

An Inflationary Model in String Theory

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We construct a model of inflation in string theory after carefully taking into account moduli stabilization. The setting is a warped compactification of Type IIB string theory in the presence of D3 and anti-D3-branes. The inflaton is the position of a D3-brane in the internal space. By suitably adjusting fluxes and the location of symmetrically placed anti-D3-branes, we show that at a point of enhanced symmetry, the inflaton potential V can have a broad maximum, satisfying the condition $V''/V \ll 1$ in Planck units. On starting close to the top of this potential the slow-roll conditions can be met. Observational constraints impose significant restrictions. As a first pass we show that these can be satisfied and determine the important scales in the compactification to within an order of magnitude. One robust feature is that the scale of inflation is low, $H = \mathcal{O}(10^{10})$ GeV. Removing the observational constraints makes it much easier to construct a slow-roll inflationary model. Generalizations and consequences including the possibility of eternal inflation are also discussed. A more careful study, including explicit constructions of the model in string theory, is left for the future.

1 Introduction

Inflation is an attractive idea that solves many important problems in cosmology. It is also in good agreement with current observational data. It is therefore important to understand if inflation can arise in string theory.

Despite several attempts no satisfactory model of inflation in string theory has been constructed as yet. This problem is closely tied to the issue of moduli stabilization. It is well known that a very flat potential is required for inflation. There are many light fields, called moduli, in string theory which to first approximation have an exactly flat potential. So it might seem at first that a slowly enough varying potential can be easily generated. However, any attempt to generate such a potential typically runs into difficulty. One finds that some very unstable direction develops, along which the potential descends much too rapidly. As a result, the required flatness condition is not met and inflation is not realized. Recently, there has been considerable progress in moduli stabilization. So one can be hopeful that these obstacles will be overcome.

In this paper we outline the construction of a model for inflation in string theory. Our construction is based on the recent developments in moduli stabilization. The setting is warped flux compactifications of type IIB string theory, in the presence of D3-branes and anti-D3-branes. Using fluxes we fix all the complex structure moduli of the Calabi-Yau space and the dilaton-axion [1]. We also use a generic superpotential of the kind which arises due to gaugino condensation to stabilize the volume modulus [2] (KKLT). Our discussion of inflation is closely related to the recent attempt in [3] (KKLMMT). In particular the inflaton in our model is a D3-brane modulus, i.e., a scalar field which corresponds to the location of the D3-brane in the internal Calabi-Yau space. In [3] it was argued that a superpotential of the kind mentioned above leads to the D3-brane moduli acquiring a mass, $m^2 = 1/6R$, where R is the curvature scalar. This mass is too big and ruins the required flatness of the potential.

We explore a small twist on this story here. Consider a Calabi-Yau space with a Z_2 symmetry¹. A mobile D3-brane is located in the vicinity of the Z_2 symmetric point and experiences an attractive Coulomb force due to two symmetrically located anti-D3-branes. In such a situation we show that by adjusting the fluxes and the brane-anti-brane separation, the Coulomb attraction can nearly cancel the effect of the curvature induced mass mentioned above. As a result the Z_2 symmetric point turns into a maximum of the potential. The near cancellation results in a broad maximum, with

¹This Z_2 symmetry need not be the one involved in the orientifold action.

$|m^2|/H^2 \ll 1$, where H , is the Hubble scale corresponding to the height of the potential at the maximum. By starting close enough to such a maximum the required conditions for slow-roll inflation can then all be met.

The observational constraints, especially the scale of density perturbations, impose stringent restrictions on the model. We show that the constraints can be met and determine the important scales in the compactification to within an order of magnitude². A robust feature of our model, independent of many details, is that the scale of inflation is low. The Hubble scale, H , is of order 10^{10} GeV, which corresponds to a cosmological constant of order 10^{14} GeV. Thus the production of tensor perturbations is highly suppressed. The observation of gravity waves by the Planck experiment would therefore rule out this model.

A more careful study of whether all the constraints can be met will require concrete constructions of the model in string theory and is left for the future. The non-perturbative superpotential we evoke, and the assumption that the full potential can be obtained by adding the brane-anti-brane interaction to the term coming from the superpotential³ also needs to be studied further. The last two issues are common to many KKLT type constructions.

As a model for inflation our construction is incomplete in three ways. We have not addressed how inflation ends, how it begins, and how the standard model can be incorporated in it. Ending inflation successfully requires adequate reheating. This depends on how the standard model is incorporated. In terms of beginning inflation, it could be that the model does not depend sensitively on initial conditions. It has been argued that a broad maximum of the kind in this model gives rise to eternal inflation. Regions where quantum fluctuations have driven the inflaton to the top of the potential hill grow exponentially more rapidly and soon dominate the universe, regardless of initial conditions. The inflationary epoch discussed in this paper then arises when fluctuations cause the scalar field to descend far enough from the top so that the classical evolution becomes dominant. This is an appealing picture but it needs to be understood better. We leave these issues for the future.

The important features of this model are quite general. They essentially depend only on the existence of a broad maximum, with $|m|^2/H^2 \ll 1$, and are independent of most details. For example, we have emphasized the role

²More accurately, our estimates of some of the important energy scales involved is uncertain by factors of order unity. Up to these uncertainties we show that the constraints can be met.

³This assumption would be correct if the brane-anti-brane interaction arise from a D -term. Evidence in support of this has been found in [42, 43]. We thank S. Kachru for bringing these references to our attention.

of a Z_2 symmetry above. But the idea works more generally, even when there is no such symmetry, for a D3-brane located between two appropriately positioned anti-D3-branes. More generally this construction can be viewed as an existence proof for broad maxima in the landscape of string theory. It seems reasonable to believe that there are many such maxima, with the complex structure moduli or Kähler moduli also playing the role of the inflaton⁴. The inflationary parameters, like the scale of density perturbations, or the tilt in the spectrum of scalar perturbations, probably take many different values at these maxima, most of which will not agree with observation. Further progress in moduli stabilization will allow us to test this grim possibility.

This paper is organized as follows. Our basic set-up is discussed in section 2.1. The positive mass for the brane moduli due to the curvature coupling, is reviewed in section 2.2. In section 2.3, we discuss the potential for a D3-brane located between two symmetrically placed anti-D3-branes, and show that a broad maximum can arise. The resulting inflationary scenario is discussed in section 2.4. The constraints on the compactification which arise are analyzed in section 2.5. We close with an extended discussion in section 3.

Before proceeding we should comment on some of the relevant literature. The idea of brane inflation was first discussed by [4]. Other related papers are [5], [6]. In particular, while not worrying about moduli stabilization, [7] showed that a flat potential could be obtained by considering symmetrically positioned branes. For a review, see [8]. Standard textbook references for inflation are [9] and [10], see also [11]. Some relevant references for moduli stabilization are [12] and [1] for general framework, and [13] for some examples. For recent progress towards meeting the conditions of the KKLT construction, see [14]. [15] explores the KKLT construction further, [16] considers inducing the anti-D3-brane charge on D7-branes. A variant of the KKLT scenario which does not require the anti-D3-brane is [17]. An investigation of de Sitter vacua using F-term potentials and additional light moduli is in [18]. A recent attempt to overcome the problems faced in KKLMNT involves the use of a shift symmetry [19]. It would be nice to see if shift symmetry is present in Calabi-Yau orientifolds or their F-theory generalizations, which are required for controlled stabilization of all moduli and preserve only $\mathcal{N} = 1$ supersymmetry. For an attempt in the context of string theory to use higher derivative terms for inflation see [20]. Inflation with a quadratic potential of the kind we obtain here was studied earlier in [21], which arrived at similar conclusions about the low energy scale during inflation etc⁵. Two related papers on brane inflation appeared while our manuscript was being readied. [44] uses additional D-terms obtained by adding the Standard

⁴S.P.T. thanks M. Douglas for emphasizing this point.

