IITM/PH/TH/2007/2
TIFR/TH/07-10

On The Stability of Non-Supersymmetric Attractors in String Theory

Suresh Nampuri[1]*, Prasanta K. Tripathy[2]† and Sandip P. Trivedi[1]‡

[1] *Department of Theoretical Physics,*
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400 005, India.

[2] *Department of Physics,*
Indian Institute of Technology Madras,
Chennai 600 036, India.

February 1, 2008

Abstract

We study non-supersymmetric attractors obtained in Type IIA compactifications on Calabi Yau manifolds. Determining if an attractor is stable or unstable requires an algebraically complicated analysis in general. We show using group theoretic techniques that this analysis can be considerably simplified and can be reduced to solving a simple example like the STU model. For attractors with $D0-D4$ brane charges, determining stability requires expanding the effective potential to quartic order in the massless fields. We obtain the full set of these terms. For attractors with $D0-D6$ brane charges, we find that there is a moduli space of solutions and the resulting attractors are stable. Our analysis is restricted to the two derivative action.

*email:suresh@theory.tifr.res.in
†email: prasanta@physics.iitm.ac.in
‡email: sandip@theory.tifr.res.in

1 Introduction and Overview

Supersymmetric black holes are well known to exhibit the attractor phenomenon. This was first observed in [1] and subsequently explored in several papers, for example, [2–8]. More recently the study of non-supersymmetric extremal black holes has gained prominence and it has been realised that these black holes are also attractors. Early investigations were carried out in, [5, 6]. Sufficient conditions for the existence of a stable attractor were discussed in terms of an effective potential in [9]. A black hole entropy function was formulated which allows higher derivative corrections to also be included in [10]. Subsequent investigations include [11–43].

In particular, of particular relevance to this note, is the investigation carried out in [14], where non-supersymmetric extremal black holes in Type IIA string theory compactified on a Calabi-Yau three-fold were analysed. An attractor corresponds to a critical point of the effective potential. For the attractor to be stable, the critical point must be a minimum of the effective potential. Flat directions are allowed, but the extremum cannot be a maximum along any direction of moduli space. For the non-supersymmetric extrema found in [14], it was noticed that the mass matrix, which governs the quadratic fluctuations about the extremum, has zero eigen values [1]. Thus it is necessary to expand the effective potential beyond quadratic order to determine whether the extrema are minima or maxima along the massless directions, or whether these directions are exactly flat. While straightforward in principle, this procedure is algebraically quite involved in practice. The purpose of this note is to show that the required algebraic manipulations can be considerably simplified by using group theoretic considerations. We find that these considerations determine the general form of the terms which can arise at the cubic, quartic etc levels upto coefficients. The coefficients can then be determined by carrying out the calculations in a simple model like the STU model.

This note is organised as follows. After some preliminaries, we first illustrate the above procedure by considering the case of a black hole which carries $D0, D4$ -brane charges and show how the mass matrix and quartic terms can be determined by a comparison with the STU case. This analysis also extends quite directly to the case of a black hole carrying, $D0, D2, D4$ charges. Next, we consider black holes which carry $D6$ brane charge. We show in general that there is a moduli space of solutions in the D0-D6 system and the resulting attractor is stable. For the $D0 - D4 - D6$ case we argue that no terms cubic in the fluctuations can appear, again by a comparison with the STU case. Throughout we work in the two derivative approximation, so our results

[1] In general hypermultiplets are not sourced by the gauge fields and will be flat directions of the effective potential in the two derivative approximation. The massless fields referred to here arise from the vector multiplets and can be lifted by corrections beyond the quadratic order even in the two derivative theory.

are applicable to "big" black holes.

Our analysis corrects some errors in earlier work in [14]. In particular the quadratic terms were not completely determined therein and it was incorrectly argued that for the $D0-D4-D6$ case a cubic term is present which renders the attractor unstable.

There are several open questions worth investigating further. The quartic correction in the $D0-D4$ case - which is the first correction along the massless directions - has an interesting structure and consists of two terms with opposite signs. Both terms depend on the triple intersection numbers of the Calabi Yau manifold and the charges carried by the black hole. It is interesting to explore whether the stable or unstable nature of the attractor can vary as this data is changed. We briefly discuss a model which illustrates this possibility towards the end of our discussion of the $D0-D4$ system. In the more general case, of a black hole which carries $D0-D4-D6$ charge, symmetries again allow the same two terms at the quartic order. It should again be a straightforward exercise to determine the coefficients by comparing, say against the STU model, but we have not done so here. Finally, an interesting general issue is the inclusion of higher derivative corrections and how they alter the required conditions for the existence of a stable attractor. These terms should be particularly important for the flat directions of the two-derivative effective potential and for the directions which are not flat but which have vanishing quadratic terms.

2 Some Preliminaries

Type IIA string theory compactified on a $CY3$ has $\mathcal{N}=2$ supersymmetry. Moduli in the resulting low-energy theory lie in vector multiplets and hypermultiplets. The vector multiplet moduli couple to the gauge fields and are fixed by the attractor mechanism. The low-energy dynamics for the vector multiplets is determined by a prepotential. If the Calabi-Yau manifold has $h(1,1)=N$, there are N vector multiplets and $N+1$ gauge fields in the low-energy theory. The prepotential (neglecting any α' corrections) is,

$$F = D_{abc}\frac{X^a X^b X^c}{X^0}. \tag{1}$$

Here $X^0, X^a, a=1,\cdots N$ are the projective coordinates of special geometry, and D_{abc} are the triple intersection numbers.

The Kahler potential is given by,

$$K = -\ln\left[i\sum_{A=0}^{N}\left(X^{A*}\partial_A F - X^A(\partial_A F)^*\right)\right]. \tag{2}$$

We will use the notation,

$$x^a = \frac{X^a}{X^0}, \tag{3}$$

in what follows, and use the projective invariance to set $X^0 = 1$. This gives a Kahler potential,

$$K = -\ln\left(-iD_{abc}(x^a - \bar{x}^a)(x^b - \bar{x}^b)(x^c - \bar{x}^c)\right) \ . \tag{4}$$

A superpotential can be defined, it depends on the charges carried by the black hole. Let $\Sigma^a, a = 1, \cdots N$ and $\hat{\Sigma}_a$ be a basis of 4-cycles and 2-cycles of the CY_3, and consider a general black hole carrying (q_0, q_a, p^a, p^0) units of $D0 - D2 - D4 - D6$ brane charge. The superpotential is then given by,

$$W = q_0 X^0 + q_a X^a - p^a \partial_a F - p^0 \partial_0 F \ . \tag{5}$$

In the gauge $X^0 = 1$ this becomes,

$$W = q_0 + q_a x^a - 3D_{abc}x^a x^b p^c + p^0 D_{abc} x^a x^b x^c \ . \tag{6}$$

In the discussion which follows, we use the notation,

$$D_{ab} \equiv D_{abc}p^c, \quad D_a \equiv D_{abc}p^b p^c, \quad D \equiv D_{abc}p^a p^b p^c. \tag{7}$$

The effective potential, which determines the existence of an attractor is given in terms of the superpotential and the Kahler potential by,

$$V_{\text{eff}} = e^K \left[g^{a\bar{b}} \nabla_a W (\nabla_b W)^* + |W|^2 \right] \ , \tag{8}$$

where $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$, $g^{a\bar{b}}$ is the inverse of $g_{a\bar{b}}$ and $\nabla_a W = \partial_a W + \partial_a K W$.

