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Classification of Mira variables based on visual light curve shape

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Summary. — We have classified 368 Mira variables of M, S and C spectral classes based on the shape of their visual light curve. The classification is essentially based on Ludendorff's (1928) scheme and the light curves used are mainly from the compilation of Campbell (1955). The distribution of light curves over period, mean amplitude, light curve asymmetry factor *f*, period variability, and spectral class at maximum are discussed.

Key words: Miras — long period variable — visual light curves-variable stars — cool stars.

1. Introduction.

Mira variables are cyclic long period variables. The shape of their visual light curves differs from one star to another, and sometimes, from on cycle to another. However, these light curves can be classified broadly into a few classes. Campbell (1925) classified 124 Mira stars into seven classes from I to VII. Ludendorff (1928), on the other hand, classified about 300 stars into three main classes, divided into ten subclasses. Visual light curve asymmetry factor *f* (rise time/period) has been found to correlate well with certain observed properties in Mira variables (Bowers and Kerr, 1977 ; Vardya, 1985 ; Vardya *et al.*, 1986). It has recently been found (Vardya, 1987a, b) that visual light curve class is also a good parameter to describe some of the observed properties of the Mira variables. Ludendorff had classified Miras based on light curve observations prior to 1928. As a large amount of new observations have accumulated for many of these stars and of newer ones, it was felt appropriate to undertake afresh classification of Mira visual light curve shapes. As a basic data, we used the comprehensive compilation of light curves given by Campbell (1955). We have also used visual observations of long period variables in the American Association of Variable Star Observers (AAVSO) Report 38 (Mattei, 1983) for a limited extent. This way we have classified 368 stars, of which about 80 are new and many others which had questionable classifications or no subclassifications.

2. Ludendorff's classification scheme.

The classification of visual light curves that we have followed here is essentially that given by Ludendorff (1928). There are three main types α, β and γ, with ten subclasses as given below :

α : The ascending branch is noticeably steeper than the decending branch, and the minimum, barring a few exceptions, is always broader than the maximum.
- α_1 - Long flat minimum (about one-third to one-half the period) and normally a steep rise in luminosity.
- α_2 - The minimum still broad but no long flat minimum and the rise very steep in most cases.
- α_3 - The minimum not as broad as in α_2 but the rise somewhat steep.
- α_4 - As α_3 but rise not that steep.

β : The light curve is basically symmetric with minor or no difference in the steepness of the ascending and descending branches.
- β_1 - The maximum narrower than the minimum.
- β_2 - The maximum as narrow or flat as minimum.
- β_3 - The maximum broader than the minimum.
- β_4 - The maximum very broad and remains flat for some time.

γ : The ascending branch has step(s) or hump, or the light curve has a double maxima.
- γ_1 - Step(s) or hump in the ascending branch.
- γ_2 - Double maxima.

3. Classification.

We have classified all the stars, whose light curves are given in the compilation of Campbell (1955), if they are Mira variables, according to Kholopov (1985) for stars in Andromeda to Orion and Kukarkin *et al.* (1969) for the remaining stars. However, values marked by a dagger (†) are from Kukarkin *et al.* (1969) as Kholopov (1985) values appear to be misprint. The visual light curve classification (l.c.c.) is listed in column 7 of table I ; also given in the table are the variable star name, period rounded to nearest day, visual light asymmetry factor f (rise time/period), mean visual amplitude $\langle \Delta M_v \rangle$, and spectral class (Kholopov, 1985 ; Kukarkin *et al.*, 1969). Where available, spectral class at light maximum has also been given in column 6 (Keenan *et al.*, 1974). In column 8, we have given the classification as given by Ludendorff (1928). Column 9 gives the classification based on observations given in Mattei (1983) ; in general, the classification in this column is somewhat uncertain as coverage of observations at different phases is inadequate. The v in the period column indicates that the period has been found variable for that star (Kholopov, 1985 ; Kukarkin *et al.*, 1969) and an asterisk in the $\langle \Delta M_V \rangle$ indicates that the mean value is not available from Kukarkin *et al.* (1969) and the current value is given.

Figure 1 gives a plot of light curve classification, based on Campbell (1955) light curves as classified in this paper against that given by Ludendorff (1928) with definite, single type sub-classification. The number in each square refers to the number of stars. Most of the stars fall on the diagonal, the rest falling essentially one subclass off, except in the case of γ_1 (1955). The reasons for stars falling off-diagonally can partly be due to i) uncertainty in classification, ii) use of different light curves in the two classifications, iii) paucity of data points to clearly discern step(s) or hump in the ascending branch in the 1928 classification and iv) change in light curve with time. As the light curves on which Ludendorff's classification is based are not available to us, it is difficult to check some of these causes. Note also that many of the α_1 (1928) have been classified by us as α_2 (1955) as we really did not find a flat minimum of length one third the period or longer. Overall, the figure shows that our classification is similar to that given by Ludendorff (1928) except that ours is based on Campbell's (1955) visual light curves. The error in classification is, in general, not greater than one subclass. Even this uncertainty can be decreased if all the light curves are normalized in amplitude and period.

Most of the stars, visual light curves of which have been given in Mattei (1983) have scanty coverage over the whole cycle. We have therefore classified only those Miras from this sample which were reasonably covered over the whole cycle. In general, these classifications are less accurate than those based on the 1955 data.

Ludendorff (1928) has classified a number of stars with double classes, i.e. $\alpha_4-\beta_2$, $\gamma_1-\gamma_2$, $\alpha_4-\gamma_1$ etc. We have refrained from this and given the dominant type only. However, a few stars, which we were not very definite about because of the nature of light curve have pec (peculiar) or a question mark after the light curve class. If part of the light curve given by Campbell is dotted (unobserved), the light curve classification (1955) is followed by a right parenthesis to denote this fact.

l.c.c. (1955)

l.c.c. (1928)	α_1	α_2	α_3	α_4	β_1	β_2	β_3	γ_1	γ_2
α_1	3	10	2			1			
α_2		7	6	1				1	
α_3		4	21	6		1			
α_4	1		8	23	1			4	
β_1			1	4	8	3	1	1	
β_2					5	16		4	
β_3						6	10	1	
γ_1					1		1	7	
γ_2									2

FIGURE 1. — Comparison of the visual light curve classification of Mira variables as given by Ludendorff (1928) and those classified in this paper based on light curve compilation of Campbell (1955). The numbers in each square denote number of stars.

4. Distribution of stars.

Figure 2 gives the distribution of 312 M, 30 S and 22 C spectral type Miras among different visual light curve classes. There are no stars with β_4 class and there are only two stars with γ_2 type, both being M Miras. Most of the Miras belong to α_2, α_3 and α_4 classes, with another peak at β_2 and another secondary peak at γ_1, and the ratio of stars in the α, β and γ classes are 55 : 35 : 10. S

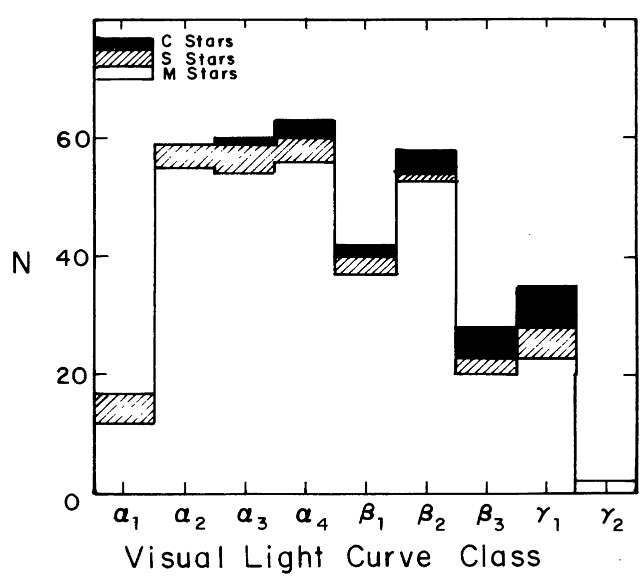

FIGURE 2. — Distribution of M, S and C Miras as a function of visual light curve class.

