POTENTIAL DISTRIBUTION IN INFINITE CONDUCTORS AND UPLIFT PRESSURE ON DAMS.

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WHEN a dam is built on porous strata such as generally exist at the beds of rivers, there is an underground flow through the porous strata under the masonry, from the upstream to the downstream. Accompanying this flow there will be a pressure acting upwards from below on the masonry floor. An accurate knowledge of this uplift pressure and the nature of its distribution is of fundamental importance for purposes of designing a dam. There have been many attempts to investigate by direct measurements from laboratory models of dams, the way in which this pressure varies with different forms of design. The usual method for model work is to build a tank of convenient size with a number of holes fitted with manometers. The tank is then filled with sand representing the sub-soil strata and the model of the dam is placed on the sand. A head of water is maintained on the upstream side of the model of the dam and the pressure distribution is indicated by the manometers. A very large number of measurements and readings are required to obtain satisfactory results. The above method is very laborious.

An accurate and quick method for studying this pressure distribution was therefore highly desirable. From the analogy between the Ohm's law for conduction of electricity and Darcy's law for the conduction of water through porous media, N. N. Pavlovsky[1] suggested as early as 1921, that an electrical method could be employed, for the purpose of studying the sub-soil pressures under dams. Some attempts were made in this direction by Lane and co-workers[2] in America, but no definite conclusion was arrived at. The experimental evidence was very unconvincing and the electrical method stood almost rejected. The question whether the electrical method gave results agreeing with direct measurements or not, is of fundamental importance. For if the potential distribution in conductors is the same as the pressure distribution in the sub-soil under the models of dams, then the potential law and the equations of Laplace can be employed for the study of this pressure distribution. If the Laplace equations do apply, then the mathematical foundation for the flow of water under dams is established. If the Potential law does not hold then the problem of sub-soil flow under dams must be attacked from a different standpoint. It was therefore highly desirable to investigate the problem very accurately and compare the results obtained by different methods, theoretical as well as experimental.

In investigating the various cases we were limited to two simple types (1) A simple impervious floor flush with the surface of the porous strata, (2) The same case with sheet-piles.* For both cases, theoretical solutions and direct measurements in models existed and so the results of the electrical method could be compared with direct observation of pressure and with theoretical solutions.

2. *Experimental.*

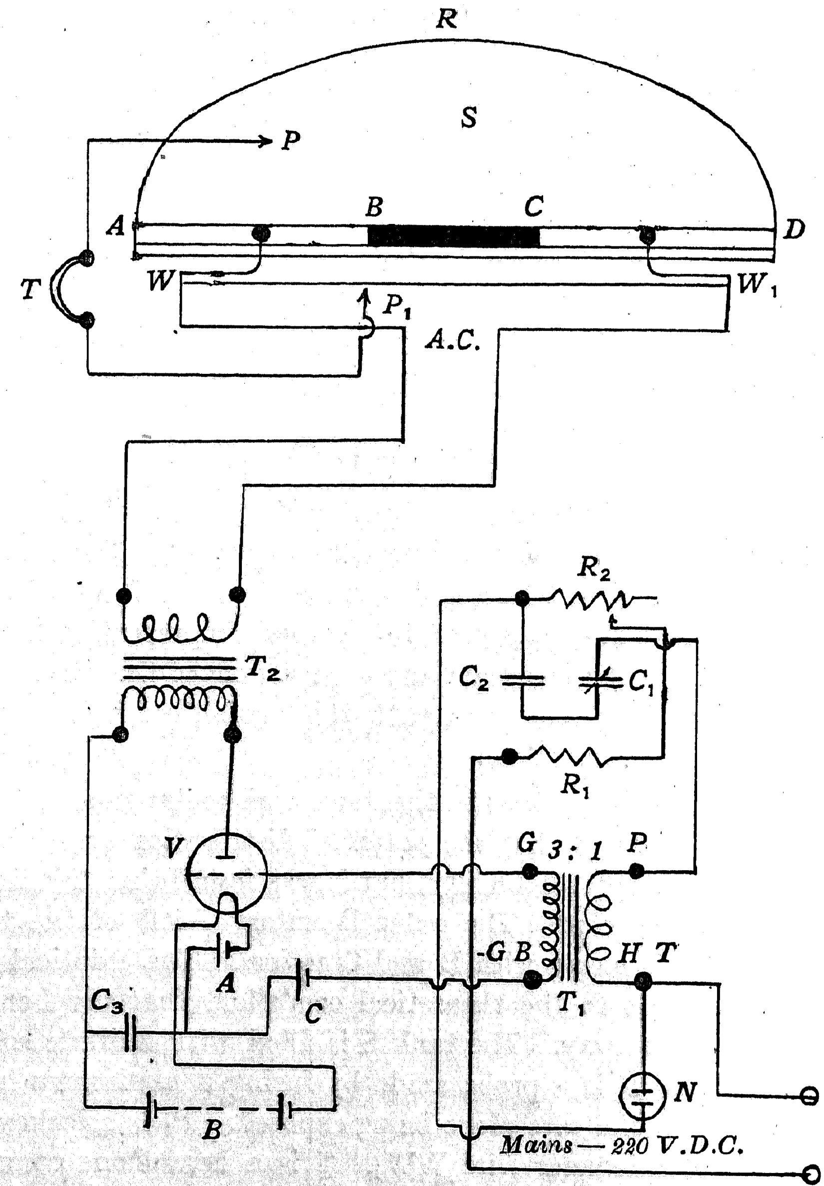
The apparatus for the electrical experiment consists of two parts, the model tank and the source of the alternating current. These are shown in Fig. 1. The tank is 4 feet long and 3 inches deep. AB and CD are two thick conducting plates of copper each 1·5 feet long representing the surfaces of the porous strata on the upstream and downstream of the dam. BC is an ebonite plate 1 foot long and flush with the conductors representing the non-porous masonry of the dam.

