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NEUTRAL OXYGEN IN LATE-TYPE STARS

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ABSTRACT

We have identified the forbidden absorption nebular and auroral lines of oxygen in the spectra of about seventy G and K stars, mostly giants. While weak, they appear in a relatively uncrowded spectral region and can be studied at high dispersion. Coming from the ground state of a dominantly neutral element, the nebular lines are insensitive to differences of model and non-equilibrium effects in line formation. They are formed over a wide range of optical depth and are therefore insensitive to the detailed structure near the boundary temperature.

Approximate model atmospheres predict the expected changes in the [O I] line strength as compared with its strength in the Sun; the models use a $T(\tau)$ relation scaled from that in the Sun and opacity tables. The depletion of the free atomic oxygen is caused mostly by CO formation which must be included in detail.

We have measured equivalent widths of forbidden oxygen lines in about one-third of the stars and estimated them in the other two-thirds. The measured strengths, from 12 to 140 mÅ, together with theoretical calculations, lead to the following conclusions:

1. Most stars have a solar abundance ratio of O/C, near 1 6, and the solar absolute abundances of both these elements.
2. The forbidden lines are observed to be strengthened in the stars with weak CN and high velocity, such as α Boo. These stars are mildly metal deficient (by a factor of 2–4), but have a larger O/metals ratio than does the Sun. The O/C ratio and O/H ratios are probably the same as in the Sun. The observed weakness of CN may be caused primarily by a low N/H ratio.
3. Stars with strong CN such as α Ser have weak [O I], an O/C ratio of about 1.2, probably caused by a slight carbon abundance enhancement. The increased CN line strength in these stars is caused primarily by enriched nitrogen. We suggest that this originates during helium flash and mixing
4. Moderately metal-deficient stars, such as HD 148897, have a large O/metals ratio. It could be that this quite weak-lined star has an oxygen abundance as high as the Sun.
5. The oxygen lines are not observed in the high-velocity, extremely metal-deficient star HD 122563. Oxygen is deficient, but perhaps not as much so as the metals.

During the early phases of nucleosynthesis in our Galaxy, the oxygen content increased at a different rate than the metals and reached its present abundance much more rapidly. This may also be true for carbon, but studies of the CN and CH bands and detailed molecular equilibria will be required. Helium burning in massive red giants, or in supermassive stars, apparently proceeded more rapidly than iron-peak element synthesis. Evolution within a star that would lead to increases of surface O and C abundances is not probable except in highly evolved stars.

I. INTRODUCTION

The usefulness of the forbidden lines of atomic oxygen for abundance analysis is limited to high-dispersion spectroscopy of relatively bright G and K stars. It provides a criterion for the oxygen/hydrogen ratio which is relatively insensitive to temperature or atmospheric models. The complications that arise stem largely from the association

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of O into CO. The important questions concern the O/C ratio and its possible variability But if we can assume an essentially fixed O/C ratio, which is not too close to unity, the analysis of the O/H ratio becomes relatively simple. We consider the complexities of the molecular equilibria largely in § II, with a coarse analysis for various H/C/N/O ratios. In § IV, an approximate model-atmosphere approach is used for two different O/C ratios. The essential results are that the O/H and O/metals ratios are nearly the same as in the Sun for normal giants, but that in the weak CN, high-velocity stars the O/metals ratio is larger, so that the O/H ratio remains unchanged. The rate of formation of oxygen in the early history of our Galaxy seems to have been more rapid than that of the metals. Special types of stars exist with abnormal [O I] line strength or weakness, probably explicable as effects of changes of the O/C ratio. Our procedure will be to predict the expected strength of [O I], and to compare it with observations described at the end of § IV.

II. OBSERVATIONAL DATA

The observational material for line strengths comes from high-dispersion spectrograms taken with the 200-inch Palomar, the 120-inch Lick, and the 100-inch Mount Wilson coudé spectrographs. The most useful data are those obtained at 2 and 3.3 Å/mm at Lick and Palomar, respectively, using largely IIaF or 103aF emulsions. As examples of the spectra, several stars in the region of λ6300 and the integrated Sun (sky) are shown in Figure 1 (Pl. 1). At the original dispersion of 2Å/mm the [O I] line is clearly resolved from the neighboring Sc II line (not marked) immediately longward. The very sharp lines appearing at different positions in the various stars (which have different velocities) are the terrestrial "b" band of O_2. Even at lower dispersion there is no danger of this feature blending with [O I], except in high-velocity stars with a large negative velocity, where caution must be exercised. The [O I] line in α Boo is seen to be enhanced relative to that in the other stars; in α Ser (less contrast because a 103aF emulsion was used) the line is relatively weak. The spectra of the stars are arranged in order of increasing color, from the top, in a decreasing temperature sequence. Reddening for these bright, nearby stars is negligible.

A large number of plates of 6.7 Å/mm, taken at Palomar by Greenstein and those already in the plate files, or kindly loaned by Dr. R. P. Kraft, could be studied; some additional, interesting stars were observed at this dispersion. Equivalent widths of all of the higher-dispersion material and a few from the 6.7 Å/mm dispersion were measured for the stars listed in Table 1. For two stars measured at both dispersions there was no appreciable difference in the equivalent width. Table 1 also lists the $V - I$ photoelectric-color measurements from Johnson (1964) where available. These are not seriously affected by differential line blanketing, and should be good temperature indicators. Eye estimates of the [O I] lines on all the 6.7 Å/mm spectra were made and are given in Table 2. The separation of Table 2 into two parts, "normal" and "weak CN and/or high velocity," was made for a reason which will become apparent later. The latter group is that defined by Roman (1952, 1955); it is mild Population II and only a few extreme halo Population II stars were studied. The stars in Table 2 represent a sample of the bright late-type stars in the sky but are not a complete list, nor a random selection, since some were observed specifically for this study. The $B - V$ colors and spectral types here and in Table 1 are from the *Bright Star Catalog* (Hoffleit 1964) or from Eggen (1966).

The eye estimates at 6.7 Å/mm in Table 2 are on a system such that:

0—λ6300 not present, $W < 10$ mÅ
1—λ6300 probably present, 10–20 mÅ
2—λ6300 present, λ6363 not present; 20–30 mÅ
3—λ6300, λ6363 present, 30–40 mÅ
4—λ6300 stronger than λ6299, 40–60 mÅ
5—λ6300 much stronger than λ6299, $W > 60$ mÅ.

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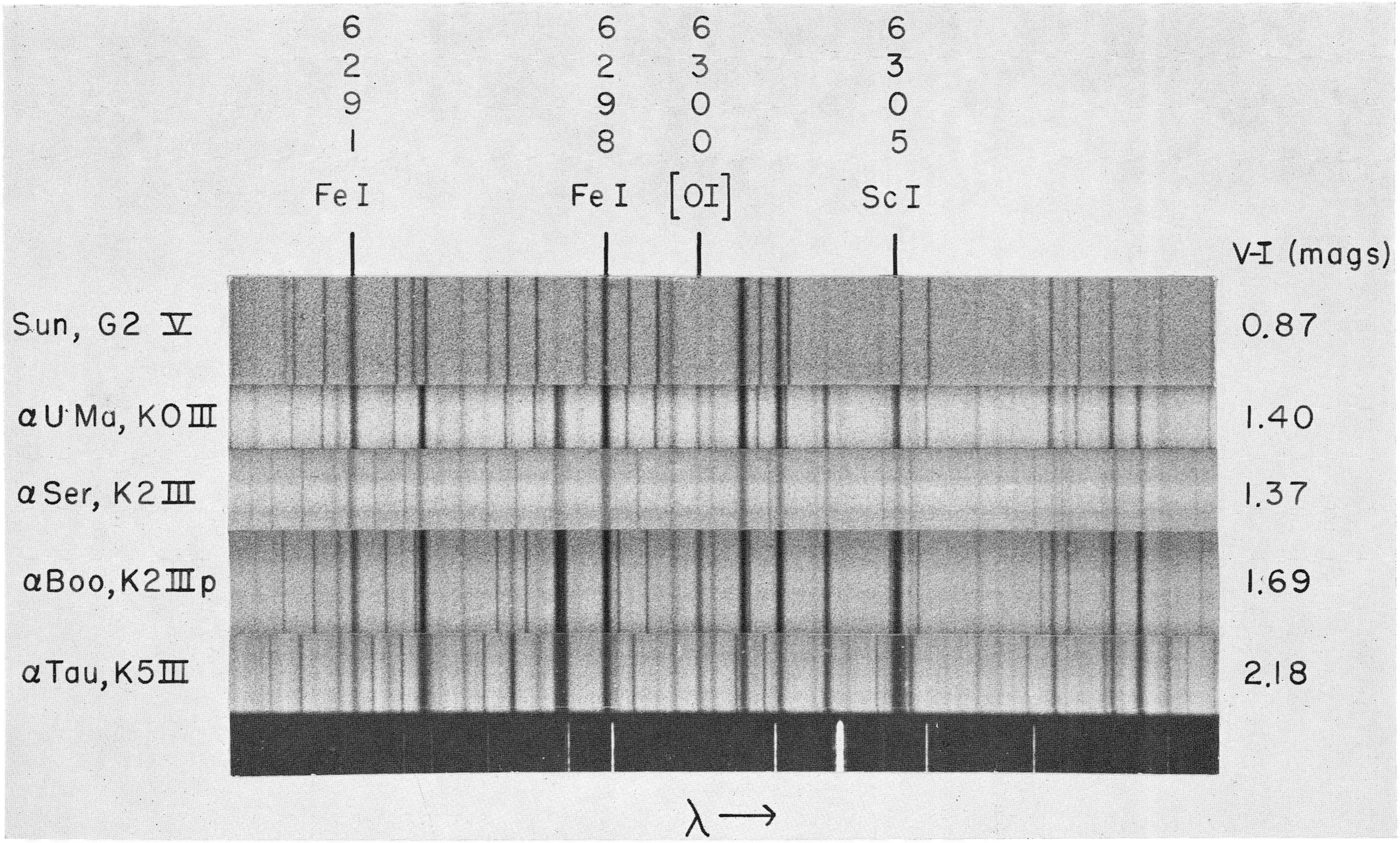

FIG. 1.—Lick coudé spectra, originals 2 Å/mm, of the Sun and some K giants. Aligned so that stellar lines coincide; the sharp atmospheric O_2 lines vary in wavelength. The central spectrum, *α* Ser, was on 103aF and the others on IIaF emulsion. The great strength of [O I] in *α* Boo is obvious. The stars are in a decreasing temperature sequence downward.