Miras are equally distributed but three fourth of the carbon stars are in the β_2, β_3 and γ_1 light curve classes and there are no α_1, α_2 and γ_2 class carbon Miras.

Figure 3 gives histograms of the fraction of M Miras as a function of period for different visual light curve classes. Peaks of α_1 and α_2 are in the range $P > 450$ days, and shifts to smaller periods as we go to α_3 and α_4. There are no α_1, α_2 and α_3 Miras with $P \leqslant 200$ days. The peak of β_1 is in the range 301-350 days, and shifts to lower periods as we go to β_2 and β_3. There are hardly any β Miras for $P > 350$ days. There are no γ_1 class stars with $P \leqslant 300$ days and the only two γ_2 stars in our sample have $P > 450$ days, 508 and 546 days to be exact.

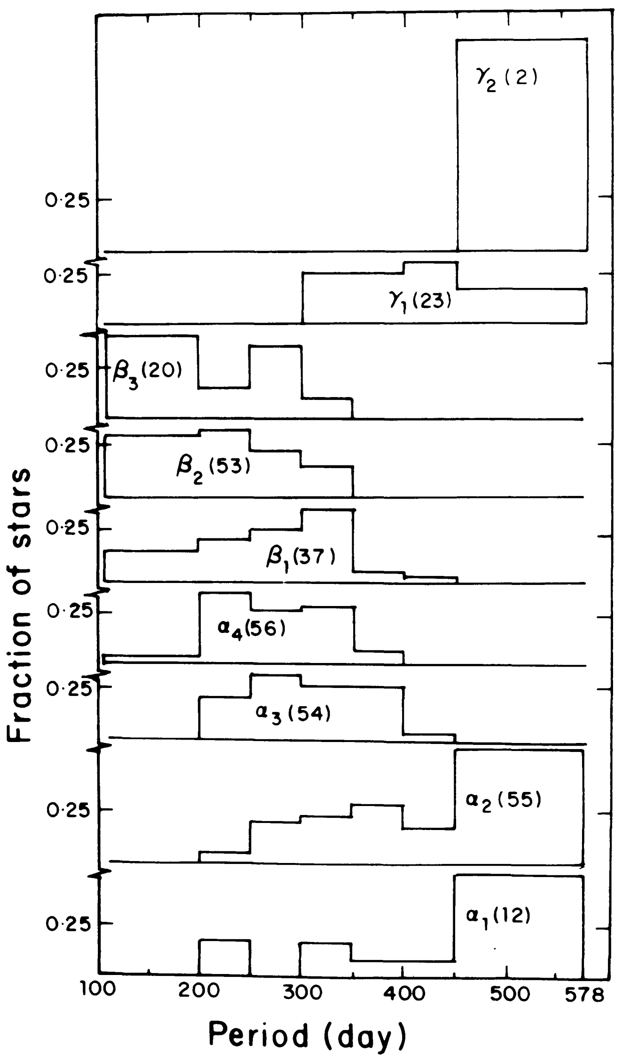

FIGURE 3. — Fraction of M Miras in different period range as a function of visual light curve class. The numbers inside the parenthesis denote the stars of that class.

Figure 4 gives histograms of the fraction of M Miras as a function of visual light curve class for different period ranges. For $P > 450$ days, α_1, α_2, γ_1 and γ_2 and for $P \leqslant 200$ days, α_4, β_1, β_2 and β_3 are the only contributors.

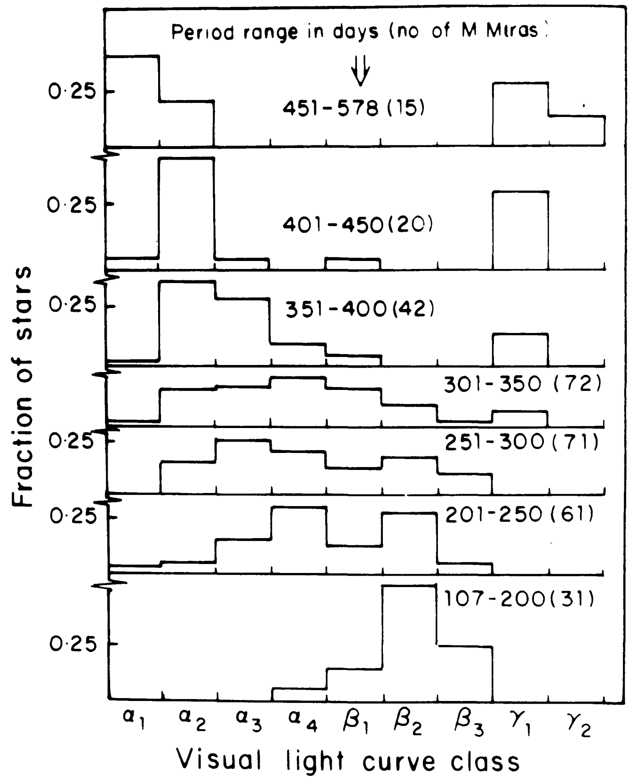

FIGURE 4. — Fraction of M Miras in different visual light curve class as a function of period range. Number of stars in each period range is given in the parenthesis.

Figure 5 gives the distribution of the mean visual amplitude, $\langle \Delta M_V \rangle$ for M, S, and C Miras. We had found for S Mira variables that the mean visual amplitude, rather than the amplitude itself, is a better quantity for correlation with other parameters (Vardya, 1985). Hence the choice of $\langle \Delta M_V \rangle$; it is a pity, therefore, that the 4th edition of the General Catalogue of Variable Stars does not give this quantity. Most of the Miras have $\langle \Delta M_V \rangle$ in the range 4-6^m. This is true for M Miras also, with steep fall for shorter or longer amplitudes. The credit for the maximum $\langle \Delta M_V \rangle$ of 9^m goes to a S Mira (RT Sco), and C Miras have, in general, smaller amplitudes than M and S stars. The peak of (ΔM_V) for M Miras lies in the range 4-5^m, for S stars 5-6^m, but for C stars 2-3^m.

Figures 6 gives the distribution of M Miras as a function of $\langle \Delta M_V \rangle$ for different visual light curve classes. The peak of the distribution for α_1, α_2, β_1, β_2 and β_3 is in the interval 4-5^m, for α_3 and γ_1 in 5-6^m and for α_4 and γ_2 in 4-6^m.