For an attractor to exists V_{eff} must have an extremum. If this extremum is a minimum, the attractor is stable. The extrema of this effective potential were analysed in [14]. For the $D0 - D2 - D4$ system the extremum is given at $x^a = x_0^a$ where by,

$$x_0^a = ip^a \sqrt{\frac{\hat{q}_0}{D}} + \frac{1}{6} D^{ab} q_b \ , \tag{9}$$

in the supersymmetric case and,

$$x_0^a = ip^a \sqrt{\frac{-\hat{q}_0}{D}} + \frac{1}{6} D^{ab} q_b \ , \tag{10}$$

in the non-susy case. Here,

$$\hat{q}_0 \equiv q_0 + \frac{1}{12} D_{ab} q^a q^b \ . \tag{11}$$

The entropy of the non-supersymmetric extremal black hole is,

$$S = 2\pi \sqrt{-D\hat{q}_0} \ . \tag{12}$$

For the $D0-D4-D6$ system the non-susy extremum is given by,

$$x_0^a = p^a(t_1 + it_2), \tag{13}$$

where t_1, t_2 are determined by the charges and given in eq.(63,64) of section 3.3 in [14]. The entropy of the non-supersymmetric extremal black hole is,

$$S = \pi\sqrt{(p^0)^2 q_0^2 - 4Dq_0}. \tag{14}$$

To determine whether the attractor is stable we now expand about the extremum. Let us define the fluctuations, $\delta\xi^a, \delta y^a$, as

$$x^a = x_0^a + \delta x^a \equiv x_0^a + \delta\xi^a + i\delta y^a. \tag{15}$$

For the $D0-D4-D6$ system, with no $D2$-brane charge, the terms quadratic in the fluctuations take the form [2],

$$\begin{aligned} S_{\text{quadr}} &= \partial_a\partial_{\bar{d}}V(\delta\xi^a\delta\xi^d + \delta y^a\delta y^d) \\ &\quad + Re(\partial_a\partial_d V)(\delta\xi^a\delta\xi^d - \delta y^a\delta y^d) - 2Im(\partial_a\partial_d V)\delta\xi^a\delta\xi^d. \end{aligned} \tag{16}$$

(Note this corrects some typos of factors of two in eq.(117) of [14]).

The mass matrix can then be read off and takes the form [3], [14],

$$M = E\left(3\frac{D_a D_d}{D} - D_{ad}\right) \otimes \mathbf{I} + D_{ab} \otimes (A\sigma^3 - B\sigma^1). \tag{17}$$

In Appendix A.1 we give the values of the coefficients, E, A, B. The mass matrix in eq.(17) is written in a tensor product notation. The labels, a, d take values, $1, \cdots N$. The $\mathbf{I}, \sigma_3, \sigma_1$ matrices act in the 2×2 space labeled by $(\delta\xi^a, \delta y^a)$, for fixed a, while $D_{ab}, D_a D_b$ matrices act in the $N \times N$ space labeled by the indices, a, b.

As was analysed in [14] the mass matrix has $N+1$ positive eigenvalues [4] and $N-1$ zero eigenvalues. The zero eigenvectors correspond to fluctuations of the type, $x^a - x_0^a = (\cos\theta + i\sin\theta)z^a$, where the z^a's satisfies the relation, $D_a z^a = 0$, and θ is defined by,

$$\cot\theta = \frac{B}{A-E}. \tag{18}$$

[2] When the $D6$-brane charge vanishes the $D2$-brane charge can be included in a straightforward manner

[3] Our conventions are that the quadratic terms are given by,

$$S_{\text{quadr}} = \frac{1}{2}M_{AB}\phi^A\phi^B,$$

where ϕ^A, ϕ^B denote all fluctuating fields.

[4] We are assuming here that the attractor values correspond to a non-singular point in the moduli space, for which the moduli space metric is non-singular.

For the $D0-D4$ system in particular, $\theta \to 0$ and the massless modes consist purely of the real parts of the fluctuations, $\delta\xi^a$, subject to the constraint, $D_a \delta\xi^a = 0$.

We now turn to some group theory. Let $A \in GL(N,R)$, be a $N \times N$ matrix which acts on the x^a variables, as

$$x^a \to A^a_b x^b. \tag{19}$$

If we also transform the charges, and D_{abc}, as follows,

$$\begin{aligned} q_a \to &\quad q_b (A^{-1})^b_a \\ p^a \to &\quad A^a_b p^b \\ D_{abc} \to &\quad D_{def} (A^{-1})^d_a (A^{-1})^e_b (A^{-1})^f_c, \end{aligned} \tag{20}$$

then we see from eq.(4), eq.(6) that the Kahler potential and the superpotential and thus V_{eff} are all left invariant. This is the central observation which will aid our discussion of the corrections to the effective potential.

The STU model is obtained from a consistent truncation of Type IIA on $K3 \times T^2$ (or Heterotic Theory on T^6). It consists of three vector multiplets, $N = 3$, and a prepotential:

$$F = -\frac{X^1 X^2 X^3}{X^0}. \tag{21}$$

This means $D_{123} = -\frac{1}{6}$ and all the other non-zero components of D_{abc} are related to this one by symmetries.

3 The $D0 - D4$ System

We will now consider the $D0 - D4$ system in more detail. Our main goal will be to use group theory considerations and determine the quadratic terms along the massless directions of the effective potential. $D2$ brane charge can be included in the analysis in a straightforward manner, but we will not do so here.