⁵We thank S. Sarkar for bringing this paper to our attention.

Model fields in the KKLMNT set up and argues numerically that inflation can be obtained, and [45] explores the possibility of inflation in the dynamics of more than one anti-brane in a K-S throat.

Our conventions are as follows. M_{10} refers to the ten-dimensional Planck scale. It is related to the string scale, α' , and the string coupling, g_s , by $\frac{1}{M_{10}^8} = \frac{1}{2}(2\pi)^7(\alpha')^4 g_s^2$. M_{Pl} refers to the four-dimensional Planck scale. It is defined by $M_{Pl}^2 = \frac{1}{8\pi G_N}$ and satisfies the relation, $M_{Pl}^2 = M_{10}^8 L^6$, where L^6 is the volume of the six-dimensional internal space. Finally, the tension of the D3-brane is given by $T_3 = \frac{1}{(2\pi)^3(\alpha')^2 g_s}$.

2 The Model

2.1 Basic Set-up

Consider IIB string theory on a six-dimensional Calabi-Yau orientifold, with the three forms H_3, F_3 turned on. More generally we can consider F-theory on an elliptically fibered Calabi-Yau fourfold. The resulting compactification is of the warped kind, [1],

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n. \quad (1)$$

The five-form F_5 is non-zero, and determined. The three-forms give rise to a superpotential for the complex structure moduli, [12],

$$W = \int G_3 \wedge \Omega, \quad (2)$$

where $G_3 = F_3 - \tau H_3$, τ is the dilaton axion field, and Ω is the homomorphic three-form on the Calabi-Yau space. This superpotential in general fixes all the complex structure moduli and the dilaton-axion.

As was discussed in [1], such a construction can provide a compactification of the Klebanov-Strassler (K-S) deformed conifold solution [22]. By tuning the fluxes the complex structure moduli can be stabilized close to a conifold singularity. An intuitive picture of the resulting compactification is as follows. Roughly speaking, the compactification contains a small three-sphere threaded by flux. The resulting backreaction is significant and causes a “throat” to develop - this is a region where the warp factor, $e^{2A(y)}$, departs significantly from unity. Unlike in the case of AdS_5 , the K-S throat terminates on a three-sphere where the warp factor acquires its minimum value. It is relevant to note for our purposes that if the Calabi-Yau manifold has

discrete symmetries, more than one small three-sphere can be present when the complex structure moduli are stabilized close to the conifold point. These three-spheres would be symmetrically located about a point of enhanced symmetry and in turn would give rise to symmetrically located throat regions where the warp factor departs significantly from unity.

In the subsequent discussion we restrict ourselves to Calabi-Yau orientifolds with one Kähler modulus, the volume. As discussed in [23], non-perturbative corrections to the superpotential, for example due to gaugino condensation on wrapped D7-branes, can arise. These are dependent on the volume and can stabilize it [2]. Additional anti-D3-branes at the bottom of one (or more) K-S throats can lift these vacua to positive cosmological constant giving rise to dS space.

Finally, mobile D3-branes can be present in the compactification. Their interaction with anti-D3-branes can be calculated. The idea explored in [3] was that the attractive potential between a mobile brane and an anti-brane might give rise to a slowly varying potential suitable for inflation. However, a detailed analysis of the resulting potential showed that when the details of volume stabilization, as mentioned above, are included, the D3-brane acquires a mass which is too big to allow for the slow-roll conditions to be met.

The new element we consider in this paper is to take a Calabi-Yau space with a Z_2 symmetry and two symmetrically located K-S throats each containing an anti-D3-brane. The mobile D3-brane is located in between in the vicinity of the Z_2 symmetric point. We will see that in such a situation the positive mass term due to the curvature coupling, can be canceled to good approximation by the brane-anti-brane potential, giving rise to a maximum in the potential energy with a small mass, $|m|^2 \ll H^2$. By starting close to the maximum the requirements for slow-roll inflation can be met.

The rest of this section is organized as follows. We first briefly sketch out how the curvature coupling and related positive mass term arises in [3]. Next we include the brane-anti-brane interaction and analyze the resulting potential. A discussion of the resulting inflationary scenario follows in the section 2.4. The constraints imposed on the compactification are discussed in section 2.5.

2.2 The Curvature Coupling: A positive mass

It is useful to consider the dynamics of the mobile D3-brane in an effective theory obtained by integrating out the complex structure moduli. This effec-

tive theory contains four complex scalar fields (with their fermionic partners to form chiral superfields). $\Phi^i, i = 1, \dots, 3$, are three complex fields related to the D3-brane location. ρ is an additional complex scalar, its real part is related to the volume r by,

$$2r = \rho + \bar{\rho} - k(\Phi^i, \bar{\Phi}^i). \quad (3)$$

(More correctly r is proportional to the volume of the Calabi-Yau manifold, in the notation⁶ of [1], $r \sim e^{4u}$). $k(\Phi^i, \bar{\Phi}^i)$ is the Kähler potential of the Calabi-Yau manifold.

The kinetic energy terms can be derived from the Kähler potential, [24],

$$K = -3 \log(\rho + \bar{\rho} - k(\Phi^i, \bar{\Phi}^i)). \quad (4)$$

The superpotential (in the absence of the anti-branes) takes the form

$$W = W_0 + Ae^{-a\rho}, \quad (5)$$

where W_0, A, a are constants in the effective theory. The first term above, W_0 , arises by replacing the complex structure moduli with their vacuum expectation values in eq. (2). The second term arises due to non-perturbative effects. These could be, for example, due to gaugino condensation on D7-branes wrapping four-cycles in the Calabi-Yau space, or due to Euclidean D3-brane instantons, [23]. The prefactor A also depends on the expectation values of the complex structure moduli. The superpotential, eq. (5), gives rise to a potential energy,

$$V^F = \frac{1}{6r} \left(\partial_\rho W \overline{\partial_\rho W} \left(1 + \frac{1}{2r} k^{i\bar{j}} k_i k_{\bar{j}} \right) - \frac{3}{2r} (\overline{W} \partial_\rho W + W \overline{\partial_\rho W}) \right). \quad (6)$$

Including the effects of the anti-branes in this theory is subtle. It can be shown (see appendix B of [3]) that the potential between a D3-brane and an anti-D3-brane in a warped background takes the form⁷

$$V^B(\vec{r}) = 2T_3 Z^4 \left(1 - \frac{1}{2\pi^3} \frac{Z^4 T_3}{M_{10}^8 |\vec{r} - \vec{r}_1|^4} \right). \quad (7)$$

The first term on the r.h.s. is really the potential energy of the anti-brane. The second term arises due to the attractive RR and gravitational potential

⁶ r in this paper is related to the field ρ , in [1], appendix A.1, after eq. (A.2), as follows: $2r = -i\rho$.