Figure 7 gives the distribution of f, the visual light curve asymmetry factor for M, S and C stars. Four-fifth of the Miras lie in the range $0.40 \leqslant f < 0.50$. However, for C Miras (22 stars), 32 % have $f \geqslant 0.50$ and none below $f = 0.40$. S Miras (20 stars) have 28 % in the range $f < 0.40$, compared to 13 % for M Miras (310 stars).

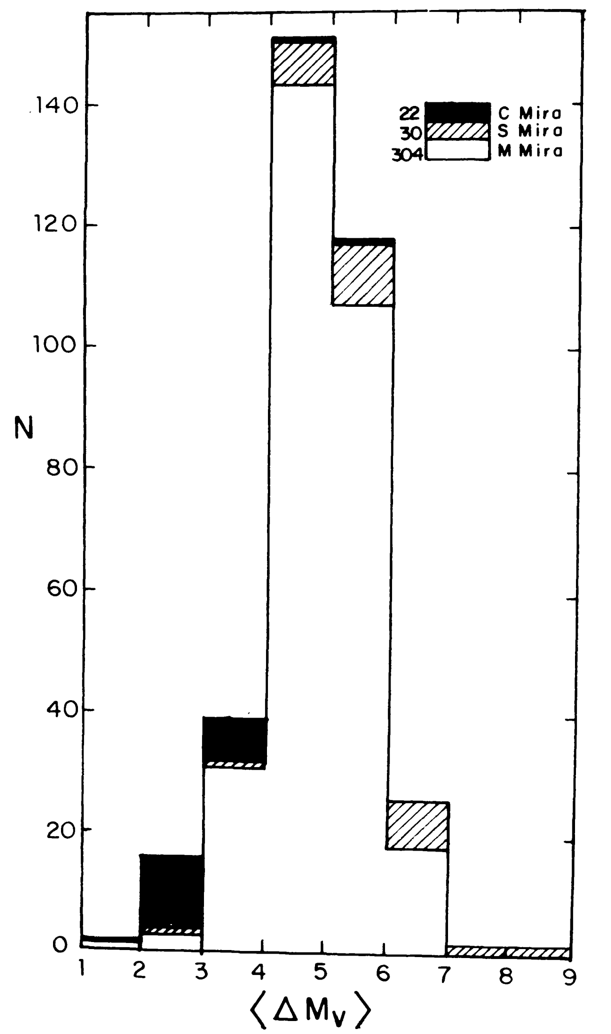

FIGURE 5. — Distribution of M, S, and C Miras as function of mean visual light amplitude, $\langle \Delta M_V \rangle$.

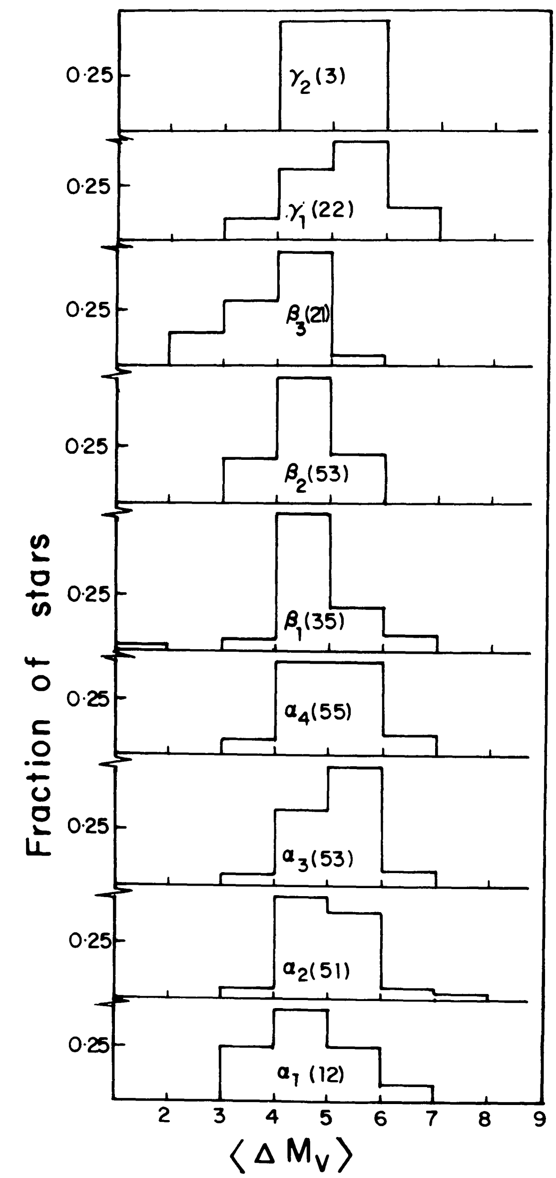

FIGURE 6. — Fraction of M Miras in different mean visual light amplitude range as a function of visual light curve class.

Figure 8 gives the distribution of M Miras as a function of f for different visual light curve type. About 70-80 % of stars of β_1, β_2 and β_3 visual light curve classes lie in the narrow range of $0.45 < f < 0.50$. The spread of f is maximum in γ_1 class.

Table II gives the mean value, dispersion and maximum and minimum value of f for different light curve classes. Note the sharp increase of $\langle f \rangle$ from (α_1, α_2, α_3) to α_4 and to (β_1, β_2, β_3).

Period has been found to be variable for a number of Mira variables and this has been denoted by « v » in front of the period (Kholopov, 1985 ; Kukarkin *et al.*, 1969). Thus, there are about 63 M Miras with variable period. However, if we compare the period as given in Table Ia with that given by Campbell (1955), we find that 92 M Miras have $|\Delta P| \geqslant 2$, increase and decrease in period being equally distributed, with W Dra having the largest increase of 23 days over Campbell's value and R Aql the largest decrease of 22 days. Note that both these Miras are α_4 type, and their visual light curve class have not changed with time considering the 1928, 1955 and 1983 classifications. The distribution of these variable period M Miras in different visual light curve class shows a fairly uniform distribution except for γ_1, where there is great excess and perhaps γ_2 (only two stars, and both have variable period). There are another about 105 stars which have period variation of about a day. Eight S Miras and seven C Miras also have period variation $\geqslant 2$ days. This implies that about 30 % of Miras show a period variation of $\geqslant 2$ days and the percentage jumps to about twice if Miras with period variation of 1 day are also included. This is not surprising as these stars, with a large rate of mass loss, are in a fairly rapid phase of evolution. A detailed evolutionary study of a few Miras with large change in periods with time may be instructive in giving clues to the transition of M stars into S and C stars.