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A comparison between the equivalent widths measured in Table 1 and the eye estimates of Table 2 provides the rough calibration for the equivalent width of λ6300 given above. Tables 1 and 2 form the basis for the observations discussed here. Individual stars of interest are discussed below, as well as correlations of line strength and color. Errors in Table 2 are: HD 47205 = ν^2 CMa and HD 50877 = o^1 CMa.

III. COARSE ANALYSIS

The dominant feature in the concentration of many simple compounds in the temperature range of the G and K stars is the association of O and C to form CO. If the abundances of C and O are nearly equal, the residual concentration of atomic O is greatly reduced. The complex interplay of these associations can best be studied in the work of Tsuji (1964), who gives molecular and atomic concentrations over a wide range of temperature (θ) and total gas pressures (P_g). Each table assumes a set of ratios of H/C/N/O; the computations are based on a recent compilation of molecular constants. This valuable source permits a rapid survey of the effect of changing composition on the expected strength of the atomic oxygen lines. Given a model atmosphere it is possible to compute the fraction of free atomic oxygen, i.e., the ratio of O'/O, by interpolation in Tsuji's tables and to integrate through the significant range of optical depth. In this section, however, for coarse analysis, we survey effects of composition changes of C and N on the free atomic oxygen at a single standard optical depth.

TABLE 1

EQUIVALENT WIDTHS IN mÅ

Star	Sp.	$B-V$	$V-I$	λ6300	λ6363	Remarks
Sun	G2 V	0 63		5	. . . . .	
α Aur . . .	G5 III	. . . . .		17	. .	G component
HD 122563	K3 II*p*	0 90		<12	.	High vel., metal-poor
HD 165195	K3 II*p*	0 93†		<10	.	Metal-poor
HD 166208	G8 III*p*	0 92	.	33	16	Very weak CN, CH
HD 18474	G8 IV*p*	. . .	. . .	18		Very weak CN, CH
ε Vir*	G9 III	0 94	1 09	22		
β Her*	G8 III	0 94	1 11	20	.	
γ Leo B	G8 III	0 94‡	. . .	35	. .	High vel , weak CN
ο Vir	G8 III	0 98	1 23	46	19	
γ Tau*	K0 III	0 99	1 20	21	. .	Hyades
β Gem*	K0 III	1 00	1 25	22	19	
α UMa*	K0 III	1 07	1 39	36	17	
γ Leo A	K1 III	1 07‡	. .	60	20	High vel , weak CN
α Ari*	K2 III	1 15	1 48	37	23	
α Ser	K2 III	1 17	1 37	18	. .	Strong CN
α Boo . . .	K2 III*p*	1 23	1 64	66	30	High vel , weak CN
HD 148897	G8 III*p*	1 27		74	33	Weak CN
TY Vir	. . . . .	1 30	. . .	36§	.	High vel , metal-poor
μ Psc* .	K4 III	1 37	1 80	60		
Gmb 1618*	M0 V	1 38		12		
SX Her	. . . . . .	1 50	. .	72§	.	High vel , metal-poor
α Tau*	K5 III	1 53	2 15	64	25	
ε Peg*	K2 Ib	1 53	1 81	108		
α Vul*	M0 III	1 50	2 18	81	.	
ν Vir .	M0 III	1 50	2 25	139	58	High vel.
β And*	M0 III	1 57	2 24	101	63	
μ UMa*	M0 III	1 60	2 24	78	42	
α Cet*.	M2 III	1 64	2 51	.	64	
μ Gem*	M3 III	1 71	2 92	68		

* Normal stars

† Unreddened

‡ Average for spectral class

§ 16 Å/mm

TABLE 2

[O I] EYE ESTIMATES

a. Normal Stars

Star	HD	Sp.	B-V	6300/63	Remarks
δ And	3627	K3III	1.30	2	
β Cet	4128	KOIII	0.98	1	St. L.
η Cet	6805	K2III	1.15	1	Like α Ser
β And	6860	MOIII	1.57	4	
θ Cet	8512	KOIII	1.07	2	Wk. L.
η Psc	9270	G8III	0.97	2	St. L.
107 Psc	10476	K1V	0.84	1	
τ Cet	10700	G8V	0.72	0	Wk. L.
γ And A	12533	K2III	1.15	3	VB
α Ari	12929	K2III	1.15	1	Like α Ser
ξ^1 Cet	13611	G8II	0.88	2	
HR 774	16458	KOp			Ba II star, carbon rich
α Cet	18884	M2III	1.64	4	
HR 969	20123	G8II	1.15	2	
HR 991	20468	K3II	1.41	3	
HR 999	20644	K3II-III	1.54	3	
ε Eri	22049	K2V	0.89	0	
δ Eri	23249	KOIV	0.93	0	Wk. L.
γ Tau	27373	KOIII	0.99	2	Hyades
δ Tau	27697	KOIII	0.98	2	Hyades
ε Tau	28305	KOIII	1.03	2	Hyades
θ^1 Tau	28307	KOIII	0.96	2	Hyades
α Tau	29139	K5III	1.53	4	
HR 1533	30504	K4II	1.43	4	
φ Aur	35620	K3p	1.39	3	Strong CN
υ^2 CMa	47205	K2IV	1.07	2	St. L.
ε Gem	48329	G8Ib	1.45	3	
θ^1 CMa	50877	K3Iab	1.72	2	Weak [O I]
ι Gem	58207	KOIII	1.03	2	Wk. L.
χ Gem	62345	G8III	0.93	1	St. L.
ζ Mon	67594	G2Ib	0.99	2	
β Cnc	69267	K4III	1.48	4	
ζ Hya	76294	KOIII	1.02	3	λ4150
α Lyn	80493	K5III	1.55	3	
α Hya	81797	K3II-III	1.47	3	
11 LMi	82885	G8IV-V	0.77	0	
μ UMa	89758	MOIII	1.60	4	
μ Hya	90432	K4III	1.48	3	
β LMi	90537	G8III-IV	0.88	1	St. L.
ψ UMa	96833	K1III	1.14	2	Wk. L.
υ UMa	98262	K3III	1.38	4	
δ CrT	98430	G8III-IV	1.11	3	Wk. L.
ο Vir	104979	G8III	0.98	4	Wk. L.
γ Com	108381	K1III-IV	1.13	1	Like α Ser
HR 5089	117440	G8III	1.19	2	Close V.B.
ξ Boo A	131156	G8V	0.76	0	
α Ser	140573	K2III	1.17	1	Strong CN
β Her	148856	G8III	0.94	2	St. L.
η Her	150997	G7III-IV	0.92	2	St. L.
π Her	156283	K3II	1.44	3	
	165195	K3IIp		0	Like HD 122563
70 Oph A	165341	KOV	0.86	0	
β Sct	173764	G5II	1.09	2	
λ Lya	176670	K3II	1.46	2	Weak [O I]
κ Cyg	181276	KOIII	0.97	1	St. L.
α Sge	185758	GOII	0.77	1	

TABLE 2 - Continued

Star	HD	Sp.	B-V	6300/63	Remarks
γ Aql	186791	K3II	1.54	4	
β Aql	188512	G8IV	0.88	0	Wk. L.
γ Sge	189319	K5III	1.58	4	
α^1 Cap	192876	G3Ib	1.08	2	
HR 7759	193092	K5II	1.64	3	
70 Aql	196321	K5II	1.60	4	
ε Cyg	197989	KOIII	1.03	1	St. L.
ξ Cyg	200905	K5Ib	1.65	4	
63 Cyg	201251	K4II	1.56	4	
ζ Cyg	202109	G8II	1.00	2	
ζ Cap	204075	G4Iab	1.01	0	Ba II star
β Aqr	204867	GOIb	0.84	2	
ε Peg	206778	K2Ib	1.53	4	
9 Peg	206859	G5Ib	1.18	2	
12 Peg	207089	KOIb	1.43	1	Weak [O I]
α Aqr	209750	G2Ib	0.98	2	
η Peg	215182	G2II-III	0.87	2	
λ Peg	215665	G8II-III	1.07	1	St. L.
HR 8726	216946	K5Ib	1.78	3	
ψ And	223047	G5Ib	1.14	3	
3 Cet	225212	K3Ib	----	4	
		b. High Velocity Stars			
ι Ari	11909	K1P	0.92	0	Wk CN, Sr II strong
HR 885	18474	G8IV		3	Wk CN, CH
θ CMa	50778	K4III	1.43	5	Wk CN
γ Leo A	89484	K1III	{0.86}	4	Wk CN
γ Leo B	89485	G8III		3	Wk CN
ν Vir	102212	MOIII	1.50	5	
HR 5270	122563	K3IIP	0.90	0	Metal poor
α Boo	124897	K2IIIP	1.23	4	Wk CN
HR 6152	148897	G8IIP	1.26	5	Wk CN, Wk L.
HR 6791	166208	G8IIIP	0.92	3	Wk CN, CH
η Ser	168723	KOIII-IV	0.94	1	Marginally high velocity
σ Dra	185144	KOV	0.80	0	Marginally high velocity
η Cep	198149	KOIV	0.92	0	Wk CN
	221170	K2IIIP	1.13	0	Like HD 122563
λ And	222107	G8III-IV	1.02	0	Wk CN, marginally high velocity.

Tables by Aller (1960) provide the mass, log $\mathfrak{m}$, above the photosphere (taken at $\tau_0 = 0.4$) as a function of surface gravity and effective temperature for a standard composition with log $A = 4.13$. Comparing a star with the Sun, we have

$$[\Gamma] = \frac{\mathfrak{m}_*}{\mathfrak{m}_\odot}. \tag{1}$$

Aller also provides the relation between P_e and P_g, the opacity $k_\lambda (P_g, \theta)$, etc. The partial pressure $P(\mathrm{O}')$ of atomic oxygen is found from Tsuji, and the number of atoms, $n(\mathrm{O}')$, or molecules, $n(\mathrm{CN})$, will be written as

$$[n(\mathrm{O}')] = [\Gamma] + [P(\mathrm{O}')] - [P_g]. \tag{2}$$

Square brackets, throughout this paper, give logarithms of ratios, object/Sun. We use the tabular data obtained for a given star:

$$[n(\mathrm{O}')] = [\Gamma] + \log P(\mathrm{O}') - \log P_g + \text{const.} \tag{3}$$

TABLE 3a

ADOPTED PARAMETERS OF THE STARS FOR ROUGH ANALYSIS

θ	log g	[Γ]	log P_g	log P_e
1.0	+2.0	+1.19	+3.54	−0.41
1.1	2.0	1.21	3.63	−0.64
1.2	2.0	1.27	3.76	−0.81
1.4	2.0	1.35	3.85	−1.20
1.0	3.0	0.78	4.15	+0.01
1.1	3.0	0.80	4.20	−0.17
1.2	3.0	0.84	4.21	−0.37
1.4	3.0	0.96	4.38	−0.79
1.0	4.0	0.33	4.67	+0.45
1.1	4.0	0.37	4.75	+0.26
1.2	4.0	0.42	4.87	+0.07
1.4	4.0	+0.52	4.92	−0.38
0.87	+4.4	0.00	+4.97	+1.05

Tsuji uses in his Table 1 as standard composition O/C = 2, O/N = 10, and O/H = 10^{-3}, and in his Table 2, a composition differing only in that O/C = 1. It is generally assumed that in the Sun O/C ≈ 1.6, so that we can interpolate either roughly for O/C = $\frac{5}{3}$ between his Tables 1 and 2 or recompute our "normal" stars. This proves, in fact, to be difficult when the association is high for O/C = $\frac{5}{3}$. We derived data for standard stars from Aller for sets of θ (from 1.0 to 1.4) and g (from 10^2 to 10^4) covering the range of giant and dwarf K stars. The constant in equation (3) was computed for the Sun with its composition (Tsuji, Table 1; hereinafter called "T1") at $\theta = 0.87$, log $g = +4.44$, log $A = +4.13$, and thereafter held fixed. For normal metal abundances, values of P_g and Γ used are given in Table 3a. From these we compute the final amount of atomic oxygen, as compared to the Sun, for various compositions, as given in Table 4a.