⁷To relate this to eq. (3.9) of [3] at $\vec{r} = 0$, we note that the redshift Z^4 is denoted by $\frac{r_0^4}{R^4}$ in [3], and from the conventions discussed in the introduction above, it follows that $\frac{1}{2\pi^3} \frac{T_3}{M_{10}^8} = \frac{R^4}{N} = 4\pi g_s (\alpha')^2$.

between the brane and anti-brane, we will refer to it as the Coulomb term below. T_3 is the tension of the 3-brane, \vec{r}, \vec{r}_1 refer to the locations of the brane and anti-brane respectively, and $Z^4 = e^{4A(\vec{r}_1)}$ refers to the redshift at the location of the anti-brane.

In eq. (7), we have assumed that the metric in the background Calabi-Yau space is flat, i.e., $g_{mn} = \delta_{mn}$, eq. (1). More generally, in the the second term on the r.h.s, the factor $1/|\vec{r}-\vec{r}_1|^4$ will be replaced by the appropriate harmonic function. If more than one brane and anti-brane are present, this formula is generalized in a straight forward manner. The first term on the r.h.s. includes the contributions of all the anti-branes with the appropriate redshift factor at their locations. The second term includes all brane-anti-brane pairs with the appropriate redshift factors and harmonic functions. We should also mention that the Coulomb term above is valid only when the brane-anti-brane separation is much bigger than the string scale, $|\vec{r}-\vec{r}_1| \gg \sqrt{\alpha'}$, and is much smaller than the size of the compactification, L , i.e., $|\vec{r}-\vec{r}_1| \ll L$. We will see in the following discussion that these conditions are indeed met⁸.

To include the effects of the anti-branes we will then simply assume that this potential due to the brane-anti-brane interactions can be added to V^F above in obtaining the full potential.

The resulting total potential then takes the form,

$$V = V^F + V^B. \quad (8)$$

Keeping only the potential energy term of the anti-brane in eq. (7) and neglecting the Coulomb term for now, we get eq. (5.14) of [3],

$$V = \frac{1}{6r} \left(\partial_\rho W \overline{\partial_\rho W} (1 + \frac{1}{2r} k^{i\bar{j}} k_i k_{\bar{j}}) - \frac{3}{2r} (\overline{W} \partial_\rho W + W \overline{\partial_\rho W}) \right) + \frac{D}{(2r)^2}. \quad (9)$$

We should point out that this equation gives the potential in four-dimensional Planck units. The second term above arises from the anti-brane potential energy, and is given by summing over all the anti-D3-branes, $2T_3 \sum_i Z_i^4$. To convert to four-dimensional Planck units we use the relation $T_3/M_{Pl}^4 \sim 1/(2r)^3$. Finally, we use the fact that the redshift factor at the bottom of a K-S throat scales like $Z^4 = e^{4A} \sim r$ for fixed integer fluxes. This gives the term $D/(2r)^2$ where the coefficient D is independent of the volume.

In the vicinity of a point in moduli space where $k(\Phi^i, \bar{\Phi}^i) = \Phi^i \bar{\Phi}^i$ one can show that a dS minimum exists at $\rho = \rho_c, \Phi^i = \bar{\Phi}^j = 0$. The potential at the

⁸If $|\vec{r}-\vec{r}_1| \sim L$, the Harmonic function in eq. (7) needs to be replaced by its compact space version.

minimum is denoted by $V = V_0(\rho_c)$. Expanding around it the potential can be written as,

$$V = \frac{V_0(\rho_c)}{(1 - \varphi\bar{\varphi}/3M_{Pl}^2)^2} \approx V_0(\rho_c) \left(1 + \frac{2}{3} \frac{\varphi\bar{\varphi}}{M_{Pl}^2}\right). \quad (10)$$

Here $\varphi = \Phi\sqrt{3/(\rho + \bar{\rho})}$ is the canonically normalized field as follows from the Kähler potential (4), and we have inserted the appropriate factor of M_{Pl} required by dimensional analysis. We see that the brane moduli acquire a positive mass. It is easy to see that $m^2 = \frac{2}{3} \frac{V_0(\rho_c)}{M_{Pl}^2} = \frac{1}{6}R$, where R is the curvature of the resulting dS space.

One more comment is in order before we proceed. The minima one gets from V^F alone are supersymmetric and have negative cosmological constant. These are lifted to positive cosmological constant because of the anti-brane contribution, the $D/(2r)^2$ term in eq. (9). At a typical dS minimum the contributions of the superpotential term and the anti-brane terms in eq. (9) are roughly comparable and each is of order $V_0(\rho_c)$.⁹ This fact will be useful to bear in mind in the next subsection.

2.3 Symmetrically Located Throats

We are now ready to consider the new twist in this paper. Consider a situation mentioned above, where there are two K-S throats symmetrically located about $\vec{r} = 0$, at $\pm\vec{r}_1$. An anti-D3-brane is located at the bottom of each throat. The D3-brane is located in the vicinity of the point of Z_2 symmetry, at $\vec{r} = 0$. See Figure 1. We are now ready to include the effects of the Coulomb interaction in eq. (7) in such a set-up¹⁰. For simplicity we assume that the metric g_{mn} is flat and this is consistent with the form of the Kähler potential assumed above in eq. (10).

The second term in eq. (7) is then given by

$$V^I = -2T_3^2 Z^8 \frac{1}{2\pi^3 M_{10}^8} \left(\frac{1}{|\vec{r} - \vec{r}_1|^4} + \frac{1}{|\vec{r} + \vec{r}_1|^4} \right). \quad (11)$$

$Z^4 = e^{4A}$ is the redshift factor at the location of either anti-brane, and by symmetry this is the same. Expanding to quadratic order, and using the

⁹The present day de Sitter phase would not be of this type, for this the two contributions would have to cancel to a very good accuracy leaving a small positive residual cosmological constant, but this is not the generic situation.

¹⁰Strictly speaking the Z_2 symmetry is not essential for this model. The idea can work for any two appropriately positioned throats, as will be discussed further in section 3, when we consider generalizations.