The extremum value for non-susy attractor is given by setting $q_a = 0$ in eq.(10) to be,

$$x_0^a = ip^a \sqrt{\frac{-q_0}{D}}. \tag{22}$$

3.1 Mass matrix and Group Theory

The mass matrix was determined in [14] by a direct calculation, and was discussed above. Here we will see that group theory allows this calculation to be carried out much more simply. The non-supersymmetric extremum is given by, eq.(22). The most general quadratic fluctuations take the form,

$$V_{\rm quadr} = \left(C_1 D_{ab} + C_2 \frac{D_a D_b}{D}\right) \delta\xi^a \delta\xi^b + \left(C_3 D_{ab} + C_4 \frac{D_a D_b}{D}\right) \delta y^a \delta y^b$$

$$+ \quad \left(C_5 D_{ab} + C_6 \frac{D_a D_b}{D} \right) \delta\xi^a \delta y^b \ . \tag{23}$$

The coefficients C_i can depend on q_0, and $D \equiv D_{abc}p^a p^b p^c$, which are the two invariants made out of the charges, under the transformation eq.(20).

For the $D0 - D4$ case the effective potential has a symmetry which is useful to bear in mind. It is invariant under the transformation, $x^a \leftrightarrow -\bar{x}^a$. Clearly the Kahler potential in invariant under this transformation, and therefore so is the metric, $g^{a\bar{b}}$. Since the superpotential is quadratic in the x^a fields, this transformation takes, $W \rightarrow \bar{W}$, and $\nabla_a \rightarrow -\nabla_{\bar{a}}$, leaving the effective potential invariant. Furthermore, since the extremum value, eq.(22), is purely imaginary, this symmetry is unbroken. Now under this symmetry, $\delta\xi^a \rightarrow -\delta\xi^a$ and δy^a is left invariant. Thus when expanding V_{eff} around the extremum no term which contains odd powers of $\delta\xi^a$ can appear. This means that C_5, C_6 must vanish.

We will now obtain the remaining coefficients in eq.(23) by comparing with the STU model. For this purpose it is enough to take the 3 $D4$-brane charges in the STU model to be all equal, $p^a = p, a = 1, \cdots 3$. The quadratic terms, eq.(23), for the STU model with these charges then become,

$$\begin{aligned} V_{\text{quadr}} &= -\frac{p}{3}C_1 \left(\delta\xi^1\delta\xi^2 + \delta\xi^2\delta\xi^3 + \delta\xi^3\delta\xi^1\right) - \frac{p}{9}C_2 \left(\delta\xi^1 + \delta\xi^2 + \delta\xi^3\right)^2 \\ & -\frac{p}{3}C_3 \left(\delta y^1\delta y^2 + \delta y^2\delta y^3 + \delta y^3\delta y^1\right) - \frac{p}{9}C_4 \left(\delta y^1 + \delta y^2 + \delta y^3\right)^2 \ . \end{aligned} \tag{24}$$

Now we directly compute the quadratic terms in the STU model. The effective potential is given by,

$$V_{\text{eff}} = -\frac{i}{(x^1 - \bar{x}^1)(x^2 - \bar{x}^2)(x^3 - \bar{x}^3)} f(x, \bar{x}) \ , \tag{25}$$

where the function $f(x, \bar{x})$ is,

$$\begin{aligned} f(x,\bar{x}) &= \Big[4q_0^2 + 2pq_0 \left(x^1x^2 + x^2x^3 + x^3x^1 + x^1\bar{x}^2 + x^2\bar{x}^3 + x^3\bar{x}^1 + \text{c.c.}\right) \\ &+ p^2 \Big\{4\left(|x^1x^2|^2 + |x^2x^3|^2 + |x^3x^1|^2\right) + 2|x^1|^2(x^2 + \bar{x}^2)(x^3 + \bar{x}^3) \\ &+ 2|x^2|^2(x^3 + \bar{x}^3)(x^1 + \bar{x}^1) + 2|x^3|^2(x^1 + \bar{x}^1)(x^2 + \bar{x}^2)\Big\}\Big] \ . \end{aligned} \tag{26}$$

Here 'c.c.' denotes the complex conjugation of all the terms inside the parenthesis. At the extremum, $x^a = x_0, a = 1, \cdots 3$, where,

$$x_0 = ip\sqrt{\frac{-q_0}{D}}. \tag{27}$$

Expanding about the extremum, we get the quadratic terms to be

$$V_{\text{quadra}} = \frac{p^2}{|x_0|} \Big(\left(\delta\xi^1 + \delta\xi^2 + \delta\xi^3\right)^2$$

$$+ \quad \left(\delta y^1+\delta y^2+\delta y^3\right)^2 - 2\left(\delta y^1\delta y^2+\delta y^2\delta y^3+\delta y^3\delta y^1\right)\Big) . \tag{28}$$

Comparing, eq.(28) and eq.(24) we then get that

$$C_1 = 0 \ , \ C_2 = -9\sqrt{\frac{-D}{q_0}} \ , \ C_3 = 6\sqrt{\frac{-D}{q_0}} \ , \ C_4 = -9\sqrt{\frac{-D}{q_0}} \ . \tag{29}$$

It is easy to see that this is in agreement with the answer obtained in [14] for the D0-D4 case, eq.(17).

3.2 Quartic terms

We see from the calculation above that since C_1 vanishes there are $N-1$ massless modes for the $D0-D4$ system on a general CY_3 ($N=3$ in the STU model). These correspond to fluctuations of the real parts, $\delta\xi^a$, subject to the constraint that $D_a\delta\xi^a=0$. The remaining modes are all heavy with positive mass. To determine if the attractor is stable we need to find the leading corrections along the massless directions. The general symmetry argument discussed above for the $D0-D4$ case tells us that there are no cubic terms in the $\delta\xi^a$ fields so we must go to quartic order. This makes the resulting calculation somewhat complicated. In particular we will need to keep terms which are both quartic in the massless degrees of freedom, and terms which are cubic involving both the massive and massless degrees of freedom. The latter, after solving for the massive fields in terms of the massless ones, will generate additional terms that are quartic in the massless variables.

To understand this better consider a simple model with one massive field Φ and one massless field ϕ. The potential around the extremum is

$$V = V_0 + \frac{1}{2}M^2\Phi^2 + \lambda_1\phi^2\Phi + \lambda_2\phi^4 \ . \tag{30}$$

Now solving for the massive field in terms of the massless one and substituting back in the potential gives, a quartic potential in ϕ of the form:

$$V_{\rm quartic} = \left(\lambda_2 - \frac{\lambda_1^2}{2M^2}\right)\phi^4 \ . \tag{31}$$

We see that cubic term in eq.(30) has given rise to an additional quartic term in eq.(31).