A change in procedure is required for stars with low metal abundance. The [Γ] of Table 3a are then incorrect, since the opacity is reduced; the P_g from Aller is also incorrect, since P_g/P_e increases when the metal abundance is reduced. If we wish to compute the behavior of oxygen when the metal/hydrogen and the O/H ratios are reduced by 10 but the C/N/O ratios are the same as in the Sun (Tsuji T6), we must recompute Γ and P_g. This was done for a reduced range of g (i.e., specifically for weak-line giants) in Table 3b. Since Aller's tables do not cover this case, we used the model atmospheres (described

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below) for the Sun (with normal abundances), and also for giants with 10 per cent of the solar metal abundance. The [Γ] and P_g obtained are given in Table 3b. These models are described by an effective temperature, θ_{eff}, but the θ used are read at $\tau_0 = 0.4$. On the other hand, Aller's θ is taken as the temperature at which the opacity and therefore [Γ] is read. Consequently, the final results in Table 4a, column T6′, are on a slightly different basis from the others.

By inspection of the models and results in Table 4a it can be seen that the numbers of oxygen atoms, O′, in the absence of significant CO association, vary nearly as $g^{1/2}$. When O/C = 2, the largest possible reduction in O′ is −0.3 in the logarithm. But when O/C = 1, CO could in principle consume most of the oxygen, and the amount of log O′

TABLE 3b

PARAMETERS FOR LOW-METAL-ABUNDANCE GIANTS BASED ON MODEL ATMOSPHERES

θ_{eff}	log g	log k_λ	[Γ]†
1 0	+2 0	−2 17	+1 43
1 1	2 0	−2 28	1 54
1 2	2 0	−2 31	1 57
1 4	2 0	−2 39	1 65
1 0	3 0	−1 67	0 93
1 1	3 0	−1 76	1 02
1 2	3 0	−1 92	1 18
1 4	3 0	−2 10	+1 36
0 87	+4 4	−0 74	

† [Γ] with respect to normal Sun $A^*/A_\odot = 10$.

TABLE 4a

NUMBER OF OXYGEN ATOMS [O′] RELATIVE TO THE SUN, BASED ON TSUJI MOLECULAR EQUILIBRIA, COARSE ANALYSIS, $\tau_0 = 0\ 4$

θ	log g	T1	T2	T6*	T11	T6′ Low Metals*	α Ser
1 0	+2 0	+1 23	+1 23	+0 23	+1 21	+0 53	+1 22
1 1	2 0	1 16	+1 00	+0 21	+1 12	+0 51	. . .
1 2	2 0	1 08	+0 77	+0 20	+0 84	+0 46	+0 80
1 4	2 0	1 08	−0 26	+0 09	−0 20	+0 37	−0 23
1 0	3 0	0 80	+0 77	−0 18	+0 77	−0 04	+0 77
1 1	3 0	0 67	+0 43	−0 23	+0 50	−0 04	. .
1 2	3 0	0 60	+0 14	−0 28	+0 17	−0 01	+0 15
1 4	3 0	0 65	−0 92	−0 35	−0 72	+0 03	−0 82
1 0	4 0	0 30	+0 25	−0 64	+0 25		
1 1	4 0	0 29	−0 15	−0 72	−0 15		
1 2	4 0	0 26	−0 57	−0 79	−0 56		
1 4	+4 0	+0 20	−1 64	−0 80	−1 50		
O/H		10^{-3}	10^{-3}	10^{-4}	10^{-3}	10^{-4}	10^{-3}
O/C		2	1	2	1	2	1
O/N		10	10	10	0 1	10	1
O abundance		Normal	Normal	Low	Normal	Low	Normal
Peculiar ratios			High C		High N,C	M/H=0 1	High C,N

* Table 6 of Tsuji does not allow for the lower opacity in the case when the metal/hydrogen ratio is also reduced by a factor of 10 Our column T6′ allows for this low opacity caused by low metal abundance

varies as $-5\Delta\theta$ (columns T2, T11, and α Ser). The association depends on P_g^2, and therefore, as g increases, the atomic oxygen drops rapidly. Table 4a shows also that O′ hardly changes with increasing N/O, even if O/C = 1 (columns T2, T11).

Since interpolation is difficult at low T, a special computation was made for O/C = $\frac{5}{3}$, with results given in Table 4b. In addition, the results of §§ IV and V give the run of θ and g for normal giants. Consequently, Table 4b gives the run of expected [O I] and CN along the giant branch. The strength of [O I] lines is so low that the equivalent width is probably always proportional to the number of atoms. Therefore, the predictions of [O′] in Table 4b can give the equivalent width of [O I]. We find that late G giants should have about 5 times as much atomic oxygen as the Sun; middle K giants lie nearly at 10 times the solar value. But [CN] in Table 5 shows order-of-magnitude differences. In addition, the lines of the blue CN bands at λλ3883, 4215 are often highly saturated. The difference in microturbulence between giants and dwarfs also controls the CN band strength. The unsaturated, red CN system would more probably follow the $W \propto n$(CN) law. The prediction for normal giants (Table 4b) shows a large increase of CN with θ, as is actually observed, and a maximum in early giant K stars. Inspection of the effect of abundance changes demonstrates a startling sensitivity of the CN strength to the O/C ratio (T2 compared to T1), with a change, Δ log n(CN) from +0.3 to +1.8 for Δ log C/O = +0.3. Similarly, for O/C = 1, the effect of adding N is enormous (T2 compared to

TABLE 4b

ATOMIC OXYGEN AND MOLECULAR CN FOR GIANT BRANCH WITH O/C = $\frac{5}{3}$, COARSE ANALYSIS*

θ	log g	[O′]	[CN]
1 0	+3 0	+0 77	+1 05
1 1	2 8	70	1 45
1 2	2 5	75	1 97
1 4	+2 0	+0 96	+1 63

* Double interpolation in Tsuji's tables is difficult when CO begins to take effect We computed a small grid of values of CO equilibria (θ, P_g) near the rough-analysis values of Table 3a, omitting all reactions except C + O $\rightleftarrows$ CO, for O/C = $\frac{5}{3}$ We later will obtain improved values of [O′] from models The CN values are very rough Values of θ and log g are discussed in §§ IV and V.

TABLE 5

PREDICTED CN BEHAVIOR ON GIANT SEQUENCE; ROUGH ANALYSIS

θ	log g	T1	T2	T9	T11	O/C=5/3
1 0	+2 0	+0.91	+1.23	+3 60	+3.18	+0 90
1 2	2 0	+2 07	+2 62	+5 04	+4 09	+2 13
1 4	2 0	+1 20	+2 84	+5 86	+3 83	+1 63
1 0	3 0	+1 07	+1 38	+3 74	+3 29	+1 05
1 2	3 0	+1 70	+2 40	+4 87	+3 73	+1.81
1 4	+3 0	+0 64	+2 47	+5 64	+3 42	+1 13
O . . .		10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
O/C		2	1	1/5	1	5/3
O/N		10	10	1/5	0 1	10
O abundance		Normal	Normal	Normal	Normal	Normal
Peculiar ratios			High C	Very high C,N	High C, very high N	

T11), where at $\theta = 1.0$, $\Delta \log n(\mathrm{CN}) = +1.9$ for $\Delta \log \mathrm{N/O} = 2.0$. Thus CN-strong giants can be expected either among those with $\mathrm{O/C} \lesssim 1$, in which case no [O I] remains, or among those with high N. The expected strength of the C_2 bands depends on which of these alternatives is adopted. The molecular equilibrium proves to be relatively straightforward, and concentrations follow the elemental abundance changes.

The observed great strength of [O I] lines in the low-metal, high-velocity giants, like α Boo or HD 148897, is not that directly predicted by column T6′ of Table 4a. The lines should be weaker than in normal giants if $\mathrm{O/H} = 10^{-4}$ in a high-velocity star. The abundance decrease of oxygen is partially cancelled by the lowered opacity in the metal-poor star, and the higher P_g does not greatly change the CO equilibrium concentration. Between T6′ and T1, at $g = 10^2$, the lines should be 5 times weaker in the mild Population II star; they are instead much stronger. Between T6′ and "normal giants" (if $\mathrm{O/C} = \frac{5}{3}$) the [O I] should be weakened by about 2 or 3. Referring to Table 3a and 3b, for θ from 1.0 to 1.4, the mass of the reversing layer is from 1.4 to 2.5 times greater than in the ordinary red giants. Association into CO, if $\mathrm{O/C} = 2$, can only reduce the atomic oxygen by a factor of 2. The observed enhancement of [O I] in the high-velocity giants suggests that they probably have $\mathrm{O/H} \approx 10^{-3}$; i.e., the "normal" solar value and that the O/metal ratio is 10 times its value in the Sun. This result is confirmed by fine analysis.

