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De Sitter universes and the emerging landscape in string theory

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Abstract. We discuss a recent proposal to construct de Sitter vacua in string theory. It is based on flux compactifications in string theory where all the moduli are stabilised and supersymmetry is broken with control. The resulting picture is that of a complicated landscape with many vacua of widely varying values for the cosmological constant.

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1. Introduction

There is increasing observational evidence that we live in an accelerating universe today [1]. This is a remarkable fact of great significance both for cosmology and for any attempt to understand physics at the smallest distance scales.

A likely explanation for the acceleration is provided by a small positive cosmological constant. It is well-known that in Einstein's theory of general relativity, gravity coupled to a positive cosmological constant gives rise to an exponentially expanding solution, called the de Sitter universe.

In this article we will discuss a proposal for how such de Sitter universes can arise in string theory. The discussion below is based on recent work in flux compactifications in string theory, and in particular on a paper written in collaboration with Kachru, Kallosh and Linde (KKLT) [2].

2. More motivation

There is a good reason, internal to string theory itself, for being interested in the question of de Sitter universes as well. There has been considerable progress in our understanding of string theory over the past few decades. Most of this has focussed on understanding the theory in situations with unbroken supersymmetry and in fact with unbroken $\mathcal{N} \geq 2$ supersymmetry. It is well-known that de Sitter space can only arise if supersymmetry is broken. Thus thinking of this question brings us to the frontiers of our understanding of string theory today.

Some attempts to construct de Sitter vacua have been made in the past. These have not been successful and have lead to various no-go theorems [3]. We will see how these can be circumvented.

Breaking supersymmetry in string theory requires us to come face-to-face with an important issue called the problem of 'moduli stabilisation'. In a string theory vacuum, with $\mathcal{N} \geq 2$ supersymmetry, there are many flat directions, or moduli. The energy as we go along these directions in field space is a constant and in fact vanishes identically. There are $\mathcal{O}(100)$ flat directions in a typical compactification.

These flat directions are bad news from the point of view of both phenomenology and cosmology. Phenomenologically, physical constants like G_{N}, α_{em}, vary along these directions. One would like to be able to compute the values of these constants in a string vacuum and compare them with the observed values. But for this one needs to be able to lift the flat directions and understand where the resulting mimima lie. Cosmologically, flat directions give rise to light and weakly coupled scalars. These cause problems in the standard model of cosmology, e.g., they ruin the successful predictions of Big Bang nucleosynthesis.

For vacua with $\mathcal{N} \leq 1$ supersymmetry, which is the case of phenomenological relevance, one expects the flat directions to be typically lifted. This is good news. Unfortunately, given our understanding of string theory today, we have a limited understanding of the resulting potential that develops. In regions of field space where the potential can be calculated with control, one finds no minima. This inability to find minima in which the flat directions are lifted is called the moduli stabilisation problem.

Recently, a new class of string compactifications, called flux compactifications, have gained prominence [4]. In these compactifications, besides curling up the extra directions present in string theory to small size, fluxes are also turned on along the compactified directions. The fluxes include higher form generalisations of magnetic flux in electromagnetism. Turning them on changes the potential in moduli space so that now minima arise in regions of field space where the potential can be calculated with control. The value of the cosmological constant in these minima can also be calculated, and those with a positive value give rise to de Sitter universes.

In the rest of this article I will describe a specific proposal to stabilise all minima, break supersymmetry, and obtain de Sitter vacua. Our strategy will be to try and proceed in a controlled manner. As a first step, we will describe how all moduli can be lifted while preserving $\mathcal{N} = 1$ supersymmetry. This will give rise to vacua with negative cosmological constant, corresponding to anti-de Sitter space. The second step will be to break supersymmetry with control. This will result in vacua with positive cosmological constant.