In the first set of experiments the tank was rectangular being about 4′×3′×3″. It was thought that the potential distribution round BC would not be affected by the outer boundary, but it was found that this introduced an error of about 5 per cent. The outer boundary ARD of the tank was then made elliptical in shape with B and C as foci. The elliptical shape of the boundary conforms to the theoretical condition, that the boundary is at an infinite distance away. The tank S is filled with a dilute solution of ammonium chloride. P is a probe made by fusing a platinum wire into a glass tube, the end of the wire alone being exposed. P_1 is a jockey moving on a ten-metre potentiometer wire WW_1. T is a headphone connected as shown in the figure.

The potentiometer wire serves as a potential divider. The ends of the potentiometer are connected in parallel with the A.C. source and with the conducting plates as is shown in the diagram (Fig. 1). The wire was accurately calibrated.

The A.C. source (Fig. 1) is obtained from a Neon oscillator with an amplifier. This arrangement which is shown completely in Fig. 1 is a very convenient source of oscillating current and a sharp null point is obtained in

* A sheet-pile is an impervious obstruction driven generally at right angles to the impervious horizontal floor into the sub-soil.



AMPLIFIER OSCILLATOR

FIG. 1.

C_3—Fixed Condenser
V —Triode Valve
T_2—Transformer
A —Filament Battery
B —Plate Battery
C —Grid Battery

N —Neon Bulb
R_1—High Resistance
R_2—Rheostat
C_1—Variable Condenser
C_2—Fixed Condenser
T_1—Transformer

the phone. A high accuracy in determining the null point is required in this experiment, because it can be seen from the arrangement that the total fall of potential between the ends of BC is equal to the fall at the ends of ten metres in the potentiometer and the problem is to determine very accurately the potential of the various points along BC. The present arrangement for determining the potential was found so good that the null points in the phone could be determined with absolute certainty. The success of the experiment was mainly due to this part of the technique. The method may be employed with advantage for measurement of resistances where an A.C. is to be used.

In the preliminary experiments where ordinary buzzers were used, no definite results were obtained.

The details of the arrangement can be understood from the diagrammatic sketch in Fig. 1.

3. *Method of Observation.*

In carrying out an experiment, the moving contact P_1 in the potentiometer (Fig. 1) is placed at 1/20th of the potential and by probing with the help of P, the series of null points corresponding to this potential are marked in the tank. P_1 is then moved to 2/20th of the potential and the observations are then repeated. A complete set of equipotential lines for decrements of 5 per cent. of the total potential drop is thus traced on the bottom of the tank. To facilitate the direct marking of the points, the bottom of the tank was made of ground glass. Tracings of the points are then made and reduced to a convenient size for comparison.

4. *Direct Measurements.*

As pointed out in the introduction, the direct measurements on model were carried out in tanks.

The size of the tank for direct measurements was the same as for the electrical method, to facilitate comparison. The details of the method for direct measurement of pressures under models are given in a memoir of the Institute.[3]

5. *Theoretical Calculations.*

The theoretical calculations for the distribution of pressures mentioned in cases 1 and 2 have been made by Weaver.[4] In the case of the simple impervious floor the solution for the pressure lines gives a confocal system of hyperbolæ with the end of the masonry as foci. This corresponds to the system of equipotential lines obtained in the case of a magnetic or electric dipole in an analogous case. The lines of force are a system of ellipses with the poles as foci, *i.e.*, in the case of flow of water the streamlines would correspond to a similar system of ellipses. In obtaining the solution, the potential law of flow was assumed. In the simple case mentioned, the solution for the pressure distribution will be an *S* curve. When a sheet-pile is introduced at one end, the system of equipotential or equipressure lines becomes very complicated and the mathematical solution difficult.