IV. DEPTH OF LINE FORMATION

The sensitivity of the residual concentration of atomic oxygen to molecular association is high. Oxygen remains neutral to large optical depths; the λλ6300, 6363 lines come from the ground state and the CO formation depends on depth and depletes oxygen near the boundary. Luminosity and population effects might be serious, especially at very small optical depth. It is therefore useful to compute the "contribution function" for a few models, to see if the effective optical depth of formation, $\bar{\tau}$, varies significantly. To simplify the problem, take the weak-line weighting function for absorption, $G(\tau)$, assume that $k_\nu = \bar{k}$ at the wavelength used, and compute the ratio of free atomic oxygen to total oxygen. In the expression

$$A_\nu = \int_0^\infty \eta_\nu G(\tau)\,d\tau\,, \tag{4}$$

we transform to pressures, using

$$\eta_\nu[\mathrm{O\,I}] = \frac{l_\nu(\mathrm{atom})}{m_{\mathrm{H}}\bar{k}}\,\frac{P(\mathrm{O}')}{P_g} = \frac{l_\nu(\mathrm{atom})}{m_{\mathrm{H}}\bar{k}}\left[\frac{P(\mathrm{O}')}{\sum_i P(\mathrm{O}_i)}\right]\frac{\mathrm{O}}{\mathrm{H}}\,. \tag{5}$$

Here $P(\mathrm{O}_i)$ is the partial pressure of any given compound or atomic form of oxygen, O/H is the initial abundance ratio of oxygen to hydrogen, and the mean atomic weight is taken as unity. The bracketed expression is the fraction of oxygen in atomic form, obtained from Tsuji's tables as a function of (θ, P_g). The behavior of the integrand is essentially that of $G(\tau)/\bar{k}$, i.e., steeply decreasing with τ except when CO formation is important. The $G(\tau)$ can be approximated as

$$G(\tau) \approx 3h\nu/kT_0(4 + h\nu/kT_0)^{-1}K_3(\tau)\,. \tag{6}$$

Then the strength of a weak line is given by

$$A_\nu = \mathrm{const.} \int_0^\infty \left(\frac{P(\mathrm{O}')}{\sum_i P(\mathrm{O}_i)}\right)\frac{K_3(\tau)}{\bar{k}}\,d\tau\,. \tag{7}$$

The constant increases by a factor of 13 per cent from the Sun to a K giant, through the change of the darkening coefficient. Call $\Delta = \log O'/O$, the bracketed factor in equations (5) and (6).

For the Sun, in the models discussed in § V, the logarithmic deficiency of atomic oxygen varies from about $\Delta = -0.12$ near $\tau = 0.1$, to -0.05 at $\tau = 0.5$ and approaches zero by $\tau = 1.0$. For a red giant with $O/C = 2$, Δ changes from -0.30 at $\tau = 0.01$, to -0.25 at $\tau = 0.5$, to -0.16 at $\tau = 1.0$, and can be neglected at $\tau = 2.5$. Thus the [O I] line is formed at slightly deeper layers than normal in the giant, because of CO association. On the other hand, CO association in a carbon-rich giant, $O/C = 1$, strikingly alters the contribution function. Thus, at $\tau = 0.01$, the atomic O is reduced by $\Delta = -1.7$, at $\tau = 0.1$ by -1.4, at $\tau = 0.5$ by -0.9, and even at $\tau = 1.0$ by -0.42. As a result, the contribution to the integral in equation (7) is nearly constant from $\tau = 0.04$ to 1.30. The actual results of the integrations were found to be, in arbitrary units, 3.4 for the Sun, 41 for the giant (if both have $O/C = 2$), and 8 for the carbon-rich giant ($O/C = 1$). Thus the increase in the expected strength of the [O I] line (weak-line theory) is a factor of 12 times in the normal giant and 2.3 in the carbon giant. Therefore, we expect large changes in the [O I] line only if the star is slightly carbon-rich. Otherwise, the sensitivity to details of a model should be small.

V. THE [O I] LINE STRENGTH PREDICTED FROM MODELS

We will treat the [O I] prediction of line strength in two phases, that without CO association and with it. The high ionization potential of oxygen makes it completely neutral at all relevant optical depths in stars of solar temperature or cooler; the λλ6300, 6363 lines have zero excitation potential. The lines are therefore relatively independent of chromospheric, superexcitation, or non-LTE effects. (The λ5577 line is, however, sensitive to non-LTE effects, and we treat it separately in the Appendix.) An extended, cool envelope of small continuous opacity would strengthen [O I]; resonance lines of neutral metals would not appear, but those of ions, like Sr II, would be enhanced. There is no evidence for such shells in our stars.

We have constructed a series of model atmospheres covering a wide range in effective temperature and gravities from late G to middle K spectral type and with fixed A. They use a single (scaled) $T(\tau)$ relation, an opacity program (Vardya 1963), and solve the equation of hydrostatic equilibrium explicitly at forty-one optical depths (at λ5000 Å) from 0.001 to 10.0. The $T(\tau)$ relation used was that of Mihalas (1965, and unpublished) for a star of solar temperature. These models, which are available but will not be reproduced here, should give reasonably accurate representations of stars in this spectral range, since the opacity source is primarily H^-. They do not, however, have strict flux constancy.

The metal abundance assumed was that of the Sun. We also constructed models using $\frac{1}{10}$ and $\frac{1}{100}$ ($[A] = 1$ and 2) times this metal abundance. Since for spectral types G and K the metals are the dominant source of electrons, the low metal-abundance cases are different. Rayleigh scattering plays a dominant role in the opacity for $[A] = 2$ and for the outer part of the models when $[A] = 1$. The assumption of a universal $T(\tau)$ is not completely safe, but since self-consistent model atmospheres have not yet been constructed for G and K stars with low metal abundance, this approximation is sufficient for the present study. Nevertheless, it is not expected that better models will change our derived conclusions. For stars cooler than spectral type K5 III the opacity source must include molecules which modify the atmospheres in unknown ways not well approximated by our approach. For this reason, we will not discuss models for M stars; Spinrad and Vardya (1966) have studied the light element abundances in M giants. For late giants, after helium flashes or thermal instabilities, it is possible that O/C abundance changes further complicate the problem of H_2O and CO formation.

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Using the model atmospheres, we predict the change in the [O I] line strength, compared to a solar type model of $\theta = 0.875$, $\log g = 4.44$. We use a weak-line weighting function and compute the contribution at each optical depth to the line strength. This should be sufficiently accurate since the observed [O I] line strengths are generally on the linear part of the curve of growth. The method used is similar to that of Cayrel and Cayrel (1963) for ϵ Vir, and that of Conti, Wallerstein, and Wing (1965) for the Hyades cluster.

For different $\log g$ and metal abundances, the expected changes in the strength of neutral oxygen lines from the Sun are shown in Figure 2. The ordinate is the (logarithmic) change $[\Gamma_0]$ in the line strength for an oxygen abundance similar to that in the Sun. Neglecting any CO association, assuming the line remains on the linear part of the curve of growth, we find, for example, that a K-giant star of $\theta_{\mathrm{eff}} = 1.0$, $\log g = 3$, should have

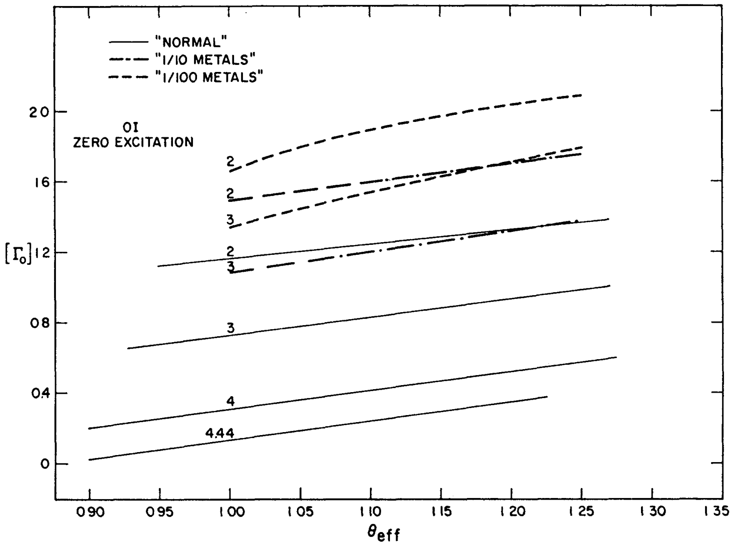

FIG. 2.—The logarithmic shifts, $[\Gamma_0]$ from model atmospheres, integrated over all optical depths with respect to the Sun (not allowing for CO formation) as a function of θ_{eff} and $\log g$ (labeled on curves). Special curves for low metal abundance (large A) are shown dashed. Note that O/H is kept at 10^{-3}, independent of A.

[O I] lines nearly five times as strong as in the Sun. The predicted line strength increases slightly with decreasing temperature, but is mainly sensitive to the effective gravity, varying roughly as

$$[\Gamma_0] \sim 1.0\Delta\theta \sim 0.4\,[g]\,. \tag{8}$$

For stars of low metal abundance, the model-atmosphere results in Figure 2 indicate that

$$[\Gamma_0] \sim 0.4[A]\,, \quad \text{for} \quad [A] \sim 1\,, \tag{9a}$$

$$[\Gamma_0] \sim 0.3[A]\,, \quad \text{for} \quad [A] \sim 2\,. \tag{9b}$$

We assume in these calculations that the O/H abundance ratio retains the solar values. For a different O/H abundance, the line strengths would vary in proportion, since oxygen does not contribute electrons. Our tentative justification of a high O/H has already been mentioned. In the coolest stars predictions of the models depart from the above approximations, as Rayleigh scattering becomes important. This effectively sets a lower bound to the opacity, or an upper bound to $[\Gamma_0]$, especially with very low metal abundance. In the hottest stars, with low metal abundance, hydrogen supplies some of the electrons, and the models depart from the above relations, as shown by the downward curvature at the left of Figure 2. The values of $[\Gamma_0]$ in Tables 3a, b agree well with them, if one remembers the difference between θ_{eff} and $\theta(\tau_0)$.

We must now allow for the effect of CO formation on the [O I] line strength, using the molecular equilibria. Figure 3 gives, as the solid lines, the logarithm of the fraction of CO; i.e., $P(\text{CO})\, N_{\text{H}}/P_g\, N_{\text{C}}$. Figure 4 gives the free, atomic-oxygen fraction $P(\text{O}')\, N_{\text{H}}/P_g N_{\text{O}}$ as a function of temperature, for a number of gas pressures. We neglect the slight contribution of H_2 to P_g which occurs only in the coolest stars considered. The pressure fraction of O′ is a measure of the residual atomic oxygen; e.g., for a value of -2.0, 1 per cent of the oxygen is in the form of free oxygen. For Figures 3 and 4, the O/C ratio is 2; Figure 5 gives the free atomic-oxygen fraction for O/C = 1. In the coolest stars of highest gas pressure, O′ decreases because of the formation of OH. This complicates the discussion of M giants and supergiants.