3. The proposal

We will work with IIB string theory (more generally F-theory). The starting point is a six-dimensional Calabi–Yau orientifold compactification. Calabi–Yau manifolds are a well-known class of string theory compactifications. The orientifold is obtained after identifying points related by a Z_2 discrete symmetry in the manifold. The moduli in this compactifcation arise due to various size and shape deformations. There is also the dilaton, $e^{-\phi}$, whose expectation value is the string coupling and its axion partner, a. Together, we will denote them as $\tau = a + ie^{-\phi}$.

IIB string theory has three-form RR, NS fluxes and the five-form RR flux respectively. We will denote these as, F_3, H_3, F_5, respectively. These will all be turned on in the compactification. In addition D7 and D3 branes can also be present. An important point to bear in mind is that for a typical Calabi–Yau space many different choices of flux are possible. A number of order 10^{100} is typical. Several important features about flux compactifications that we will discuss later are tied to this vast number of choices.

It is well-known that turning on flux gives rise to a superpotential at tree-level that depends on the shape moduli and the dilaton–axion. This takes the form [5,6],

$$W_{\text{tree}} = \int G_3 \wedge \Omega,$$

where $G_3 = F_3 - \tau H_3$, and Ω is the holomorphic three-form in the Calabi–Yau space. The dependence on the dilaton–axion is explicit in this superpotential. The dependence on the shape moduli enters through Ω. The requirements of supersymmetry typically lift all these moduli.

Corrections to the superpotential can arise at the non-perturbative level. These were explored in [7] and related works. There are two ways in which these could happen: First, due to Euclidean D3 branes wrapping 4-cycles and second, due to gaugino condensation or more generally strong coupling dynamics in the world volume gauge theory of coincident D7 branes. Both corrections take the form

$$W_{\text{NP}} = A e^{ia\rho_i}\,.$$

Here ρ_i (more correctly its imaginary part) is a size modulus, the prefactor A depends in general on the shape moduli, and a is determined by the non-perturbative effect which gives rise to the correction. These non-perturbative effects could stabilise all the size moduli.

Thus all the shape and size moduli, related axion fields, and the dilaton–axion could be lifted in the presence of flux.

3.1 *An example*

It is worth examining a toy model in more detail. We consider the case where there is only one size modulus, which we denote as ρ. The shape moduli, we saw above, get lifted due to tree-level effects. Therefore for a large volume these will be heavier than the size modulus. After integrating them out one gets an effective theory involving only ρ. The superpotential in this theory takes the form:

$$W = W_0 + A e^{ia\rho}\,.$$

W_0 is a constant which arises from the tree-level superpotential, W_{tree}, and the second term arises from W_{NP}. The Kahler potential for ρ if of the no-scale type,

$$K = -3\ \log(-i(\rho - \bar{\rho}))\,.$$

The potential can be calculated from the superpotential and the Kahler potential using standard supergravity formulae.

We simplify things by setting the axion in the ρ modulus to zero, and letting $\rho = i\sigma$. To simplify things further, we assume that A, a, and W_0 are all *real*. It is easy to see that this potential does have a supersymmetric critical point at negative W_0.

$$DW = 0 \quad \rightarrow \quad W_0 = -A\mathrm{e}^{-a\sigma_{\mathrm{cr}}}(1 + 2/3a\sigma_{\mathrm{cr}})\,.$$

The potential at the minimum is negative and equal to

$$V_{\mathrm{AdS}} = (-3\mathrm{e}^K W^2)_{\mathrm{AdS}} = -\frac{a^2 A^2 \mathrm{e}^{-2a\alpha_{\mathrm{cr}}}}{6\alpha_{\mathrm{cr}}}\,,$$

which shows that we have a supersymmetric AdS minimum in this case. It is important that the AdS minimum is quite generic. As an example we can take, $a = 1; W_0 = -1; A = 20$ for which $\sigma_{\mathrm{cr}} \sim 113$, and the value of the potential (in Planck units) at the AdS minimum is $\sim -2.5 * 10^{-3}$.

Thus we see concretely in this example that all the moduli can indeed be stabilised while preserving supersymmetry.

