arXiv:1002.4498v3 [hep-th] 24 May 2010

On The Stability Of Non-Supersymmetric AdS Vacua

Prithvi Narayan, and Sandip P. Trivedi,

Tata Institute for Fundamental Research
Mumbai 400005, India

Email: prithvi.narayan@gmail.com, trivedi.sp@gmail.com

ABSTRACT: We consider two infinite families of Non-Supersymmetric AdS_4 vacua, called Type 2) and Type 3) vacua, that arise in massive IIA supergravity with flux. We show that both families are perturbatively stable. We then examine non-perturbative decays of these vacua to other supersymmetric and non-supersymmetric AdS_4 vacua mediated by instantons in the thin wall approximation. We find that many decays are ruled out since the tension of the interpolating domain wall is too big compared to the energy difference in AdS units. In fact, within our approximations no decays of Type 2) vacua are allowed, although some decays are only marginally forbidden. This can be understood in terms of a "pairing symmetry" in the landscape which relate Type 2) vacua with supersymmetric ones of the same energy.

Contents

---

1. Introduction

String Theory has a rich and complicated landscape of vacua. Non-supersymmetric anti-deSitter (AdS) vacua are an interesting class amongst these. Lacking supersymmetry, they are not as well understood as supersymmetric spacetimes. But being time independent, they should be easier to understand than deSitter spacetimes.

In this paper we construct a class of non-supersymmetric AdS vacua which are perturbatively stable and investigate their non-perturbative stability.

Our construction is based on massive IIA theory compactified on a particular Calabi-Yau manifold. After suitably orientifolding and adding fluxes one obtains vacua where all moduli are stabilized and supersymmetry is broken with a negative cosmological constant. The Calabi-Yau manifold is obtained by blowing-up a $T^6/(Z_3 \times Z_3)$ orbifold. This model was studied in considerable depth by [1]. Here we carefully include the effects of the blow-up modes and related fluxes and analyze the stability of the resulting non-susy vacua in detail.

We find two classes of non-susy vacua which are perturbatively stable. These are called Type 2) and Type 3) vacua in our terminology (Type 1 vacua are supersymmetric). Both classes contain an infinite number of vacua.

Next we turn to a study of the non-perturbative stability of these vacua. In fact, such an investigation was one of the main motivations for this paper. At first sight one might expect that a non-supersymmetric AdS vacua in the landscape would always have some non-zero rate to decay to other vacua with lower vacuum energy. Such a decay rate, if it is small enough, would not have very drastic consequences for an observer in AdS space.

However, from the point of view of a dual CFT the small decay rate in the bulk leads to a divergence and has dramatic repercussions. For, consider AdS_{d+1} in Poincare coordinates:

$$ds^2 = (\frac{r}{R})^2(-dt^2 + \sum_{i=1,\cdots,d-1} (dx^i)^2) + (\frac{R}{r})^2 dr^2. \qquad (1.1)$$

Let the decay rate per unit volume in the bulk be Γ. The corresponding decay rate per unit volume in the boundary is obtained by integrating the bulk decay rate in

the radial direction. Taking the boundary metric to be flat, the decay rate per unit volume in the boundary theory is,

$$\Gamma_{boundary} = \int \sqrt{g} dr \Gamma \sim r_{boundary}^{d+1} \Gamma, \qquad (1.2)$$

where $r_{boundary}$ is the radial location of the boundary. As $r_{boundary}$ is taken to infinity we see that this diverges.

Thus an arbitrarily small decay rate in the bulk leads to an infinitely fast decay in the boundary. The putative CFT dual meets with an instantaneous end and cannot exist. This consequence of a bulk decay was noted in [2] where the decay rate in a non-susy AdS spacetime, obtained by taking an orbifold of $AdS_5 \times S^5$, was discussed in some detail [1] [2].

We see then that our expectation that the non-susy AdS vacua are unstable non-perturbatively suggests that non-supersymmetric CFT's which admit a gravity dual are unlikely to exist [3]. If true, this is an important consequence since holography has emerged as a major tool with which to study strongly coupled conformal field theories.

The two large families of perturbatively stable non-susy vacua mentioned above provide us with a laboratory in which we can investigate this issue of non-perturbative stability. Somewhat surprisingly, our analysis reveals that in a large number of cases the decays to vacua with lower energy are in fact ruled out.

The essential reason for this is the geometry of AdS space. The dynamics of a decay is governed by competition between the volume gain in bulk energy and the surface cost due to the tension of the interpolating domain wall. For a system in flat space, say water and steam[4], the volume grows more rapidly than the surface area, so eventually the volume gain always wins out and an instanton always exists which allows a transition to the more stable phase. However in AdS space the volume and area grow at the same rate. This means in AdS space a decay can only happen if the tension of the interpolating domain wall is small enough compared to the difference in energies between the two vacua. Explicit calculations show that in several cases the tension turns out to be much too large thereby forbidding the decay.

---

[1]Another way to state this result is that the CFT has no scale in it so the only answer for a decay rate could be either zero or infinite. Non-zero Γ leads to an infinite result.

[2]In the discussion leading upto eq.(1.2) above we have kept the boundary volume fixed and not scaled it with the cut-off $r_{boundary}$. This is the standard convention in studying AdS/CFT, e.g., in global coordinates the boundary S^3 is taken to be of fixed radius. However, for purposes of studying the decay in the boundary theory, it might be more transparent to scale the volume with $r_{boundary}$. The discussion above then suggests that the boundary theory also has a finite decay rate per unit volume, and the divergence in $\Gamma_{boundary}$ is really a consequence of summing over the volume in the boundary theory which diverges as $r_{boundary} \to \infty$. We thank A. Sen for these comments.

[3]Of course non-supersymmetric CFT's clearly do exist. The worry here is about CFT's which admit a gravity dual.

[4]In this case it is the difference in Free energy rather than energy that is relevant.

In fact it turns out that working within our approximations, Type 2) vacua are stable and can at best decay marginally. Type 3) vacua, in contrast, do indeed decay to some other Type 3) and Type 2) vacua. We find no decays of the non-susy vacua to susy ones are allowed.

It is important to go beyond our approximations especially for decays which are marginal at leading order. Subleading corrections should tip the marginal cases one way or another and this would determine the stability of the Type 2) vacua. We leave such an investigation for the future.

Among our approximations one of the more significant ones is the thin wall approximation, as described in the classic paper by Coleman and Deluccia [3]. The domain walls involved cause a jump in flux and thus carry D-brane charges. At first sight one would expect them to be just D-branes and therefore well described in the thin wall approximation. However the change in flux also causes the moduli to vary and these typically have a mass of order the AdS scale. As a result the domain walls are no longer thin [5]. To work within the thin wall approximation then we must restrict ourselves to cases where the moduli variation contributes little, compared to the D-brane contribution, to the tension of the domain wall. This allows the domain wall to be approximated as being just a D-brane, which is indeed thin. Another limitation comes from not having explored the full set of non-susy vacua [6]. There could be other non-susy vacua which we have not constructed which are allowed end points of decays.

Despite these limitations in our analysis, we find it significant that a large number of possible decays are in fact ruled out. This indicates that perhaps stable non-susy AdS vacua and associated CFT's might exist after all.

We should also mention that the stability of Type 2) vacua can be understood in terms of a "pairing symmetry" in the landscape. By reversing the sign of the four-form flux in these vacua one obtains susy vacua with the same vacuum energy. The stability of the Type 2) vacua then follows from the stability of their partner susy vacua. While we have established the argument which relates the stability of the Type 2) vacua to that of their susy partners only within our approximations it could have a greater range of validity.

The basic strategy we employ in studying the non-perturbative stability is as follows. The tension of the interpolating domain wall satisfies a lower bound in terms of the jump in the superpotential caused by the domain wall. By comparing this lower bound against an upper bound which must be met for the decay to be allowed in AdS space, several decays can be ruled out.

---

[5]Fat would not be PC.

[6]In particular we have only investigated choices of flux for which the Calabi-Yau manifold is a slightly blown-up version of the $T^6/(Z_3 \times Z_3)$ orbifold. General non-susy vacua, where the moduli could be stabilized far away from the orbifold point, have not been analyzed.

Such a bound on the tension of domain walls is familiar in supersymmetric settings. It might seem puzzling at first that it arises in our study of non-susy vacua. The essential reason is that even in the non-supersymmetric case the compactification can be well approximated to be a Calabi-Yau space. The ratio of the size of the internal space to the AdS radius goes to zero for large flux in these compactifications making them of non Freund-Rubin type. This means that the main effect of the fluxes in these cases is to stabilize the moduli while the shifts of the Kaluza Klein modes is small. The tension of the domain wall, which is fixed to good approximation by the geometry of the Calabi-Yau space, is then determined by supersymmetric data and can be bounded by the jump in the superpotential. We expect that a similar strategy should be useful in other flux compactifications as well, where the internal compactification can continue to be well approximated as a supersymmetric one. It might also be useful in going beyond the thin wall approximation.

Let us end this introduction by discussing some related literature. The idea of reversing the sign of fluxes to obtain non-susy vacua from susy ones, which we have used for Type 2) cases, is an old one in the sugra literature. See [4] and references therein for a discussion of "skew whiffing", and also in the context of black holes, [5], [6]. For other recent constructions of non-susy AdS_4 vacua see [7], [8]. A general review for flux compactifications is [9], which contains several references. Early papers on IIA compactifications include [10] which developed the 4D framework, and [11], [12], which discuss Moduli stabilization. Also see, [13], [14], [17], [18], [19]. A recent discussion on non-perturbative decays, especially decays of Minkowski nearly susy vacua to AdS space and related topics, is in [15]. Aspects of metastability in AdS space are discussed in [16].

This paper is organized as follows. In section II we review the Model discussed in [1] and construct the non-susy vacua. In §3 we briefly review the discussion of vacuum decay in [3] with particular emphasis on AdS spacetimes. In §4 we turn to non-perturbative decays and analyze decays mediated by $D4$-branes. This is a long section, subsections §4.1-§4.3 contain some of the main points. More general decays are briefly discussed in §5. Appendices A-D contain important supplementary details.

2. The Model

We will consider a simple compactification of massive Type IIA theory [13] on a slightly blown up $T^6/(Z_3 \times Z_3)$ orbifold [20]. By turning on flux one can get non-susy AdS vacua with all moduli being stabilized. This model was discussed extensively in [1]. We will follow their notation for the most part and only discuss the essential features of this compactification, referring the reader to [1] for further details.

We will show in this section that for appropriate choices of flux the vacua do not have any tachyons lying below the BF bound and thus are perturbatively stable. This

will set the stage to consider their possible non-perturbative decays in the following sections.

Let us begin by describing the $T^6/(Z_3 \times Z_3)$ orbifold. This is an orbifold limit of a Calabi-Yau three-fold. Let $z_i = x_i + iy_i, i = 1 \cdots 3$ be three complex coordinates on the T^6. They satisfy the periodicity conditions,

$$z_i \simeq z_i + 1, \quad z_i \simeq z_i + \alpha, \alpha = e^{\frac{\pi i}{3}}. \tag{2.1}$$

The first Z_3, called T, identifies points related by the transformation,

$$(z_1, z_2, z_3) \rightarrow (\alpha^2 z_1, \alpha^2 z_2, \alpha^2 z_3). \tag{2.2}$$

The resulting orbifold has 27 fixed points. The second Z_3 has the generator,

$$(z_1, z_2, z_3) \rightarrow (\alpha^2 z_1 + \frac{1+\alpha}{3}, \alpha^4 z_2 + \frac{1+\alpha}{3}, z_3 + \frac{1+\alpha}{3}). \tag{2.3}$$

This leaves 9 fixed points.

The resulting compactification has no complex structure moduli. The six -torus is a product manifold, $T^6 = T^2 \times T^2 \times T^2$. The resulting orbifold has three Kahler moduli corresponding to the sizes of the three T^2's. Each T^2 also gives rise to a zero mode for B_2 giving rise to three axions; together these give rise to three complex moduli. The dilaton and an axion which arises from C_3, give rise to one more complex modulus. Finally there are nine complex moduli which arise from metric and B_2 moduli associated with the 9 blow up modes.

We will need to consider a further Z_2 orientifold of this orbifold. This is obtained by modding out by $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma$, where Ω_P is world sheet orientation reversal, F_L is left-moving fermion number, and σ is reflection,

$$\sigma : z_i \rightarrow -\bar{z}_i, i = 1, 2, 3. \tag{2.4}$$

There is a single 0_6 plane which fills the non-compact directions and wraps a 3-cycle which is the locus of fixed points of the σ reflection symmetry. The resulting compactification now has $\mathcal{N} = 1$ supersymmetry. The three T^2 moduli, the dilaton-axion, and the nine blow up modes, all survive the orientifolding and form the bosonic components of chiral superfields.

We now turn to incorporating the effects of flux. To begin, we discuss the effects of flux in the orbifold limit. Subsequently we will include the blow-up modes and related fluxes as well.

2.1 Fluxes, Superpotential and Potential

A basis of two-forms on the three T^2's is given by,

$$\omega_i = (\kappa\sqrt{3})^{1/3}idz^i \wedge d\bar{z}^i, i = 1 \cdots 3. \tag{2.5}$$

κ which appears in the normalization above is defined later in terms of the triple intersection number in eq.(2.31). A basis for dual 4-forms is,

$$\tilde{\omega}_i = (\frac{3}{\kappa})^{1/3}(idz^j \wedge d\bar{z}^j)(idz^k \wedge d\bar{z}^k). \tag{2.6}$$

The holomorphic three-form is ,

$$\Omega = 3^{1/4}idz_1 \wedge dz_2 \wedge dz_3. \tag{2.7}$$

Its real and imaginary parts are α_0, β_0,

$$\Omega = \frac{1}{\sqrt{2}}(\alpha_0 + i\beta_0). \tag{2.8}$$

Under the Z_2 orientifold symmetry α_0, β_0 are respectively even and odd.

The three- form H_3 has odd intrinsic particy under $\mathcal{O}$. Therefore a three-form flux,

$$H_3 = -p\beta_0, \tag{2.9}$$

with p constant, is allowed by the equations of motion and Bianchi identities and the symmetries. Similarly the form form flux F_4 can be expanded as[7]

$$F_4 = e_i\tilde{\omega}_i. \tag{2.10}$$

After accounting for the presence of Cherns- Simons terms the tadpole condition for the C_7 potential is given by,

$$m_0 p = -2\sqrt{2}\kappa_{10}^2\mu_6. \tag{2.11}$$

Here m_0 is the Romans parameter, and μ_6 is the tension of a single D_6 brane (an O_6 plane has tension $2\mu_6$). We note that in our conventions, $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$, $\mu_6 = (2\pi)^{-6}(\alpha')^{-7/2}$.

The metric takes the form,

$$ds^2 = \sum_{i=1}^{3} \gamma_i dz^i d\bar{z}^i. \tag{2.12}$$

It is convenient to work with the moduli,

$$v_i = \frac{1}{2}\frac{1}{(\kappa\sqrt{3})^{1/3}}\gamma_i. \tag{2.13}$$

below. These combine naturally with the axions coming from B_2 to give the bosonic components of chiral superfields as was mentioned above. For now we suppress the dependence on the B_2 axions and also the dependence on the axion coming from C_3.

---

[7](we are ignoring the possibility of turning on F_4 with components along 4-cycles dual to the blow up modes for now, these will also be incorporated subsequently.)

The resulting potential in the 4 dimensional Einstein frame effective theory is then,

$$V = \frac{p^2}{4} \frac{e^{2\phi}}{(vol)^2} + \frac{1}{2}(\sum_{i=1}^{3} e_i^2 v_i^2)\frac{e^{4\phi}}{(vol^3)} + \frac{m_0^2}{2}\frac{e^{4\phi}}{vol} - \sqrt{2}|m_0 p|\frac{e^{3\phi}}{vol^{3/2}}. \tag{2.14}$$

where vol, which is related to the volume of the compactification[8], is defined to be

$$vol = \int_{T^6/(Z_3)^2} \sqrt{g_6} = \kappa v_1 v_2 v_3. \tag{2.15}$$

The four terms on the rhs of the potential above arise from the $|H_3|^2$, $|F_4|^4$, m_0^2 and the tension of the O_6 planes respectively.