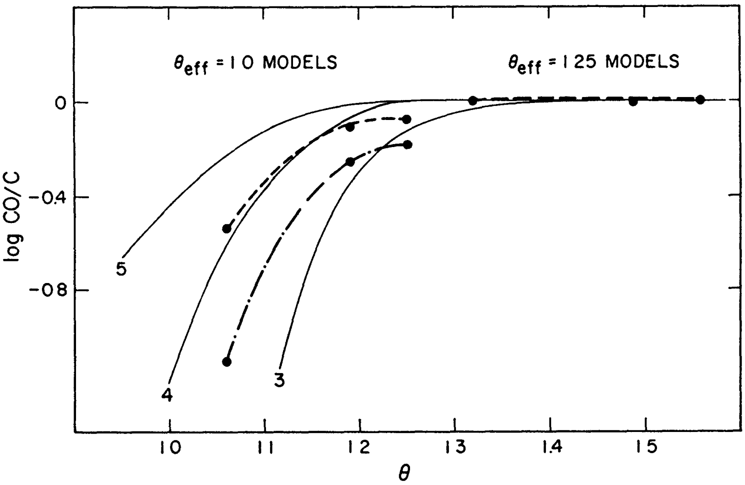

FIG. 3.—The fractional consumption of the atoms of C and O by CO formation increases toward cooler stars. The ratio of numbers of CO/C is plotted for O/C = 2, as a function of θ and P_g The stellar models give $T(\tau)$ and P_g (τ), and the dashed lines show log CO/C over the region in which the line is formed, from $\tau = 0.01$ (at right end) to $\tau = 0\ 5$ (left end). The log $g = 3$ models are dashed lines and the log $g = 2$ models are dash-dot lines. Both $\theta_{\text{eff}} = 1\ 0$ and 1.25 are shown. The abundances are $\text{O/H} = 10^{-3}$, O/C = 2, N/C = 0.1.

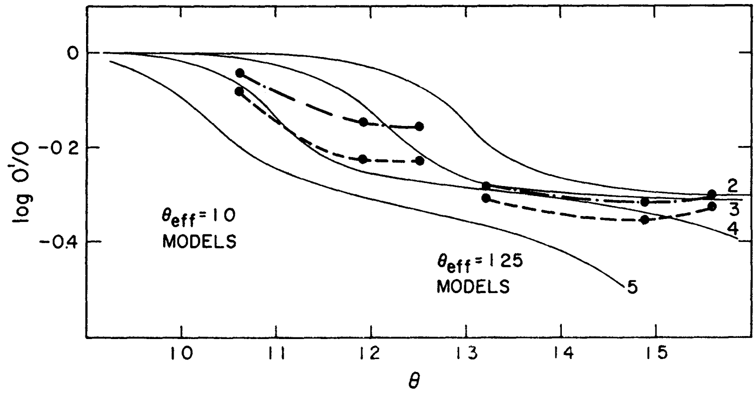

FIG. 4.—Solid lines, from Tsuji's tables, show log O′/O, the fraction of oxygen remaining in atomic form at various temperatures, θ, for log P_g's as marked. Models and symbols as in Fig. 3.

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The dotted lines in Figures 3–5 indicate the important regions of line formation (τ = 0.01 to 0.5) for models with θ_{eff} = 1.00 and 1.25, and log g = 3 and 2, corresponding to K0 III and K4 III stars. It can be seen that for both O/C ratios, the CO association is complete for the K4 III star and partial for the K0 III star. Earlier stars would have little or no CO association. The same conclusion holds for stars in which A is 10 times the solar value. The O′/O ratio is somewhat different in each abundance case because the O/C ratio controls this parameter. In Figures 4 and 5 it can be seen that the O′/O ratio changes with depth.

To obtain a temperature scale related to observable quantities, we adopt the $B - V$, θ_{eff} relation of Johnson (1964) for giants and dwarfs. We see that in a K0 III star about half of the carbon is associated in the line-forming region. For a "solar" O/C value of 1.6, then, O′/O ~ 0.7. For complete association, in the K4 III star the solar O/C gives O′/O ~ 0.4. We arbitrarily interpolate between these limits for intermediate spectral types. For a G5 III star we estimate that there is no CO association and O′/O ~ 1.0. We are now in a position to revise the predictions of line strength shown in Figure 2, for the solar O/C value, a correction which cannot amount to a reduction by more than a factor of 2 if association is complete.

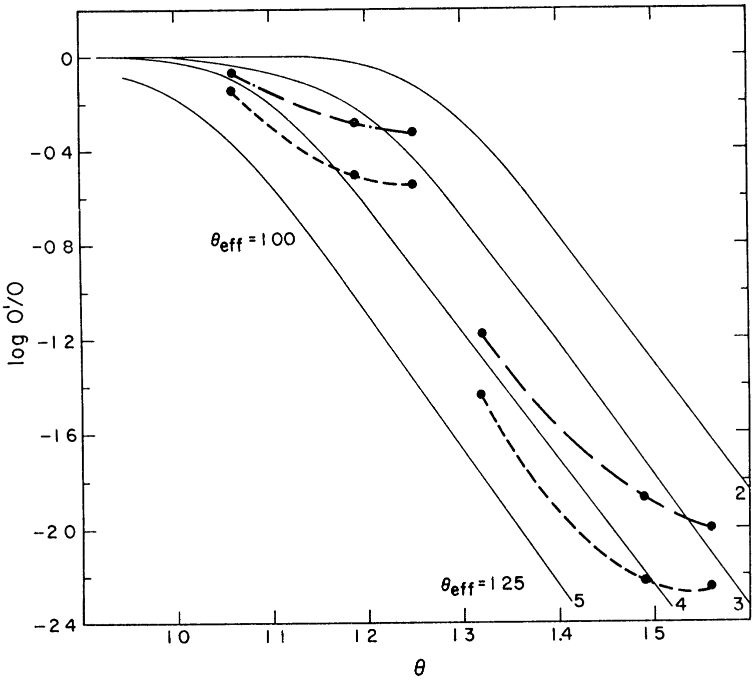

FIG. 5.—The amount of atomic oxygen as a function of θ and P_g, with models indicated as in Fig. 3 However, the O/C ratio is unity, so that the atomic oxygen is almost completely consumed by CO in the low-temperature model, θ_{eff} = 1 25

Let us adopt as the solar equivalent width of λ6300 the value 5 mÅ. Figure 6 shows the equivalent widths of λ6300 predicted as a function of $B - V$. With constant gravity, the expected equivalent width decreases with decreasing temperature, caused by the increasing CO association, from type G8 III to K4 III. However, the effective gravity in giant stars decreases toward later spectral type. The relation between effective gravity and spectral type (Aller 1963) gives the dashed line in Figure 6.

The observed relation between measured equivalent widths and colors of the stars of Table 1 is also shown in Figure 6. We find that the normal stars (*open circles*) fall very closely along the predicted (*dashed*) line. The (*solid circles*) high-velocity stars or weak

CN stars, from Table 2b, have equivalent widths which are about a factor of 2 stronger than expected for their colors (or spectral type). No extremely metal-poor star is included. The strong CN star, α Ser, has a λ6300 line too weak for its temperature. The choice of the $B = V$, $\theta_{\rm eff}$ relation is not crucial because of the slight dependence on temperature. The line blanketing in metal-rich stars may cause their $B - V$ colors to be too red, as compared with black-body, or $V - I$, colors; however, for such high-velocity stars as are plotted here, the differential line-blanketing effect cannot be large. Moreover, to remove the difference between the two groups would require a misclassification of the high-velocity stars by half of a spectral class. For example, Arcturus, K2p, has a color near K3, and temperature near K3.5; to have the same [O I] observed line strength as Population I stars, it would have to be moved to near M0, and be later in type than α Tau, which is quite implausible. The colors in Table 1 show that α Tau has $V - I = +2.15$ mag and α Boo, $+1.64$ mag; such $V - I$ colors can hardly be affected by differential line blanketing.

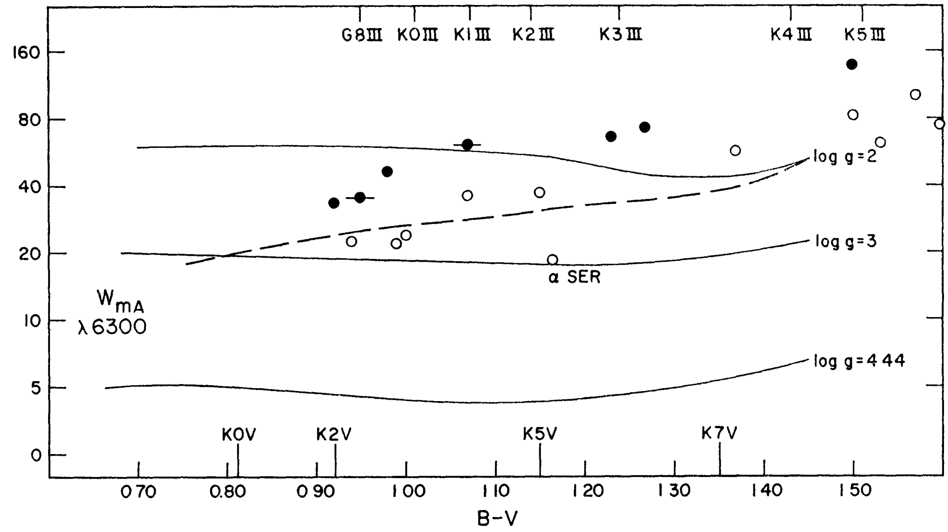

FIG. 6.—Predicted relation between equivalent width, W of λ6300 and $B - V$ colors for various surface gravities. *Dashed line:* the prediction for normal giants; *open circles:* normal stars; *filled circles:* weak CN, high-velocity stars.