We are working within the framework of low-energy supergravity above. There are two sources of corrections to the potential we have used. First there are corrections due to the α' expansion. These will be small as long as the volume (or σ_{cr}) is large. We see from the discussion above that small W_0 will give rise to large σ_{cr}. The second source of corrections is due to the g_s expansion. These corrections are small if g_s is small, and this can be arranged by suitably choosing the RR and NS three-form flux. The requirement of small W_0 does impose a restriction on the choice of flux. However, since there are many possible values of flux to begin with $\sim O(10^{100})$ as mentioned above, this should still leave many vacua with the volume stabilised at a large value.

4. Supersymmetry breaking and de Sitter vacua

We now turn to breaking supersymmetry. This will be done by introducing an anti-D3 brane in the compactification. One additional feature of flux compactifications will be important in controlling the resulting potential and preventing the AdS minimum discussed above from destabilising completely. This is the fact that flux gives rise to warping. This feature is familiar from the study of the $\mathrm{AdS}_5 \times S^5$ solution in string theory where the warping gives rise to an ‘infinite’ throat. Here we will be interested in situations where the resulting throat is finite and terminates at a minimum value of the red-shift, which we will denote by Z. A good example of this is provided by the Klebanov–Strassler solution [8]. One considers a Calabi–Yau space close to a conifold point. The Calabi–Yau space has a small non-vanishing S^3 which is threaded by the RR three-form flux. In addition NS three-form flux and F_5 flux are also excited. The resulting warping is significant in the vicinity of the small S^3 and gives rise to a finite throat.

By introducing an anti-D3 brane at the bottom of this throat there is an additional contribution to the potential energy given by

$$V_{\rm antibrane} = \frac{2T_3 Z^4}{\sigma^2}\,.$$

Adding this to the potential obtained in the previous section one finds that for a suitable range of values for the AdS minimum, the minimum is uplifted to a minimum with positive cosmological constant.

By changing the value of Z, and the other parameters, W_0, a, A, the resulting cosmological constant can take different values, of both positive and negative signs.

A few comments are worth making about the de Sitter vacua. The effects of flux and also supersymmetry breaking, both go to zero as the volume tends to infinity. This means there is a supersymmetry preserving vacuum at infinite volume. As a result the de Sitter vacuum is only metastable. It can decay in two ways, either due to a Coleman–Delucia instanton which can be thought of as tunelling under the barrier, or due to a Hawking–Moss instanton which can be thought of as going over the barrier due to the finite temperature of de Sitter space [9]. Both effects give rise to a rate which is bigger than the Poincaré recurrence rate associated with the de Sitter space due to its entropy [10]. One expects that at least in some fraction of the vacua the transition rate of the de Sitter vacuum can be made much smaller than the age of the universe, so that the metastability has no observable consequences.

5. The landscape and conclusions

The fact that string theory allows for de Sitter universes is good news.

As was emphasised above though, there is a huge number of choices for flux that are allowed. Even after imposing the restrictions of large volume and small string coupling one therefore expects many many vacua with widely varying values of the cosmological constant. Other constants of nature would also take varying values in these vacua.

The picture which emerges is that of a complicated landscape in string theory [11], with about $\sim$100 directions and between 10^{100} and 10^{1000} vacua. This embarrassment of riches is bad news from the point of view of predicting the standard model from string theory and raises questions which are currently eliciting much discussion.

What chooses the vacuum we live in? Are the other possibilities realised, either in different parts of this universe as in the eternal inflation scenario, or in different branches of the wave function describing this universe, or not at all? Do we have to give up on Einstein's dream of understanding all the constants of nature based on a few fundamental principles? Can string theory at least predict some of the important constants of nature? And so on.

It is the author's belief that whereas discussing these questions is worthwhile, any conclusions at this stage are at best speculative. The study of supersymmetry breaking and cosmology in string theory is at its earlier stages. One can be sure that there will be many surprises as our understanding progresses. Hopefully, string theory will provide answers to some of the questions mentioned above. But the answers and even the precise nature of the questions will be understood only within the context of the developments to come.

Let us be patient. Time will tell.

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