In the $D0-D4$ system the terms which are quartic to begin with in the light fields (analogue of the $\lambda_2\phi^4$ terms in eq.(30)) were calculated in [14] and are,

$$V_{\rm quartic1} = -\frac{9}{2D}\left(\frac{-D}{q_0}\right)^{\frac{3}{2}}\left(D_{ab}\delta\xi^a\delta\xi^b\right)^2 . \tag{32}$$

(More correctly we need to evaluate this term subject to the constraint that $D_a\delta\xi^a = 0$ to get the quartic terms along the massless directions.) However, the terms which originate from cubic terms involving the heavy fields were left out in the analysis in [14]. We turn to determining these next.

Since the massless fields arise from $\delta\xi^a$, we are interested in cubic terms involving either two $\delta\xi^a$'s and one δy^a, or three $\delta\xi^a$'s. However the latter vanish due the symmetry which prevents odd powers of $\delta\xi^a$ from appearing. The most general terms involving two $\delta\xi^a$'s and one δy^a, from group theoretical considerations must take the form,

$$V_{\text{cubic}} = \frac{1}{q_0}\left(C_1 D D_{abc} + C_2 D_{ab} D_c + C_3 D_a D_{bc} + C_4 \frac{D_a D_b D_c}{D}\right)\delta\xi^a\delta\xi^b\delta y^c \ . \tag{33}$$

At first it might seem that other terms can also appear. For example, a term of the type,

$$D^{ab}D_{abc}D_{de}\delta\xi^c\delta\xi^d\delta y^e \ ,$$

is allowed by the symmetries. However in this term the D^{ab} tensor is fully contracted with D_{abc}, and one can see that such a term cannot arise when expanding V_{eff}. D^{ab} can only appear through $g^{a\bar{b}}$ in the potential. But, since $g^{a\bar{b}}$ appears in the term $g^{a\bar{b}}\nabla_a W\overline{\nabla_b W}$ of the potential, and the D_{abc} tensor would have to arise either from $D_a W$ or from $\overline{D_b W}$, it cannot be fully contracted with D^{ab}. A similar argument also rules out other possible terms from appearing. Thus eq.(33) is the most general cubic term containing two massless and one massive field.

Now, to determine the coefficients C_i, we compare with STU model. Once again we choose $p^a = p, a = 1, \cdots 3$. As discussed in Appendix A.2 one finds that:

$$C_1 = 3 \ , \ C_2 = -9 \ , \ C_3 = 18 \ , \ C_4 = 27 \ . \tag{34}$$

We can now solve for the massive modes and obtain the quartic terms for the massless fields. Since the massless directions correspond to the $\delta\xi^a$ fields, subject to the constraint that $D_a\delta\xi^a = 0$, we need only keep the first two terms in eq.(33), with coefficients proportional to C_1, C_2. Instead of solving for all the heavy fields we will here only solve for the δy^a fields in terms of the $\delta\xi^a$ fields. We will then need to restrict the fluctuations in $\delta\xi^a$ to satisfy the constraint $D_a\delta\xi^a = 0$, to get the final quartic terms along the massless directions.

Setting $D6$-brane charge, $p^0 = 0$, in eq.(17) we see that the δy^a fields have a mass term,

$$V_{mass} = \frac{1}{2}M_{ab}\delta y^a\delta y^b = E\left(\frac{3D_a D_b}{D} - 2D_{ab}\right)\delta y^a\delta y^b \ . \tag{35}$$

As discussed in Appendix A.2 solving for δy^a in terms of $\delta\xi^a$ then gives a quartic term,

$$V_{\text{quartic2}} \ = \ -\frac{3}{8}\left(\frac{-D}{q_0}\right)^{3/2}\left(D^{ab}D_{alm}\delta\xi^l\delta\xi^m D_{bpq}\delta\xi^p\delta\xi^q\right)$$

$$+\left(\frac{27}{8D}\right)\left(\frac{-D}{q_0}\right)^{3/2}\left(D_{lm}\delta\xi^l\delta\xi^m\right)^2. \quad (36)$$

Combining, eq.(32), eq.(36) we then get the full quartic contribution to be,

$$\begin{aligned} V_{\text{quartic}} &= -\frac{3}{8}\left(\frac{-D}{q_0}\right)^{3/2}\left(D^{ab}D_{alm}\delta\xi^l\delta\xi^m D_{bpq}\delta\xi^p\delta\xi^q\right) \\ &\quad -\frac{9}{8D}\left(\frac{-D}{q_0}\right)^{3/2}\left(D_{ab}\delta\xi^a\delta\xi^b\right)^2. \end{aligned} \quad (37)$$

It is important to again emphasise that in the above expression we must constrain the δx^a fields to satisfy the constraint $D_a\delta\xi^a = 0$, in order to get the required quartic contribution along the massless directions.

It is also useful to rewrite eq.(37) as follows. The metric on the vector multiplet moduli space at the extremum, eq.(22), is given by,

$$g_{a\bar{b}} \equiv \partial_a\partial_{\bar{b}}K|_{x^a=x_0} = -\frac{3}{2Dq_0}\left(\frac{3}{2}D_aD_b - DD_{ab}\right). \quad (38)$$

Inverting this, we get the relation,

$$D^{ab} = \frac{3}{D}p^ap^b + \frac{3}{2q_0}g^{a\bar{b}}. \quad (39)$$

Substituting in eq.(37) gives,

$$V_{\text{quartic}} = \frac{9}{4D}\left(\frac{-D}{q_0}\right)^{3/2}\left[-\left(D_{lm}\delta\xi^l\delta\xi^m\right)^2 + \frac{1}{4}\left(\frac{-D}{q_0}\right)\left(g^{a\bar{b}}D_{alm}\delta\xi^l\delta\xi^m D_{bpq}\delta\xi^p\delta\xi^q\right)\right]. \quad (40)$$

Now for a non-supersymmetric attractor $(\frac{-D}{q_0}) > 0$, and for a solution where the attractor value is non-singular, $g^{a\bar{b}}$ is non-degenerate with positive eigenvalues, thus we see that the two terms within the square brackets come with a relative opposite sign. If the net resultant contribution is positive the attractor is stable, else it is unstable. In some cases, and we will see an example of this shortly, the two terms can cancel against each other identically.