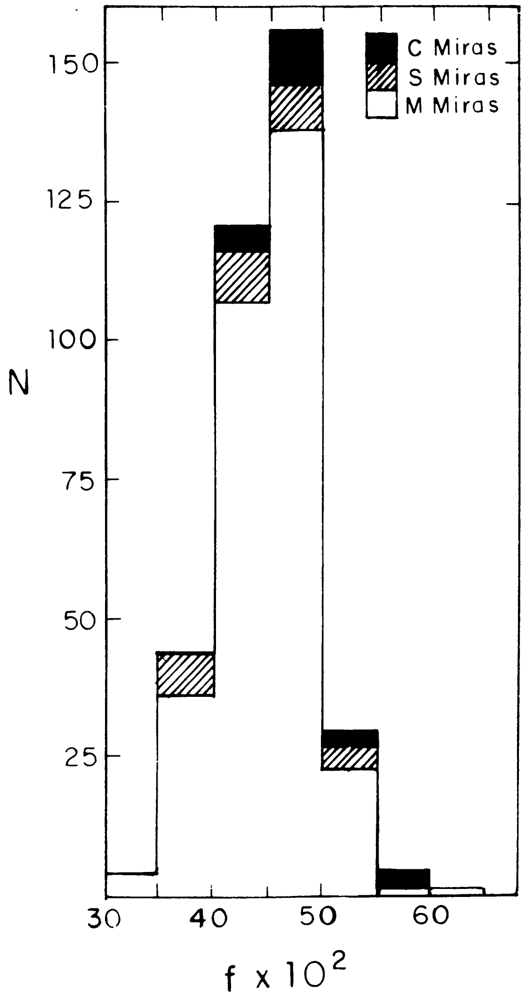

FIGURE 7. — Distribution of M, S and C Miras as a function of visual light asymmetry factor *f*.

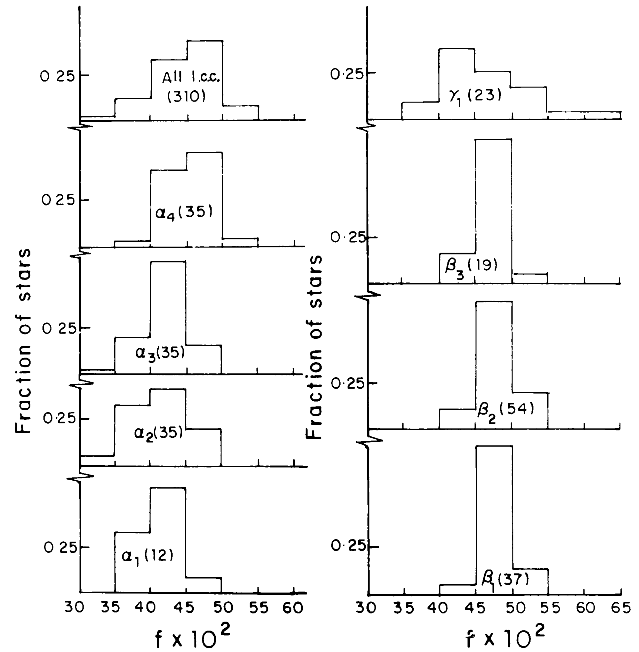

FIGURE 8. — Fraction of M Miras in different *f*-range as a function of visual light curve class. Number of stars is denoted in parenthesis.

For about 180 M Miras, spectral class at visual light maximum has been given in table I from Keenan *et al.* (1974). The distribution of spectral class at maximum shows that for α_1, α_2 and α_3 class light curves, 90 % of the stars are M5 or later at maximum. This shifts to ~ 50 % for α_4. In the case of β_1, ~ 90 % of stars at maximum are M4 or later, but this shifts to less than 10 % for β_3. Except for one star (S Cet), all Miras with γ_1 light curves have spectral class at maximum of M6 or later. Mean spectral types at maximum, with number of stars used in averaging in the parenthesis, for different light curve class Miras are : $\alpha_1(6)$: M5.5, $\alpha_2(26)$: M5.8, $\alpha_3(30)$: M5.7, $\alpha_4(36)$: M5.0, $\beta_1(24)$: M5.2, $\beta_2(39)$: M4.6, $\beta_3(15)$: M.2.9, and $\gamma_1(13)$: M6.3.

5. Conclusions.

The distribution of visual light curve classes is not properly understood. Miras exhibit variations of luminosity in the photosphere due to pulsation. Their nature as indicator of circumstellar matter has already been shown (Vardya, 1987a, b). A deeper understanding should be far more revealing and is bound to increase the utility of the empirical findings.

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TABLE Ia. — *Visual light curve classification of M Miras.*

Star name	Period (day)	fx10^2	<ΔM$_V$>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
T And	281	46	5.3	M4e-7.5e	M5.5e	β_1	β_1	α_4:
U	347	41	4.4	M6e		α_3	α_1	α_2:
V	258	44	4.9	M2e-3e	M3e	β_2	β	β_2:
Y	221	48	5.0	M3e-4.5e	M3e	α_4	α_4	β_1:
SV	316	38	5.0	M5e-7e	M5.5e	α_2	α_2	
TU	317v	48	4.0	M5e	M6e:		$\alpha_4-\beta$?	γ_1:
UZ	314	39	4.8	M7e-9e	M7e:	α_2	α_1	α_2:
T Aps	261	43	5.6	M3e		α_4		
R Aqr	387	42	3.8	M5e-8.5ep	M7e+?	α_1	α_1	
S	279	39	5.8	M4e-6e	M5.5e	α_4	α_3	
T	202	48	5.4	M2e-5.5e	M3e	β_2	β_2	
W	381	42	5.3	M6-8e		α_3	α_2?	
Y	382	43	5.4	M6.5e-9	M6e:	α_2	α_3?	
RR	182	45	4.4	M2e-4e	M2e	β_2	β	
RS	215	49	4.0	Me		β_2		
RT	246			M5e-6e			β	
R Aql	284v	42	5.4	M5e-9e	M6.5e	α_4	α_4	α_4
X	347	46	6.0	M6e	M6e:	α_3	α_4	
Z	129	47	4.9	M3e		α_4	$\alpha_4-\beta_2$	
RR	395	30	4.9	M6e-9	M7e:	α_3	α_2	
RS	410	48	5.5	M5e-8		α_2	α_1?	
RT	327	42	5.6	M6e-8e(S)	M7e:	α_3	α_3	α_3:
RU	274	42	4.6	M5e	M5e	α_3	α_3	
RV	219	47	5.2	M2e-7:e	M4e:	α_4	$\alpha_4-\beta_1$	
SY	356	37	4.9	M5e-7e	M6e:	α_3	α_4	
TV	243v			M4e			α_4?	

Star name	Period (day)	fx10^2	<ΔM$_V$>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
U Ara	225	44	5.2	M3IIep-5e		α_3	?	
R Ari	187	45	5.0	M3e-6e	M3e	β_3	β_3	β_2:
S	292	45	4.3	M4e-6.5e	M4.5e:	α_2	α_1?	
U	371	40	6.5	M4e-9.5e	M5.5e	α_2	α_2	
R Aur	458	51	5.6	M6.5e-9.5e	M6.5e	γ_1	γ_1	γ_1
U	408	39	5.5	M7e-9e	M7e	α_2	α_1	α_3:
W	274	41	5.4	M3e-8e	M4e:	β_3		
X	164	50	4.1	M3e-7e	M3.5e	β_2	β_3	β_2
RU	466	40	4.9	M7e-9e		α_1		
R Boo	223	46	5.1	M3e-8e	M4.5e	β_1	β_1	β_2
S	271	44	4.9	M3e-6e	M5.5e	β_1	$\alpha_4-\beta_1$	β_2
Z	281	39	5.5	M5e-6e	M6-e	α_3	α_3	α_3
RR	195	48	3.9	M2e-6e				α_4
R Cae	391	41	5.2	M6e		α_2	α_2	
V Cam	522	31	5.5	M7e		α_2	α_1	α_2
X	144	49	4.5	K8-M8e	M3e	β_3	β_3	[illegible]
R Cnc	362	47	4.4	M6e-9e	M6.5e	α_4	β_1	
U	305	40	4.7	M2e		α_2	α_1	
W	393	40	5.9	M6.5e-9e	M7e	α_3	α_2	α_3
R CVn	329v	46	4.2	M5.5e-9e	M6.5e	β_2	$\alpha_4-\beta_2$	β_2:
T	290	42	2.3	M6.5e		β_3		
U	346		>3.7*	M7e				α_2

TABLE Ia (*continued*).