The important point for the present analysis is that the potential is an even function of the fluxes e_i. Thus the minimum value of the potential and the location of the minimum in moduli space will only depend on the absolute values of e_i and not on their signs. We emphasize this because the conditions for supersymmetry do care about signs. These conditions take the form,

$$\text{sign}(m_0 e_i) < 0, \text{sign}(m_0 p) < 0. \tag{2.16}$$

The second condition is automatically met once the tadpole condition eq.(2.11) is satisfied. It follows from eq.(2.16) that the e_i's must have the same sign to preserve susy.

This gives an easy way to construct non-supersymmetric minima. Starting with the supersymmetric case, we can change the sign of some or all of the e_i's (while keeping m_0, p fixed). The tadpole condition eq.(2.11) will continue to be met but the susy conditions will not be. However since the potential is an even function of the e_i fluxes, the susy minimum in moduli space will continue to be a minimum even for these non-susy choice of fluxes.

Now that we have understood the basic idea behind the construction of the non-supersymmetric vacua we turn to exploring them in more detail below.

2.2 The Superpotential

To begin, we discuss the general case of massive IIA on a CY_3 orientifold, and then specialize to the compactification at hand. Let $\omega_a, a = 1, \cdots h^{1,1}$ be a basis of $(1, 1)$ forms in the CY_3. Let $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma$ be the Z_2 orientifold symmetry, with σ being an antiholomorphic involution of the Calabi-Yau manifold. The space of $(1, 1)$ forms splits into $H^{1,1} = H_-^{1,1} + H_+^{1,1}$ forms which are odd and even under σ, with dimension $h_-^{1,1}, h_+^{1,1}$ respectively. Only the moduli coming from H_- survive

---

[8]After the additional Z_2 orientifolding, the volume of the internal space becomes $\frac{vol}{2}$. For a more complete discussion of our conventions see subsection §2.2.2.

the orientifold projection [9]. Let $J_c = B_2 + iJ$ be the complexified Kahler two-form. Then we can expand J_c in terms of a basis of odd two-forms,

$$J_c = \sum_{a=1}^{h_-^{1,1}} t_a \omega^a, \tag{2.17}$$

Here,

$$t_a = b_a + iv_a. \tag{2.18}$$

are the complexified Kahler moduli. The Kahler potential is given by,

$$K^K(t_a) = -\log(\frac{4}{3}\int J \wedge J \wedge J) = -\log(\frac{4}{3}\kappa_{abc}v_a v_b v_c), \tag{2.19}$$

where the κ_{abc} are the triple intersection numbers,

$$\kappa_{abc} = \int \omega_a \wedge \omega_b \wedge \omega_c. \tag{2.20}$$

In general there are also moduli which arise from complex structure deformations and related axions. We do not discuss them in detail here, since these are absent in the compactification being considered in this paper.

An single axion arises from C_3 in the compactification of interest to us,

$$C_3 = \xi \alpha_0, \tag{2.21}$$

where α_0 is defined as the real part of Ω in eq.(2.8). Susy pairs this with the 4$-$dimensional dilaton defined by,

$$e^{2D} = \frac{e^{2\phi}}{vol}. \tag{2.22}$$

The resulting Kahler potential for this modulus is

$$K^D = 4D. \tag{2.23}$$

Let $\tilde{w}_a$ be a basis for $H_+^{2,2}$. These are are dual to the $(1,1)$ forms ω_a which are a basis of $H_-^{1,1}$ eq.(2.17). Then since F_4 is even under $\mathcal{O}$ we can expand it in this basis,

$$F_4 = e_a \tilde{w}_a. \tag{2.24}$$

The full superpotential has two terms in it,

$$W = W^Q + W^K, \tag{2.25}$$

with

$$W^Q = -p\xi - \sqrt{2}ipe^{-D}, \tag{2.26}$$

---

[9]This is easy to see from their partner B_2 moduli which are odd under $\mathcal{O}$

and

$$W^K = e_a t_a - \frac{m_0}{6}\kappa_{abc}t_a t_b t_c. \tag{2.27}$$

For future use, we mention below that including e_0 units of F_4 flux turned on along the non-compact directions (or equivalently dual F_6 flux turned on along all compact directions), and m_a units of F_2 flux turned on along compact two -cycles gives the more general superpotential for Kahler moduli,

$$W^K = e_0 + e_a t_a + \frac{1}{2}\kappa_{abc}m_a t_b t_c - \frac{m_0}{6}\kappa_{abc}t_a t_b t_c, \tag{2.28}$$

while keeping W^Q unchanged.

We now restrict restrict ourself to the orbifold limit, working with only the untwisted Kahler moduli and the the F_4 fluxes eq.(2.10). There are three complexified Kahler moduli from the untwisted sector,

$$t_i = b_i + iv_i, i = 1, \cdots 3. \tag{2.29}$$

The v_i were introduced above in eq.(2.13). The b_i arise from B_2.

$$B_2 = \sum_{i=1}^{3} b_i \omega_i. \tag{2.30}$$

The triple intersection number on $T^6/(Z_3)^2$ is given by

$$\kappa_{123} = \int_{T^6/(Z_3)^2} \omega_1 \wedge \omega_2 \wedge \omega_3 = \kappa. \tag{2.31}$$

The resulting Kahler potential for these moduli is then [10]

$$K^K = -\log(8\kappa v_1 v_2 v_3). \tag{2.32}$$

Now with the superpotential, eq.(2.26), eq.(2.27), and the Kahler potential eq.(2.23), eq.(2.32), one gets a potential

$$
\begin{aligned}
V =\ & \frac{e^{2D}p^2}{4vol} + \frac{m_0^2 vol\, e^{4D}}{2} + e_i{}^2 v_i{}^2 \frac{e^{4D}}{2vol} + \sqrt{2}e^{3D}m_0 p + e^{4D}\frac{(b_i e_i - p\xi)^2}{2vol} \\
& + \frac{m_0^2 vol\, e^{4D}}{2}\left(\frac{b_1^2}{v_1^2} + \frac{b_2^2}{v_2^2} + \frac{b_3^2}{v_3^2}\right) - \frac{m_0 e^{4D}\kappa b_1 b_2 b_3}{vol}\left(\frac{e_1 v_1^2}{b_1} + \frac{e_2 v_2^2}{b_2} + \frac{e_3 v_3^2}{b_3}\right) \\
& - \frac{m_0 \kappa b_1 b_2 b_3 e^{4D}}{vol}(b_i e_i - p\xi) + \frac{m_0^2(\kappa b_1 b_2 b_3)^2 e^{4D}}{2vol}\left(\frac{v_1^2}{b_1^2} + \frac{v_2^2}{b_2^2} + \frac{v_3^2}{b_3^2}\right) \\
& + \frac{m_0^2 e^{4D}(\kappa b_1 b_2 b_3)^2}{2vol}.
\end{aligned}
\tag{2.33}
$$

---

[10]The Kahler potential for the moduli which survive after the orientifolding is done, is inherited from the parent $\mathcal{N}=2$ theory.

Setting all the axions, $b_i, \xi = 0$ it is easy to see gives the potential in eq.(2.14). Minimizing eq.(2.14) gives,

$$v_i = \frac{1}{|e_i|}\sqrt{\frac{5}{3}|\frac{e_1 e_2 e_3}{\kappa m_0}|}, \tag{2.34}$$

$$e^D = |p|\sqrt{\frac{27}{160}|\frac{\kappa m_0}{e_1 e_2 e_3}|}, \tag{2.35}$$

and,

$$e^\phi = \frac{3}{4}|p|\left(\frac{5}{12}\frac{\kappa}{|m_0 e_1 e_2 e_3|}\right)^{\frac{1}{4}}. \tag{2.36}$$

The potential at the minimum takes the value,

$$V_0 = -\frac{2|e_1 e_2 e_3|}{3\kappa v}e^{4D}. \tag{2.37}$$

where $v_i = \frac{v}{|e_i|}$. In terms of fluxes this is,

$$V_0 = -\sqrt{\frac{4}{15}}\left(\frac{27}{160}\right)^2 \frac{p^4 \kappa^{\frac{3}{2}}|m_0|^{\frac{5}{2}}}{|e_1 e_2 e_3|^{\frac{3}{2}}}. \tag{2.38}$$

Keeping the terms in action which are quadratic in the axions gives,

$$S_{axion} = \frac{1}{2}\int d^4x\sqrt{-g}\left(-\sum_i \frac{1}{2}\partial_\mu \tilde{b}_i \partial^\mu \tilde{b}_i - e^{4D}(m_0^2 vol\ \tilde{b}_i^2 - 2m_0\tilde{b}_1\tilde{b}_2\tilde{b}_3\frac{e_i v_i}{\tilde{b}_i})\right.$$
$$\left. -\frac{1}{2}\partial_\mu x \partial^\mu x - \frac{e^{4D}}{vol}(\tilde{b}_1 e_1 v_1 + \tilde{b}_2 e_2 v_2 + \tilde{b}_3 e_3 v_3 - \frac{p}{\sqrt{2}}e^{-D}x)^2)\right). \tag{2.39}$$

where $\tilde{b}_i = \frac{b_i}{v_i}, x = \sqrt{2}e^D \xi$ are the canonically normalized axion fields. The resulting mass matrix for the axions is then,

$$M_{ij}^2 = 2|m_o|e^{4D}v\begin{pmatrix} \frac{34}{15} & \frac{3}{5}s_1 s_2 - s_3 & \frac{3}{5}s_1 s_3 - s_2 & \frac{4}{5}s_1 \\ \frac{3}{5}s_1 s_2 - s_3 & \frac{34}{15} & \frac{3}{5}s_2 s_3 - s_1 & \frac{4}{5}s_2 \\ \frac{3}{5}s_1 s_3 - s_2 & \frac{3}{5}s_2 s_3 - s_1 & \frac{34}{15} & \frac{4}{5}s_3 \\ \frac{4}{5}s_1 & \frac{4}{5}s_2 & \frac{4}{5}s_3 & \frac{16}{15} \end{pmatrix} \tag{2.40}$$

with $i, j, = 1, \cdots 3$ being the $\tilde{b}_i$ directions, and $i, j = 4$ being x, and where $s_i = \text{sign}(m_0 e_i)$.

There are two distinct cases which arise for the eigenvalues of the mass matrix.

• When $\text{sign}(m_0 e_1 e_2 e_3) = -1$ it turns out that all eigenvalues are positive. This includes the susy case where $\text{sign}(m_0 e_i) = -1$ for each value of i. But it also includes non-susy cases where the condition eq.(2.16), is not met and the condition $\text{sign}(m_0 e_1 e_2 e_3) = -1$ still holds.

• When $\text{sign}(m_0 e_1 e_2 e_3) = +1$ and susy is necessarily broken, there is one negative eigenvalue and thus one tachyon. Its mass is given by, $M^2 = -\frac{4}{15}|m_0|e^{4D}v$. The BF bound is $M_{BF}^2 = -\frac{3}{4}V_{min}$. From eq.(2.37), we see that

$$\frac{M^2}{M_{BF}^2} = \frac{-(\frac{4}{15})|m_0|e^{4D}v}{-(\frac{3}{10})|m_0|e^{4D}v} = \frac{8}{9}. \tag{2.41}$$

Thus we see that the mass lies above the BF bound and hence the resulting vacua are stable with respect to these axionic directions.

2.2.1 Some General Comments

Some important features of the vacua which arise from eq.(2.34)- eq.(2.38) are worth emphasizing at this stage. For the purpose of scalings, in this section we work in string frame with string scale set equal to one. We will be interested in vacua where the four-form flux $|e_i| \sim e \gg 1$. Note that the tadpole condition eq.(2.11) imposes no constraint on the four form flux e_i, which is allowed to get arbitrarily large. From eq.(2.36) we see that the dilaton $e^\phi \sim e^{-3/4} \to 0$, so that for large flux one has a weakly coupled theory.

For $e \gg 1$ we find parametrically that $\gamma_i \sim e^{1/2}$, so that the size of the internal space scales like $l \sim e^{1/4}$. In contrast the potential scales like $e^{3/2}$ and $M_{pl}^{(4)} \sim e^{3/2}$ so that the radius of AdS space goes like $R_{AdS} \sim e^{3/4}$. Thus we find that both l, R_{AdS} become parametrically large as $e \gg 1$. As a result higher derivative corrections in the α' expansion will be suppressed. This still leaves corrections which involve higher powers of the field strengths without additional derivatives. The biggest worry are terms involving higher powers of F_4. It is easy to see that the leading term which goes like F_4^2 scales like $O(1)$, with two powers of e coming from F_4 and four powers of $e^{-1/2}$ coming from the inverse metric. The correction due to F_4^4 terms are then also of $O(1)$. However, as noted in [1] these are suppressed by an additional power of $g_s^2 \sim e^{-3/2}$ making them subdominant.

Note that the ratio $l/R_{AdS} \sim \frac{1}{\sqrt{e}} \to 0$, so that the compactification is not of Freund-Rubin type. The non Freund-Rubin nature of the compactification actually simplifies the analysis when it come to checking for possible tachyons. The KK modes have positive $(mass)^2$ in the absence of flux, including the effects of flux cannot make them tachyonic because of the parametric separation of scales. Thus it is sufficient to look for possible tachyons among the moduli, which are massless in the absence of flux.

In the discussion above we saw that the F_4^2 terms scale like $O(1)$ in string units. In susy breaking vacua the flux sets the scale of susy breaking and one might therefore worry that our starting point, which is a Calabi-Yau orientifold with supersymmetry, is itself inconsistent. However this is not true. The gravitational backreaction of the flux is parametrically suppressed for large e, as we argued in the previous paragraph since $l/R_{AdS} \to 0$. From a ten dimensional point of view this follows from the fact

that the gravitational back reaction is suppressed by g_s^2 which is small. Corrections to the Calabi-Yau metric can be systematically calculated in an expansion in $1/e$. Such corrections arise in the supersymmetric case as well and including them in the susy case alters the internal metric so that it is no longer Calabi-Yau but instead is a half-flat metric with $SU(3)$ structure. For some discussion of this see, [21], [22].

2.2.2 Conventions

Since numerical factors will be sometime important in the following discussion it is worth discussing our conventions in some detail.

We essentially follow the conventions of [1]. More specifically, the volume of the internal space after the additional Z_2 orientifolding is given by $\frac{vol}{2}$. To go to 4 dimensional Einstein frame from the 10 dim string frame we use the relation

$$g_{\mu\nu} = \frac{e^{2\phi}}{vol} g_{\mu\nu}^E. \tag{2.42}$$

In addition we set $2\kappa_{10}^2 = 1$. This gives the E-H term and potential term to be

$$S = \int d^4x \sqrt{-g_E}(\frac{1}{2}R - V). \tag{2.43}$$

where V is given in eq.(2.33). The potential term above agrees with eq.(3.21) of [1] after we set $\kappa_{10}^2 = 1$ in their formula. Similarly other formulae in [1] can be related to ours after setting $\kappa_{10}^2 = 1$ in those formulae.

2.3 The Blow-up Modes

So far we have ignored the blow-up modes. We now include them and check if there are any unacceptable tachyons which arise from the blow-up moduli of their axionic partners. It is convenient for us to seek vacua where the Calabi-Yau manifold is close to its orbifold limit [11]. There are 9 blow-up modes, turning on a blow-up mode replaces the corresponding singularities with a P^2 of non-vanishing size. We will introduce additional F_4 flux threading each of these P^2's.

