , \tag{6.17}$$

with Λ_L (Λ_R) the scale of the first (second) group respectively (defined such that the coefficients of the last two terms in (6.17) are unity), and A is a Lagrange multiplier. Upon perturbing (6.17) with a maximal rank Yukawa coupling, $\delta W = \lambda^{IA} Y_{IA}$, one finds that there is a SUSY preserving runaway vacuum along $X \to \infty$. This runaway direction corresponds to a flat direction present in the classical theory. One could try to stabilize the X runaway direction, by adding another term, proportional to X, to the superpotential. But in this case the theory has a SUSY preserving vacuum without any runaway behavior.

7   Conclusions.

In this paper, we have studied a large class of $N = 1$ supersymmetric theories with gauge group $SU(N) \times SU(N - M)$ and fundamental matter content. We used duality to elucidate their nonperturbative dynamics. We showed that the odd-N, $M = 2$ theories break supersymmetry after adding appropriate Yukawa couplings. We expect that a similar analysis is applicable in other classes of product-group theories as well, e.g. the $SU(N) \times SP(M)$ theories [11]. Our

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[5]We are grateful to M. Dine and especially Y. Shirman for discussions in this regard.

results provide more examples of theories that break supersymmetry and suggest that this phenomenon is fairly common in chiral supersymmetric gauge theories. We hope that these examples will prove of use in the construction of phenomenologically relevant models, and will contribute to a systematic understanding of supersymmetry breaking.

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