Star	name	Period (day)	$fx10^2$	$<\Delta M_V>$	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
S	CMi	333	49	5.1	M6e-8e	M7e	β_1	β_1	γ_1:
T		328v	47	3.5	M4Se-M8		α_2	α_4 ?	
U		414	52	4.2	M4e		γ_1	γ_1	
V		366	39	6.2	M4e-10	M6.5e	α_3	α_3	
T	Cap	269	44	4.4	M2e-8.2	M5e:	α_3	$\alpha_4-\beta_1$?	
U		203	46	3.7	M5.5e		α_1		
V		276	42	6.0*	M5e-8.2	M5.5e	α_2	α_3?	
W		210	51	3.1	M5e-8		α_4 pec		
X		218	48	3.7	M2.0e-7.5		α_3		
Y		206†	44†	3.2			α_3		
Z		181	48	4.5	M2e-7.0	M3e:	β_1	β?	
RR		278	40	5.2	M5e-8.5	M5e:	α_2	α_3?	
RU		347	36	5.4	M9e		α_3	α_3	
R	Car	309	48	5.0	M4e-8e	M5e	β_1	$\alpha_4-\beta_1$	α_4
S		149	51	2.8	K5e-M6e	MOe	β_3	γ_1	
Z		384	37	4.5	M6e		α_3)	?	
RV		366	39	4.9	M6e		α_2)		
RW		319	47	5.7	M4e-7e	M6e:	γ_1	α_4	
RZ		273	42	5.4	M4e-8e	M6e:	α_3	α_3?	
R	Cas	430	40	5.6	M6e-10e	M7e	γ_1	α_4	γ_1:
T		445	56	4.0	M6e-9e	M7.5e	γ_1	$\gamma_1-\gamma_2$	γ_1
V		229	48	4.3	M5e-8.5e	M5.5e	β_2	β_2	β_2
Y		413	43	4.7	M6e-8.5e	M7e:	α_2	α_3	
Z		496	39	4.7	M7e	M7e:	α_2	α_1	
RR		300	48	3.5	M5e		β_3		

Star	name	Period (day)	$fx10^2$	$<\Delta M_V>$	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
RV	Cas	332	38	5.8	M4.5e-9.5e	M6,Se	γ_1	α_3	α_3
SS		141	48	3.3	M3e-8e	M5e	β_1	β?	β_1
R	Cen	546		5.3	M4e-8IIe	M4.5e	γ_2	γ_2	
U		220	47	5.2	M3II:e-5IIe	M3e	β_2	β	
W		202	47	4.7	M3e-8(III)e		β_1	β	
X		315	41	5.4	M5e-6.5e	M6e:	α_4	α_4	
RS		164	46	4.8	M1Ibe-5(III)e		β_2	$\alpha_4-\beta_3$	
RT		255v	47	3.7	M6II:e		β_3		
RX		328	38	>6.3*	M5e		α_2		
T	Cep	388v	54	4.3	M5.5e-8.8e	M6.5e	γ_1	γ_1	γ_1
X		535	42	6.3	M4.5e-7e	M4.5e:	α_2	α_1?	
Y		333	40	5.5	M5e-8.2e	M5e:	α_3	α_3	α_3
Z		279	48	4.6	M2e-8.2		α_3		
RR		384	41	4.5	M5e-8.8e		α_2		α_2:
R	Cet	166	43	4.9	M4e-9	M4.5e	α_4	$\alpha_4-\beta$	
S		320	47	6.0	M3e-6.5e:	M3.5e:	γ_1	$\alpha_4-\beta_1$	
U		235	44	5.1	M2e-6e	M4e	α_4	α_4	
V		258	45	4.9	M3e		α_3	α_3?	
X		177	49	3.5	M2e(S)-6e	M3e	β_3	β_3	
Z		185	48	4.6	M1e-6-5e	M5e	β_2	β	
o		332	38	5.6	M5e.9e	M5.5e	α_3	α_3	α_3
R	Cha	335	41	5.1	M4e-7e	M4.5e	α_4	α_4	

TABLE Ia (*continued*).

Star	name	Period (day)	fx10^2	<ΔM_V>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Col	328	39	5.4	M3e-7		α_1	α_1 ?	
S		326	46	4.5	M6e-8		α_3		
T		226	50	4.4	M3e-6e	M5-e	β_2	β	
R	Com	363	38	5.7	M5e-8ep	M6e	α_2	α_2	α_2:
U	CrA	148	42	4.4*	M2Ibe	M2e:		β?	
S	CrB	360	33	5.6	M6e-8e	M6.5e	α_2	α_3 pec	α_2
W		238	45	5.0	M2e-5e	M4.5e	α_4	α_4-γ_1	β_2:
X		241	46	4.5	M5e-7e	M6e:	β_1	β_2	
Z		251	42	4.6	M4e-5e	M4.5e	α_4	α_4	
R	Crv	317	41	6.3	M4.5e-9:e	M5e	β_1	β_1	
U	Cru	343		3.7*	M4e-6e			α_3	
Z	Cyg	264	45	4.6	M5e-9e		α_2	α_1	α_3
RT		190	44	4.5	M2e-8.8eIb	M3e	β_3	β_3	β_2
ST		337	48	4.0	M5.5e-8.0e		α_4	α_4-β_1	
SX		411	41	5.3	M7e	M7-e	α_2	α_2?	
TU		219	49	4.8	M3e-6e	M3.5e:	β_2	α_4-β	
TW		341	48	4.5	M6.5-10ep		α_4		
TY		349	48	5.1	M6e-8e	M6e:	β_1	β_1	α_3:
UX		565v	40	7.5	M4e-6.5e	M5e:	α_1	α_1?	
BG		288v	50	3.8	M7e-8e				β_1

Star	name	Period (day)	fx10^2	<ΔM_V>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Del	285	45	5.0	M5e-6e	M6-e	α_4	α_4	
S		278	52	3.2	M5e-8	M5.5e	β_2?	β	α_4:
T		332	45	5.5	M3e-6e		α_3	β_1	α_3:
V		534	42	5.4	M4e-6e	M6-e	α_1	α_2?	α_1:
X		281	42	5.1	M4e-6e	M5.5e	α_3	α_2	
Y		468	43	4.1	M8e		α_1		
R	Dra	246	45	4.8	M5e-9eIII	M5e	α_4	α_4	α_4
U		316v	47	4.3	M6e-8	M6e	β_1	β_1	β_1
V		278	50	4.3	M4e		β_2	α_4-β_1	
W		279v	43	4.8	M3e-4e		α_4	α_4-β	α_3:
X		257	44	4.7	M5e:		α_3		
Y		326	45	5.3	M5e		α_2	α_1	
SV		256	49	4.6	M7e		α_4		
R	Equ	261	44	5.2	M3e-4e	M4e	α_3	α_3	
T	Eri	252	45	4.8	M3e-5e	M4e:	α_4	α_4 ?	β_2:
U		275	49	5.4	M4e	M4e	α_4	α_4	
W		377	40	5.2	M7e-9		α_3	α_3	
S	Gem	293	42	5.2	M4e-8e	M6e	α_3	α_2	α_3
V		275v	45	5.7	M4(S)e-8		β_2	α_3	
X		264	49	5.0	M5e-8e	M6e	β_2	β_2	β_2:
R	Gru	332	42	6.3	M5e-7II-IIIe		α_4	α_4	
S		402	43	6.7	M5e-8IIIe		γ_1	α_4 ?	
T		136	48	2.9	M1Iae-2Ibe	M1-e	β_3	β_3	

TABLE Ia *(continued)*.