The complexified Kahler two-form is now given by[12].

$$J_c = \sum_a t_a \omega_a = \sum_{i=1}^{3} t_i \omega_i + \sum_{A=1}^{9} t_A \omega_A, \tag{2.44}$$

with

$$t_A = b_A + iv_A. \tag{2.45}$$

---

[11]A full analysis considering vacua far away from the orbifold point is left for the future.

[12]In our notation the index A which denotes the basis elements of $H_-^{1,1}$ takes values $i = 1, 2, 3, a = 1, \cdots 9$.

ω_A are elements of $H^{1,1}$ dual to the blow-up two cycles, and v_A, b_A are the blow-up moduli and corresponding axions.

The Kahler potential in the Kahler moduli space is[13].

$$K^K = -\log(8\kappa v_1 v_2 v_3 + \frac{4}{3}\beta \sum_{A=1}^{9} v_A^3).$$ (2.46)

Let us note that to stay within the Kahler cone, $v_A < 0$, [1]. This agrees with the intuition that as v_A increases the total volume, which is the argument of the logarithm in the Kahler potential, decreases.

The four- form flux is,

$$F_4 = e_i \tilde{\omega}_i + e_A \tilde{\omega}_A.$$ (2.47)

The superpotential is

$$W = -p\xi - i\sqrt{2}pe^{-D} + e_i t_i + e_A t_A - m_0(\kappa t_1 t_2 t_3 + \frac{\beta}{6}\sum_A t_A^3).$$ (2.48)

To keep the blow-up modes small and the Calabi-Yau close to the orbifold point, we take the extra flux e_A to satisfy the condition,

$$\frac{|e_A|}{|e|} \ll 1,$$ (2.49)

where $e \sim e_i$ denotes a generic flux along the $T^2 \times T^2$ four-cycles. We will see below that the resulting expectation value for

$$v_A \sim \sqrt{|\frac{e_A}{m_0}|}.$$ (2.50)

As a result the ratio

$$\frac{v_A}{v_i} \sim \sqrt{\frac{|e_A|}{|e_i|}} \ll 1,$$ (2.51)

so that the blow up moduli have a comparatively small value and the Calabi-Yau moduli will then be stabilized close to the orbifold point.

For discussion below it is convenient to introduce the variable,

$$\delta = \sqrt{|\frac{e_A^3}{e_1 e_2 e_3}|}.$$ (2.52)

To calculate the value of the blow-up modes it is enough to expand the potential and keep only the first two terms in an expansion in δ. Keeping in mind the expected relation, eq.(2.51), this means keeping terms upto order $(\frac{v_A}{v_i})^3$.

---

[13]κ, β take values, $\kappa = 81, \beta = 9$, according to [1], but we will not need these explicit values below.

We saw above that the leading order potential gives an acceptable extremum as far as the untwisted moduli are concerned. The Kahler moduli and dilaton are given by, eq.(2.34), eq.(2.36), and the axions are zero. All masses lie above the BF bound. The first corrections will have a small effect on the untwisted moduli masses and they will continue to be safely above the BF bound. Also, the shifts in the values at the extremum for the Kahler moduli and the dilaton due to the first order correction can be ignored at this order. In contrast, for the blow-up modes and their axions the first corrections provide the dominant potential. In the analysis below we set the untwisted Kahler moduli and dilaton to their minimum values, eq.(2.34), eq.(2.36), and examine the effects of the first order corrections on the blow-up modes and their axionic partners.

First let us set the axions to be all zero. This gives,

$$V = V_0 + \sqrt{\frac{15\kappa}{4\beta}}|V_0| \sum_A \sqrt{\frac{|e_A|^3}{|e_1e_2e_3|}} \left\{ \frac{1}{2x_A} - \frac{3}{10}\text{sign}(e_A) \sum_i \text{sign}(e_i)x_A \right.$$
$$\left. + \frac{3x_A^3}{200}(1 + 2\sum_{<ij>} \text{sign}(e_ie_j) \right\}, \tag{2.53}$$

where

$$x_A = -\sqrt{\frac{\beta|m_0|}{|e_A|}}v_A. \tag{2.54}$$

We note that an explicit minus sign has been introduced in the definition of x_A since the allowed values of $v_A < 0$. Also in the last summation on the rhs of eq.(2.53) the indices $i, j = 1, \cdots 3$ must take different values, and each distinct pair $< ij >$ appears once.

Let us now introduce the axion dependence in the potential which has so far been suppressed. From the superpotential, eq.(2.48) it follows that if the sign of all the axions is reversed, keeping the Kahler moduli and the dilaton the same, then $W \to -\bar{W}$ and therefore the potential is invariant. This means that the first term in a power series in the axions must be quadratic and therefore the extremum we find by setting them to zero is also an extremum once their dependence is included. We will examine whether this extremum is free of tachyonic modes lying below the BF bound in the following discussion. As was mentioned above, the leading term in the potential already provides an acceptable extremum as far as the untwisted axions are concerned.

The quadratic terms for the blow-up axions in the potential are,

$$V_{b_A} = \frac{1}{40}\sqrt{\frac{15\kappa}{4\beta}}|V_0| \sum_A \sqrt{\frac{|e_A|^3}{|e_1e_2e_3|}} \frac{b_A^2}{(|e_A|/\beta|m_0|)} \left\{ -\frac{20\text{sign}(m_0e_A)}{x_A} \right.$$
$$\left. + x_A(29 + 9\sum_i \text{sign}(m_0e_i)) \right\}. \tag{2.55}$$

For a solution to preserve supersymmetry eq.(2.16) must hold. In addition, since the fluxes e_A are also now turned on we have the conditions,

$$\text{sign}(m_0 e_A) < 0. \tag{2.56}$$

We are now ready to discuss the different cases which arise when the signs of various fluxes are varied. The relative sign between m_0, p is fixed by the tadpole condition eq.(2.11). The different case will arise as we change the relative signs between m_0 and the fluxes e_0, e_A. As was explained above, our main concern as we scan over various choices of signs for the fluxes are the blow-up modes and their axionic partners.

• Case 1): $\text{sign}(e_i m_0) = \text{sign}(e_A m_0) = -1$. This case preserves susy. The potential has a minimum when the blow up modes take the value,

$$x_A = \sqrt{\frac{10}{3}}. \tag{2.57}$$

All axions are non-tachyonic including the b_A axions.

• Case 2): $\text{sign}(m_0 e_i) = \text{sign}(m_0 e_A) = +1$. Susy is broken. The extremum of the potential is at the same value, eq.(2.57). In this case there are no tachyons from the blow-up modes including the x_A directions and the blow-up axions.

• Case 3): $\text{sign}(m_0 e_i) = +1$, $\text{sign}(m_0 e_A) = -1$ Susy is broken, now the extremum of the potential lies at,

$$x_A = \sqrt{\frac{10}{21}}. \tag{2.58}$$

Again there are no tachyons from the blow-up modes and blow up axions.

Before proceeding let us note that there are 9 blow up modes. From eq.(2.53), eq.(2.55) we see that the potential for the blow-up modes and their axionic partners decouple to leading order in $(\frac{e_A}{e_i})^{3/2}$ from each other. Thus with $\text{sign}(m_0 e_i) = 1$ there are actually 2^9 cases with $\text{sign}(e_A m_0)$ for $A = 1, \cdots 9$ being ± 1. Depending on the sign the discussion of Case 2) or Case 3) applies for each blow-up mode and its axionic partner independently of the others.

• Case 4): $\text{sign}(m_0 e_i) = -1$, $\text{sign}(m_0 e_A) = +1$. Susy is broken. Minimum is at the same value for x_A as in Case 3). Now the b_A axion is tachyonic with a mass below the BF bound, see eq. (8.8) of appendix A.

• Case 5): $\text{sign}(m_0 e_1) = \text{sign}(m_0 e_2) = -\text{sign}(m_0 e_3) = 1$, $\text{sign}(m_0 e_A) = +1$. There are three cases of this type with e_3 being exchanged with e_1, e_2. In this cases there is no extremum for the blow-up mode at small values where the condition $|\frac{v_A^3}{v_i^3}| \ll 1$ is met. We have not looked for minima away from this region in moduli space.

• Case 6): $\text{sign}(m_0 e_1) = \text{sign}(m_0 e_2) = -\text{sign}(m_0 e_3) = -1$, $\text{sign}(m_0 e_A) = -1$. Again there are three cases of this type, with e_3 exchanged with e_1, e_2. And once again there is no extremum close to the orbifold limit.

• Case 7): $\text{sign}(m_0 e_1) = \text{sign}(m_0 e_2) = -\text{sign}(m_0 e_3) = 1$, $\text{sign}(m_0 e_A) = -1$. There are three cases of this type obtained by permuting the e_i's. These have a point of inflection at

$$x_{A*} = \sqrt{\frac{10}{3}}. \tag{2.59}$$

The first two derivatives with respect to v_A vanish at this point and the potential locally has the form $V \sim (x_A - x_{A*})^3$. To determine whether this extremum is stable we now need to calculate the next correction in the potential to order $\left(\frac{e_A}{e_i}\right)^3$. Depending on the sign of the resulting corrections a minima will arise close to the value eq.(2.59) or not. We have not carried out this calculation.

The b_A axion in this case has a positive mass at the point, eq. (2.59). So if the minimum for the v_A modulus does lie close to this point the axion will also be non-tachyonic.

• Case 8): $\text{sign}(m_0 e_1) = \text{sign}(m_0 e_2) = -\text{sign}(m_0 e_3) = -1$, $\text{sign}(m_0 e_A) = +1$ This case is very similar to case 7). There is a point of inflection again at eq.(2.59). And the axion b_A has positive mass.

We close this section by noting that the ground state energy for case 2), where $(m_0 e_i) > 0, (m_0 e_A) > 0$, is given by,

$$V = V_0 + V_0 \sqrt{\frac{2\kappa}{\beta}} \sum_A \frac{|e_A|^{3/2}}{\sqrt{|e_1 e_2 e_3|}}, \tag{2.60}$$

and for case 3), where $(m_0 e_A) < 0, (m_0 e_i) > 0$ by,

$$V = V_0 - V_0 \sqrt{\frac{50\kappa}{7\beta}} \sum_A \frac{(|e_A|)^{3/2}}{\sqrt{|e_1 e_2 e_3|}}. \tag{2.61}$$

These results for the ground state energy are correct [14] to order δ. We have neglected the shift in the untwisted Kahler moduli due to the blow-up fluxes, this contributes at order δ^2, since the first corrections in the energy about the minimum are second order in the values of the moduli shifts.

Further details leading to the results presented above are in Appendix A, which also contains Table 1 which summarizes the different cases.

2.4 More General Fluxes

In this subsection we consider what happens when F_2 and F_6 flux are also activated. The fluxes are specified, in terms of the basis of two-forms ω_a by,

$$e_0 = \int F_6, \quad F_2 = -m_a \omega_a. \tag{2.62}$$

---

[14]By case 2) and 3) we mean here cases where all $e_A > 0$ or $e_A < 0$ respectively. In the mixed case where some e_A are positive and negative the terms within the sum in eq.(2.60), eq.(2.61) have to be changed appropriately.

After they are turned on, W^Q remains unchanged, eq.(2.26). W^K now becomes, eq.(2.27). The full superpotential is therefore,

$$W = e_0 + e_a t_a + \frac{1}{2} \kappa_{abc} m_a t_b t_c - \frac{m_0}{6} \kappa_{abc} t_a t_b t_c - p\xi - \sqrt{2} i p e^{-D}. \tag{2.63}$$

The third term on rhs, which is quadratic in t_a contains the effects of the m_a flux. By shifting t_a by

$$t_a \rightarrow t_a - \frac{m_a}{m_0}, \tag{2.64}$$

and ξ by

$$\xi \rightarrow \xi - \frac{e_0}{p} - \frac{e_a m_a}{p} - \frac{1}{3} \kappa_{abc} \frac{m_a m_b m_c}{m_0^2 p}, \tag{2.65}$$

one can can reexpress the superpotential in terms of the shifted variables as,

$$W = \hat{e}_a t_a - \frac{m_0}{6} \kappa_{abc} t_a t_b t_c - p\xi - \sqrt{2} i p e^{-D} \tag{2.66}$$

with

$$\hat{e}_a = e_a + \frac{\kappa_{abc}}{2} \frac{m_b m_c}{m_0}. \tag{2.67}$$

Notice that m_a and e_0 have both disappeared in this superpotential which is the same as the sum of eq.(2.26) and eq.(2.27). The shift, eq.(2.64), eq.(2.65), changes the real part of the chiral superfields and thus does not change the Kahler potential which is expressed in terms of the imaginary part of the chiral superfields, eq.(2.23), eq.(2.46). Thus we see that the theory can be mapped into the one we had studied earlier, without any m_a and e_0 flux.

The solutions in terms of the shifted variables then are the same as those in eq.(2.34) - eq.(2.38), and similar equations with the blowup fluxes also activated in §2.3, with e_a replaced by $\hat{e}_a$.

The conditions for supersymmetry also, it turns out, can be obtained by replacing e_a with $\hat{e}_a$ and are given by

$$\text{sign}(m_0 p) < 0, \quad \text{sign}(m_0 \hat{e}_a) < 0. \tag{2.68}$$

The discussion in the previous subsections now carries over and we see that non-susy solutions of Type 2) and Type 3) can be constructed with $\text{sign}(m_0 \hat{e}_a) > 0$ in the first case and $\text{sign}(m_0 \hat{e}_i) < 0$, $\text{sign}(m_0 \hat{e}_A) > 0$ in the second. These have vacuum energies gives in eq.(2.60) and eq.(2.61) respectively with $e_a \rightarrow \hat{e}_a$.

One final comment is worth making regarding this case. The compactification has gauge symmetries under which the flux and moduli transform, these can be thought of as the analogue of the $\tau \rightarrow \tau + 1$ subgroup of $SL(2, Z)$ which arises on a torus [1]. Under this symmetry, Configurations related by these symmetries are not

distinct but should be identified. One set of such symmetries involve the shift in the b_a axions by integer units,

$$t_a \to t_a - u_a, \tag{2.69}$$

$$m_a \to m_a - m_0 u_a, \tag{2.70}$$

$$e_a \to e_a + \kappa_{abc} m_b u_c - \frac{m_0}{2} \kappa_{abc} u_b u_c, \tag{2.71}$$

$$e_0 \to e_0 + \frac{1}{2} \kappa_{abc} m_a u_b u_c - \frac{m_0}{6} \kappa_{abc} u_a u_b u_c + e_a u_a, \tag{2.72}$$

for integer u_a. The other involves the shift in ξ axion,

$$\xi \to \xi - u, \tag{2.73}$$

$$e_0 \to e_0 - pu. \tag{2.74}$$

Using these any m_a which is an integer multiple of m_0 and e_0 which is integer multiple of p can be set to zero. Since the m_a and e_0 fluxes satisfy quantization conditions, this only leaves a few physically distinct cases where these fluxes are non-vanishing.

3. Vacuum Decay in the Thin wall Approximation

In this section we review the classic discussion of the non-perturbative decay of an unstable vacuum in [3]. Consider an unstable vacuum, called the false vacuum, which can decay to another state, the true vacuum. The decay is mediated by the nucleation of a bubble of true vacuum inside the false vacuum. This nucleation is a quantum tunneling process, and gives rise to a probability for decay per unit volume per unit time of the form

$$\Gamma/V = A e^{-B/\hbar}. \tag{3.1}$$

In the semi-classical approximation one seeks a solution to the Euclidean action which can interpolate between the false and true vacua. Given such a solution, which is called the bounce, the coefficient in the exponent above is given by,

$$B = S_E - S_{False}, \tag{3.2}$$

where S_E is the euclidean action of the bounce and S_{False} is the action of the false vacuum.