Let us close this section on the $D0-D4$ system with some more comments on the STU model. In this case $D_{123} = -\frac{1}{6}$, and all other non-zero components of D_{abc} are related to it by symmetries. Setting all the p^a's equal [5], $p^a = p, a = 1\cdots 3$, and evaluating eq.(37) one gets,

$$V_{\text{quartic}} = \frac{1}{4p}[(\delta\xi^1\delta\xi^2 + \delta\xi^2\delta\xi^3 + \delta\xi^1\delta\xi^3)^2 - \{(\delta\xi^1\delta\xi^2)^2 + (\delta\xi^2\delta\xi^3)^2 + (\delta\xi^1\delta\xi^3)^2\}]. \quad (41)$$

[5] This entails no loss of generality since the p^a's can be bought to this form by rescaling the x^a's.

Recall that for the quartic terms of the massless fields alone, we need to evaluate this expression after imposing the constraint, $D_a \delta\xi^a = 0$. For the STU model this takes the form,

$$\delta\xi^1 + \delta\xi^2 + \delta\xi^3 = 0. \tag{42}$$

On imposing this conditions among the $\delta\xi^a$ fields in eq.(41) one finds that the quartic term identically vanishes.

In fact one can show that the two massless directions for the STU model are exactly flat directions of the effective potential. Let $x_0^a = \xi_0^a + iy_0^a$ denote the critical value for the field x^a. Now, solving for the general non-susy critical point of the effective action, eq.(25), one finds that ξ^a, y^a must satisfy the four equations,

$$\begin{aligned} q_0 - p(\xi^{a\ 2} + y^{a\ 2}) &= 0, a = 1, 2, 3 \\ q_0 \sum_a \xi^a + p \prod_a \xi^a &= 0 \end{aligned} \tag{43}$$

These four equations admit a 2 real dimensional moduli space of solutions. The moduli space can be parametrised by the real parts, ξ^a, subject to the constraint $q_0 \sum_a \xi^a + p \prod_a \xi^a = 0$. At the linearised level this constraint takes the form, $\sum_a \xi^a = 0$. This agrees with the constraint, $D_a \delta\xi^a = 0$, we found earlier that the massless fields had to satisfy at the quadratic level. We will see in the next section that the existence of these flat directions for the STU model follows from duality and the existence of flat directions for the $D0 - D6$ system in general.

At we have pointed out earlier, the quartic terms can make the attractor either stable or unstable. In the following we demonstrate it by considering an explicit example. Consider the model [46] with a prepotential: $F = (aX^{1^3} - X^1X^2X^3)/X^0$. In this case, it is quite straightforward to compute the quartic term. We again take $p^a = p, 1 \cdots 3$, then, $D = p^3(a-1)$, and for a non-susy attractor to exist, $(\frac{-D}{q_0}) = \frac{p^3(1-a)}{q_0} > 0$. Now denote $\delta\xi^a$, subjected to the constraint $D_a\delta\xi^a = 0$, to be the massless directions. We find, on solving for $\delta\xi^3$ in terms of $\delta\xi^1, \delta\xi^2$ that,

$$V_{\text{quartic}} = \frac{a}{2(1+3a)} \frac{p^2}{q_0} \left(\frac{-D}{q_0}\right)^{1/2} \left(\delta\xi^1 - \delta\xi^2\right)^2 \left((3a-2)\delta\xi^1 - \delta\xi^2\right)^2 . \tag{44}$$

It can be seen that the quartic term diverges for $a = -1/3$. For all other a it has one flat direction and one other linearly independent direction which becomes stable or unstable depending on the values of q_0, a, p. For example, if $0 < a < 1$ and $p, q_0 > 0$, the attractor is stable, while if $p, q_0 < 0$, it is unstable. If on the other hand, $-\frac{1}{3} < a < 0$, and $p, q_0 > 0$, the attractor is unstable, while if $p, q_0 < 0$, it is stable. This example illustrates that the quartic terms have considerable structure in them, we will leave a more detailed study of these terms and their implications for the future.

4 Adding $D6$ Branes

We now turn to considering black holes which carry $D6$-brane charge. First we consider the $D0-D6$ system and then discuss the case with $D0-D4-D6$ brane charges.

4.1 The $D0-D6$ Attractor

A black hole with $D0-D6$ brane charges breaks supersymmetry. Here we show that the effective potential at the supersymmetry breaking extremum has flat directions.

A non-susy extremum for the case with $D0-D4-D6$ brane charges was given above in eq.(13), eq.(50), eq.(51). From there we can obtain a solution for the $D0-D6$ case by taking a limit where the $D4$ brane charge goes to zero. Some care must be exercised in taking this limit. Let us start with the $D0-D4-D6$ charges chosen so that we are in the branch where, $s/p_0 > 1$. This means $D/q_0 < 0$. Now we take the limit of vanishing $D4$ brane charge by scaling all the p^a's to go to zero at the same rate, i.e. we take $p^a \rightarrow \lambda p^a$ and take the limit as $\lambda \rightarrow 0$. In this limit it is easy to see that $t_1 \rightarrow \frac{2}{|p_0|}$, and since the real part, $\xi^a = p^a t_1$, and p^a goes to zero, we find that $\xi^a \rightarrow 0$. On the other hand, $t_2 \rightarrow \left(\frac{q_0}{-D}\right)^{\frac{1}{3}} \left(\frac{1}{|p_0|}\right)^{\frac{1}{3}}$, this means the imaginary part,

$$y^a = p^a t_2 = p^a \left(\frac{q_0}{-D|p_0|}\right)^{\frac{1}{3}}, \tag{45}$$

stays finite in this limit, since $|D|^{1/3}$ goes to zero at the same rate as p^a goes to zero. From eq.(45) we see that the resulting values for the y^a's satisfy the equation [6] ,

$$D_{abc} y^a y^b y^c = -|\frac{q_0}{p_0}|. \tag{46}$$

So we see that by setting the ξ^a fields to zero and choosing any set of y^a's which satisfies the relation eq.(46) we get an extremum of the effective potential for the $D0-D6$ case. This means there is a moduli space of non-susy solutions for the attractor equations in the $D0-D6$ case. For a CY_3 with N vector multiplets the moduli space is $N-1$ real dimensional.

One can also directly analyse the conditions for an extremum of the effective potential in the $0-6$ case. This leads to the same result, that any choice of ξ^a, y^a where $\xi^a = 0$ and y^a satisfies the constraint, eq.(46) extremises the entropy function. Some steps are indicated in Appendix A.3. For the specific case of the STU model one can go further and show that these are in fact all the solutions to the attractor conditions.

[6] The attractor value for the volume of the CY_3 in the $D0-D4-D6$ system we start with is proportional to $V \propto -Dt_2^3$. Thus $D < 0$. Since we are also working with charges for which $D/q_0 < 0$ this means $q_0 > 0$.