VI. DERIVED O/C RATIOS AND OXYGEN ABUNDANCES

a) Normal Stars

We see from the predicted march of line strength in Figure 6 that the normal giant stars have about the expected strength of λ6300 for solar oxygen abundance. Before proceeding further, let us examine the evidence in support of our values of the effective gravity. Unfortunately, reliable spectroscopic gravities for most of these stars have not been determined. The absolute bolometric magnitude, temperature, and mass of a star yield the gravity from

$$[g] = [M] + 4[T_e] - [L] . \qquad (10)$$

Several stars in Table 1 are sufficiently near to have reliable trigonometric parallax (i.e., $>0\rlap{.}''040$). We can also use absolute magnitudes derived from the K emission-line width found by Wilson and Bappu (1957). Using the temperature scale and bolometric corrections of Johnson (1964), together with the absolute magnitude found by either of the two above methods (or for γ Tau from its Hyades-cluster membership) we derive Table 6. Two very metal-deficient stars TY Vir and SX Her, are discussed below. The largest uncertainty in Table 6 is the mass; we adopt 1 $M_\odot$ for the high-velocity stars, and for the normal giants, 2.0–2.5 $M_\odot$. We derive gravities for the normal stars (marked by daggers in the table) close to those given by Aller (1963) and shown by the dashed line in Figure 6. Therefore we may conclude that the O/C ratio in the normal stars is the same as the adopted solar value. An unexpectedly large O/C ratio, combined with an O/H ratio lower by a factor of 2, would leave the [O I] unchanged. This seems improbable because a molecular band involving carbon, notably CN, would be abnormally weak; carbon for CN would be lacking because it would be locked up as CO.

b) Weak CN, High-Velocity Stars

In this group, typical stars are α Boo and γ Leo A and B. They have [O I] line strengths too strong by about a factor of 2 for their spectral type and color, as compared to the other giants. A detailed analysis of α Boo by Gratton (1954) indicated that this star is deficient in metals, and that A was about 2.5 the solar value. Gratton found a similar metal deficiency for γ Leo A, but more recent work by Helfer and Wallerstein (1966) indicates a deficiency by a factor of 4 in γ Leo A. This is probably more accurate. We therefore adopt $A \approx 3\, A_\odot$ for such stars including α Boo, but the exact value does not affect our main conclusion.

TABLE 6

TEMPERATURE AND GRAVITY

Star	$\theta_{\rm eff}$	M_v	Source of M_v*	$M_{\rm bol}$	$\mathfrak{M}/\mathfrak{M}_\odot$	log g	Estimated $A/A_\odot$
Sun†	0 875	+4 63	1	+4 63	1 0	+4 44	1
γ Leo B	(1 02)	+1 54	2	+1 31	1 0	2 8	3
ϵ Vir†	1 02	+1 40	2	+1 15	2 0	3 0	1
γ Tau†	1 05	+0 68	3	+0 41	2 5	2 8	1
β Gem†	1 06	+1 05	1	+0 78	2 0	2 8	1
γ Leo A	(1 09)	+0 44	2	−0 11	1 0	2 3	3
α Ari†	1 13	+0 16	1	−0 25	2 5	2 4	1
α Ser ..	1 14	+0 95	1	+0 54	2 5	2.7	1
α Boo	1 21	−0 15	1	−0 71	1 0	1 7	3
HD 148897 .	1 24	+0 2	2	−0 2	1 0	1 8	10?
TY Vir	1 28	−2 0	4	−2 4	1 0	1 0	50:
SX Her	1 38	−2 0	4	−2 4	1 0	0 8	63:
α Tau†	1 33	−0 75	1	−1 81	5 0	+1 8	1

* Source of M_v: 1, trigonometric parallax ($\pi > 0''.040$); 2, K-line width (Wilson, private communication); 3, Hyades-cluster magnitude; 4, analogy with globular-cluster giants

† "Normal" stars

From Table 6 we see that these stars have gravities slightly smaller than in a normal giant of the same color, by about a factor of 2, from the expected mass difference. From our model calculations, as well as the relations (8) and (9), the metal deficiency in these stars reduces the opacity by a factor 1.6, and the low gravity an additional factor of 1.3, with respect to normal giants. If the O/metal ratio is preserved, then the line strengths would be $\frac{1}{3} \times 1.6 \times 1.3 \sim 0.7$ that in normal giants, in these stars. Instead the [O I] lines are about twice as strong. We conclude that the O/metal ratio must be abnormally large in the high-velocity giants. A different choice of A in these weak CN, high-velocity stars cannot alter the conclusion that the O/metals ratio is high.

It might be argued that O/C must be high. Otherwise, as much carbon would be free as in a normal star, to associate in CH or CN. With lower opacity, the CH lines would be stronger than in a normal giant, and the CN lines (depending on the N/H ratio also) could be strong. Weak CN and strong CH could be expected if the N/H ratio was low, but the C/H normal. Such an interpretation of some weak CN stars was suggested by Greenstein and Keenan (1958). However, this conclusion needs a detailed restudy; these bands are, in fact, sensitive to details of the temperature profile near the boundary.

Does the oxygen abundance in these weak CN stars have exactly the solar value? The answer depends critically on A, the exact gravity, and the O/C ratio. Without detailed analysis, we cannot comment further on the first two parameters. In α Boo, CO is completely associated and the predictions of Figure 6 use the solar O/C ratio. We have already seen that O/C may be high; this would have the effect of increasing the O′/O ratio. Recent work on α Boo by Gasson and Pagel (1966), who measure some metallic lines in addition to the [O I] lines, indicates that the oxygen abundance is solar and the metals and carbon are low by a factor of 2. The line ratio of [O I]/Sc II (Fig. 1), which shows α Boo anomalously high, also substantiates this conclusion. This line ratio is insensitive to the gravity and depends only slightly on the temperature, since the Sc II line is from an excited state.

We note in Tables 1 and 2 that the G8 III star, o Vir, also has an [O I] line strong for its spectral type. It has the relatively high velocity of 61 km/sec according to Roman (1952), but was not in Roman's (1955) *Catalogue of High Velocity Stars*. We conclude that it, too, has a high value of A and O/metals. Weak CN lines have been reported by Griffin and Redman (1960). Other stars in Tables 1 and 2 share the same peculiarities as the individual stars we will discuss below.

c) The Strong CN Star, α Ser

The λ6300 line in α Ser (Fig. 1) is weakened by about a factor of 2 for its spectral type and color. The gravity, according to Table 6, is roughly normal for its type. There is no firm evidence to suggest a change of A. These facts suggest a real anomaly, then, in the O/C ratio (and/or a low O/H). If O/C were smaller than 1.6, nearly complete CO association would reduce the O′/O ratio and weaken the [O I] lines. However, the O/C ratio cannot be as small as unity. From Figure 6 and a rough interpolation, $\theta_{\text{eff}} = 1.14$, we see that for O/C = 1, O′/O $\sim$ 0.1. Hence, the abundance of carbon in α Ser increases by less than a factor of 2 and O/C lies between 1.6 and 1.0.

We find that α Ser has weak [O I] but strong CN. How large a change in O/C is required? For this star we made a calculation of the equilibrium $C + O \rightleftarrows CO$. We pick a midpoint in the α Ser atmosphere, at $\tau = 0.4$, which corresponds to $\theta = 1.25$, $\log P_g = 4.04$. With $\Delta = \log O'/O$, if O/C = 1.4, $\Delta = -0.55$; if O/C = 1.6, $\Delta = -0.22$. Thus we get nearly a factor of 2 reduction in [O I] by increasing the C/O ratio by 15 per cent. If we convect C^{12}-enriched material through a hydrogen-burning shell, which would transmute C^{12} largely to N^{14}, and mix this to the surface in proportions such that the C abundance goes up 15 per cent, we would weaken [O I] as observed. We increase the CN strength by an amount which could be large if N^{14}/C^{12} is as large as the equilibrium ratio. The rough computations of Tables 4b and 5 show that the CN increase is roughly proportional to the added N. The strong CN stars could be interpreted as giants in a later evolutionary stage, perhaps after the helium flash.

d) HD 148897

According to Eggen (1965) HD 148897 is a member of the Wolf 630 group of moderately low space velocity. If it were, HD 148897 would have $(U, V, W) \approx (-24, -33, +21)$ km/sec. The group resembles M67, with only a small $U - B$ excess. Its luminosity M_v is +0.3 from group membership and +0.2 from emission line-widths; i.e., the spectroscopic giant characteristics fit group membership. The star has greatly weakened metallic lines, so that we do not believe HD 148897 is a member of the Wolf 630 group. The nearly normal spectra of M67 stars (and of most of the stars with spectral types listed by Eggen) suggest less line weakening in the Wolf 630 group than in HD 148897. If HD 148897 is not a group member, but is a globular-cluster giant, its space motion is large but not impossibly so. If, instead, it is a star like those in M67, the strong oxygen line is very interesting, since with nearly normal metal abundance and opacity, the strong forbidden line would indicate a large excess of oxygen. The temperature indicated by $B - V$ colors is the same as α Boo; since the lines are weaker, the smaller blanketing means even lower temperature. Therefore, the metal abundance is much lower than in α Boo, although higher than in HD 122563. The [O I] line in HD 148897 is slightly stronger than in α Boo (Table 1) and a similar argument demands an essentially solar value of O/H. However, the lack of information on surface gravity and the value of A prevents a more quantitative conclusion.

e) TY Vir and SX Her

These two stars have been studied by Preston and Wallerstein (1963), who found $\log A/A_\odot = 1.7$ and 1.8, respectively, with a possible error as large as ±0.5. They also noted the λ6300 [O I] line and concluded that the O/Fe ratio is the same as in the Sun, since the line falls on the Fe II curves of growth, in these stars and in the Sun. Their analysis neglected the effect of CO formation. If these stars have absolute visual magnitudes of −2.0, by analogy with globular-cluster giants, and masses of 1.0 $M_\odot$, we can derive $\log g$ values. Using these numbers and [A] found by Preston and Wallerstein and their temperatures (Table 6), we derive the [Γ_0] values given in Table 7. From the equivalent widths in Table 1 we find [Y], the shift of abscissa in the curve of growth, and derive [O/H] from [O] = [Y] − [Γ_0]. Then [O/metals] is given from [A] for these stars. The results, still neglecting possible CO association, differ from those of Preston and Wallerstein, possibly because of the use of a different gravity and excitation temperature. It is possible that Preston and Wallerstein's value of the O′/Fe ratio is more accurate. However, their neglect of possible CO association will modify their conclusion that O/Fe is normal. For example, if O/C were 1.6, complete association in these stars would imply that O′/O ∼ 0.4 and that [O/metals] ∼ 0.4, from their results, and [O/metals] ≈ 0.7, from our Table 7. If these are correct, the O/H ratio is less than the solar value in these very metal-deficient stars, and the O/metals ratio is slightly increased. This is based on a value of [C/metals] also ∼ 0.4. If we assert that O/C is so large that there is no CO association, O/metals would be normal but C/metals would be low. We cannot decide which of these, or other possibilities, is correct without further knowledge of the C/metals or C/H ratio from other molecules.

TABLE 7

REANALYSIS OF WEAK-LINED VARIABLES

Star	[Γ]	[Y]	[O/H]	[A]	[O/Metals]
TY Vir	+2.3	+0.9	−1.4	1.7	+0.3
SX Her	+2.5	+1.2	−1.3	1.8	+0.5

f) An Extremely Metal-deficient Star, HD 122563

This well-known star has been studied by Wallerstein, Greenstein, Parker, Helfer, and Aller (1963), who pointed out its extraordinary low metal content, and also by Pagel (1965). Pagel's reanalysis will be used here; he gives $\theta_{\rm eff} = 1.15$, $\log g = 1.2 \pm 0.7$, and $A/A_\odot = 300$. We use these numbers and our models to predict the [Γ_0] in this star, with a slight interpolation for gravity and extrapolation for A; we find [Γ_0] = +2.4.