We will work in the thin wall approximation in this paper. In this approximation the bounce solution or the bubble has three parts. The inside where the solution is well approximated by the true vacuum, the outside which is the false vacuum, and the bubble wall which interpolates between the two. In the thin wall approximation, the thickness of this wall is much smaller than all the other length scales in the problem. These include the radius of the bubble and the radii of curvature of the inside and outside spacetimes. Once these conditions are met, the tension of the bubble wall

can be calculated by taking it to be a flat wall in flat space-time, neglecting both the curvature of spacetime and the curvature of the bubble wall. This simplifies the analysis considerably.

Before we proceed a comment about notation is in order. In the conventions of [3] the EH term and potential are given by,

$$S = \int d^4x \sqrt{-g}[\frac{1}{2\kappa}R - U]. \tag{3.3}$$

These can be related to our conventions eq.(2.43) by seeting $\kappa = 1$ and taking $U \rightarrow V$.

The Euclidean metric of the bounce solution can be taken to be S^3 symmetric and of form,

$$ds^2 = d\xi^2 + \rho(\xi)^2(d\Omega)^2, \tag{3.4}$$

where $(d\Omega)^2$ is the volume element on a unit S^3. We will be interested in decays where the inside and outside spacetime are both AdS. We denote the vacuum energy of the false vacuum, which is outside the bubble, and the true vacuum, which is inside, as U_+ and U_- respectively. Both are negative. The bubble wall lies at $\rho = \bar{\rho}$.

The bounce action gets contributions from the three parts of the solution, the inside, the wall and the outside,

$$B = B_{inside} + B_{Wall} + B_{outside}. \tag{3.5}$$

Since the outside region is essentially identical to the false vacuum, $B_{outside} = 0$. For a wall with tension S_1

$$B_{wall} = 2\pi^2 \bar{\rho}^3 S_1. \tag{3.6}$$

Finally the inside region contributes,

$$B_{inside} = 12\pi^2 \left[\frac{\left(1 - \frac{1}{3}\bar{\rho}^2 V_-\right)^{\frac{3}{2}} - 1}{V_-} - \frac{\left(1 - \frac{1}{3}\bar{\rho}^2 V_+\right)^{\frac{3}{2}} - 1}{V_+} \right]. \tag{3.7}$$

The value of $\bar{\rho}$ can be calculated by extremizing B, i.e. requiring,

$$\frac{dB}{d\bar{\rho}} = 0. \tag{3.8}$$

For small values of $\bar{\rho}$, $B_{inside} \simeq -\frac{\pi^2}{2}\epsilon\bar{\rho}^4$, where

$$\epsilon = V_+ - V_-. \tag{3.9}$$

is the energy difference between the two vacua. Therefore for small $\bar{\rho}$,

$$B \simeq 2\pi^2 \bar{\rho}^3 S_1 - \frac{\pi^2}{2}\epsilon\bar{\rho}^4. \tag{3.10}$$

This is exactly what we would get in the absence of gravity. The bulk gain in energy grows more rapidly with increasing $\bar{\rho}$ than the surface tension and there is always a solution to the minimization condition eq.(3.8).

However, now notice from eq.(3.7) that with $U_\pm < 0$, for large $\bar{\rho}$

$$B_{inside} = -\frac{4}{\sqrt{3}}\pi^2\bar{\rho}^3(\sqrt{|V_-|} - \sqrt{|V_+|}), \qquad (3.11)$$

so that the bulk gain in energy only grows like $\bar{\rho}^3$ and not as $\bar{\rho}^4$. This is a consequence of the fact that the volume and the area both grow in the same fashion in AdS space. The wall contribution which goes like the area also grows like $\bar{\rho}^3$ with a positive coefficient, eq.(3.6). At small $\bar{\rho}$ the leading contribution to B comes from B_{wall} and is positive. Thus a sufficient condition for an extremum value of B to exists is that the net coefficient of the $\bar{\rho}^3$ dependence at large $\bar{\rho}$ is negative. For $|V_-| > |V_+|$ this yields the condition,

$$S_1 < \sqrt{\frac{4}{3}}[\sqrt{|V_-|} - \sqrt{|V_+|}]. \qquad (3.12)$$

We will see below that this is also a necessary condition for an extremum solving eq.(3.8) to exist.

We now proceed with a more detailed analysis of eq.(3.8). From eq.(3.5), eq.(3.6, eq.(3.7), this gives the condition,

$$\left(1 - \frac{1}{3}\bar{\rho}^2 V_-\right)^{\frac{1}{2}} - \left(1 - \frac{1}{3}\bar{\rho}^2 V_+\right)^{\frac{1}{2}} = \frac{\bar{\rho}S_1}{2}. \qquad (3.13)$$

Multiplying both sides by $\left(1 - \frac{1}{3}\bar{\rho}^2 V_-\right)^{\frac{1}{2}} + \left(1 - \frac{1}{3}\bar{\rho}^2 V_+\right)^{\frac{1}{2}}$ and simplifying, we get

$$\left(1 - \frac{1}{3}\bar{\rho}^2 V_-\right)^{\frac{1}{2}} + \left(1 - \frac{1}{3}\bar{\rho}^2 V_+\right)^{\frac{1}{2}} = \frac{2}{3S_1}\bar{\rho}\left(V_+ - V_-\right). \qquad (3.14)$$

Some more algebra then gives,

$$2\left(1 - \frac{1}{3}\bar{\rho}^2 V_-\right)^{\frac{1}{2}} = \frac{\bar{\rho}S_1}{2} + \frac{2}{3S_1}\bar{\rho}\left(V_+ - V_-\right), \qquad (3.15)$$

$$2\left(1 - \frac{1}{3}\bar{\rho}^2 V_+\right)^{\frac{1}{2}} = -\frac{\bar{\rho}S_1}{2} + \frac{2}{3S_1}\bar{\rho}\left(V_+ - V_-\right). \qquad (3.16)$$

Note that eq.(3.16) gives the requirement that $\frac{\epsilon}{3S_1} > \frac{S_1}{4}$. In particular this requires that $\epsilon > 0$ so that the inside vacuum has a lower energy than the outside one, as one would expect physically. Solving eq.(3.16) we get,

$$\bar{\rho} = \frac{1}{\sqrt{\frac{V_+}{3} + \left(\frac{\epsilon}{3S_1} - \frac{S_1}{4}\right)^2}}. \qquad (3.17)$$

For a bounce solution to exist it is necessary that $\bar{\rho}$ is finite. This gives the condition

$$\left(\frac{\epsilon}{3S_1} - \frac{S_1}{4}\right) > \sqrt{\frac{|V_+|}{3}}, \tag{3.18}$$

which can also be expressed as

$$\frac{S_1^2}{4} + \sqrt{\frac{|V_+|}{3}}S_1 - \frac{\epsilon}{3} < 0. \tag{3.19}$$

It is easy to see that this condition can be met iff eq.(3.12) is true.

Let us end this section with a few comments. If the outside vacuum is flat, with $U_+ = 0$, there continues to be a condition on the tension,

$$S_1 < \sqrt{\frac{4}{3}}|V_-|^{1/2}, \tag{3.20}$$

which must be met for the bounce solution to exist. Looking back at eq.(3.7) one finds that in this case the large $\bar{\rho}$ behaviour of B_{inside} continues to be of the form $\bar{\rho}^3$, and is driven by the AdS vacuum inside. As a result the tension must again be smaller than a critical value for the decay to proceed.

If the outside vacuum is deSitter, with $V_+ > 0$, it follows from eq.(3.17) that $\bar{\rho}$ is always finite. For the decay to be allowed the condition $\epsilon > 0$ must be met of course. However there seems to be an additional condition which arises from eq.(3.16) that gives the requirement $\frac{\epsilon}{3S_1} > \frac{S_1}{4}$. In fact this requirement does not have to be met[23]. It turns out that there are additional possibilities for the decay of dS space which arise because Euclidean deSitter is a sphere and therefore a compact manifold. Allowing for these additional possibilities removes this requirement.

4. $D4$-Brane Mediated Decays

In this section we consider non-perturbative decays mediated by a domain wall that carries only $D4$ brane charge. More general brane configurations carrying other charges as well will be discussed later.

The $D4$ brane wraps a two-cycle in the internal space and extends along two of the 3 spatially non-compact directions of AdS_4 thereby giving rise to a domain wall which separates the true vacuum from the false one. This causes the four-form flux F_4, along the 4-cycle dual to the two-cycle wrapped by the $D4$, to jump. And this change in F_4 cause a change in the cosmological constant.

We will work within the thin wall approximation below. For this approximation to hold the domain wall must have a a thickness which is much smaller than all the other relevant length scales, namely, the radius of the S^3, and the AdS radii of the true and false vacua. At first sight it would seem that this condition is obviously met since a $D4$-brane is much thinner than all these distance scales, when supergravity

is valid. However there is an important caveat, which was also mentioned in the introduction. In the situations at hand a change in flux also causes the vacuum expectation value of the moduli to change. As a result the moduli also begin to vary across the domain wall. Now the moduli have a mass of order the R_{AdS}^{-1}, see Appendix B. Thus their variation results in a wall with thickness of order R_{AdS}, which is not thin.

To stay within the thin wall approximation we will only consider decays where the change in the moduli from one vacuum to the other is sufficiently small. The moduli contribution to the tension will then be much smaller than the $D4$ brane contribution and can therefore be neglected. The domain wall can then be well approximated by a $D4$-brane which is indeed thin. The precise conditions ensuring that the moduli contribution is small will be worked out for various cases as we proceed.

This section is quite long. The essential arguments can be found in the beginning few subsections, §4.1 - §4.3 and also in §4.8.

4.1 Non-Susy to Susy Decay

We begin by considering the decay of a non-susy vacuum to a susy one. We will see below that for all these decays the tension of the domain wall is larger than the energy difference between the two vacua, resulting in the decays being forbidden in the thin wall approximation [15]. This mismatch is parametric in the flux, therefore in this subsection we do not need to keep track of precise numerical factors.

To begin, we work in the orbifold limit, neglecting the blow-up modes and the related fluxes, e_A. This leaves three two-cycles, namely the three T^2's, and three fluxes, $e_i, i = 1, 2, 3$. The essential argument will become clear if we take all the three fluxes e_i to be of the same order, $e_i \sim e$. For the supergravity description to be valid $|e| \gg 1$.

Now consider a single $D4$ brane which wraps the first T^2. Its tension arises from the Nambu-Gotto action,

$$\text{Action} = \mu_4 e^{-\phi} \int \sqrt{-g} d^3 \xi_i dx^1 dy^1 \sim \gamma_1 \mu_4 e^{-\phi} \int \sqrt{-g} d^3 \xi_i. \tag{4.1}$$

where $\xi_i, i = 1, \cdots 3$ are the 3 directions in AdS space along which it extends, and we have used eq.(2.12) and done the integral over the T^2.

We will work in 4 dim Einstein frame below. This is related to the string frame by

$$g_{\mu\nu} = \frac{e^{2\phi}}{vol} g_{\mu\nu}^E. \tag{4.2}$$

---

[15]More correctly the necessary condition eq.(3.18) will not be met.

Accounting for this, gives the Einstein frame tension for a single $D4$ wrapping the first T^2 to be

$$S_1 \sim \gamma_1 \mu_4 e^{-\phi} (\frac{e^{2\phi}}{vol})^{3/2}, \tag{4.3}$$

where γ_1 is the size of the T^2.

The domain wall of interest to us is obtained by wrapping all three T^2's in general, since all three fluxes must reverse sign. Its tension is of order

$$S_1 \sim \gamma \mu_4 e^{-\phi} (\frac{e^{2\phi}}{vol})^{3/2} |\delta e|, \tag{4.4}$$

where $\gamma \sim \gamma_i$ is the size of the 3 T^2's and $\delta e \sim \delta e_i$ is the change in flux. Using eq.(2.13) and eq.(2.34), eq.(2.36), eq.(2.35), we get

$$S_1 \sim \frac{|\delta e|}{|e|} (\frac{1}{|e|})^{9/4}, \tag{4.5}$$

where $\delta e \sim \delta e_i$ is the change in the flux.

Now let us take into account the conditions imposed by the thin wall approximation. From Appendix B we see that the moduli make a contribution to the tension

$$T_{mod} \sim M(\Delta\Phi)^2. \tag{4.6}$$

Here $M \sim 1/R_{AdS} \sim 1/|e|^{9/4}$, is the mass of canonically normalized moduli field, and

$$\Delta\Phi \sim \frac{\delta v_i}{v_i}. \tag{4.7}$$

is the total change in the vacuum expectation value of the canonically normalized field across the domain wall. The value of the v_i moduli in any vacuum depend on the absolute value of the flux. This gives

$$\Delta\Phi \sim \frac{\delta|e|}{|e|}. \tag{4.8}$$

Substituting in eq. (4.6) yields

$$T_{mod} \sim \frac{(\delta|e|)^2}{|e|^2} (\frac{1}{|e|})^{9/4}. \tag{4.9}$$

For the thin wall approximation to hold, S_1 must dominate over T_{mod}. This gives

$$\frac{|\delta e|}{|e|} \gg \frac{(\delta|e|)^2}{|e|^2}. \tag{4.10}$$

Now this condition can be met if the susy vacuum has fluxes which are opposite in sign but approximately the same in magnitude as the non-susy vacuum we start with. That is,

$$e_i^{susy} \sim e^{susy} \sim -e. \tag{4.11}$$

Then the non-susy vacuum we start with and the susy vacuum it could decay to, lie in approximately the same region of moduli space, but are very far apart in flux space. As a result

$$|\delta e| \sim 2|e|, \tag{4.12}$$

and eq.(4.10) becomes,

$$1 \gg \frac{(\delta|e|)^2}{|e|^2}. \tag{4.13}$$

From eq.(4.11), $|e^{susy}| \sim |e|$ and therefore $\delta|e| = |e| - |e^{susy}|$ is small and this condition is indeed met [16].

Having ensured that the decay process lies within the thin wall approximation let us now see why it is not allowed. The important point is that since the absolute value of the flux in the non-susy and susy vacua are close, their energy difference is also small. From eq.(2.37),

$$\epsilon \sim \frac{\delta|e|}{|e|}(\frac{1}{|e|})^{9/2}. \tag{4.14}$$

For the decay to proceed, a necessary condition which follows from eq.(3.18) is that $\frac{\epsilon}{3S_1} > \frac{S_1}{4}$. From, eq.(4.5) and eq.(4.14), this condition becomes,

$$\frac{\delta|e|}{|e|} \geq (\frac{|\delta e|}{|e|})^2 \sim O(1), \tag{4.15}$$

where the last relation follows from eq.(4.12). We see now that the condition in eq.(4.15) is incompatible with eq.(4.13).

Thus we see that the decay of a non-susy vacuum to a susy vacuum is not allowed in the thin wall approximation.

So far we have neglected the blow-up modes and also neglected the related blow-up fluxes. After including these one can have perturbatively stable non-susy vacua of Type 2) or Type 3) as discussed in the §2. The obstruction we found above disallowing a non-susy to susy decay was parametric in the e_i fluxes for large $|e_i|$. Including the blow-up fluxes cannot overcome this parametric obstruction as long as the blow-up fluxes are small and meet the condition, eq.(2.49). Therefore we conclude that non-susy vacua of Type 2) and 3), which arise when the flux meets the condition eq.(2.49), cannot decay to susy vacua in the thin wall approximation.

4.2 Decays From Non-Susy to Other Non-Susy Vacua

We now turn to examining whether a non-susy vacuum can decay to other non-susy vacua. We will need to calculate the tension of a $D4$ brane wrapping a two-cycle in the internal space. The $D4$-brane causes a jump in the flux to occur and therefore a

---

[16]We cannot take $|\delta e| \gg 2|e|$ because then $|\delta e| \sim \delta|e|$ and thus eq.(4.10) will not be met.

jump in the superpotential. It is well known that the tension of the resulting domain wall satisfies a lower bound

$$T \geq T_L, \tag{4.16}$$

where T_L is given by

$$T_L = 2e^{K/2}|\Delta W|, \tag{4.17}$$

with

$$\Delta W = \delta e_a v_a, \tag{4.18}$$

being the change in the superpotential caused by the jump in the flux.