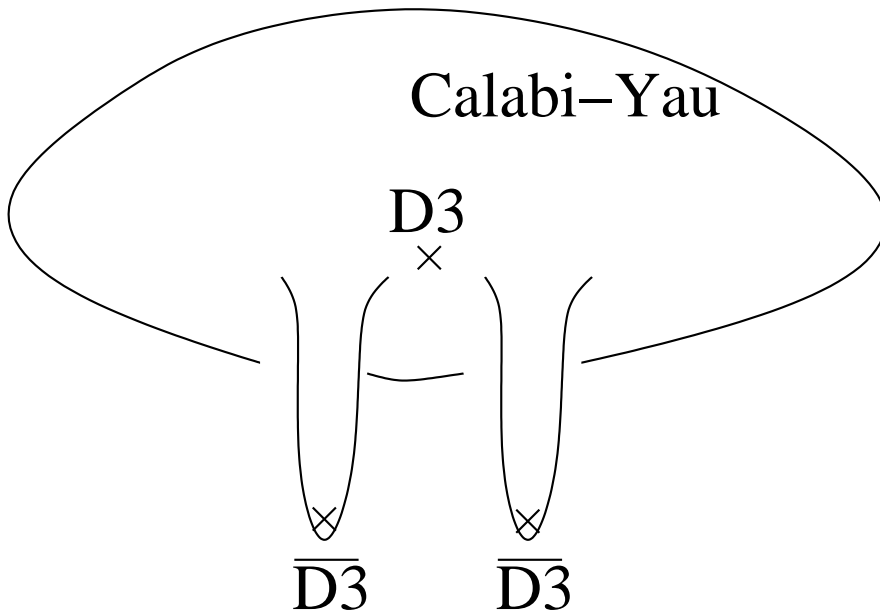


Figure 1: Two symmetrically located Klebanov-Strassler throats in Calabi-Yau space. Anti-D3-branes are at the bottom of each throats and mobile D3-brane is in between.

relation, $\frac{T_3^2}{M_{10}^8} = \pi$ we get,

$$V^I = -\frac{2Z^8}{\pi^2 r_1^4} \left(1 + \frac{2}{r_1^2} \left(\frac{6r_{1m}r_{1n}}{r_1^2} - \delta_{mn} \right) r_m r_n \right). \quad (12)$$

The first term on r.h.s. gives a correction to the vacuum energy. We will see below that this correction is small. The second term is quadratic in the displacement, \vec{r} , and gives a contribution to the mass. If the anti-branes are located along the y^1 direction, it takes the form,

$$V_{quad}^I = -\frac{4Z^8}{\pi^2 r_1^6} (5(y^1)^2 - (y^2)^2 - (y^3)^2 - (y^4)^2 - (y^5)^2 - (y^6)^2). \quad (13)$$

In particular the mass term associated with the y^1 direction is negative, as would be expected from the attractive nature of the force.

The kinetic energy terms for brane moduli can be derived from the DBI action,

$$L = -T_3 \int d^4x \sqrt{-g} \frac{1}{2} g^{\mu\nu} \partial_\mu y^i \partial_\nu y^i. \quad (14)$$

From this we see that the canonically normalized field $\varphi^i = \sqrt{T_3} y^i$.

Requiring the negative mass term along the y^1 direction to approximately cancel the curvature induced term discussed in the previous section then gives the condition,

$$\frac{2}{3} \frac{V_0(\rho_c)}{M_{Pl}^2} \simeq \frac{40Z^8}{\pi^2 r_1^6} \frac{1}{T_3}. \quad (15)$$

As was mentioned above, up to a factor of unity, the de Sitter vacuum energy is of order the contribution of the anti-branes, $V_0(\rho_c) \sim T_3 Z^4$. Dropping factors of order unity we then get,

$$\frac{T_3^2 Z^4}{M_{Pl}^2} \sim \frac{Z^8}{r_1^6}. \quad (16)$$

Using the relations, $M_{Pl}^2 = M_{10}^8 L^6$, and $T_3^2/M_{10}^8 = \pi$, this leads finally to the condition,

$$r_1 \sim Z^{2/3} L. \quad (17)$$

Note that since $Z < 1$, eq. (17) is consistent with requiring that the brane-anti-brane separation is less than size of the compactification, $r_1 < L$.

Let us make two comments before closing this subsection. First, as was mentioned above, Z in eq. (17) scales with the volume. E.g., for a K-S throat, $Z^4 = e^{4A} \sim \frac{L^4}{(\alpha')^2} \exp(-\frac{8\pi K}{3g_s M})$, where, M and K are two integers which specify the flux of F_3 and H_3 threading the vanishing S^3 and its dual three-cycle respectively [1, 22]. Including this in eq. (17) and requiring $r_1 < L$, gives rise to the condition,

$$e^{-\left(\frac{4\pi K}{9g_s M}\right)} < \left(\frac{\sqrt{\alpha'}}{L}\right)^{2/3}, \quad (18)$$

which can be met for $L > \sqrt{\alpha'}$ by choosing appropriate integers K, M . Thus we see that by appropriately choosing the fluxes, the two contributions to the mass, from the curvature coupling and the brane-anti-brane interaction, can be made to approximately cancel. Second, we mentioned above that the first term in eq. (12) makes a small contribution to the vacuum energy. We can now verify this. We had mentioned above that the vacuum energy is of order the anti-brane potential energy, i.e., $V_0(\rho_c) \sim T_3 Z^4$. So the required condition is, $T_3 Z^4 \gg \frac{Z^8}{r_1^4}$. From eq. (17) this takes the form, $\frac{1}{g_s} \frac{L^4}{(\alpha')^2} \gg Z^{4/3}$, where we have used the relation, $T_3 \sim \frac{1}{g_s(\alpha')^2}$. This is obviously met when, $g_s < 1$, $\frac{L}{\sqrt{\alpha'}} > 1$ and $Z \ll 1$.

In the following discussion we will assume that the fluxes etc have been chosen so that eq. (17) is met and the two contributions to the mass for the inflaton approximately cancel, leaving a small residual negative (mass)², for

motion along the y^1 direction. The point $\vec{r} = 0$, is then a maximum of the potential along this direction. In its vicinity the potential is given by,

$$V = V_0 - \frac{1}{2}m^2\phi^2, \quad (19)$$

where $V_0 \equiv V_0(\rho_c)$, $\phi \equiv \sqrt{T_3}y^1$ is the canonically normalized field, and m^2 is the small residual mass. We will examine the inflationary scenario that results when one starts close to this maximum next. Along all the other directions the potential is stable. For simplicity we will assume that during inflation the brane is at rest along these directions, at $y^i = 0, i = 2, \dots, 6$. Before proceeding let us note that inflation with a potential of the form eq. (19) was studied earlier in [21]. This paper noted that the scale of inflation would have to be low and also the tilt would be negative in accord with our analysis in the next subsection.

2.4 The Resulting Inflationary Scenario

The potential for the inflaton was described in the previous section, eq. (19). In the standard classification of inflationary models this is a canonical example of “small field” inflation (in the classification of [27], see also [28]).

The Hubble scale during inflation is,

$$H^2 = \frac{V}{3M_{Pl}^2} \simeq \frac{V_0}{3M_{Pl}^2}. \quad (20)$$

The last approximation follows from the fact that $\phi^2 \ll \frac{V_0^2}{m^2}$, during inflation, as we will see shortly. The two slow roll parameters are given by

$$\epsilon \equiv \frac{1}{2} \left(\frac{V'}{V} \right)^2 M_{Pl}^2 = \frac{1}{18} \left(\frac{m^2}{H^2} \right)^2 \left(\frac{\phi^2}{M_{Pl}^2} \right), \quad (21)$$

and

$$\eta \equiv \frac{V''}{V} M_{Pl}^2 = \frac{m^2}{3H^2}. \quad (22)$$

The condition for slow-roll inflation is that $\epsilon, \eta \ll 1$. We see that by taking

$$m^2 \ll H^2 \quad (23)$$

and starting at a small enough value of ϕ , both these conditions can be met.