Expanding around any non-singular point in this moduli space, the general analysis of the mass matrix in [14] shows that all the $N+1$ massive fields (which are the real fields ξ^a and one combination of the y^a's) have a positive mass. We have seen above that the $N-1$ massless fields correspond to flat directions and thus are not lifted at cubic or higher order in the expansion around the critical point. Thus the solutions we have found are stable attractors.

One more comment is worth making. For the STU model $N=3$, so there are two exactly flat directions in this case when the black hole carries $D0-D6$ brane charges. This agrees with the number of flat directions we had found for this model in the $D0-D4$ case. The agreement in fact follows from duality. The STU model corresponds to taking Type IIA on $K3 \times T^2$. The duality group is $O(6,22) \times SL(2)$. There is only one duality invariant, the entropy of the black hole [7]. This means a black hole with $D0-D6$ charges can be turned after duality transformation into a $D0-D4$ black hole with the same entropy. Thus duality tells us that the number of flat direction of the effective potential in the two cases needed to match. More generally using duality one can relate the $D0-D6$ black hole in the STU model to a $D0-D2-D4-D6$ black hole. Thus the non-susy extremum of the effective potential must have two exactly flat directions in this more general case as well.

4.2 The $D0-D4-D6$ Black Hole

Finally let us consider a black hole which carries $D0-D4-D6$ brane charges. From the discussion in section 2 we know that for the non-supersymmetric extremum there are $N-1$ massless directions. In the $D0-D4$ case the effective potential has a symmetry under the exchange, $x^a \leftrightarrow -\bar{x}^a$, This symmetry is now broken by the terms in the superpotential dependent on the $D6$ brane charge, p_0. Thus there is no direct argument which says that odd powers of the massless fields cannot appear in the expansion about the non-supersymmetric extremum.

We are interested in the higher order corrections along the massless directions in order to decide if the attractor is stable. The first correction which is now allowed by the symmetries is cubic in the massless fields. Along the massless directions we can write $x^a - x_0^a = (\cos\theta + i\sin\theta)\alpha^a$, where the angle θ was defined in eq.(18), and the α^a's satisfy the constraint, $D_a\alpha^a = 0$. Group theory considerations tell us that the most general cubic term along the massless directions must take the form,

$$V_{cubic} = C D_{abc}\alpha^a\alpha^b\alpha^c \tag{47}$$

The coefficient C can depend on q_0, p^0 and $D = D_{abc}p^a p^b p^c$, which are the three invariants under the $GL(N,R)$ transformation, eq.(19), eq.(20), that can be made from the

[7] Since we are dealing with the two derivative action we can take the duality groups to be valued in Reals. More generally the duality groups are valued in Integers and there are extra invariants.

charges and the intersection numbers. Therefore by calculating C in the STU model and expressing the answer in terms of q_0, p^0 and D, we can obtain the value of the cubic term in general.

In fact, we already know from the duality argument given at the end of the last subsection that the two massless directions in the $D0 - D4 - D6$ case for the STU model must be exactly flat and thus no cubic term can appear in the STU model. This means that C must identically vanish as a function of q_0, p^0, D, and thus there will be no cubic term for the case of a general Calabi-Yau compactification.

For good measure we have checked this conclusion by directly calculating the cubic term in the STU model [8]. We have found that the cubic term does indeed vanish. We have also carried out an analytic calculation to first order in p^0 and found that once again the cubic term vanishes.

These considerations correct the earlier results reported in [14] where it was stated that the cubic term is in fact non-vanishing.

Since the cubic term vanishes one must now go to the quartic order. Once again group theory tells us that only two terms can appear. These have the same tensor structure as in the $D0 - D4$ case eq.(37), with the $\delta\xi^a$ fields now being replaced by the α^a fields which (after imposing the constraint $D_a\alpha^a = 0$) are the massless directions. The coefficients can be obtained by a comparison with the STU model. We know from duality, as has been argued above, that in this case the massless directions are flat, so that no quartic term can appear either. This imposes one relation between the two coefficients of the quartic terms. The allowed quartic terms then take the form,

$$V_{quart} = C_1 \left[D^{ab}(D_{alm}\alpha^m\alpha^n)(D_{bpq}\alpha^p\alpha^q) + \frac{3}{D}(D_{ab}\alpha^a\alpha^b)^2 \right] . \tag{48}$$

The coefficient C_1 can be obtained by a direct calculation in the STU model. This calculation is straightforward in principle, but we do not carry it out here and leave it for the future. One thing can be said, since we know that the massless directions are exactly flat in the $D0 - D6$ system, C_1 must vanish in the limit when the $D4$ brane charge vanishes, and more generally when, $\frac{D}{(p^0)^2 q_0} \to 0$.

Acknowledgments

We would like to thank Rudra Jena for collaboration at an early stage in this work. We are also grateful to Suresh Govindarajan and especially Atish Dabholkar for helpful discussions. P.K.T. is indebted to the Institute of Physics, University of Neuchatel for kind hospitality. We thank the organisers of the conference, "ISM06", held in December 2006 in Puri, where some of this research was done. This research is supported by

[8]This calculation was carried out using Mathematica.

the Government of India. S.P.T. acknowledges support from the Swarnajayanti Fellowship, DST, Govt. of India. P.K.T. acknowledges support form the IC&SR (IITM) Project No. PHY/06-07/17/NFSC/PRAS. Most of all we thank the people of India for generously supporting research in String Theory.

Appendix

A.1 Some more details

In this appendix we give some more details regarding the non-susy extrema in the $D0-D4-D6$ case. These results are taken from [14].

In the $D0-D4-D6$ case the non-susy extremum is located at, $x^a = x^a_0 = p^a(t_1+it_2)$. The values of t_1, t_2 are determined by the charges. There are in fact two branches for the solution. It is useful to define a variable $s > 0$ given by,

$$s = \sqrt{(p^0)^2 - \frac{4D}{q_0}}. \tag{49}$$

The two branches correspond to $|s/p^0| < 1$ and $|s/p^0| > 1$ respectively. t_1 is given by

$$t_1 = \begin{cases} \frac{2}{s}\frac{\left(1+\frac{p^0}{s}\right)^{1/3}-\left(1-\frac{p^0}{s}\right)^{1/3}}{\left(1+\frac{p^0}{s}\right)^{4/3}+\left(1-\frac{p^0}{s}\right)^{4/3}} & |\frac{s}{p^0}| > 1 \\ \frac{2}{p^0}\frac{\left(1-\frac{s}{p^0}\right)^{1/3}+\left(1+\frac{s}{p^0}\right)^{1/3}}{\left(1-\frac{s}{p^0}\right)^{4/3}+\left(1+\frac{s}{p^0}\right)^{4/3}} & |\frac{s}{p^0}| < 1 \end{cases} \tag{50}$$

and t_2 by:

$$t_2 = \begin{cases} \frac{4s}{(s^2-(p^0)^2)^{1/3}\left((s+p^0)^{4/3}+(s-p^0)^{4/3}\right)} & |\frac{s}{p^0}| > 1 \\ \frac{4s}{((p^0)^2-s^2)^{1/3}\left((|p^0|+s)^{4/3}+(|p^0|-s)^{4/3}\right)} & |\frac{s}{p^0}| < 1 \end{cases} \tag{51}$$

In these expressions the branch cuts are chosen so that all fractional powers are real.