Star	name	Period (day)	fx10^2	$<\Delta M_v>$	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Her	318	39	5.8	M6e		γ_1	α_3	
S		307	47	5.0	M4,Se-7.5,Se	M5,Se	β_2	α_4-γ_1	
T		165	47	4.8	M2.5e-8e	M4e	β_3	β_3	β_2
U		406	40	5.0	M6.5e-9.5e	M7-e	γ_1	α_4	α_2:
W		280	45	5.2	M3e-5e	M4.5e	β_1	β_2	β_2
RS		220	47	4.6	M4e-8:	M4.5e	β_2	β_3	β_2
RT		298	40	5.6	M4e	M5e:	α_2	α_3	α_2
RU		485	43	5.7	M6e-9	M7-e	γ_1	γ_1	γ_1:
RV		205	44	4.7	M2e	M2.5e	α_3		α_3
RY		221	44	4.8	M4e-6e	M5e:	β_2	β_2	
RZ		329	44	5.4	M5e-6e	M5e	β_2	α_1	
SS		107	48	3.2	M0e-5e	M2.5e	β_2		β_2
SV		239	46	4.6	M5e		α_4		
TV		304	37	4.8	M4e		α_2		α_2
R	Hor	408	40	7.0	M5e-8eII-III	M6.5e:	α_3	α_3	α_3:
T		218	48	5.0	M5IIe		β_2	β_3	
R	Hya	389v	49	5.0	M6e-9eS	M6.5e	γ_1	β_1	
S		257	49	4.9	M4e-8e	M4e	β_3	β	
T		299v	49	4.8	M3e-9:e	M4e	β_3	β	
X		301	42	4.4	M7e-8.5e	M7.5e	α_3	α_4-β	
RR		343	50	5.1	M3e-8e		α_4	?	
RS		339	45	4.1	M6e		β_1)	β_1	
RU		332v	35	5.6	M6e-8.8e	M6.5e	α_2		
R	Ind	216	47	5.9	M2e-4(II)e		β_1	α_4?	
S		400	41	6.8	M6e-8eII-Ib		α_2)	α_1?	

Star	name	Period (day)	fx10^2	$<\Delta M_v>$	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Lac	300†	41	5.3	M5e-8.5e	M5e	α_3	α_3	
S		242v	46	4.8	M4e-8.2e	M5.5e	β_2	β_2	
R	Leo	310v	43	4.2	M6e-9.5IIIe	M7e	α_4	α_4	
S		190v	47	3.8	M3e-6e:		β_1		
V		273	44	4.6	M5e	M5.5e	α_2	α_2	
W		392v	35	4.4	M5.5e-7e		α_2	α_2?	α_2
R	LMi	372	41	5.5	M6.5e-9e	M7e	α_4	α_4	α_4:
S		234	42	5.3	M2e-8.2e	M5-e	α_3	α_3?	
T	Lep	368	47	4.6	M6e-9e		β_1	β_1	
R	Lib	242	44	4.5	M5e		α_3		
S		193v	49	3.6	M1e-6e	M1.5e	β_3	β_3	
T		238	41	4.3	M4e-5.5e		α_3		
U		227	44	4.8	M3e-8e		α_4	α_3?	
V		255	42	5.0	M5e-8e		α_2		
W		206v	45	3.9			α_4		
X		160v	40	3.5	M4e		β_2		
Y		276v	41	3.5	M5e-8.2e	M5.5e:	α_3	α_3	
RR		277v	47	5.6	M4e-8e	M4e:	α_2	α_4	
RS		218	48	4.5	M4e-8.5e	M7-e	β_2	β_2	
RT		251v	45	5.3	M2.5pe-8.2e	M4e var	α_4	α_3	α_3:
RU		317v	46	5.9	M5e-6e	M5e:	α_4	α_4-β_1	
Y	Lup	397v	37	5.3	M7e		α_2	α_1	

TABLE Ia (*continued*).

Star	name	Period (day)	fx10^2	< M_v>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
S	Lyn	296v	45	4.7	M6e-8.2e	M6e:	α_2	α_1	
U		434v	42	4.9	M7e-9.5e	M7-e	γ_1		
V	Lyr	374	33	5.1	M7e		α_2	α_2	
W		198v	48	4.3	M2e-8e	M4.5e	β_1	γ_1	β_2
Z		292v	49	4.7	M4e-5.5e:e	M4.5e	β_2		
RS		301v	48	4.8	M5e		β_3	α_4?	
RT		254v	45	4.5	M5e	M5-e	α_3	α_3	
RU		372v	45	4.5	M6e:-8e		α_3		
RW		504	38	4.3	M7e		α_1	α_1	
RX		248v	45	5.1	Me		α_2	α_3	
RY		326	40	4.9	M5e-6e	M6e	α_4		α_4:
SS		346		5.6*	M5IIIe			$\alpha_4-\gamma_1$??	
R	Mic	139v	46	4.2	M4e		β_2	β_2	
S		210	43	4.8	M3e-5.5	M4e	α_4	β?	
U		334	39	5.2	M5e-7e		α_2	α_3	
V		381		4.6*	M3e-6e			α_3?	
V	Mon	341v	46	6.1	M5e-8e	M5e	α_4	β_1	
Y		228v	49	4.8	M4e-8.2e	M4e	β_1	β pec	
R	Nor	508v		5.0	M3e-6II		γ_2	γ_2	
T		241v	41	5.8	M3e-6e	M4e:	α_4	α_3	
R	Oct	405	44	4.5	M5.5e	M5.5e:	α_2	$\alpha_4-\gamma_1$	
S		259	42	5.1	M4(II)e-M5e		α_4	$\alpha_4-\beta_1$	
T		219	51	4.8	M2e-4(II:)e			α_4?	
U		308v	47	5.7	M4e-6(II-III)e		β_2	$\alpha_4-\beta$	