Let us examine the observational constraints imposed in more detail now. We will see that they determine the scale of inflation, independent of the string coupling constant and the size of Calabi-Yau space in this model.

The slow roll conditions stop holding in our model when the subleading corrections to the potential become significant. We denote the value of the inflaton when this happens by, ϕ_f . There are two sources for these corrections. From eq. (11), we see that there are corrections¹¹ of $\mathcal{O}(\frac{\phi}{\sqrt{T_3 r_1}})^2$. These will become important when

$$\phi \sim \sqrt{T_3 r_1}. \quad (24)$$

From eq. (10), we see that there are also corrections of $\mathcal{O}(\frac{\phi}{M_{Pl}})^2$, these become significant when $\phi \sim M_{Pl}$. It is easy to see that eq. (24) is more restrictive. This follows from eq. (17) after noting that, $\sqrt{T_3} r_1 \sim \sqrt{T_3} Z^{2/3} L \ll M_{Pl} = M_{10}^4 L^3$, for $g_s < 1$, $Z \ll 1$ and $L/\sqrt{\alpha'} > 1$. So we learn that inflation comes to an end when $\phi_f \sim \sqrt{T_3} r_1$. This condition of course makes good physical sense. The anti-brane is located at $\phi = \sqrt{T_3} r_1$, and we expect inflation to have ended by the time the brane gets to the vicinity of the anti-brane.

Two comments are worth making at this stage. The two conditions, $\phi \leq \phi_f \ll M_{Pl}$, and, eq. (23), imply that $\phi^2 \ll \frac{V_0}{m^2} \simeq \frac{H^2}{m^2} M_{Pl}^2$, as was mentioned above eq. (21). Also, using these two conditions in eq. (21) and eq. (22), we learn that $\epsilon \ll \eta$. The observed value of the tilt, as seen below in eq. (28), then tells us that

$$\eta = \frac{m^2}{3H^2} \sim -10^{-2} < 0. \quad (25)$$

It is easy to see that the value of the inflaton, ϕ , N_e e -foldings before the end of inflation, is given by

$$\log\left(\frac{\phi_f}{\phi}\right) = \frac{|m|^2}{3H^2} N_e. \quad (26)$$

The scale of the adiabatic density perturbations, [10], [11], is given by

$$\delta_H = \frac{1}{\sqrt{75}\pi} \frac{1}{M_{Pl}^3} \frac{V^{3/2}}{|V'|} = \frac{3}{5\pi} \frac{H^3}{|m^2 \phi|}. \quad (27)$$

Data tells us that $\delta_H = 1.9 \times 10^{-5}$, see for example, [25]. The tilt in the spectrum is given by

$$n = 1 - 6\epsilon + 2\eta \simeq 1 + 2\eta. \quad (28)$$

From eq. (25), we see that this is less than unity in our model. Observational data indicates a non-zero tilt, with, $n \simeq 0.97$, [25], [26].

¹¹Here we are using the fact that the canonically normalized field $\phi = \sqrt{T_3} r$.

We are now ready to consider the constraint imposed by the density perturbations, eq. (27). The observed anisotropy arises due to perturbations which leave the horizon about 60 e -foldings before the end of inflation¹². From eq. (26), eq. (25), we learn that the value of the inflaton, 60 e -foldings before the end of inflation, is given by, $\phi \sim \phi_f/1.8 \sim \sqrt{T_3}r_1/1.8$. Substituting this¹³ along with eq. (25), in eq. (27), then gives us,

$$\frac{H}{\sqrt{T_3}r_1} \sim \frac{1}{1.8} \times 10^{-4} \frac{|m|^2}{H^2} \sim \frac{3}{1.8} \times 10^{-4} |\eta|. \quad (29)$$

Some more algebraic manipulation leads to a determination of the Hubble scale during inflation. Using the relation, $V_0(\rho_c) \sim T_3 Z^4$, and eq. (20), one gets,

$$\frac{1}{\sqrt{3}r_1 M_{Pl}} \frac{Z^2}{M_{Pl}} \sim \frac{3}{1.8} \times 10^{-4} |\eta|. \quad (30)$$

Next substituting eq. (17), in eq. (30), leads to the relation

$$\frac{1}{\sqrt{3}LM_{Pl}} \frac{Z^{4/3}}{M_{Pl}} \sim \frac{3}{1.8} \times 10^{-4} |\eta| \quad (31)$$

which determines the redshift factor, Z , in terms of the scale of compactification L and M_{Pl} . Finally, putting this condition into the expression for the Hubble constant yields,

$$\frac{H^2}{M_{Pl}^2} \sim \frac{T_3 Z^4}{3M_{Pl}^4} \sim 0.8 \times 10^{-11} \frac{T_3}{M_{10}^4} |\eta|^3 \sim 1.4 \times 10^{-17} \left(\frac{|\eta|}{0.01} \right)^3, \quad (32)$$

where we have used the relation, $M_{Pl}^2 = M_{10}^8 L^6$, and $\frac{T_3}{M_{10}^4} = \sqrt{\pi}$. Note that various model dependent features like the scale of the compactification of Calabi-Yau space and the value of g_s drop out in this expression. The resulting value of the Hubble scale is indeed low in this model. Eq. (32) gives,

$$H \sim 9.2 \times 10^9 \left(\frac{|\eta|}{0.01} \right)^{3/2} \text{ GeV}. \quad (33)$$

This corresponds to an energy scale

$$\Lambda \sim 2.0 \times 10^{14} \left(\frac{|\eta|}{0.01} \right)^{3/4} \text{ GeV}, \quad (34)$$

¹²The number of e -foldings depends on the reheat temperature. While we leave this matter for further study, it seems likely that the reheat temperature in this model will turn out to be low, since the Hubble scale is quite low, $H = 10^{10}$ GeV. This decreases N_e , but does not change the estimates below significantly.

¹³Precisely speaking, this N_e e -foldings suppression factor $1.8 \simeq \exp(N_e |\eta|)$ is dependent on η , but this η -dependence is very weak so we will neglect this dependence in the following discussion.

which is small compared with the SUSY GUT scale, 10^{16} GeV.

The power in gravity wave perturbations is given by,

$$\mathcal{P}_{grav} = \frac{1}{2\pi^2} \frac{H^2}{M_{pl}^2} \sim 7 \times 10^{-19} \left(\frac{|\eta|}{0.01} \right)^3. \quad (35)$$

It is clear that the production of gravity waves in this model is greatly suppressed, much below the level of detection in future experiments.

To summarize, this model gives rise to an example of small field inflation¹⁴. The inflaton varies by much less than the Planck scale during inflation. The slow roll parameter ϵ is extremely small and the tilt is determined by η which is negative. The Hubble scale during inflation is quite low, of order 10^{10} GeV, and the corresponding vacuum energy is of order 10^{14} GeV. As a result, the observed anisotropy arises almost entirely due to adiabatic density perturbations and the production of gravity waves is highly suppressed.