Our basic strategy will be to compare T_L with an upper bound in terms of the energy difference between the two vacua. This will allow us to rule out various decays.

Let us note here that the formula, eq.(4.17) is true more generally as well, when the D brane carries other charges too. It arises because the tension of the domain wall is only determined by the geometry of the Calabi-Yau space, in our approximations. In fact, in the absence of fluxes, the Calabi-Yau manifold preserves supersymmetry and the lower bound for the tension, in terms of the jump in the superpotential, is really a BPS bound. Branes which saturate the bound preserve supersymmetry, in the absence of flux.

In the $D4$-brane case the lower bound follows from the fact that the Kahler form on the Calabi-Yau is a calibration [17]. For the sake of clarity let us pause to quickly review how this comes about. This will also allow us to fix the normalization constant in eq.(4.17).

Consider a two- cycle in the Calabi-Yau manifold. Let $\sigma, \bar{\sigma}$ be holomorphic and anti-holomorphic coordinates on the world volume. And let the Pull back of the Kahler form of the Calabi-Yau onto the world volume be,

$$P[J] = K_{\sigma\bar{\sigma}}d\sigma d\bar{\sigma}. \tag{4.19}$$

Also let $P[g]$ be the induced metric on the world volume (we are supressing indices here). The area element is then given by,

$$\sqrt{det(P[g])}d\sigma \wedge d\bar{\sigma}. \tag{4.20}$$

Now since the Kahler form of the Calabi-Yau is a calibration we know that for any two-cycle,

$$|K_{\sigma\bar{\sigma}}| \leq \sqrt{det(P[g])}. \tag{4.21}$$

The equality is met only when the cycle is either holomorphic, or antiholomorphic. In the holomorphic case $z^i(\sigma)$ where z^i are coordinates of the Calabi-Yau manifold; in

---

[17]For an early reference see [24]. For a pedagogical discussion see, [25].

the antiholomorphic case, $z^i(\bar{\sigma})$. In these cases the $D4$ brane wrapping the two-cycle is supersymmetric [18].

The tension of the resulting domain wall is given by

$$T = \mu_4 e^{-\phi} \int d^2\sigma \sqrt{det(P[g])}. \tag{4.22}$$

Using eq.(4.21) we get a lower bound on the tension,

$$T \geq T_L \equiv \mu_4 e^{-\phi} | \int P[J] |. \tag{4.23}$$

Now if γ_a is a basis of two-cycles and ω_a a basis of dual-two forms, and if the two-cycle wrapped by the $D4$ brane is $\gamma = \delta n_a \gamma_a$ then we have

$$\int P[J] = \delta n_a v_a, \tag{4.24}$$

where the Kahler moduli v_a are defined in eq.(2.17). This leads to the lower bound

$$T_L = \mu_4 e^{-\phi} |\delta n_a v_a|. \tag{4.25}$$

We now go to 4 dim Einstein frame using eq.(4.2). In addition we relate the winding numbers δn_a to the jump in the four form flux δe_a by

$$\delta e_a = 3^{2/3} \sqrt{2} \kappa^{1/3} \mu_4 \delta n_a, \tag{4.26}$$

as discussed in Appendix C. Finally we also include an extra normalization factor having to do with our definition of the Kahler two-form, which is also discussed in Appendix C. Altogether we then get eq.(4.25), with eq.(4.18).

Let us now turn to evaluating the upper bound on the tension. We saw in §3 that for the decay to be allowed it must meet the condition, eq.(3.12). Using the definition of ϵ in eq.(3.9), and working to leading order in ϵ we get,

$$T \leq T_U \equiv \frac{\epsilon}{\sqrt{3|V_0|}}. \tag{4.27}$$

The justification for working to leading order in ϵ comes from the thin wall approximation, as we will see below.

In the discussion below we will ask if the lower bound T_L is bigger than the upper bound T_U. If this is true the decay will not be allowed. In cases where $T_L < T_U$ there will be interpolating $D4$ branes.e.g. wrapping susy cycles which saturate the lower bound, which will lead to allowed decays.

Before proceeding let us make one more comment. When we calculated T_L above we assumed that the moduli are fixed and calculated the tension of the $D4$ brane in

---

[18]For a more extensive discussion see §4.9.

this fixed moduli background. Actually the change in flux caused by the $D4$ brane also causes the moduli to change. But as long as the fractional change in expectation value of the moduli is small, i.e.,

$$|\frac{\delta v_a}{v_a}| \ll 1, \tag{4.28}$$

the resulting effect on the $D4$ brane tension can be neglected. In the thin wall approximation the variation of the moduli must make a smaller contribution to the domain wall tension than the $D4$-brane makes, this requirement gives rise to the condition, eq.(4.28), as we will see below.

4.3 Decays in the Orbifold limit

To begin let us set the flux e_A along the blow-up 4-cycles to be zero. Only the e_i fluxes are then activated and we only consider $D4$ branes which wrap the T^2 two-cycles and cause these fluxes to jump.

The limitations imposed by the thin wall approximation can be understood in terms of a discussion analogous to the one in §4.1. The important difference here is that since the true and false vacua are non-susy of Type 2) or 3) the $e_i, i = 1, \cdots 3$ fluxes have the same sign in both of them. To save clutter we set $m_0 > 0$ below. Then $e_i > 0$ in these vacua. To begin let us consider a case where all the fluxes are comparable, $e_1 \sim e_2 \sim e_3 \sim e$, and where the change in flux caused by the domain wall is also comparable, $\delta e_i \sim \delta e$. A $D4$ brane which changes the flux by amount δe contributes a tension,

$$T_{brane} \sim |\frac{\delta e}{e}| \frac{1}{|e|^{9/4}}. \tag{4.29}$$

The moduli contribute a tension which is now,

$$T_{moduli} \sim |\frac{\delta e}{e}|^2 \frac{1}{|e|^{9/4}}. \tag{4.30}$$

For $T_{brane} \gg T_{moduli}$ we get,

$$|\frac{\delta e}{e}| \ll 1. \tag{4.31}$$

From eq.(2.34) we see that the moduli change in response to the flux by

$$\frac{\delta v_i}{v_i} \sim \frac{\delta e}{e}. \tag{4.32}$$

Thus eq.(4.28) follows from the condition, eq.(4.31). From eq.(2.37) the potential at the minimum changes by

$$|\frac{\epsilon}{V_0}| \sim |\frac{\delta e}{e}| \ll 1. \tag{4.33}$$

This justifies working to leading order in ϵ in eq.(4.27).

If the three e_i fluxes and/or their changes are not comparable, a similar argument goes through with the factor $|\frac{\delta e}{e}|$ being replaced by that for the flux with the largest fractional change, i.e. the largest values of $|\frac{\delta e_i}{e_i}|$. Once again, both eq.(4.28), and eq.(4.27) follow.

We now calculate both T_U and T_L for such decays. The energy difference ϵ can be calculated in terms of the change in fluxes δe_i from eq.(2.38). This gives,

$$T_U = \frac{3}{2}(\sum_i \frac{\delta e_i}{e_i})\sqrt{\frac{|V_0|}{3}}. \tag{4.34}$$

From eq(2.15), eq(2.34) and eq(2.37), we get

$$\frac{\sqrt{|V_0|}}{e_i} = v_i\sqrt{\frac{2}{3}}\frac{e^{2D}}{\sqrt{vol}}. \tag{4.35}$$

Eq.(4.34) then becomes,

$$T_U = \frac{e^{2D}}{\sqrt{2vol}}\sum_i \delta e_i v_i. \tag{4.36}$$

For the decay to occur $\epsilon > 0$, this means $\sum_i \delta e_i v_i > 0$. As a result we get,

Next we calculate T_L. Since $\Delta W = \delta e_i v_i$, and Kahler potential K is as given in eq(2.32), using eq(2.34), we get

$$T_L = 2e^{\frac{K}{2}}|\Delta W| = \frac{e^{2D}}{\sqrt{2vol}}\sum_i \delta e_i v_i. \tag{4.37}$$

Comparing eq(4.36) and eq(4.37), we see that $T_U = T_L$.

This means the decay can at best be marginally allowed. The marginal cases arises when the $D4$ brane wraps a supersymmetric cycle. Since supersymmetry is broken one expects that corrections to the approximation we are working in will result in the marginal case becoming either allowed or disallowed [19]. We will incorporate some of these corrections in the following discussion and also comment on which cases remain marginal after including some of these corrections further below.

In the discussion above we have set the e_A fluxes to vanish. If they are turned on but are small so that δ defined in eq.(2.52) is small, then for $D4$ branes which only wrap the T^2 two-cycles the calculations above still give the leading answers in δ for T_U, T_L. In the discussion below we will now turn to including $D4$ branes which can cause a change in the e_A fluxes.

4.3.1 Explicit Example of a Disallowed Decay

The advantage of working in the orbifold limit is that one can explicitly calculate the size of the T^2 two-cycles and associated tension of branes. This allows us to give

---

[19]The marginal case corresponds to a no-force condition on the $D4$ brane and a flat direction in the AdS_4 theory. Such a flat direction should get lifted without susy.

a simple example of a situation where the brane tension is too big, because the cycle is not holomorphic resulting in the decay being disallowed.

Consider a $D4$ which wraps the first two T^2's. Let $\sigma, \bar{\sigma}$ be the holomorphic, and antiholomorphic coordinates on the world volume, and let the mapping from the world volume to the first T^2 be linear and holomorphic, $z^1(\sigma) = \sigma$, and to second T^2 be linear and anti-holomorphic, $z^2(\bar{\sigma}) = \bar{\sigma}$. The resulting cycle is clearly not holomorphic, and the resulting wrapping numbers for the two T^2's are $+1$ and -1 respectively. The tension of the resulting $D4$ brane is

$$T = \frac{e^{2D}}{\sqrt{2vol}}\delta|e_1|(v_1 + v_2). \tag{4.38}$$

From the discussion above we have,

$$|T_U| = T_L = \frac{e^{2D}}{\sqrt{2vol}}\delta|e_1||(v_1 - v_2)|. \tag{4.39}$$

Thus we see that $T > T_U$ and the decay is not allowed.

4.4 General Decays With Blow-up Fluxes

Let us first examine the conditions imposed by the thin wall approximation on the allowed change in the blow-up fluxes. From the Kahler potential eq.(2.46) it is easy to see that a change in canonically normalized blow-up modes is,

$$\Delta\phi_{bu} \sim \sqrt{\delta}\frac{\delta v_A}{v_A}, \tag{4.40}$$

where δv_A is the change in the blow-up moduli. It then follows that the blow up modes also have a mass,

$$M_{bu} \sim \sqrt{|V_0|} \sim R_{Ads}^{-1}, \tag{4.41}$$

and their contribution to the tension is

$$T_{bu} \sim M_{bu}\Delta(\phi_{bu})^2 \sim \sqrt{|V_0|}\delta(\frac{\delta e_A}{e_A})^2, \tag{4.42}$$

where we have used the fact that the vacuum expectation value of $v_A \sim \sqrt{|e_A|}$.

The $D4$ brane wrapping the dual two-cycle which causes this jump in flux has a tension,

$$T_{brane} \propto |\delta e_A v_A|. \tag{4.43}$$

Inserting the correct proportionality factors and converting to Einstein frame as in the previous subsection now gives,

$$T_{brane} \sim \sqrt{|V_0|}\delta|\frac{\delta e_A}{e_A}|. \tag{4.44}$$

Thus comparing eq.(4.42), eq.(4.44), gives the condition,

$$|\frac{\delta e_A}{e_A}| \ll 1,$$ (4.45)

which must be met for the thin wall approximation to hold.

We now turn to various different cases. In the discussion in §4.5-§4.7 that follows we set $m_0 > 0$ for simplicity. The case $m_0 < 0$ can be obtained by changing the sign of all fluxes.

4.5 Type 2) to Type 2) Decays

The vacuum energy is given in eq.(2.60). In this case, $e_A, e_i > 0$. The change in blow up fluxes δe_A contributes to the difference in energy density ϵ and thus to T_u,

$$\delta T_U = -\sqrt{\frac{3}{2\beta}} \sqrt{|V_0|} \sqrt{\frac{e_A^3}{e_1 e_2 e_3}} \sum_A \frac{\delta e_A}{e_A}.$$ (4.46)

Using eq(2.37),eq(2.15), eq(2.34) and the fact that for Type 2 vacuum $v_A = -\sqrt{\frac{10e_A}{3\beta|m_0|}}$, we get

$$T_U = \frac{1}{\sqrt{2}} \frac{e^{2D}}{\sqrt{vol}} (\sum_i \delta e_i v_i + \sum_A \delta e_A v_A).$$ (4.47)

In obtaining this formula we had also added the contribution due to the change in the e_i flux which was obtained in eq.(4.36) above.

Since $\Delta W = \delta e_i v_i + \delta e_A v_A$, and the Kahler potential eq(2.46) can be approximated as eq(2.32) to leading order in δ, we get

$$T_L = 2e^{\frac{K}{2}}|\Delta W| = \frac{e^{2D}}{\sqrt{2vol}} |\sum_i \delta e_i v_i + \sum_A \delta e_A v_A|.$$ (4.48)

For the decay to occur the rhs of eq.(4.47) must be positive, thus we see that again $T_U = T_L$. Therefore the decays can again be at most marginal.

In the calculation above the effects due to the jump in the non-blow up fluxes were calculated as in the previous subsection and thus are correct only to leading order in δ. Thus we are assuming that

$$|\delta e_i v_i| \delta \ll |\delta e_A v_A|.$$ (4.49)

Using the relation that $|\frac{v^A}{v^i}| \sim |\frac{e_i}{e_A}| \delta$ this gives,

$$|\frac{\delta e_i}{e_i}| \ll |\frac{\delta e_A}{e_A}|.$$ (4.50)

In fact the Type 2) vacua are stable with at best marginal decays upto a high order of approximation. This is due to their being related (after a change in the sign of all fluxes) with supersymmetric vacua, as will be explained in section §4.8.

4.6 Type 3) to Type 3) Decays

Here we consider the analogous decays for Type 3) vacua. In this case $e_A < 0$, $e_i > 0$. The ground state energy is given in eq. (2.61). Including a contribution due to the change in the e_i flux gives,

$$T_U = \frac{1}{\sqrt{2}} \frac{e^{2D}}{\sqrt{vol}} \left(\sum_i \delta e_i v_i + 5 \sum_A \delta e_A v_A \right). \tag{4.51}$$

Note that the contribution proportional to δe_A on the rhs comes with a coefficient 5. T_L continues to be given by eq(4.48). Therefore now there can be situations where $T_U > T_L$.

   As an example consider the case where δe_i vanishes, and one of the $\delta e_A \neq 0$. For the energy difference to be positive, $\epsilon > 0$, which means $\delta e_A < 0$, since $v_A < 0$. As we will argue below, in this case there is a susy cycle which saturates the lower bound $T = T_L$. Thus $T < T_U$ and the decay will proceed.

   The argument establishing that there is a susy cycle with $\delta e_A \neq 0, \delta e_i = 0$ is as follows. Blowing up the orbifold slightly gives rise to a P^2 at every fixed point. There is [20] a $P^1 \subset P^2$. It is easy to see that this P^1 is a holomorphic cycle and is non-trivial in homology [21]. Its size, a, is proportional to v_A the blow-up modulus. Now being holomorphic a must be proportional to the resulting jump in the superpotential. This can only happen if the δe_i coefficients vanish for this cycle, since $v_i \gg |v_A|$ [22].