Pagel did not discuss the oxygen abundance nor did he compute [Γ_0]. We can check this number by predicting Sr II lines. These behave like [O I], in this temperature regime, since Sr is wholly ionized and the lines have zero excitation. We can write the derived abundance [Sr II] = $[Y]_{\text{Sr II}} - [\Gamma_0]$. Pagel gives values for the observed shift of the two Sr II lines and the derived abundance, implying that his value of $[\Gamma_0]$ was +2.4, in agreement with our model.

The upper limit to the equivalent width of λ6300 in HD 122563 is 12 mÅ or $[Y] \leq +0.4$. Hence, for no CO association, $[O] \leq -2.0$. If we arbitrarily adopt the O/C ratio of 1.6 (half of the O associated) we conclude that $O'/O \sim 0.7$; $[O] \leq -1.8$. With no a priori reason to select a solar O/C for HD 122563, there remains a family of solutions with different combinations of O/H and O/C. A final choice can be made only when a carbon abundance can be derived. [O/H] can be low, or normal, in this extreme case.

In these peculiar stars, it has obviously been necessary to assume knowledge of the O/C ratio in very old (or evolved) stars. It will, however, eventually be feasible to determine the various parameters. We observe metallic lines, the [O I] line, the CH and CN bands, and sometimes C_2. With four (or five) observables and the unknowns O/C, N/C, O/H, H/metals it should be possible to improve the situation if g is better known. In high-velocity, weak CN stars, Greenstein and Keenan (1958) state that the abundance of carbon had essentially the solar value, while that of nitrogen showed the same deficiency as the metals (about a factor of 6). In G subdwarfs (usually of much higher metal deficiency), the results are internally contradictory; Aller and Greenstein (1960) and Danziger (1966) find from CH lines that C/metals is low but Baschek (1962) finds it normal. While not affecting our main conclusion, it is clear that a study of molecular lines in the K giants and in the G dwarfs of moderate metal deficiency will be of great importance in clarifying the early value of the O/C ratio. The use of the [O I] line provides the extra parameter making the problem solvable.

VII. NUCLEAR HISTORY OF RED GIANTS

In this section we explore the possibility that the oxygen or carbon content in K giants has been modified by mixing with material that has undergone thermonuclear processing. While the normalcy of the O/metals ratio in most stars indicates that this is unlikely, the newly established high ratio of O/metals in mildly metal-poor stars makes such an investigation important.

Computed sequences of evolutionary models for stars near 1 $M_\odot$ show no indication of oxygen enhancement in the atmosphere. Not only does the CNO cycle *deplete* oxygen for $T_6 \geq 20$ but even if helium-flash products are mixed to the surface, carbon rather than oxygen would be enhanced.

Thus the present O/metals ratio in the mild Population II stars must be representative of the ratio at the time the stars were formed. This implies that the galactic oxygen content was enhanced at a more rapid rate than the metal content.

In computed sequences of evolutionary models of Population I stars, of 3–5 $M_\odot$, there has been no suggestion that red giants should alter their surface oxygen content in any substantial fashion (Iben 1965, 1966). Can any events increase the C/O ratio at the surface? During the normal giant phase at the time when the core hydrogen is exhausted, the central temperature is roughly the same as in the hydrogen-burning shell. Although convection occurs, it is not supposed to reach very deep layers: Li is usually, but not always, depleted; Be is not destroyed, for example, in β Gem (Conti and Danziger 1966), at this phase or at earlier phases in older stars. But even if convection mixed the products of the CNO cycle to the surface this could not produce a substantial increase of the C/O ratio. With $N^{14}(p,\gamma)O^{15}$ as the determining, slow reaction in this cycle, the C^{12}/N^{14} ratio always drops, although at higher temperature the C^{12}/O^{16} ratio may rise. In Table 8 we show the initial, normal abundances in column (1), in (2a) and (2b) the values after CNO equilibrium has been established, at low T_c and also at high T_c, and the H is exhausted. Reaction rates used are given by Caughlan and Fowler (1962). At $T_6 = 20$, the C^{12}/O^{16} ratio is lowered and at $T_6 = 60$ substantially decreased. But if we attempt to derive a mixture of surface material with nuclear-processed material (using $T_6 = 60$), such that we would obtain as high a ratio as $C^{12}/O^{16} \approx 1$, as shown in column (3), we

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must take an essentially pure sample of nuclear-processed material. As a result, the surface and the envelope become He-rich and there is an enormous excess of N^{14}. This is certainly not observed. The dependence of CN bands on N/C in our Table 5 is steep. In addition, it is clear that so helium-rich a surface composition (≈ 90 per cent He) is incompatible with a red-giant inhomogeneous structure unless the core is C^{12} or heavier; such an evolutionary stage would be very short-lived.

We might imagine that (1) the CNO ratios should be those of incomplete burning, or (2) the N^{14} $(p,\gamma)O^{15}$ reaction rate is incorrect, or (3) the CNO cycle is not the source of C/O enhancement. The last alternative requires C^{12} production from the triple-alpha reaction. As to alternative (1), the unsaturated CNO-cycle abundances are given by Caughlan (1965) in her Figures 4 and 5. At $T_6 = 20$, the C/O ratio is very low, about 0.01 for a few protons consumed per initial nucleus, and rises toward the saturation value of 0.4, at an exposure of 1000 protons per nucleus. At $T_6 = 50$, the C/O rises from 0.1 to about 6, with increased proton exposures; in our Table the equilibrium values were 0.3 and 8.5, in reasonably good agreement. Therefore, incomplete burning does not help to increase the C/O ratio. If (alternative [2]) the C^{12} $(p,\gamma)N^{13}$ reaction is slower than N^{14} $(p,\gamma)O^{15}$, the CNO cycle would provide higher C^{12}/O^{16} ratios.

TABLE 8

TRANSFORMATIONS OF ABUNDANCES UNDER VARIOUS ASSUMED CYCLINGS
(Log_{10} abundances)

Element	Initial (1)	CNO-Equil		Mixed $T_6 = 60$ (3)	Mixed		
		$T_6 = 20$ (2a)	$T_6 = 60$ (2b)		$x_{12} = 1$ 0 (4a)	0 1 (4b)	0 001 (4c)
H .	12 0		.	12 00	12 00	12 00	12 00
He^4 .	11 2	11 61	11 61	12 45	11 20	11 21	11 78
C^{12}	8 6	6 96	7 64	8 81	8 78	8 78	8 78
C^{13}	6 6	6 36	7 03	7 86	6 58	6 58	6 58
N^{14}	8 2	9 03	9 03	9 92	8 57	8 57	8 57
O^{16}	8 8	7 45	6 71	8 80	8 78	8 78	8 78

Finally, if the triple-alpha process produces C^{12} in the core, say, at $T_6 = 200$, and if it is brought to the surface through the hydrogen-burning zone, say, at $T_6 = 20$, with an average proton consumption of less than one-half per C^{12} nucleus, the N^{14}/C^{12} ratio can be held below unity, so that one-half or more of the C^{12} survives. Then the C^{12}/He^4 ratio will depend on how much C^{12} has initially been produced. Let us take this pre-circulation core abundance as x_{12}, with $x_4 = 1 - x_{12}$, and $x_1 = 0$; the new values are $x'_{12} = x_{14}' = \frac{1}{2}x_{12}$ after passing through the hydrogen-burning zone. How large does x_{12} need to be to give $C^{12}/O^{16} = 1$ at the surface? Column (4a) is computed with $x_{12} = 1.0$, i.e., an unrealistic, complete He^4 burning to C^{12}. The shell production of N^{14} results in only a modest increase of N^{14} after mixing to the surface, so that column (4a) would give an acceptable surface composition. If, on the other hand, the carbon core is less developed and $x_{12} = 0.1$, only a negligible He^4 increase can be noted in column (4b). This gives an acceptable surface composition, in which only C^{12} has been added. The model is extremely unrealistic, permitting C^{12} synthesis from the triple-alpha process to occur long before model interiors suggest it is possible, and requiring efficient circulation to the surface. In contrast, column (4c) computed with $x_{12} = 0.001$, i.e., a very small C^{12} production, requires so much total material to be mixed with surface layers that the entire star is He^4-rich. (Note that in cols. [3] and [4a]–[4c] we have kept $H = 10^{12}$.) While columns (4a) or (4b) are acceptable, they seem to require a very improbable series of events. A carbon core with any of the envelope composition of columns (4a)–(4b) would probably result in sufficient inhomogeneity to retain the red-giant characteristic. But none of these internal

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sequences of events is required by our present knowledge of stellar evolution. A sequence such as leads to column (3) might occur at a late stage and produce a slightly carbon-rich, relatively hydrogen-poor star like R CrB or HD 182040.

However, certain stars, like α Ser, apparently are slightly enriched by carbon (and/or nitrogen), but otherwise resemble normal giants. It appears that there is some mixing to the surface at some epochs in the giant phase, perhaps at helium burning or carbon burning, which might occur during second traversals of the giant region. The barium stars, the S stars, and the carbon stars may represent such short-lived stages where carbon production is accompanied sometimes by neutron production and *s*-process. The few, slightly carbon-rich, peculiar stars we have observed, like HD 16458, have weak oxygen lines as would be expected.