Let us end this section with some comments. The qualitative features of the inflationary scenario in this model mainly arise from the broad maximum, with $m^2/H^2 \ll 1$, and are quite insensitive to various details and approximations. For example, we assumed at various points in the discussion that the internal metric, eq. (1), is trivial, $g_{mn} = \delta_{mn}$. This is of course an approximation valid only locally, since Calabi-Yau spaces are not flat. For a non-trivial metric the Z_2 symmetric point will still be an extremum. It will always be a maximum of the attractive Coulomb potential eq. (11). In addition if it is a minimum for the terms, eq. (9), the basic idea will work. By adjusting the redshift factor, Z , as a function of the brane-anti-brane separation, as in eq. (17), one can arrange a near cancellation, leading to the condition $m^2/H^2 \ll 1$. It could be that the corrections to the metric causes inflation to end for a smaller value of ϕ than we estimated above. It follows from eq. (29), that the resulting Hubble scale of inflation will be lower than our estimate above, order 10^{10} GeV.

It is interesting to compare the inflationary parameters obtained above, with those obtained in the KKLMNT model, with the curvature induced mass term set to zero by hand, appendix C, [3]. In the latter case after setting $N_e = 60$, one finds that δ_H directly determines the energy scale during inflation, Λ , (C.12), [3]. Remarkably, the resulting value is the same as that obtained above, 10^{14} GeV. The tilt parameter is directly determined by N_e , and with $N_e = 60$, is in good agreement with the data, (C.19), [3]. In contrast in the model above, we saw that the energy scale during inflation and the tilt are relatively insensitive to N_e . Fixing the the value of m^2/H^2 ,

¹⁴This is an ‘‘A’’ type model in the classification of [26].

so that the tilt agrees with observations, we found that δ_H determines the energy scale during inflation.

2.5 Parameter Constraints

2.5.1. Meeting the Various Constraints:

There are four important parameters which characterize the compactification, r_1 - which governs the distance between the two K-S throats, Z , the redshift at the bottom of each K-S throat, L , the size of the compactification, and g_s the string coupling. Here we will examine the various constraints imposed on them. As we will see these turn out to be very stringent and will restrict some of the parameters to within an order of magnitude or so. Our estimates are uncertain by factors of order unity because of our lack of knowledge about how to define some of the scales precisely. Within these uncertainties, we will find that all the constraints can be met. This establishes, as a first pass, that this model is viable. However, given the stringent nature of the constraints one would like to do better. This requires an improved estimate of the numerical factors in the constraints and is not easy. For some of the constraints one will probably need explicit string theory constructions of the model. We leave this for the future.

The constraints on the parameters arise in three ways. First, the low-energy supergravity theory, within which our analysis has been carried out, must be valid. Second, the initial brane-anti-brane separation, r_1 , must satisfy some conditions so that the form of the potential we assumed in eq. (11) is valid and the resulting maximum is broad. Third, the observational constraints discussed in section 2.4 must be met. We take these up in turn now.

For the low-energy supergravity approximation to be valid, the α' and g_s , expansions must hold. These give rise to the conditions

$$L^6 \gg (\alpha')^3, \quad (36)$$

and

$$g_s \ll 1, \quad (37)$$

respectively. In addition, the scale of supersymmetry breaking must be small compared to the string scale. We take this condition to be

$$T_3 Z^4 \ll \frac{1}{(2\pi)^3 (\alpha')^2}. \quad (38)$$

The l.h.s. above arises because the scale of supersymmetry breaking is set by the anti-brane tension. The r.h.s. has been chosen as follows. We expect the supersymmetry breaking scale to be unacceptably large when $g_s = Z = 1$. The r.h.s. is the value of the D3-brane tension for this choice¹⁵ of g_s, Z . Eq. (39) can be then re-expressed as a condition on Z, g_s alone, and takes the form

$$Z^4 \ll g_s. \quad (39)$$

The initial brane-anti-brane separation r_1 must satisfy three conditions. First, it must be big enough so that no tachyon is present at the start of inflation. The mass of the tachyon m_T^2 as a function of the initial separation, r_1 , is given by

$$m_T^2 = \frac{r_1^2}{(2\pi\alpha')^2} - \frac{1}{2\alpha'}. \quad (40)$$

The requirement that no tachyon is present then takes the form,

$$\frac{r_1}{\sqrt{\alpha'}} \gg \sqrt{2\pi}. \quad (41)$$

Second, r_1 must be small enough compared to the size of the compactification, L , so that the corrections to the harmonic function due to compact nature of the internal space can be neglected. Third, the resulting maximum should be broad, this gives eq. (17). In the analysis below we will assume that eq. (17), is the more restrictive of the latter two constraints. In the particular examples we consider we will see this is true.

The observed anisotropy gives the condition, eq. (31). Also, the tilt, eq. (28), determines η and thereby m^2/H^2 by eq. (25). It is worth repeating here, that eq. (31) completely fixes the energy scale during inflation as eq. (34), or equivalently, the SUSY breaking scale up to a factor of order unity.

To summarize the four parameters, r_1, Z, L, g_s , mentioned above, must meet the constraints, eq. (36), eq. (37), eq. (39), eq. (41), eq. (17), and, eq. (31).

We are now ready to analyze these constraints in more detail.

2.5.2. Analysis of the Constraints:

We can view, eq. (31) as determining Z in terms of L , and then eq. (17) as determining r_1 in terms of L . This leaves L and g_s undetermined. They

¹⁵The numerical factor, $\frac{1}{(2\pi)^3}$, is less than unity and therefore makes eq. (38) more stringent.

must satisfy the remaining four constraints, eq. (36), eq. (37), eq. (39), and eq. (41).

Since

$$M_{Pl}^2 = M_{10}^8 L^6, \quad (42)$$

eq. (31) takes the form

$$Z^4 \simeq \frac{3^{9/2}}{(1.8)^3} \times 10^{-12} (M_{10} L)^{12} |\eta|^3. \quad (43)$$

Eq. (39) then imposes the condition,

$$(LM_{10})^{12} \ll \frac{(1.8)^3}{3^{9/2}} \times 10^{12} \frac{g_s}{|\eta|^3}. \quad (44)$$

Using the relation

$$\frac{1}{M_{10}^8} = \frac{1}{2} (2\pi)^7 (\alpha')^4 g_s^2, \quad (45)$$

eq. (36) takes the form,

$$(LM_{10})^{12} \gg \frac{1}{(2^6 \pi^7)^{3/2}} \frac{1}{g_s^3}. \quad (46)$$

Similarly, using eq. (45) and eq. (17), eq. (41) gives

$$(LM_{10})^{12} \gg \frac{(1.8)^2 \sqrt{\pi}}{3^3 \cdot 2} \frac{10^8}{|\eta|^2 g_s}. \quad (47)$$

It is easy to see that for reasonable values of g_s , eq. (47) is more restrictive than eq. (46).

So we see that L , the size of the compactification, and g_s , the string coupling, must satisfy the conditions, eq. (44), eq. (47), and eq. (37).

We can express eq. (44) and eq. (47) together as,

$$1 \times 10^{12} \left(\frac{g_s}{0.1}\right)^{-1} \left(\frac{|\eta|}{0.01}\right)^{-2} \ll (M_{10} L)^{12} \ll 4 \times 10^{15} \left(\frac{g_s}{0.1}\right) \left(\frac{|\eta|}{0.01}\right)^{-3}. \quad (48)$$

Since the upper and lower bounds in the above inequality are somewhat far apart, for reasonable values of g_s and η , we see that the required constraints on L can be met.