Star	name	Period (day)	fx10^2	< ΔM_v>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Oph	307v	45	5.7	M4e-6e	M5e:	α_4	α_4	
S		234	45	5.0	M5e	M5e	α_4		
T		367	36	4.2	M6.5e		α_2		
W		333v	41	4.6	M8e		γ_1		
X		329v	53	2.0	M5e-9e	M6.5e	β_1	β_1	β_1
Z		349v	40	4.6	K3ep-M7.5e	K3ep	α_4	α_4	
RR		292	46	5.7	M3e-7	M4.5e	α_4	$\alpha_4-\beta_1$	
RT		426v	36	5.5	M7e(C)		α_2		α_2
RU		202v	49	4.5	M3e-5e	M4e:	β_2		
RY		150	46	5.0	M3e-6e	M4.5e	β_2	$\alpha_4-\beta_2$	
SS		181v	47	4.8	M5e	M5+e	β_1	β_1?	
S	Ori	414v	48	4.5	M6.5e-9.5e	M7-e	β_1	γ_1	
U		368v	38	5.7	M6e-9.5e	M6.5e	α_3	$\alpha_4-\gamma_1$	α_3:
V		264v	49	4.7	M3e-8e	M3.5e:	β_1	β	β_2:
R	Pav	230	48	4.5	M4e-5e	M5e	β_1	$\alpha_4-\beta$?	
T		244	43	5.8	M4e		α_3	α_3	
U		290		>3.4*	M4e			α_3	
W		283	40	5.1	M4e-7e		α_4	α_3	
R	Peg	378	44	5.4	M6e-9e	M7e	α_3	$\alpha_4-\gamma_1$	
S		319	47	5.0	M5e-8.5e	M6.5e	β_1	$\alpha_4-\beta_1$	
T		373v	49	5.4	M6e-7e		γ_1	β_1	
V		302	44	5.7	M3e-6e	M6e:	α_3	α_2	
W		345	46	4.5	M6e-8e	M7-e	β_2	β_1	
X		201	49	4.4	M2e-5e	M4e:	β_2		
Y		207	45	4.4	M3e		α_2	α_3	

TABLE Ia *(continued)*.

Star	name	Period (day)	fx10^2	<ΔM_V>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
Z	Peg	325	50	4.8	M6e-8e	M7-e	β_2	β_2	
RR		264	43	4.9	M4e-6e	M5e:	α_2	α_3	
RS		412	44	5.0	M6e-9e		α_2	α_2:	α_2
RT		216	44	4.6	M3e-6e	M4.5:	β_2	pec	
RV		390	38	4.7	M6e		α_2	α_2?	
RW		208	48	4.3	K3e:-M6.5e	M1e var	β_3	β	
R	Per	210	49	5.3	M2e-5e	M4e	β_2	β	β_2:
U		321v	46	3.2	M5e-7e	M5.5e	β_3	α pec	β_3:
RR		390	45	5.2	M6e-7e	M6e:	α_3	α_1	
R	Phe	268	49	6.1	M3e-4e		β_1	β	
T		281	38	4.8	M5e		α_3	α_2	
V		257	47	4.8	M4e		β_1	?	
S	Pic	427	36	5.7	M6.5-8e(III-II)	M7e	α_2	α_4	
T		201	49	5.5	M6IIIe		β_2	β_2	
R	Psc	344	44	6.1	M3e-6e	M5-e	α_3	α_3	
S		405	42	5.4	M5e-7e		α_2	α_2?	α_2:
U		173	49	3.4	M4e		β_2		
R	PsA	293	38	5.5	M4(II)-5IIe		α_3)	α_4?	
S		272	42	>5.4*	M3e-5IIe		α_3)	α_3?	
U	Pup	317	41	4.3	M5e-8e		α_2	?	
W		120	47	4.0	M3e		β_2	β_3	
Z		510	38	6.4	M4e-9e	M5e:	α_1	α_1?	

Star	name	Period (day)	fx10^2	<ΔM_V>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
S	Pyx	206	45	4.9	M3e-5e	M4e:	β_1	?	
R	Ret	278	48	5.7	M4e-6.5e	M5e:	α_4	α_4	
W	Sge	278			M4e			α_4	
R	Sgr	269	46	5.2	M4e-6e	M4.5e	β_2	β	
S		231	45	4.6	M4e:	M4e:	β_2	β?	
Z		450	47	7.4	M4e-6(Se)	M5e:	α_2	α_1 ?	
RR		335	43	6.4	M5e-7e	M6e:	α_4	α_4	
RT		305	47	6.3	M5e-7e		α_4	α_4-β_1	
RU		240	43	5.6	M3e-6e	M5e:	α_4	α_4	
RV		318	47	6.3	M5e		β_1	β	
RX		334	49	4.1	M5e		β_1)	β	
SW		290	51	>4.0*	M5e		β_1	?	
TY		325	44	5.2	M3e		α_4)	α_1?	
R	Sco	223	46	4.6	M(3)e		α_2		
S		177	50	4.1	M(3)e		β_2		
W		221	48	3.1			α_4?		
X		200	50	3.3	M2e	M2e:	β_2		
Y		352	44	3.7			α_2)		
Z		350v	50	4.2	M5.5e:-7e	M5.5e:	β_1	β_2	
RR		279	47	5.9	M6II-IIIe-8(II)e	M6e	β_2	β_2	
RS		320	42	5.2	M5e-8e	M6e	α_3	α_4	
RU		369v	54	4.0	M7II-IIIe		γ_1	γ_1	
RW		390	40	6.2*	M5e		α_2)	α_1?	
RZ		156v	45	3.4	M3e-4e	M4e:	β_2	β_3	

TABLE Ia (continued).

Star	name	Period (day)	fx10^2	<ΔM$_V$>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
SV	Sco	256	47	5.0	M3e		β_3	β	
SW		261		3.2	M5e			α_4-β?	
S	Scl	365	48	6.2	M3e-8e	M6-e	γ_1	β_2	
T		202	49	3.8	M3e		β_3	β_3?	
U		334	39	5.3	M5e		α_2		
V		296	48	4.7	M4e-6e	M5.5:	β_1	α_4?	
X		261	50	3.6:			β_1		
R	Ser	356	41	6.5	M5e-9e	M6.5e	α_4	α_4-γ_1	α_4
S		369	43	4.8	M5e-6e		γ_1	γ_1	
T		341v	47	5.3	M7e		β_1		
U		238	47	4.9	M4e:-6e	M4e:	α_4	α_4-β_1	
S	Sex	263	50	4.3	M3e-5e	M3.5e:	β_2		
R	Tau	324	41	5.6	M5e-9e	M6e	γ_1	α_2	α_3:
S		373	43	5.1	M6.5e:-7e	M6.5e:	α_2		
V		170	45	4.5	M0e-4e	M3e	β_2	β_2	
RU		568	62	4.7	M3.5e		γ_1		
RX		335	47	4.4	M7e		α_4	α_4	
R	Tel	462	43	6.2	M5IIe-7e		γ_1	α_2 ?	
U		445		>2.4*	M7e			α_1 ?	
W		304		>3*	M5e-8e			α_3?	
R	Tri	266	44	5.5	M3e-8.5e		β_2	β_2	γ_1:

Star	name	Period (day)	fx10^2	<ΔM$_V$>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Tuc	286	41	5.3	M5e		α_3	α_4	
S		241	44	5.2	M3e-5e	M4e	α_4	α_4	
T		251	46	5.1	M3IIe-6IIe		β_2	β	
U		259	46	5.5	M3e-5e	M4e:	β_2	β_2	
R	UMa	302	39	5.5	M3e-9e	M5e	α_3	α_3	α_3
T		257	41	5.2	M4e-7e	M4e	α_3	α_3	α_3
X		249	43	4.7	M4e		α_3	α_4	α_3
RS		259	42	5.3	M4.5e-6e	M4.5e	α_4	α_3	
S	UMi	326v	50	3.6	M7e-9e	M7e	β_2	β_2	β_2
T		314v	45	4.8	M5.5e	M5.5e	β_1	β_1	
U		327v	50	3.8	M6e-8e	M6e	β_2	β_2	β_1
W	Vel	395	44	4.8	M8IIIe		α_3		
Y		445	40	4.3	M8e-9.5		α_1	α_1	
Z		422	46	5.3	M9e		γ_1		
R	Vir	146v	50	4.6	M3.5e-8.5e	M4.5e	β_2	β_3	β_2
S		377v	45	5.7	M6e-9.5e	M6.5e	α_4	α_4	
T		339	36	4.6	M6e	M6e	α_2	α_2 ?	
U		207	47	4.9	M2e-8e	M3.5e	β_3	β	
V		250	42	5.4	M3e-6e	M5e	α_1	β ?	
Y		218	46	4.2	M3e-5e	M5e:	α_4	α_4	
Z		307	39	4.5	M5e	M5e:	α_1		
RR		218	47	3.4			α_4		
RS		353	37	5.8	M6e-7e		α_3	α_3	
RV		268	41	4.1	M5e		α_2		
SU		210	48	4.2	M2e-5.5e	M3.5e	α_4	α_4	α_4
S	Vol	396	54	5.0	M4e		α_1		
R	Vul	136	49	4.5	M3e-7e	M4e	β_2	β_2	β_2:

TABLE Ib. — *S spectral type Miras.*

Star	name	Period (day)	fx10^2	<ΔM_V>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	And	409	38	7.4	S3,5e-8,8e	S4,6e	α_3	α_3	α_3
W		396	42	6.3	S6,1e-9,2e	S8,2e	α_3	$\alpha_3-\gamma_1$	α_2:
X		346	37	5.8	S2,9e-5,5e	S2,9e:	α_3	α_3	α_3:
RR		328	52	6.0	S6.5,2e	S6,2:e	β_2	β_1	
RW		430	36	6.1	S6,2e	S6,2e	α_2	α_1?	α_2:
X	Aqr	312	42	6.1	S6,3e:		α_3	α_2?	
W	Aql	490	37	5.7	S3,9e-6,9e	S3,9e	α_2		
R	Cam	270v	45	4.9	S2,8e-8,7e	S2,8e	β_3	β_3	
T		373	47	5.8	S4,7e-8.5,8e	S5,7.5e	γ_1	γ_1	γ_1:
V	Cnc	272	46	4.9	S0e-7,9e	S1,9e	α_4	α_4	
RY	Car	424v	45	3.0	S7,8e		α_1)		
S	Cas	612	43	5.1	S3,4e-5,8 e	Se	γ_1	pec	
U		277	44	6.4	S3,5e-8,6e	S4.5,5e	α_4	$\alpha_4-\beta$	α_4
W	Cet	351		6.8	S6,3e-9,2e	S6,3e	γ_1	α_4	
R	Cyg	426	35	6.4	S2.5,9e-6,9e	S3,9e	α_3	α_4	α_2
S		323v	50	5.7	S2.5,1e	S5,5e:	β_1	pec	
X		408	41	8.2	S6,2e-10,4e	S7,2e	γ_1	γ_1	γ_1:
Z	Del	304	48	5.7	S5,2.5e-7,2e:	S5,2.5e	β_1	α_4	

Star	name	Period (day)	fx10^2	<ΔM_V>	Spectral Type	Spectral type at maximum	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
R	Gem	370	36	6.4	S2,9e-8,9e	S3,9e	α_4	α_4	α_3:
T		288	50	5.3	S1.5,5e-9,5e	S2.5,5e	β_3	β	
S	Lup	340v	52	4.4	Se		α_1		
R	Lyn	379	44	5.9	S2.5,5e-6,8e:	S3,9e	α_4	$\beta_2-\gamma_1$	α_4
S	Lyr	438v	40	4.4	SCe		α_1	α_2?	
RR	Mon	395v	39	5.6	S7,2e-8,2e		α_2	α_2?	
RZ	Per	354	47	4.3	S4,9e		γ_1		
T	Sgr	392	47	4.6	S4.5,8e-5.5,8e	S4.5,8e	β_1	β	
ST		395	44	6.2	S4,3e-9,5e		α_1	α_2?	
RT	Sco	449	39	9.0:	S7,2		α_2)	α_1	
Z	Tau	491v	41	4.1	S7.5,1e:		α_1		
S	UMa	226v	47	3.9	S0.5,9e	S0.5,9e	β_3	pec	β

TABLE Ic. — *C spectral type Miras.*

Star name		Period (day)	fx10^2	< Δ M_V>	Spectral Type	l.c.c. (1955)	l.c.c. (1928)	l.c.c. (1983)
V	Aur	353	52	2.9	C6,2e(N3e)	γ_1	β_2	β_2:
R	CMi	338	48	3.0	C7,1Je(CSep)	β_3	β_3	
R	Cap	345	44	3.0	Ne	α_3		
W	Cas	406	46	3.0	C7,1e	γ_1	β_3	β_2
X		423v	55	2.4	C5,4e(N1e)	β_1	β_2	β_3
RV	Cen	446	56	5.8	N3e	β_3	$\beta_3-\beta_4$	
S	Cep	487	55	2.9	C7,4e(N8e)	β_2	$\beta_3-\beta_4$	
V	CrB	358	41	3.5	C6,2e(N2e)	γ_1	$\alpha_4-\beta_3$	α_3
V	Cru	377		3.5*	Ce(Ne)		β?	
U	Cyg	463v	48	3.5	C7,2e-9,2(Npe)	β_3	β_3	β_2
V		421	46	3.7	C5,3e-7,4e(Npe)	α_4	$\alpha_4-\beta_2$	α_3:
WX		410	48	2.9	C8,2JLi(N3e)	β_3	$\beta_3-\beta_4$	β_2:
T	Dra	422	44	2.7	C6,2e-8,3e(N0e)	α_4	β_1	β_1:
R	For	389	52	3.3	C4,3e(Ne)	β_3		
R	Lep	427v	55	3.0	C7,6e(N6e)	γ_1	β_2	
U	Lyr	452v	52	2.5	C4,5e(N0e)	γ_1	β_2	
V	Oph	297v	48	2.7	C5,2-7,4e(N3e)	β_1	β	
R	Ori	377v	40	3.5	C8,2e(Ne)	β_2	β	
RZ	Peg	439	44	4.0	C9,1e	α_4		α_4
Y	Per	253	48	1.9	C4,3e	β_2	β_2	β_2
RU	Vir	437	49	3.3	C8,1e	γ_1	β	
SS		355v	48	2.1	C5,3e(Ne)	β_2	β	
R	Vol	450	48	4.5	Ce	γ_1		

TABLE II. — *Dependence of f on visual light curve classification.*

l.c.c (1955)	n	<f x10^2>	σ_{n-1}	f_{max}/f_{min}
α_1	12	40.75	2.34	46/38
α_2	55	40.69	3.91	48/31
α_3	55	41.78	3.22	46/30
α_4	55	44.71	2.85	51/39
β_1	37	47.57	2.35	54/41
β_2	54	47.33	2.30	52/40
β_3	19	47.26	2.5	51/41
γ_1	23	46.43	6.22	62/38