The mass matrix for quadratic fluctuations was given in eq.(17). In this formula,

$$\begin{aligned} E &= 12De^{K_0}\left(Y_1^2 + \frac{1}{D^2t_2^2}X_1^2\right) \\ A &= 12De^{K_0}\left(\frac{1}{D^2t_2^2}X_2^2 - Y_2^2 - 2Y_1Y_2\right) \\ B &= 24De^{K_0}\frac{(X_2-X_1)}{Dt_2}Y_2, \end{aligned} \tag{52}$$

with,

$$\begin{aligned}
X_1 &= q_0 + 3Dt_2^2(1-p^0t_1) - Dt_1^2(3-p^0t_1) \\
X_2 &= q_0 - Dt_2^2(1-p^0t_1) - Dt_1^2(3-p^0t_1) \\
Y_1 &= -p^0t_2^2 - 3t_1(2-p^0t_1) \\
Y_2 &= -p^0t_2^2 + t_1(2-p^0t_1). \qquad (53)
\end{aligned}$$

Here K_0 is the Kahler potential evaluated on the solution, eq.(4), and t_1, t_2 are as given above.

A.2 Determining The Cubic and Quartic Terms

In this appendix we give some more details of the steps leading to the determination of the quartic terms as discussed in section 3.2

First we begin with cubic term involving two massless and one massive field. The general structure of such terms is given in eq.(33). Evaluating this expression for the STU model, with $p^a = p$, we get,

$$\begin{aligned}
V_{\rm cubic}^{\rm STU} &= \left(\frac{C_1p^3}{3q_0}\right)(\delta y^1\delta\xi^2\delta\xi^3 + \delta y^2\delta\xi^1\delta\xi^3 + \delta y^3\delta\xi^1\delta\xi^2) \\
&+ \left(\frac{C_2p^3}{9q_0}\right)(\delta\xi^1\delta\xi^2 + \delta\xi^1\delta\xi^3 + \delta\xi^2\delta\xi^3)\left(\delta y^1 + \delta y^2 + \delta y^3\right) \\
&+ \left(\frac{C_3p^3}{18q_0}\right)(\delta y^1(\delta\xi^2+\delta\xi^3) + \delta y^2(\delta\xi^1+\delta\xi^3) + \delta y^3(\delta\xi^2+\delta\xi^1))(\delta\xi^1+\delta\xi^2+\delta\xi^3) \\
&- \left(\frac{C_4p^3}{27q_0}\right)(\delta y^1+\delta y^2+\delta y^3)(\delta\xi^1+\delta\xi^2+\delta\xi^3)^2. \qquad (54)
\end{aligned}$$

The effective potential for the STU model was given in eq.(25) and eq.(26). Expanding this directly gives,

$$\begin{aligned}
V_{\rm cubic}^{\rm STU} &= \left(\frac{p^3}{2q_0}\right)\left[-4\{\delta y^1\delta\xi^1(\delta\xi^2+\delta\xi^3) + \delta y^2\delta\xi^2(\delta\xi^1+\delta\xi^3) + \delta y^3\delta x^3(\delta\xi^1+\delta\xi^2)\}\right. \\
&- \left. 2\{\delta y^1(\delta\xi^1)^2 + \delta y^2(\delta\xi^2)^2 + \delta y^3(\delta\xi^3)^2\}\right]\ . \qquad (55)
\end{aligned}$$

Equating coefficients, gives the result, eq.(34).

To obtain the quartic terms we need to solve for the δy^a fields in terms of the $\delta\xi^a$ fields. From the cubic terms,

$$V = \frac{1}{q_0}\left(C_1DD_{abc} + C_2D_{ab}D_c\right)\delta y^a\delta\xi^b\delta\xi^c\ , \qquad (56)$$

and the mass terms, eq.(35), we get that,

$$\delta y^a = -M^{ab}\left[\frac{C_1D}{q_0}D_{bcd}\delta\xi^c\delta\xi^d + \frac{C_2}{q_0}D_bD_{cd}\delta\xi^c\delta\xi^d\right]\ . \qquad (57)$$

Here M^{ab} is the inverse of the mass matrix, eq.(35), and is given by,

$$M^{ab} = \frac{1}{2E}\left(\frac{3}{D}p^a p^b - D^{ab}\right) \ . \tag{58}$$

Substituting eq.(57) for δy^a in the cubic terms, eq.(56), then gives the contribution to the quartic term for the light fields, eq.(36).

A.3 The $D0 - D6$ System

Here we present some more details in the analysis for the $D0 - D6$ case, showing that there is a moduli space of solutions to the extremum conditions of the effective potential. The superpotential in this case is given by,

$$W = q_0 + p_0 D_{abc} x^a x^b x^c \tag{59}$$

For the STU model it is easy to see that if x_0^a is a solution to the attractor equations, then so is $\lambda^a x_0^a$, where the λ^a's satisfy the condition, $\lambda^1\lambda^2\lambda^3 = 1$. Using this fact we can set the three x_0^a's to be equal, $x^a = x_0, a = 1 \cdots 3$. Putting this ansatz into the effective potential and solving for x_0 one finds that the only solution is of the form, $x_0 = iy$, with, $y^3 = |\frac{q_0}{p^0}|$. More generally then a solution to the attractor conditions takes the form,

$$D_{abc} y^a y^b y^c \equiv -y^1 y^2 y^3 = -|\frac{q_0}{p^0}|. \tag{60}$$

For a general CY_3 we have

$$V_{eff} \;=\; e^K |W|^2 \left[\frac{M}{6}\left(M^{ab} - \frac{3(x^a-\overline{x^a})(x^b-\overline{x^b})}{M}\right)\left(-\frac{3M_a}{M} + \frac{\partial_a W}{W}\right)\left(\frac{3M_b}{M} + \overline{\frac{\partial_b W}{W}}\right) + 1\right], \tag{61}$$

where,

$$\begin{array}{rcl} M_{ab} & = & D_{abc}(x^c - \overline{x^c}) \\ M_a & = & D_{abc}(x^c - \overline{x^c})(x^b - \overline{x^b}) \\ M & = & D_{abc}(x^a - \overline{x^a})(x^b - \overline{x^b})(x^c - \overline{x^c}) \\ g_{a\bar{b}} & = & \frac{3}{M}\left(2M_{ab} - \frac{3}{M}M_a M_b\right) \\ g^{a\bar{b}} & = & \frac{M}{6}\left(M^{ab} - \frac{3(x^a-\overline{x^a})(x^b-\overline{x^b})}{M}\right) \ . \end{array} \tag{62}$$