To summarize, we saw in the analysis above that r_1 and Z can be expressed in terms of L and $|\eta|$, using the relations eq. (17) and eq. (31). The remaining two parameters, g_s and L must then satisfy the condition eq. (37) and eq. (48). We saw above that these can be met.

2.5.3. Explicit Examples with Conclusions:

It is worth examining some explicit examples which meets all the constraints in more detail.

Scenario I

We take $(M_{10}L)^{12} = 10^{13}$, with $g_s = 0.1$ and $|\eta| = 0.01$. Note that $(M_{10}L)^{12} = 10^{13}$ lies between the two bounds in eq. (48). Using, eq. (42), we now obtain, $M_{10} = 1.4 \times 10^{15}$ GeV, $1/L = 1.1 \times 10^{14}$ GeV. The redshift factor is then given by eq. (43). For $|\eta| = 0.01$ we get, $Z^4 = 2.4 \times 10^{-4}$, $Z = 0.12$. The brane-anti-brane separation, r_1 given by eq. (17), to be $1/r_1 = 4.5 \times 10^{14}$ GeV. Using eq. (45), with $\alpha' = l_s^2$, we get the string scale as $1/l_s = 3.5 \times 10^{15}$ GeV. Finally, as we mentioned above, the SUSY breaking scale, $(T_3 Z^4)^{1/4}$, is of order the vacuum energy during inflation, 2.0×10^{14} GeV. These different energy scales are summarized as follows:

	physical quantities	Scale (GeV)
Table I	M_{pl}	2.4×10^{18}
	$1/l_s$	3.5×10^{15}
	M_{10}	1.4×10^{15}
	$1/r_1$	4.5×10^{14}
	$\Lambda = (T_3 Z^4)^{1/4}$	2.0×10^{14}
	$1/L$	1.1×10^{14}
	H	0.9×10^{10}

warped factor: $Z = 0.12$

$$g_s = 0.1, |\eta| = 0.01$$

Scenario II

To get an idea of how these parameters change, we also consider the another scenario where $(M_{10}L)^{12} = 10^{14}$ and $g_s = 0.1$. For $|\eta| = 0.01$, the resulting values for the parameters are summarized in Table II. One sees that qualitatively the energy scales look similar, although some of them become a little bit smaller.

	physical quantities	Scale (GeV)
Table II	M_{pl}	2.4×10^{18}
	$1/l_s$	2.0×10^{15}
	M_{10}	7.7×10^{14}
	$1/r_1$	1.4×10^{14}
	$\Lambda = (T_3 Z^4)^{1/4}$	2.0×10^{14}
	$1/L$	5.2×10^{13}
	H	0.9×10^{10}

warped factor: $Z = 0.22$

$$g_s = 0.1, |\eta| = 0.01$$

Finally we note that some limited amount of variation is allowed for g_s . In general reducing g_s tightens the constraints in eq. (48), increasing the lower bound and decreasing the upper bound. E.g., taking $g_s = 0.01$, the lower bound in eq. (48) becomes 1×10^{13} , and the upper bound be 4×10^{14} .

From Table I and II, we see that the parameters are quite tightly restricted by the constraints. While all the inequalities we discussed above are indeed met, several relevant scales are close together. Thus, our analysis does not conclusively establish that the required conditions have been met. For example in Table II, $1/r_1 = 1.4 \times 10^{14}$ GeV, while $1/l_s = 2.0 \times 10^{15}$ GeV, so that $r_1/(\sqrt{2}\pi\sqrt{\alpha'}) = 3.1 > 1$, as is needed for the absence of a tachyon. However, since our estimate of r_1 in eq. (17) is uncertain by a factor of order unity, and the above ratio is not much bigger than unity, a more careful estimate is needed to conclusively establish this point.

As we mentioned at the beginning of this subsection, a more careful study is therefore needed to establish that the constraints are satisfied. This would include a better understanding of the requirements for the low-energy supergravity approximation to be valid. It would also need to be done in the context of concrete constructions in string theory. In such constructions one can hope to calculate the numerical factors in eq. (17) and eq. (31), and also understand whether the brane-anti-brane potential is well approximated by eq. (11). Finally, a better observational determination of the tilt parameter, and therefore of η , will also help.

Let us end with some comments. From Table II we see that $r_1/L \simeq 1/(2.7)$. Corrections to the potential, eq. (11), due to compact nature of the internal space can be estimated roughly as arising due to “images”. These contributions are small and do not affect eq. (16) since the harmonic function falls like $1/r^4$ in six dimensions.

We see from Table I and II that $1/L \gg H$, so that the internal space has a size much bigger than the Hubble scale. This is consistent with our four-dimensional description of inflation.

It is worth pointing out that the string scale in our model is close to the SUSY GUT scale but slightly lower. This connection is interesting to explore further. It will be interesting if successful inflation necessarily requires a string tension of order the SUSY GUT scale.

We have assumed above that the parameters, Z, r_1, L, g_s , can in effect be tuned independently to meet all the constraints. This needs to be checked in explicit models. Here we only note that given the large number of fluxes which we can turn on and dial, this seems plausible.

The important observational constraint in the analysis above comes from the scale of density perturbations eq. (31). Requiring a small scale of density perturbations make the r.h.s. of eq. (31) smaller, and the upper bound in eq. (48) bigger. As a result the constraint eq. (48) becomes less severe¹⁶. It therefore seems likely that even after a more detailed analysis along the lines mentioned above, our model will remain viable as an example of slow-roll inflation in string theory. By varying the different fluxes etc one can probably implement the model in many different ways in string theory, giving rise to inflationary scenarios with different values for the density perturbations, tilt etc. Hopefully, some of these will agree with the data, although the vast majority will probably not.

3 Discussion

1. Putting the Proposal on Firmer Footing:

Some important issues need to be studied further to put this model on a firmer footing. Two of these are common to many KKLT type constructions. First, non-perturbative effects, like gaugino condensation, responsible for volume stabilization need to be understood better. Second, in deriving the full potential we assumed that the brane-anti-brane potential can be simply added to the contribution coming from the superpotential. This would be true if the former arises through a D -term. For evidence in support of this see [42, 43]. Finally, as mentioned above, explicit examples of Calabi-Yau

¹⁶One way to make it easier to get the required density perturbations would be to end inflation earlier. Since $\delta_H \sim H^3/(m^2\phi)$, smaller ϕ gives bigger δ_H . This can be done if the moving D3-brane encounters a D7-brane or some other feature which obstructs its motion before it gets close to the anti-D3-brane.

spaces need to be constructed, where all the constraints discussed above can be met simultaneously. For all these reasons the construction discussed in this paper should be thought of as a proposal rather than a concrete model at the moment.

For recent progress towards meeting some of the requirements of the KKLT construction see, [14].

2. Building a More Complete Model:

As a model of inflation our construction is incomplete in three important ways. We have not addressed how inflation ends, and how it begins. And we have not asked if the standard model can be satisfactorily incorporated in this construction. Let us briefly comment on some of these issues here.