   Let us estimate the decay rate which results in this case from changing δe_A. Since the tension

$$T = T_L \sim T_U = \frac{\epsilon}{\sqrt{3|V_0|}} \tag{4.52}$$

we see that $\frac{\epsilon}{T} \sim \sqrt{|V_0|}$, and so from eq.(3.17) the size of the bubble is

$$\bar{\rho} \sim \frac{1}{\sqrt{|V_0|}}. \tag{4.53}$$

The bounce action is $B \sim B_{wall} \sim \bar{\rho}^3 T$. Substituting for $T, \bar{\rho}$ from eq.(4.52), eq.(4.53), we then see that the bounce action is

$$B \sim \frac{\epsilon M_{Pl}^4}{V_0^2} \tag{4.54}$$

---

[20]We thank S. Kachru for suggesting this possibility and I. Biswas and N. Nitsure for extensive discussions.

[21]In the coordinates used in [26], eq.(3.1), this cycle is given by setting $z_2 = w = 0$, so it is clearly holomorphic. To include the point at infinity, $z_1 \to \infty$ a second patch is needed. The Kahler form integrates to a non-zero value on this cycle so it is clearly non-trivial in homology.

[22]Ideally we should have calculated the intersection numbers of this cycle with the P^2 divisor and the other four-cycles from first principles and shown that these are of the required form. We will not attempt this here.

where we have reinstated the dependence on the four dimensional Planck scale M_{Pl} on dimensional grounds.

The rate of decay goes like $\Gamma \sim e^{-B}$, so the fastest decays are those with the smallest jumps in flux. Working out the resulting discharge of a particular vacuum due to all the competing decays is a fascinating question that we leave for the future [23].

4.7 Type 3) to Type 2) Decays

Here we discuss the decays of Type 3) to Type 2) vacua. The former have $e_A < 0$ while the latter have $e_A > 0$. Thus the e_A flux must reverse sign in such a decay and we see that condition eq.(4.45), which we had imposed so that the variation of the v_A moduli does not contribute significantly to the domain wall tension, cannot be met. This situation is analogous to §4.1 where we dealt with decays from non-susy to susy vacua. The expectation value of the v_A moduli depend actually on the absolute value of e_A. So to meet the thin wall approximation we can now adjust $|e_A|$ so that the variation in v_A is small and therefore its contribution to the domain wall tension can be neglected. To illustrate this we infact adjust $|e_A|$ so that this variation vanishes.

Using $v_A = -\sqrt{\frac{10}{3}}\sqrt{\frac{|e_A|}{\beta|m_0|}}$ for Type 2) and $v_A = -\sqrt{\frac{10}{21}}\sqrt{\frac{|e_A|}{\beta|m_0|}}$ for Type 3, we learn that for v_A to be the same,

$$e_{A \text{ type 3}} = -7e_{A \text{ type 2}}. \tag{4.55}$$

Using this, we can calculate the difference in energy

$$\epsilon = V_{\text{type 3}} - V_{\text{type 2}} = \frac{3}{2}\sum \frac{\delta e_i}{e_i}|V_0| + c|V_0|\sqrt{\frac{\kappa}{\beta e_1 e_2 e_3}}\sum_A |e_{A \text{ type 2}}|^{\frac{3}{2}}. \tag{4.56}$$

where $c = \sqrt{2} + 7\sqrt{50}$. Note that for the decay to be possible $\epsilon > 0$. Using eq(4.35) T_U can be calculated to be

$$T_U = \left(\delta e_i v_i + 24\sum_A e_{A \text{ type 2}} |v_A|\right) \frac{e^{2D}}{\sqrt{2vol}}. \tag{4.57}$$

Note that for $\epsilon > 0$ the term in the brackets in the above equation is greater than zero.

T_L can be calculated to be,

$$T_L = |\delta e_i v_i + \sum_A \delta e_A v_A|\frac{e^{2D}}{\sqrt{2vol}}. \tag{4.58}$$

---

[23]For this we also need to take into account the fact that inside the bubble is a negatively curved FRW universe which ends in a big crunch.

Now $v_A < 0$ and from eq(4.55) we know that $\delta e_A = -8 e_A$ type 2, therefore

$$T_L = |\delta e_i v_i + 8 \sum_A e_A \text{ type 2} |v_A|)| \frac{e^{2D}}{\sqrt{2vol}}. \tag{4.59}$$

It is now clear that as long as $\delta e_i v_i > 0$, $T_U > T_L$. A decay will be allowed if $T < T_U$. Like in the Type 3) -Type 3) case, concrete examples can be given where this is true. For example in the case where $\delta e_i = 0$ the $D4$ brane tension saturates the lower bound with $T = T_L$, since it is a holomorphic cycle, leading to an allowed decay.

We can also ask about the possibility of Type 2) vacua decaying to Type 3). Running the above argument again the coefficient 24 in the second term on the rhs of eq.(4.57) and 8 in the second term of eq.(4.59) both reverse sign making both these terms negative. Since $\epsilon > 0$ for the decay to happen, we find that $T_U < T_L$. This shows that such decays are disallowed.

We have adjusted the fluxes so that the v_A moduli have the same value in the two vacua, thereby ensuring that the moduli contribution to the domain wall tension is small. Our conclusions will remain unchanged if the fluxes took different values, allowing for a variation in v_A, as long as one stays in the thin wall approximation.

4.8 Supersymmetric Partners in the Landscape and Marginality

We have seen above that Type 2) vacua are stable and can at most decay marginally, within our approximations. We will now see that this stability is quite general and can be understood by relating these vacua to supersymmetric ones.

We had seen in §2.1, eq.(2.14), that when only e_i fluxes are excited the potential energy is invariant under a change in sign of the four-form fluxes, $e_i \to -e_i$, as long as the axions, b_a all vanish. In fact this is more generally true and follows directly from the IIA supergravity action where the F_4 dependence arises in the term,

$$S_{IIA} = -\frac{1}{2} \int d^{10}x \sqrt{-g} |\tilde{F}_4|^2 + \cdots \tag{4.60}$$

with,

$$\tilde{F}_4 = F_4 - F_2 \wedge B_2 - \frac{m_0}{2} B_2 \wedge B_2, \tag{4.61}$$

As long as B_2 vanishes[24], taking

$$F_4 \to -F_4, \tag{4.62}$$

gives the same action.

In contrast the conditions for supersymmetry *do* care about the sign of the fluxes, as we have discussed extensively above. Now in the Type 2) vacua all the four- form flux has a sign opposite to that required by supersymmetry. This means that starting

---

[24]More correctly we mean the axions which arise from B_2 should vanish.

with a vacuum of Type 2) we can construct a susy vacuum with the same energy by reversing all the F_4 fluxes. This susy vacuum will also have the same expectation values for the Kahler moduli and the dilaton.

Now consider a possible decay of a Type 2) vacuum to a susy vacuum of this type. By reversing the sign of all the fluxes we can relate this to the decay of a susy vacuum to a non-susy Type 2) vacuum. The vacuum energies of the initial and final vacua in the first decay and its partner decay are the same. The domain wall in the second case carries charges opposite to the first one. If the first decay is mediated by a $D4$ brane wrapping some cycle, the partner is mediated by the anti $D4$ brane wrapping the same cycle. Thus the two domain walls must also have the same tension. It then follows that the first decay of the non-susy vacuum can be allowed iff the partner susy vacuum can decay. But on general grounds one expects the susy vacua to be stable. We therefore conclude that the Type 2) non-susy vacuum we started with also cannot decay.

It is clear that a similar argument would also work if instead of considering the decay of the Type 2) vacuum to a susy vacuum we considered its decay to another Type 2) or a Type 3) vacuum. In both of these cases the axions are not turned on. By reversing all the four- form fluxes we can relate this to the decay of the susy vacuum to a susy vacuum in the first case, or the decay of a susy vacuum to the partner of a Type 3) vacuum in the second case. Both should not occur, given the stability of susy vacua. The partner for the Type 3) case is a vacuum with $e_i < 0, e_A > 0$. This is case 4) in the classification of §2. Here the b_A axions are tachyonic and lie below the BF bound, but this does not invalidate the argument above.

How general is this argument which ensures the stability of the Type 2) vacua by relating it to partner susy vacua? Our discussion above is based on the thin wall approximation in supergravity. And holds if the true and false vacuum have vanishing values for the axions. In the thin wall approximation only the $D4$ brane contribution to the domain wall tension is important, and this is clearly the same in the non-susy vacuum decay and its partner. Going beyond, one can argue that the domain wall tension continues to be equal in the two cases if the moduli contribution is included in the tension, as long as the axions are not activated in the domain wall. This follows from the fact that the potential energy and Kinetic energy terms all respect the flux reversal symmetry in the absence of axions. Since the axions vanish in both the true and false vacuum there is no reason as such for them to get activated, but for thick enough walls where the moduli under go big excursions this could happen anyways as a way of reducing the tension. If so, eventually for a thick enough wall the argument would break down.

Even for decays which are well described by the thin wall approximation sub-leading correction are important in the marginal case. We had found above that decays of Type 2) to Type 2) vacua are marginal if the $D4$ brane wraps a susy cycle. This result is easy to understand in light of the above discussion, since the partner

susy decay would be now mediated by a BPS domain wall. However in the non-susy Type 2) decay case, one expects that the marginal nature is only approximate and eventually corrections lead to the decay being either allowed or disallowed. The corrections responsible for this might arise as corrections to the thin wall approximation itself, as we have mentioned above, or they might require going beyond the sugra approximation and including α' and g_s corrections. We leave an exploration of such questions for the future.

Finally the argument above applies only for decays of the Type 2) vacua to others where the axions are not turned on. All the stable vacua we have explored in this paper are of this type, but there could be other vacua where the b_A axions have non-zero expectation values. The argument above says nothing about the possible decays of Type 2) vacua in such cases and this would have to be examined on a case by case basis.

4.9 More on Supersymmetric Domain Walls

Let us end this section with some more comments on susy domain walls. We had mentioned in the discussion around eq.(4.21) that $D4$ branes which wrap holomorphic or antiholomorphic cycles preserve susy. More accurately if we take Type IIA on the Calabi-Yau manifold without flux the $D4$ brane wrapping such a cycle will preserve half the supersymmetries, i.e., $\mathcal{N} = 1$. If we now turn on flux to preserve $\mathcal{N} = 1$ susy then only one of the two cases, either the holomorphic or antiholomorphic cycle, preserves the surviving $\mathcal{N} = 1$ susy [27]. That only one of the two cases could preserve susy at best is easy to understand from the requirement of force balance. The antiholomorphic case can be thought of as the anti D4 wrapping the same cycle. If the attractive gravitational force cancells the RR repulsion for the brane it will not cancell for the anti-brane and vice-versa.

It is easy to see that a susy brane leads to a marginal decay. In this case the tension is given by T_L and the energy difference, $\epsilon = -3e^K\Delta|W|^2$. It is then easy to see that the condition for marginality,

$$\frac{\epsilon}{3T} - \frac{T}{4} = \sqrt{\frac{|U|}{3}}, \tag{4.63}$$

is met, where U is the cosmological constant. The tension is given by T_L in the probe approximation. Going beyond would require including changes in the moduli which arise because the brane causes the flux to jump. One expects the susy branes to continue to be marginal even then. Susy domain walls where moduli fields vary have been discussed in [28], [29], [30], [31], where it was shown that the walls are indeed marginal [25]. In this analysis the fluxes (which are parameters in the superpotential)

---

[25]This is true only when the superpotential does not vanish in between the two vacua, otherwise the wall tension is too big. In our case, starting with the probe approximation and including corrections, the superpotential will not vanish. However for larger changes the resulting analysis might be more involved.

were held fixed. One could try to include the changes of flux in the analysis of these authors as well, but we leave this for the future.

Finally, in practice given the charges carried by the domain wall it is not always easy to decide whether a corresponding supersymmetric cycle exists. As a special case we can consider the orbifold theory and linear branes, for which the z^i coordinates are linear functions of $\sigma, \bar{\sigma}$. Even in this simple case, the existence of a supersymmetric cycle translates into a fairly intricate number theoretic constraint on the wrapping numbers of the $D4$ brane, as discussed in [27]. Things simplify if the integers δn_i are large, $|\delta n_i| \gg 1$. Now, upto fractional corrections, which are of order $1/\sqrt{|n|}$, we can approximate, $\delta n_i \simeq \pm m_i^2$, to be a perfect square. The only obstruction to having a susy brane then arises due to the signs of the δn_i. If the δn_i's all have the same sign then a susy cycle exists, else it does not exist.

5. More General Decays

In this section we consider domain walls which carry more general charges.

The general vacuum with all fluxes turned on was discussed in §2.4. The ground state energy for different vacua can be calculated by replacing e_a in formulae obtained in the case with $m_a = 0$, with eq.(2.67).

Our discussion of domain walls will follow that in §4 above. Given a domain wall with some charges, the change in the superpotential provides a lower bound on its tension. Below we will then calculate this lower bound, T_L and compare with an upper bound T_U defined in eq.(4.27).

We calculate T_L by keeping the moduli which appear in the superpotential to be fixed. We will come back to justifying this probe approximation below when we also discuss the validity of the thin wall approximation. For the superpotential, eq.(2.63), the change due to a domain wall carrying charges, $(\delta e_0, \delta e_a, \delta m_a)$ is,

$$\Delta W = \delta e_0 + \delta e_a t_a + \kappa_{abc} \delta m_a t_b t_c. \tag{5.1}$$

It is useful to express this in terms of the real and imaginary parts of $t_a = b_a + iv^a$, and in terms of $\hat{e}_a$,

$$\delta \tilde{e}_a = \delta e_a + m_0 \kappa_{abc} \delta m_b b_c. \tag{5.2}$$

This gives,

$$\Delta W = \delta e_0 + \delta e_a b_a + \frac{\kappa_{abc}}{2} \delta m_a (b_b b_c - v_b v_c) + i \delta \hat{e}_a v_a. \tag{5.3}$$

From here using eq.(4.17) we get,

$$T_L = 2e^{K/2}|\Delta W| = \frac{e^{2D}}{\sqrt{2vol}} \sqrt{(\delta \hat{e}_a v_a)^2 + ([\delta e_0 + \delta e_a b_a + \frac{\kappa_{abc}}{2} \delta m_a b_b b_c] - \frac{\kappa_{abc}}{2} \delta m_a v_b v_c)^2}. \tag{5.4}$$

The axion fields b_a which appear above have a vacuum expectation value,

$$b_a = \frac{m_a}{m_0}. \tag{5.5}$$

It is worth pausing to discuss the physics behind this expression. The 6-brane component of the domain wall gives rise to an induced 4 brane component because the axions b_a are now-non-zero. This is responsible for the shift in δe_a, eq.(5.2). The second term within the square root arises from a $D2$ brane and a $D6$ brane component. The $D2$ brane component included a contribution due to induced $D2$ brane charge which arises from the $D4$ brane and $D6$ brane components in the presence of the axions. These together with e_0 account for the term within the square brackets.

Let us now compare this with T_U.

For a Type 2) - Type 2) decay this is given by replacing $(\delta e_i, \delta e_a)$ in eq.(4.47) by their hatted counterparts giving,

$$T_U = \frac{e^{2D}}{\sqrt{2vol}} |\delta \hat{e}_a v_a|. \tag{5.6}$$

Thus we see that for this case now $T_L \geq T_U$ and the decay is at best marginal. The marginal case arises when the second term within the square root in eq.(5.4) vanishes. For this to happen the sum total of the $D2$ brane charge and $D6$ brane must vanish. In addition the tension must equal the lower bound, this would require the brane configuration to be supersymmetric.

For Type 3) -Type 3) we obtain T_U by replacing the δe_a by $\delta \hat{e}_a$ in eq.(4.51). This gives,

$$T_U = \frac{e^{2D}}{\sqrt{2vol}} |\delta \hat{e}_i v_i + 5\delta \hat{e}_A v_A|. \tag{5.7}$$

Now we see that T_U can be greater than T_L. For example this can happen if the sum of the D2 and D6 brane charges vanish and the δe_i and δe_A fluxes have opposite sign [26]. In such cases if the tension is equal to T_L or close to it the decay will occur.