Setting the real parts to zero, $x^a = iy^a$ we now look for a solution to the extremum of the effective potential, of the form,

$$D_{abc} y^a y^b y^c = C. \tag{63}$$

We find that this ansatz satisfies the equations of motion if

$$C = -|\frac{q_0}{p^0}|. \tag{64}$$

References

[1] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. D **52**, 5412 (1995) [arXiv:hep-th/9508072].

[2] A. Strominger, Phys. Lett. B **383**, 39 (1996) [arXiv:hep-th/9602111].

[3] S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1514 (1996) [arXiv:hep-th/9602136].

[4] S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1525 (1996) [arXiv:hep-th/9603090].

[5] S. Ferrara, G. W. Gibbons and R. Kallosh, Nucl. Phys. B **500**, 75 (1997) [arXiv:hep-th/9702103].

[6] G. W. Gibbons, R. Kallosh and B. Kol, Phys. Rev. Lett. **77**, 4992 (1996) [arXiv:hep-th/9607108].

[7] F. Denef, JHEP **0008**, 050 (2000) [arXiv:hep-th/0005049].

[8] F. Denef, B. R. Greene and M. Raugas, JHEP **0105**, 012 (2001) [arXiv:hep-th/0101135].

[9] K. Goldstein, N. Iizuka, R. P. Jena and S. P. Trivedi, arXiv:hep-th/0507096.

[10] A. Sen, [arXiv:hep-th/0506177].

[11] P. Kraus and F. Larsen, arXiv:hep-th/0508218.

[12] R. Kallosh, arXiv:hep-th/0509112.

[13] R. Kallosh, JHEP **0512**, 022 (2005) [arXiv:hep-th/0510024].

[14] P. K. Tripathy and S. P. Trivedi, JHEP **0603**, 022 (2006) [arXiv:hep-th/0511117].

[15] A. Giryavets, JHEP **0603**, 020 (2006) [arXiv:hep-th/0511215].

[16] K. Goldstein, R. P. Jena, G. Mandal and S. P. Trivedi, JHEP **0602**, 053 (2006) [arXiv:hep-th/0512138].

[17] M. Alishahiha and H. Ebrahim, JHEP **0603**, 003 (2006) [arXiv:hep-th/0601016].

[18] B. Chandrasekhar, S. Parvizi, A. Tavanfar and H. Yavartanoo, JHEP **0608**, 004 (2006) [arXiv:hep-th/0602022].

[19] S. Bellucci, S. Ferrara and A. Marrani, Phys. Lett. B **635**, 172 (2006) [arXiv:hep-th/0602161].

[20] S. Parvizi and A. Tavanfar, arXiv:hep-th/0602292.

[21] R. Kallosh, arXiv:hep-th/0603003.

[22] B. Sahoo and A. Sen, JHEP **0609**, 029 (2006) [arXiv:hep-th/0603149].

[23] S. Ferrara and R. Kallosh, Phys. Rev. D **73**, 125005 (2006) [arXiv:hep-th/0603247].

[24] M. Alishahiha and H. Ebrahim, JHEP **0611**, 017 (2006) [arXiv:hep-th/0605279].

[25] S. Ferrara, E. G. Gimon and R. Kallosh, Phys. Rev. D **74**, 125018 (2006) [arXiv:hep-th/0606211].

[26] D. Astefanesei, K. Goldstein, R. P. Jena, A. Sen and S. P. Trivedi, JHEP **0610**, 058 (2006) [arXiv:hep-th/0606244].

[27] R. Kallosh, N. Sivanandam and M. Soroush, Phys. Rev. D **74**, 065008 (2006) [arXiv:hep-th/0606263].

[28] P. Kaura and A. Misra, Fortsch. Phys. **54**, 1109 (2006) [arXiv:hep-th/0607132].

[29] G. L. Cardoso, V. Grass, D. Lust and J. Perz, JHEP **0609**, 078 (2006) [arXiv:hep-th/0607202].

[30] J. F. Morales and H. Samtleben, JHEP **0610**, 074 (2006) [arXiv:hep-th/0608044].

[31] S. Bellucci, S. Ferrara, A. Marrani and A. Yeranyan, arXiv:hep-th/0608091.

[32] D. Astefanesei, K. Goldstein and S. Mahapatra, arXiv:hep-th/0611140.

[33] A. Dabholkar, A. Sen and S. P. Trivedi, JHEP **0701**, 096 (2007) [arXiv:hep-th/0611143].

[34] B. Chandrasekhar, H. Yavartanoo and S. Yun, arXiv:hep-th/0611240.

[35] R. D'Auria, S. Ferrara and M. Trigiante, JHEP **0703**, 097 (2007) [arXiv:hep-th/0701090].

[36] R. G. Cai and D. W. Pang, arXiv:hep-th/0701158.

[37] A. Ceresole and G. Dall'Agata, JHEP **0703**, 110 (2007) [arXiv:hep-th/0702088].

[38] L. Andrianopoli, R. D'Auria, S. Ferrara and M. Trigiante, arXiv:hep-th/0703178.

[39] K. Saraikin and C. Vafa, arXiv:hep-th/0703214.

[40] M. R. Garousi and A. Ghodsi, arXiv:hep-th/0703260.

[41] M. R. Garousi and A. Ghodsi, arXiv:0705.2149 [hep-th].

[42] J. H. Cho and S. Nam, arXiv:0705.2892 [hep-th].

[43] S. Ferrara and A. Marrani, arXiv:0705.3866 [hep-th].

[44] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova and W. K. Wong, Phys. Rev. D **54**, 6293 (1996) [arXiv:hep-th/9608059].

[45] B. de Wit, P. G. Lauwers and A. Van Proeyen, Nucl. Phys. B **255**, 569 (1985).

[46] K. Behrndt, Phys. Lett. B **396**, 77 (1997) [arXiv:hep-th/9610232].