2a) Ending Inflation: This model is in fact an example of hybrid inflation. When the brane-anti-brane separation gets to be of order the string scale, a tachyon develops. What happens next is currently a matter of active study. See for example [29, 30, 31, 32, 33, 34]. It seems reasonable to believe that the energy in the brane-anti-brane pair is eventually converted to light closed string modes like the graviton. Whether this energy can be efficiently transferred to the standard model degrees of freedom, reheating the universe satisfactorily, is dependent on how the standard model couples to the degrees of freedom in the inflationary throat. This is tied to another incomplete aspect of the model mentioned above, namely how the standard model is incorporated in it, and needs to be studied further.

Cosmic strings, both of D and F type will be produced in this model at the end of inflation due to D3 anti-D3-brane annihilation [35], [36]. If these strings are metastable [37], their tension must meet the condition, $G_N T_1 \leq 10^{-7}$, to avoid generating unacceptably large anisotropies. A D1-brane at the bottom of the K-S throat has tension, $T_{D1} = \frac{1}{2\pi g_s \alpha'} Z^2$, where the Z dependence arises due to the redshift. The tension of the anti-D3 brane is given by $T_3 Z^4$ and is of order the vacuum energy during inflation $(2 \times 10^{14} \text{ GeV})^4$. Using the relation $T_{D1} = \sqrt{2\pi T_3 Z^4 / g_s}$, $T_{F1} = \sqrt{2\pi T_3 Z^4 g_s}$, we get $G_N \sqrt{T_{D1} T_{F1}} \simeq 6.5 \times 10^{-10}$. This is lower than the bound mentioned above. Interestingly, future observations might be sensitive to such low values of the tension, [37], [38].

A small positive cosmological constant is needed after the end of inflation to account for the acceleration of the present epoch. This could arise as follows. The brane is drawn towards one of the throats and annihilates the anti-brane at the bottom of it. But the anti-brane in the second throat survives after inflation ends. The final resulting cosmological constant will get

a contribution from this anti-brane's tension and the superpotential terms, eq. (6). These can nearly cancel leaving behind a small positive value.

2b) The Initial Conditions and Eternal Inflation

The scale of inflation in this model, $\Lambda \sim 10^{14}$ GeV, is considerably smaller than the ten-dimensional Planck scale, and makes the question of initial conditions all the more important.

We can only offer a few speculations about this here. It is possible that the universe did not begin in a big bang, with temperatures of order the Hagedorn temperature. Instead as has been suggested in [39, 40], it might have begun by a tunneling event in the vicinity of the maximum of the potential.

It could also be that the question of initial conditions is not very significant in this model. It has been argued that a potential with a broad maximum, of the kind we have here, will give rise to eternal inflation. If the inflaton is close enough to the top of the potential of eq. (19) with $\phi < \phi_c \sim H^3/m^2$, quantum fluctuations can drive it up the hill at a faster rate than the classical gradient term allows it to descend. Since the regions where the cosmological constant is bigger also grow exponentially more rapidly, soon the universe will be dominated by regions where the inflaton is at the top of the hill, making the initial conditions irrelevant. The observed universe in this picture would arise when fluctuations cause the inflaton to descend far enough from the top so that the classical evolution discussed in section 2.4, becomes valid. This is an attractive and fairly plausible picture, but it is somewhat speculative at the moment and needs to be understood better.

3. Generalizations of the Model:

3a) Asymmetric Throats:

As was mentioned towards the end of section 2.4, the inflationary scenario implemented in this note is mainly dependent on the small curvature at the maximum and independent of many details of the model. For example, we have considered the Z_2 symmetric model in our discussion above, but the Z_2 symmetry is not essential. In general, the curvature couplings discussed in section 2.2 will result in a potential for the inflaton. Suppose this potential has a minimum at some point in the compactification. Two K-S-like throats containing anti-branes at the bottom, not necessarily symmetrically located, whose distance from the minimum is adjusted relative to the red-shift factors and the number of anti-branes at the bottom of the throats, can then nearly cancel the resulting mass giving rise to a maximum with a small curvature. The resulting inflationary scenario will then be qualitatively unchanged, in

particular the scale of inflation will continue to be quite low.

3b) More Symmetry:

Conversely we can have situations with higher symmetry, with more than two throats symmetrically located with respect to a point of enhanced symmetry. The term $(\frac{6r_{1m}r_{1n}}{r_1^2} - \delta_{mn})$ in eq. (12), would then be replaced by the quadrupole moment of the resulting configuration, Q_{mn} . As long as Q_{mn} does not vanish, it will have some positive eigenvalues and a similar discussion can go through. We note here that higher symmetries, e.g., a Z_4 symmetry, could lead to positive eigenvalues of Q_{mn} along more than one direction. As a result additional directions would be unstable in the potential and during inflation the brane can roll along these additional directions as well.

4) The Landscape and Inflation:

4a) Broad Maxima:

It seems quite likely that maximum of the kind found in this paper, which are broad with a small curvature, are more generally present in the string theory landscape [41]. The unstable directions could be complex structure moduli or Kähler moduli as well. This possibility can be examined further as our understanding of moduli stabilization progresses. For complex structure moduli in the KKLT type constructions, one needs a better understanding of the non-perturbative corrections that are involved¹⁷. As far as Kähler moduli are concerned, it seems difficult to get the required small mass with only one Kähler modulus, since the canonically normalized field is logarithm of the volume. But this might be possible with more than one Kähler moduli, again this requires further developments in our understanding of moduli stabilization.

Generically broad maxima in the landscape, which agree with observational constraints, will have a low scale of inflation¹⁸ $H \lesssim 10^{12}\text{GeV}$. If

¹⁷In particular these contributions go like $W_{NP} = A(z_i)e^{-a\rho}$. The prefactor, $A(z_i)$, depends on the complex structure moduli, and one needs to know this dependence.

¹⁸To see this, let us write the potential as follows,

$$V = \Lambda^4 f(\phi/M_{Pl}) = \Lambda^4 - m^2\phi^2 + \mathcal{O}(1)\frac{\Lambda^4}{M_{Pl}^4}\phi^4 + \dots, \quad (49)$$

where the ellipses denote higher powers of (ϕ/M_{Pl}) . Typically the slow roll conditions stop holding when the quartic terms come into play, i.e., when $\phi \sim M_{Pl}\frac{m}{H}$. Imposing $\eta = m^2/3H^2 \sim 10^{-2}$, and the observed value of the density perturbations, eq. (27), then leads to $H \sim 10^{12}$ GeV. The resulting power in gravity waves is $\mathcal{P}_{grav} \sim 10^{-14}$, which is quite small even for future observations. In specific models, like the one considered in this paper, inflation can end earlier and the resulting Hubble scale can be lower.

inflation arises due to such a maximum in the landscape, gravity waves are highly suppressed and will be difficult to observe in future experiments.

4b) The Landscape and Various Inflationary Scenarios:

So far in our discussion we have used the observational data and deduced various constraints on the compactification. Once explicit constructions become possible, we can turn this around and ask instead whether string theory makes any predictions about the observational data. While it is premature to speculate on this of course, it seems likely that in general the landscape will have many broad maxima of various different types, and the resulting values for the Hubble scale, H and the mass, m^2/H^2 , will take various different values, resulting in many different possibilities for the scale of density perturbations and the tilt. Most of these will probably not agree with observation.

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