Similarly one finds that decays of Type 3) to Type 2) can indeed occur. And also one finds that decays of Type 2) to Type 3) cannot occur because $T_L > T_U$. We skip some of the details here.

Let us end with three comments. First, in this section we have not discussed the constraints imposed by the thin wall approximation. This requires that the contribution the moduli make to the domain wall tension is smaller than the D brane contribution. It is straightforward to evaluate the moduli contribution and then impose this constraint; we will spare the patient reader the tedious details.

---

[26]Remember that v_i, v_A have opposite signs, so when this happens the two contributions actually add.

Second, one of the conclusions that follows from our analysis above is that Type 2) vacua continue to be stable even after decays involving the most general kind of brane are considered. We had argued in §4 that there was a pairing symmetry which related the Type 2) vacua to susy vacua and this explained their stability. However this symmetry required that the b_A axions are not turned on. Now, with the most general kind of interpolating brane, m_a will in general undergo a change so that the b_a fields will become non-zero even if they vanish to begin with.

It turns out that while there is no exact symmetry which relates Type 2) vacua to susy ones in general, there is an approximate symmetry of this type to leading order in the change in fluxes, as is discussed in Appendix D. The calculations above have been carried out only at leading order. For example we have calculated the change in vacuum energy density ϵ and thus T_U to leading order in δe_a and δm_a. Thus this approximate symmetry provides an explanation for the stability of the Type 2) vacua in our approximations. Subleading corrections beyond first order would not be important for decays which are ruled out at leading order. However they are important in the marginal case. The fact that the symmetry is only an approximate one for the general case suggests that Type 2) vacua might well decay by nucleating a brane which includes a $D6$ brane component once such subleading effects are incorporated. We leave a detailed investigation of this question for the future.

Finally, one can also consider $D8$ and $NS5$ brane mediated decays. These lie outside the thin wall approximation. This too is discussed in Appendix D.

6. Discussion

• We have constructed two explicit classes of non-susy AdS vacua in this paper, denoted as Type 2) and Type 3). Both are perturbatively stable. We have found that several possible decays of these vacua to other susy and non-susy vacua with lower energy are disallowed since the tension of the interpolating domain wall is much too big. The underlying reason for this is the geometric nature of AdS space where volume and surface area grow at the same rate for a large bubble.

The Type 3) vacua do have allowed decays to some other Type 3) and Type 2) vacua. The Type 2) vacua were found to be stable in our approximations, although some decays are only marginally disallowed. It is important to go beyond our approximations to decide what happens in these marginal cases. By changing the sign of all the four-form fluxes the Type 2) vacua are turned into susy vacua with the same energy. We argued, within our approximations, that the stability of the susy vacua then ensures the stability of their Type 2) partners. This protection mechanism might well be more robust and perhaps extends even beyond leading order, but one expects it to eventually fail, tipping the marginal decays one way or another. We leave an analysis of this for the future.

• Our analysis was carried out by considering a specific model of IIA theory on the blown-up $T^6/(Z_3 \times Z_3)$, after including the effects of flux and a further orientifolding. However some of our conclusions are more general and apply to IIA on any Calabi-Yau manifold with fluxes. E.g., Type 2) vacua, obtained by flipping the sign of all the four-form fluxes exist as extremum of the potential in general, since in the absence of axions coming from the B_2 field, IIA sugra will continue to have the symmetry, eq.(4.62). However their perturbative stability is not guaranteed in general, since some of the axions could lie below the BF bound in these vacua.

• A small bulk rate of decay leads to a diverging decay rate in the boundary as we had discussed in the introduction. What is the dual description of this in the boundary CFT? In the bulk the divergence arises after integrating over all radial locations of the instanton, due to the diverging bulk volume. It is tempting to speculate that in the boundary there is a corresponding one-parameter family of instantons, parametrised by their size. And summing over the different sizes then gives rise to this divergence, which arises in the CFT due to instantons of very small size [27].

It might seem that the the divergence mentioned above can be controlled by introducing a cut-off at a large and finite radial location in the bulk. Conformal invariance would not be exact now but would be an approximate symmetry in the deep IR. However a more detailed analysis is needed, depending on the kind of instability one is dealing with, before one can be sure. It could be that the detailed nature of the boundary conditions at the cut-off play a significant role even in the IR [28]. We leave a detailed understanding of this divergence in the boundary theory and related issues about controlling it also for the future.

7. Acknowledgements:

---

[27]We are grateful to G. Mandal, S. Kachru and S. Minwalla for a discussion of these issues.

[28]This is more of a worry, in our minds, for the kind of decays discussed in this paper nucleated by a D brane rather than decays in non-susy orbifolds [2] nucleated by an instanton analogous to the one responsible for the decay of the KK vacuum [32]. In the D-brane case the RR repulsion dominates over gravitational attraction and results in a runaway $-\phi^6$ potential arising in its world volume action. This could potentially cause an instability in the quantum theory whose cure depends delicately on the correct boundary conditions.

8. Appendix A : Blow up modes

In this appendix we give the details of the calculations involved in §2.3. We give the potential after including the blow up modes and discuss its minimization. We work in the regime where $|\frac{e_A}{e_i}| \ll 1$. The potential can be then expanded in the small parameter $\frac{|e_A|^{3/2}}{|e_i|^{3/2}}$.

The superpotential W and Kahler potential K are given as in eq(2.46) and eq(2.48). We have already found in eq(2.40) that with $e_A = 0$, the axions of the untwisted sector which vanish at the extrema lie above the BF bound. Now their masses will receive corrections of order $(|e_A|/|e_i|)^{3/2}$ but these are small and so their masses will continue to lie above the B.F bound.

The various Kahler derivatives to the leading order are as follows

$$K_{t_i \bar{t}_i} = \frac{1}{4v_i^2} \left(1 - \frac{\beta \sum v_A^3}{3vol} \right). \qquad K_{t_i \bar{t}_i} = -\frac{\beta \sum v_A^3}{24vol} \frac{1}{v_i v_b}.$$

$$K_{t_A \bar{t}_B} = -\frac{\beta v_A}{4vol} \delta_{AB}. \qquad K_{t_A \bar{t}_i} = \frac{\beta v_A^2}{8vol v_i}. \qquad (8.1)$$

Their inverses are

$$K^{t_i \bar{t}_i} = 4v_i^2 (1 + \frac{\beta \sum v_A^3}{12vol}). \qquad K^{t_i \bar{t}_i} = -4v_i v_b (\frac{\beta \sum v_A^3}{12vol}).$$

$$K^{t_A \bar{t}_B} = -\frac{4vol}{\beta v_A} \delta_{AB}. \qquad K^{t_A \bar{t}_i} = 2v_i v_A. \qquad (8.2)$$

The derivatives of superpotential as

$$D_{t_i} W = \partial_{t_i} W + K_{t_i} W = \left(e_i + \frac{m_0 vol}{v_i} - \frac{W}{2iv_i} (1 - \frac{\beta \sum v_A^3}{6vol}) \right).$$

$$D_{t_A} W = \partial_{t_A} W + K_{t_A} W = e_A + \frac{m_0 \beta v_A^2}{2} (1 - i\frac{b_A}{v_A})^2 - \frac{\beta v_A^2}{4ivol} W. \qquad (8.3)$$

Using these we can calculate the potential from

$$V = e^K \left(K^{t_a \bar{t}_b} D_{t_a} W D_{\bar{t}_b} \bar{W} - 3|W|^2 \right). \qquad (8.4)$$

We keep terms in the potential to leading order in $\frac{|e_A|}{|e_i|}^{\frac{3}{2}}$. Our analysis will show that

$v_A, b_A \sim \sqrt{|e_A|}$. Keeping all terms to this order then gives,

$$
V = \frac{e^{4\phi}}{2vol^3} \sum_i e_i^2 v_i^2 + \frac{p^2}{4} \frac{e^{2\phi}}{vol^2} + \sqrt{2} m_0 p \frac{e^{3\phi}}{vol^{\frac{3}{2}}} + \frac{m_0^2}{2} \frac{e^{4\phi}}{vol}
$$
$$
- \sum_A \frac{e_A^2}{\beta v_A vol} \frac{e^{4\phi}}{2vol} + \sum_A (\frac{e_A v_A}{vol}) \sum_i (e_i v_i) \frac{e^{4\phi}}{2vol^3}
$$
$$
- \frac{\beta \sum v_A^3}{vol} \left(\frac{5e^{4\phi}}{24vol^3} \sum_i e_i^2 v_i^2 + \frac{e^{4\phi}}{12vol^3} \sum_{i \neq j} e_i v_i e_j v_j \right.
$$
$$
\left. - \frac{p^2 e^{2\phi}}{12vol^2} + \sqrt{2} \frac{e^{3\phi} m_0 p}{vol^{\frac{3}{2}}} + \frac{m_0^2 e^{4\phi}}{12vol} \right)
$$
$$
+ \sum_A \frac{b_A^2}{vol} \left\{ \frac{e^{4\phi}}{2vol} \frac{m_0 e_A}{v_A} - \frac{3e^{4\phi}}{8vol^2} \sum_i (e_i v_i) m_0 \beta v_A + \frac{3 m_0 p e^{3\phi}}{4\sqrt{2} vol^{\frac{5}{2}}} \beta v_A \right.
$$
$$
\left. - \frac{e^{4\phi}}{8vol} m_0^2 \beta v_A \right\} . \tag{8.5}
$$

The minima of the zeroth order potential were given in eq(2.34),eq(2.35) and eq(2.36)[29]. Plugging these into the eq(8.5), we get

$$
V = V_0 + |V_0| \sqrt{\frac{15\kappa}{4\beta}} \sum_A \sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}} \left\{ \frac{1}{2x_A} - \frac{3 x_A \operatorname{sign}(e_A) \sum_i \operatorname{sign}(e_i)}{10} \right.
$$
$$
+ \frac{3x_A^3}{200} (1 + 2 \sum_{i \neq j} \operatorname{sign}(e_i e_j)) + \frac{y_A^2}{40} \left[-\frac{20 \operatorname{sign}(m_0 e_A)}{x_A} \right.
$$
$$
\left. \left. + x_A (29 + 9 \sum_i \operatorname{sign}(m_0 e_i)) \right] \right\} , \tag{8.6}
$$

where $x_A = -v_A \sqrt{\frac{\beta|m_0|}{|e_A|}}$ and $y_A = b_A \sqrt{\frac{\beta|m_0|}{|e_A|}}$. Thus the minimization of the blow up modes will depend upon the signs of various fluxes turned on.

Table(1) lists the different cases which arise from the different choices for signs of fluxes. For convenience we take $m_0 > 0$. For the other choice, $m_0 < 0$, results can be obtained by changing the sign of all the e_a fluxes. In the Table the nature of the extremum, if present, along the x_A direction is discussed in the second last column. The potential obtained for the y_A field after setting x_A to its extremum value is in the last column.

• Case 1 is the susy solution found by [1], who also discuss some aspects of the non-susy solutions.

---

[29]These may receive $\frac{|e_A|^{\frac{3}{2}}}{e_i}$ corrections, but they are immaterial for the potential to the leading order.

Case	e_1, e_2, e_3	e_A	Extrema of x_A, nature	Potential for y_A
1.	$-,-,-$	$-$	$\sqrt{\frac{10}{3}}$, minima	$\frac{1}{\sqrt{2}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2$
2.	$+,+,+$	$+$	$\sqrt{\frac{10}{3}}$, minima	$\frac{25}{4\sqrt{2}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2$
3.	$+,+,+$	$-$	$\sqrt{\frac{10}{21}}$, minima	$\frac{7\sqrt{7}}{4\sqrt{2}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2$
4.	$-,-,-$	$+$	$\sqrt{\frac{10}{21}}$, minima	$-\frac{5}{\sqrt{14}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2$
5.	$+,+,-$	$+$	No extrema	
6.	$-,-,+$	$-$	No extrema	
7.	$+,+,-$	$-$	$\sqrt{\frac{10}{3}}$, Inflection	$\frac{11}{2\sqrt{2}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2$
8.	$-,-,+$	$+$	$\sqrt{\frac{10}{3}}$, Inflection	$\frac{7}{4\sqrt{2}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2$

Table 1: Different choices of signs with $m_0 > 0$. The $\pm$ denotes the sign of the corresponding flux.

All the other cases below break supersymmetry.

• Case 2) and Case 3) are perturbatively stable vacua.

• Case 4 : The blow up mode has a tachyon. To see whether it is acceptable we compare its mass with the BF bound. Note that in this case the action for y_A is

$$S = \int dx^4 \left(K_{t_A \bar{t}_A} \partial_\mu t^A \partial^\mu \bar{t}^A - \frac{5}{\sqrt{14}}\sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}}|V_0|y_A^2 \right)$$
$$= \sqrt{\frac{\kappa}{\beta}}\sqrt{\frac{|e_A|^3}{|e_1 e_2 e_3|}} \int dx^4 \left(\frac{3\sqrt{2}}{20\sqrt{7}}\partial_\mu y_A \partial^\mu y_A - \frac{5}{\sqrt{14}}|V_0|y_A^2 \right). \tag{8.7}$$

Therefore mass of the blow up axion in this case is

$$M_{\text{axion}} = -\frac{50}{3}|V_0|. \tag{8.8}$$

Since the BF bound is $M_{BF} = -\frac{3}{4}|V_0|$, the BF bound is violated.

• Case 5 and 6 have no extrema at all (near the orbifold point).

• Case 7. and Case 8. have extrema for x_A at a point of inflection (first and second derivatives vanish). Therefore in these cases the next correction must be calculated to find if there is a minimum close to the point of inflection. This is determined by the sign of the the next order correction. We do not carry out this calculation here.

9. Appendix B: Moduli Tension

In this section we estimate the contribution the moduli make to the domain wall

tension. Let ϕ be a canonically normalized scalar field with action,

$$S_{scalar} = \int d^4x [-\frac{1}{2}(\partial\phi)^2 - U(\phi)]. \tag{9.1}$$

Consider a domain wall in flat space which is translationally invariant along x, y directions. The scalar field varies along the z direction in the domain wall. The tension of the wall receives a contribution from the gradient term and the potential energy term,

$$T = \int dz [\frac{1}{2}(\frac{d\phi}{dz})^2 + U(\phi)]. \tag{9.2}$$

The scalar profile is obtained by balancing these two. If the scalar has a mass M, the spatial variation is of order $1/M$. For two vacua which are separated by $\Delta\phi$ in field space the resulting tension is then of order,

$$T \sim M(\Delta\phi)^2. \tag{9.3}$$

Strictly speaking in making the estimate, eq.(9.3), we are assuming that the two vacua separated by the domain wall are "close by" in field space and that the mass M for fluctuations about both of them is approximately the same. As we discuss in the main text the thin wall approximation within which we work requires that the moduli contribution to the tension is much smaller than the D-brane contribution. This requirement often leads to the condition that the total change in the scalar, $\Delta\phi$, is sufficiently small, and therefore that the vacua are close by. Even when this is not true and the vacua are not sufficiently close by, the estimate in eq.(9.3) for the tension is often parametrically a good one.

In our case the domain walls which arise actually have two components, a D-brane component which is thin and a component which arises because the moduli vary across them. In such a situation the potential seen by the moduli jumps as one crosses the thin D brane, due to the jump in flux. Despite this complication, as we will see below, eq.(9.3) continues to be a good approximation to the tension.