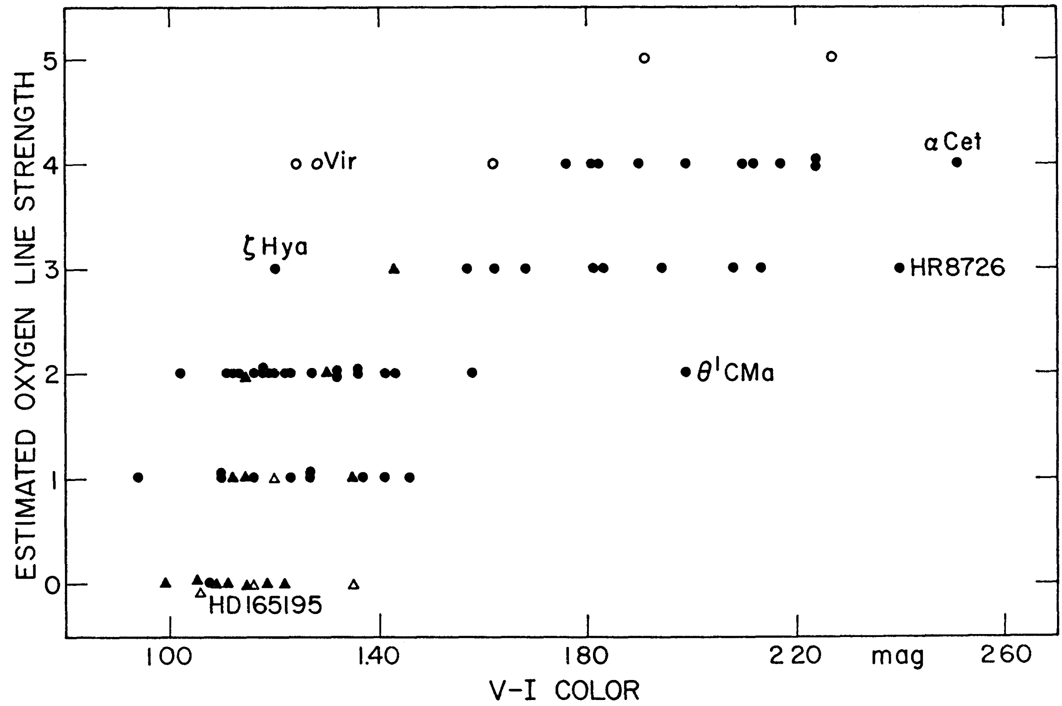

FIG. 7.—Eye estimates of the strength of [O I] in stars with known $V - I$ color. High-velocity stars are marked by open circles for giants or brighter, open triangles for main-sequence or subgiants. Solid figures, normal stars. The group of stars resembling α Ser have $I = 1$, and $V - I \approx 1.40$, all below the normal relation. All the weak CN line stars are on the upper boundary. Since color is a function of both temperature and surface gravity, the expected intensity is a complicated function of location. HD 165195 with very weak metallic lines, has weak [O I]; it is a high-velocity giant, marked with an open triangle.

VIII. CONCLUSION

The normal stars show about the same oxygen/hydrogen ratio as does the Sun, and the higher-velocity giants have the same value of O/H, which means a higher O/metals ratio than the Sun. In addition, marginal evidence suggests that very metal-poor halo stars have a smaller O/H than the Sun. Certain stars may be self-enriched in carbon, and if the strong CN stars belong to this group, their O/C ratio is reduced. Although limited to high dispersion, the use of the forbidden oxygen lines seems a very promising tool for the study of the early phases of nucleosynthesis.

Although we limited our discussion to stars with equivalent widths measured at very high dispersion, for which we have estimated temperatures and surface gravities, simpler methods can give interesting, if less quantitative results. In Figure 7 we plot our eye estimates of [O I] strength as a function of infrared color, $V - I$. These colors are obtained from the résumé of available infrared data by Johnson, Mitchell, Iriarte, and Wiśniewski (1966) This color scale should be insensitive to differential line blanketing, and the TiO bands are not strong before M0 III. Just as in Figure 6, the high-velocity, weak CN stars lie along the upper boundary of Figure 7; α Ser and similar stars lie at the lower edge. Many interesting stars which do not have any known peculiarity lie far from the normal relation. Peculiar effects are to be expected at the end of stellar evolutionary tracks, including changes in the O/C ratio. Exploratory observations of [O I] in many peculiar types of stars can also provide information on late phases of nucleosynthesis.

We wish to thank Miss Carol Webb for identifying the oxygen lines on the Lick spectrograms.

APPENDIX

THE INTERLOCKING EFFECTS IN λ5577 AND λ6300

A relatively simple and interesting non-LTE problem arises in connection with the expected strength in absorption of λ5577, a forbidden line arising from the common upper level of the λλ6300, 6363 forbidden lines. Note that if recombination of O^+ or collision of O and H would populate this upper level, the excitation potential is sufficiently low that one would expect $T_{\text{exc}} \lessapprox T_{\text{ion}} \approx T_{\text{kin}}$ in LTE. But we will find collision rates in giants to be low; the ionization of O is also negligible, like that of H. In consequence, we set up the three-state problem for [O I]. To simplify it, we will consider only the $^3P_2$ portion of the two ground states, and call the line from this single state (λ6300) "line #1." The other states are singlets. The transauroral line, λ2972 can be included as a source of population of level 3. We neglect stimulated emission, since exp $(-h\nu/kT)$ is less than 1 per cent. Table A1 shows the adopted parameters. The line λ6363 is neglected in the transition $2 \longrightarrow 1$.

TABLE A1

DATA FOR THE THREE-STATE POPULATION PROBLEM

Transition	λ(Å)	A (sec^{-1})	Exc Lower State (eV)	$h\nu$ (eV)
2→1	6300	0 0069	0	1 95
3→2	5577	1 28	2 21	2 21
3→1	2972	0 078	0	4 16

The steady-state equations for the populations N_1, N_2, N_3, can be written in convenient form if one takes the radiation field within the line, $I_\nu = WB_\nu(T)$; i.e., a thermal continuum depressed by a fraction W by the presence of the absorption line itself. Then we find

$$-N_1W_{12}\frac{g_2}{g_1}A_{21}e^{-u_{21}}+N_2A_{21}+N_3A_{31}-N_1W_{13}\frac{g_3}{g_1}A_{31}e^{-u_{31}}=0\,. \qquad \text{(A.1a)}$$

$$-N_2W_{23}\frac{g_3}{g_2}A_{32}e^{-u_{32}}-N_2A_{21}+N_3A_{32}+N_1W_{12}\frac{g_2}{g_1}A_{21}e^{-u_{21}}=0\,, \qquad \text{(A.1b)}$$

$$+N_2W_{23}\frac{g_3}{g_2}A_{32}e^{-u_{32}}-N_3A_{32}-N_3A_{31}+N_1W_{13}\frac{g_3}{g_1}A_{31}e^{-u_{31}}=0\,, \qquad \text{(A.1c)}$$

where $u_{jk} = (11600/T)\,h\nu_{jk}$, with $h\nu_{jk}$ in electron volts as in Table A1. The Einstein coefficients B_{kj} have been eliminated. If we insert $W = 1$ in equations (A.1a)–(A.1c) we obtain the

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Boltzmann populations. If we examine the size of the coefficients for the special case of $T = 4200°$ ($\theta = 1.2$) the equations (A.1a)–(A.1c) become

$$-N_1\,(W_{12}\,10^{-4\ 50} + W_{13}\,10^{-6\ 81}) + N_2\,10^{-2\ 16} + N_3\,10^{-1\ 11} = 0\,, \qquad \text{(A.2a)}$$

$$+N_1\,W_{12}\,10^{-4\ 50} - N_2\,(10^{-2\ 16} + W_{23}\,10^{-3\ 24}) + N_3\,10^{+0\ 11} = 0\,, \qquad \text{(A.2b)}$$

$$+N_1\,W_{13}\,10^{-6.18} + N_2\,W_{23}\,10^{-3.24} - N_3\,10^{+0\ 13} = 0\,. \qquad \text{(A.2c)}$$

It can be seen that there is an enormous range in the size of the coefficients; there are three equations and five unknowns N_2/N_1, N_3/N_1, W_{12}, W_{13}, W_{23}. But W_{12} and W_{13} depend on N_1, and W_{23} on N_2 through the line-formation equation and the curve of growth. Therefore, it is not possible to solve the general problem easily. In fact, several approximate, perturbation-to-LTE solutions failed, giving negative populations or $W > 1$. If we omit level 3, the problem is very simple, equations (A.2a) and (A.2b) giving

$$N_2/N_1 = W_{12} 10^{-2\ 34}\,. \qquad \text{(A.3)}$$

This is the Boltzmann population reduced by W_{12} ($0 < W_{12} < 1$). The population of level 2 is reduced as λ6300 strengthens, and if λ6300 ($1 \longrightarrow 2$) is black over the level width arising from Doppler broadening, $N_2 \longrightarrow 0$ and λ5577 ($2 \longrightarrow 3$) disappears. Returning to the three-level problem, since λ5577 is always weak, $W_{23} \longrightarrow 1$. We may see, approximately, what effect typical values, $W_{12} = 0.5$ and $W_{13} = 0.1$, would have. We find $\log N_2/N_1 = -2.64$ and $\log N_3/N_1 = -5.99$; the Boltzmann formula gives -2.34 and -5.70, respectively Again the quantity W_{12} dominates the computation, so that a reasonable guess at the three-level solution agrees with equation (A.3). When λ6300 has an equivalent width exceeding the Doppler width, $W_\lambda > \Delta\lambda_D$ and $W_{12} \lessapprox 1 - e^{-1}$ ($\approx 0\ 6$), i.e., enough to suggest that λ5577 can easily be weakened by a factor of 2, or more, in stars where non-LTE, radiative excitation dominates. It is interesting to evaluate the collisional rate of population of state 2 as compared to the radiative rate. No cross-section for the neutral hydrogen excitation of neutral oxygen is known Assume that $\sigma \approx 10^{-17}$ cm^2 (small interaction for neutral atoms). Then the collisional excitation rate is given from the gas pressure and temperature:

$$(dN/dt)_{\text{coll}} = N_1 N_{\text{H}} \langle \sigma v \rangle = 7 \times 10^4 N_1 P_g/T \text{ cm}^{-3} \text{ sec}^{-1}\,. \qquad \text{(A.4)}$$

The radiative rate is measured by the radiation within the Doppler width, i.e., for a moderately strong λ6300.

$$4\pi J_\nu \Delta\nu_D W_{12} N_1 B_{21} \approx 4\pi B_\nu(T)\,\Delta\nu_D \tfrac{1}{2} N_1 A_{21} \frac{g_2}{g_1} \frac{c^2}{2h\nu^3}\,, \qquad \text{(A.5a)}$$

$$(dN/dT)_{\text{rad}} \approx 2\pi \Delta\nu_D N_1 A_{21} \exp\left(-\frac{h\nu}{kT}\right) \approx 3 \times 10^5 \text{ cm}^{-3} \text{ sec}^{-1}\,. \qquad \text{(A.5b)}$$

The value of P_g/T depends on the optical depth and on the surface gravity. It is interesting to find that for a dwarf, $(dN/dt)_{\text{coll}} > (dN/dt)_{\text{rad}}$ and the excitation is Boltzmann. But for $\log g = 2$, $P_g/T = 1.3$ and for $\log g = 3$, $P_g/T = 5.1$, so that the collisional rate has dropped close to or below the radiative rate, permitting the non-LTE effects described above to operate and to weaken λ5577 in giants. This would be an interesting luminosity discriminant but difficult to observe.

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