Let us construct the scalar profile for such a domain wall in more detail. We take the D-brane, which is thin, to be located at $z = 0$ and the two vacua to be close together in field space for simplicity. Then the resulting potential can be approximated as,

$$\begin{aligned} U(\phi) &= U_- + M_-^2 (\phi - \phi_-)^2 \quad z < 0 \\ U(\phi) &= U_+ + M_+^2 (\phi_+ - \phi)^2 \quad z > 0. \end{aligned} \tag{9.4}$$

Here $U_\pm$ are the values of the minimum values of potential to the left and the right of the wall, and $\phi_\pm$ are the vacuum expectation values of the scalar field and $M_\pm^2$ are the masses about these minima. Solving for the scalar with the correct boundary

conditions gives

$$\phi(z) - \phi_- = Ae^{M_-z} \quad z < 0$$
$$\phi(z) - \phi_+ = -Be^{-M_+z} \quad z > 0. \tag{9.5}$$

Continuity of ϕ and its first derivative at $z = 0$ then leads to,

$$\phi_- + A = \phi_+ - B$$
$$M_-A = M_+B. \tag{9.6}$$

From these we get that

$$A = \frac{(\phi_+ - \phi_)M_+}{M_+ + M_-} \quad B = \frac{(\phi_+ - \phi_)M_-}{M_+ + M_-}. \tag{9.7}$$

If $\Delta\phi = \phi_+ - \phi_-$ is small we can approximate $M_+ \sim M_-$ to leading order and then it is easy to see that the resulting tension is of order eq.(9.3).

10. Appendix C: More On Four-Form flux and Normalization of Kahler form

10.1 The Jump in the Four Form

Let us consider the orbifold limit and take a single $D4$-brane which wraps the first T^2, spanned by the (x_1, y_1) directions. The first Z_3 symmetry (denoted by S in §2) has fixed points. We take the $D4$-brane to be located, in the remaining internal directions (x_2, y_2, x_3, y_3), away from these fixed points. The $Z_3 \times Z_3$ orbifold action then does not result in any self identifications on the world volume of the $D4$-brane. Such a $D4$-brane corresponds to winding number $\delta n^1 = 1$ in our notation. We would like to calculate the related jump in the F_4 flux.

The $D4$-brane is electrically charged with respect to F_6 which is governed by the action

$$S = -\frac{1}{2\kappa_{10}^2} \int dx^{10}\sqrt{-g}F_{6ABCDEF}F_6^{ABCDEF} + \sqrt{2}\mu_4 \int C_5. \tag{10.1}$$

The last term arises from the world volume action of the $D4$, and the extra factor of $\sqrt{2}$ in front of this term is because we use the notation of [1] in which $C_{RR} = \frac{C_{RR}^{Polch}}{\sqrt{2}}$, where C_{RR}^{Polch} refers to the conventions of [33]. We will use this action to calculate the jump in F_6 and from there the jump in F_4.

The non-compact directions are those of AdS_4 with metric

$$ds^2 = dr^2\frac{R^2}{r^2} + \frac{r^2}{R^2}(-dt^2 + d\mu_1^2 + d\mu_2^2). \tag{10.2}$$

We work in the orbifold limit and the internal space has the metric eq.(2.12).

The $D4$ brane extends along (t, μ_1, μ_2) of the non-compact directions and is located at $r = r_0$. It results in a change in the $(tr\mu_1\mu_2x_1y_1)$ component of the six-form. For simplicity we take the $D4$ brane to be smeared out in the remaining internal directions, i.e. the (x_2, y_2) and (x_3, y_3) directions. Then the six-form sourced by the $D4$ brane will be independent of all the internal directions. Its jump can be calculated by carrying out the integral over the internal space in eq.(10.1) and working with the resulting 4 dimensional action.

This gives,

$$S = -\frac{1}{2}(\frac{1}{9})(\frac{\sqrt{3}}{2})^3(\gamma_1\gamma_2\gamma_3)\int d^4x\sqrt{-g_4}F_{6\ tr\mu_1\mu_2x_1y_1}F^{6\ tr\mu_1\mu_2x_1y_1} \tag{10.3}$$

$$+\sqrt{2}\mu_4\frac{\sqrt{3}}{2}\gamma_1\int C_{5\ t\mu_1\mu_2x_1y_1}dtd\mu_1d\mu_2. \tag{10.4}$$

The coefficient of the first term on the rhs above arises as follows. The factor of $\frac{1}{9}$ arises because the $T^6/(Z_3)^2$ orbifold has a volume which is smaller than that of T^6 by a factor of $1/9$. In addition after including the Z_2 orientifolding the volume has a factor of $1/2$, this cancels against the factor of κ_{10}^2 in our conventions since $2\kappa_{10}^2 = 1$.

From this action, we get the equation of motion,

$$\frac{\gamma_2\gamma_3}{12}\partial_r\left(\sqrt{-g_4}F_6^{012rx_1y_1}\right) = \sqrt{2}\mu_4\delta(r - r_0). \tag{10.5}$$

Integrating with respect to r then gives the jump in F_6

$$\Delta F_{6\ 012rx_1y_1} = \frac{r^2}{R^2}\frac{12\sqrt{2}\gamma_1}{\gamma_2\gamma_3}\mu_4. \tag{10.6}$$

In our normalization the related component of the Four form flux is given by,

$$F_4 = 4\left(\frac{3}{\kappa}\right)^{\frac{1}{3}}e_1dx_2 \wedge dy_2 \wedge dx_3 \wedge dy_3. \tag{10.7}$$

Using the duality,

$$F_{6\ 012rx_1y_1} = \sqrt{g}\,\epsilon_{012rx_1y_1x_2y_2x_3y_3}\,F_4^{x_2y_2x_3y_3}$$
$$= \left(\frac{r}{R}\right)^2\left(\frac{3}{\kappa}\right)^{\frac{1}{3}}\left(\frac{4e_1\gamma_1}{\gamma_2.\gamma_3}\right), \tag{10.8}$$

then gives,

$$\Delta F_{6\ 012rx_1y_1} = \frac{r^2}{R^2}\left(\frac{4\gamma_1}{\gamma_2\gamma_3}\right)\left(\frac{3}{\kappa}\right)^{\frac{1}{3}}\delta e_1. \tag{10.9}$$

Using the jump in F_6 calculated in eq(10.6) then gives the jump in F_4 to be,

$$\delta e_1 = 3^{\frac{2}{3}}\sqrt{2}\kappa^{\frac{1}{3}}\mu_4. \tag{10.10}$$

For a $D4$ brane which wraps the first T^2 δn_1 times the rhs would be multiplied by the factor δn_1.

More generally consider a $D4$ brane which wraps a two-cycle with wrapping numbers δn_a. This results in a jump of the fluxes δe_a. Now δe_a and the winding numbers δn_a will be related by the factor on the rhs of eq.(10.10) for each a.

The discussion above should actually have been carried out more correctly by including various Cherns Simons terms, and working with the gauge invariant four-form $\tilde{F}_4$ and its dual $\tilde{F}_6$. It is easy to see that doing this gives the same results.

10.2 Normalization of the Kahler Form

Consider the two-cycle of the previous subsection which extends along the first T^2 and is located at a point in the other two T^2's. The $Z_3 \times Z_3$ orbifold symmetry does not result in any self-identifications on the two-cycle, which has nine images under this symmetry.

Now in our notation the Kahler form is defined so that its pull back onto the two-cycle after integration gives,

$$\int P[J] = v_1. \tag{10.11}$$

The actual size of the T^2 from the metric eq.(2.12) is

$$\int \gamma_1 dx_1 dy_1 = \frac{\sqrt{3}}{2}\gamma_1. \tag{10.12}$$

Using the relation eq.(2.13) one gets,

$$\int \gamma_1 dx_1 dy_1 = 3^{2/3}\kappa^{1/3}\int P[J]. \tag{10.13}$$

This shows that with our definition the integral of the Kahler form must be multiplied by the factor $3^{3/2}\kappa^{1/3}$ to get the correct area of the two-cycle. In effect this means that the relation between the Pull-back of the metric and the Kahler form has this extra factor on the lhs of eq.(4.21). The right hand side of eq.(4.25) must therefore be multiplied by this extra factor also to get the correct lower bound T_L.

11. Appendix D

11.1 Approximate Symmetry between Type 2) vacua decays and susy vacua decays

Consider a general Type 2) vacua with all fluxes activated. This vacuum meets the conditions, $\text{sign}(m_0\hat{e}_a) > 0$ for all a. Now consider a partner susy vacuum with $\hat{e}_a$ reversed in sign and the same values for all the other fluxes, m_0, p and m_a. The

ground state energy only depends on $|\hat{e}_a|$. So the partner vacuum has the same vacuum energy density.

Now consider a Type 2) vacuum decay mediated by a brane with charges, $(\delta e_0, \delta e_a, \delta m_a)$. The resulting final vacuum has charges, $e'_a = e_a + \delta e_a$ etc. The corresponding values of

$$\hat{e}'_a = e'_a + \frac{\kappa_{abc}}{2m_0}(m_b + \delta m_b)(m_c + \delta m_c). \tag{11.1}$$

Let us relate this to decay of the partner susy vacuum mediated by the corresponding anti-brane which carries opposite charges, $(-\delta e_0, -\delta e_a, -\delta m_a)$. The domain walls in the two decays have the same tension.

A little algebra shows that values of the flux $\hat{e}_a$ in the resulting final vacuum which we denote by $\tilde{\hat{e}}'_a$ is given by,

$$\tilde{\hat{e}}'_a = -\hat{e}'_a + O(\delta m_a)^2. \tag{11.2}$$

Thus the two values in eq.(11.1), eq.(11.2) are the same upto corrections of $O(\delta m_a)^2$. This is significant because as was mentioned above the ground state energy only depends on $|\hat{e}_a|$. Since this is the same for the two final vacua at leading order, and since the Type 2) vacuum and its susy partner also have the same ground state energy we find that the energy difference ϵ is the same for the two decays.

So to summarize, we see that for a Type 2) vacuum there is a partner susy vacuum. Any decay of the Type 2) vacuum maps to a corresponding decay of the susy vacuum, where the tension of the brane mediating the two decays is the same and the energy difference between the true and false vacuum for the two decays is also the same to leading order. It then follows that the first decay can occur iff the second one does. Since the susy vacuum is stable one finds that the Type 2) vacuum will also not decay, more correctly it can at best decay marginally.

11.2 D8-brane and NS5-brane mediated decays

Finally we consider the possibility of the domain wall also containing a D8 and NS5 brane component. We will find that such decays do not lie within the thin wall approximation. The $D8$ brane wraps all six directions of the internal space and causes a jump in F_0, i.e. in m_0. The $NS5$ brane causes a jump in H_3, i.e. in p. Both must change in conjunction so that the tadpole condition eq.(2.11) continues to hold.

A $D8$ brane has tension,

$$T_8 \sim vol\, e^{-\phi}\left(\frac{e^{2\phi}}{vol}\right)^{\frac{3}{2}} \sim |\hat{e}_1 \hat{e}_2 \hat{e}_3|^{-\frac{3}{4}}. \tag{11.3}$$

A $NS5$ brane has tension which is comparable,

$$T_{NS5} \sim e^{-2\phi}\sqrt{vol}\left(\frac{e^{2\phi}}{vol}\right)^{\frac{3}{2}} \sim |\hat{e}_1 \hat{e}_2 \hat{e}_3|^{-\frac{3}{4}}. \tag{11.4}$$

The fractional change in m_0 that results is of order unity, i.e., $\frac{\delta m_0}{m_0} \sim 1$. Similarly in p. This causes a fractional jump in the v_i moduli $\frac{\delta v_i}{v_i}$, also of order unity giving, for the moduli contribution to the domain wall for the v_i moduli,

$$T_{v_i \text{mod}} \sim (\frac{\delta v_i}{v_i})^2 M \sim |\hat{e}_1 \hat{e}_2 \hat{e}_3|^{-\frac{3}{4}}.\tag{11.5}$$

We see that this is comparable to the $D8$ and $NS5$ contributions. Thus the thin wall approximation is not good.

References

[1] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, JHEP **0507**, 066 (2005) [arXiv:hep-th/0505160].

[2] G. T. Horowitz, J. Orgera and J. Polchinski, Phys. Rev. D **77**, 024004 (2008) [arXiv:0709.4262 [hep-th]].

[3] S. R. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).

[4] M. Duff, arXive: hep-th 021062.

[5] A. Dabholkar, Phys. Lett. B **402**, 53 (1997) [arXiv:hep-th/9702050].

[6] P. K. Tripathy and S. P. Trivedi, JHEP **0603**, 022 (2006) [arXiv:hep-th/0511117].

[7] J. Polchinski and E. Silverstein, arXiv:0908.0756 [hep-th].

[8] D. Gaiotto and A. Tomasiello, JHEP **1001**, 015 (2010) [arXiv:0901.0969 [hep-th]].

[9] M. R. Douglas and S. Kachru, Rev. Mod. Phys. **79**, 733 (2007) [arXiv:hep-th/0610102].

[10] T. W. Grimm and J. Louis, Nucl. Phys. B **718**, 153 (2005) [arXiv:hep-th/0412277].

[11] J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, Nucl. Phys. B **715**, 211 (2005) [arXiv:hep-th/0411276]; G. Villadoro and F. Zwirner, JHEP **0506**, 047 (2005) [arXiv:hep-th/0503169].

[12] S. Kachru and A. K. Kashani-Poor, JHEP **0503**, 066 (2005) [arXiv:hep-th/0411279].

[13] L. J. Romans, Phys. Lett. B **169**, 374 (1986).

[14] D. Lust and D. Tsimpis, JHEP **0502**, 027 (2005) [arXiv: hep-th/0412250]

[15] M. Dine, G. Festuccia and A. Morisse, arXiv:0901.1169 [hep-th]

[16] D. Harlow, arXiv:1003.5909 [hep-th].

[17] D. Lust, F. Marchesano, L. Martucci and D. Tsimpis, arXiv:0807.4540.

[18] Ulf H. Danielsson, Sheikh Shajidul Haque, Gary Shiu, Thomas Van Riet, arXiv:0907.2041v4 [hep-th].

[19] P. Koerber and S. Kors, arXiv:1001.0003.

[20] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B **261** (1985) 678; A. Strominger, "Topology Of Superstring Compactification," in *Unified String Theories*, M. Green and D. Gross editors, World Scientific (1986).

[21] S. Chiossi and S. Salamen, math.dg/0202282.

[22] B. S. Acharya, F. Benini and R. Valandro, JHEP **0702**, 018 (2007) [arXiv:hep-th/0607223].

[23] J. D. Brown and C. Teitelboim, Nucl. Phys. B **297**, 787 (1988).

[24] J. Gutowski, G. Papadopoulos and P. K. Townsend, Phys. Rev. D **60**, 106006 (1999) [arXiv:hep-th/9905156].

[25] S. S. Gubser, arXiv:hep-th/0201114.

[26] O. J. Ganor and J. Sonnenschein, JHEP **0205**, 018 (2002) [arXiv:hep-th/0202206].

[27] O. Aharony, Y. E. Antebi and M. Berkooz, JHEP **0802**, 093 (2008) [arXiv:0801.3326 [hep-th]].

[28] S. Weinberg, Phys. Rev. Lett. **48**, 1303 (1982).

[29] M. Cvetic, S. Griffies and S. J. Rey, Nucl. Phys. B **389**, 3 (1993) [arXiv:hep-th/9206004].
M. Cvetic, S. Griffies and S. J. Rey, Nucl. Phys. B **381**, 301 (1992) [arXiv:hep-th/9201007].

[30] M. Cvetic and H. H. Soleng, Phys. Rept. **282**, 159 (1997) [arXiv:hep-th/9604090].

[31] A. Ceresole, G. Dall'Agata, A. Giryavets, R. Kallosh and A. D. Linde, Phys. Rev. D **74**, 086010 (2006) [arXiv:hep-th/0605266].

[32] E. Witten, Nucl. Phys. B **195**, 481 (1982).

[33] J. Polchinski, String Theory, Volume II, Cambridge University Press.