Weaver[4] has worked out a theoretical solution for the case of one sheet-pile and reference may be made to that paper for details of the calculations. For the sake of clearness, one diagram from his paper giving a graphical representation of the mathematical calculation is reproduced in Fig. 2. For ordinary cases of weirs which are generally very complicated, the mathematical solutions have not yet been obtained and they are very difficult. Only experimental solutions for the pressure distribution for the complicated cases are thus possible.

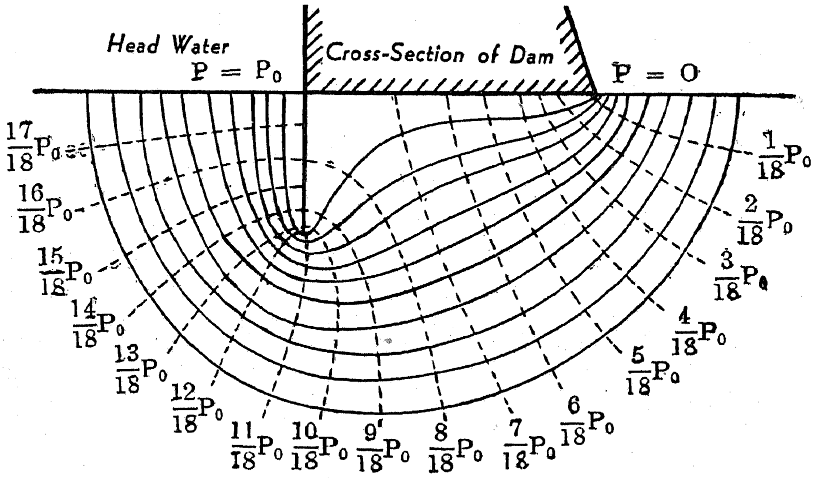


FIG. 2. Lines of Seepage Flow and Lines of Equal Pressure under Dam with Sheet-Piling.

(Reproduced from the Journal of Mathematics and Physics, June 1932.)

6. *Results.*

The results are represented graphically in Figs. 3 and 4.

CASE I.

A simple impervious floor flush with the surface of the porous strata.—The complete system of potential lines obtained from the electrical method, direct measurements and theoretical calculations are shown in Fig. 3, only one half of the system being shown. The difference between the results of the electrical method and that of theoretical calculations was generally less than 0·2%. This small error was due to resistances at the contacts and of the leads. The difference between the results of the direct measurements and those of the electrical method was about 3%. This agreement should be taken as remarkably good when the various sources of error and difficulties of measurements in the direct method are considered.

The potential distribution on the non-conductor corresponding to the pressure distribution along the bottom of the masonry is compared in Curve 1, Fig. 4. The points obtained by the two methods and those of the theoretical

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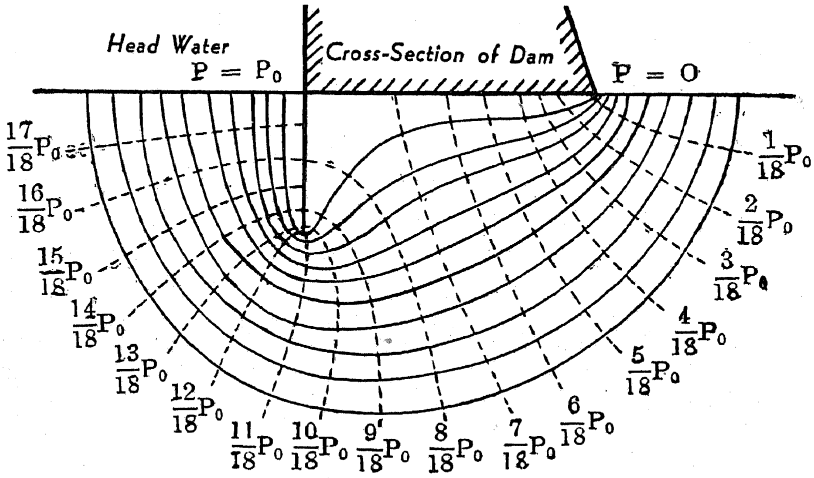

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The potential distribution on the non-conductor corresponding to the pressure distribution along the bottom of the masonry is compared in Curve 1, Fig. 4. The points obtained by the two methods and those of the theoretical

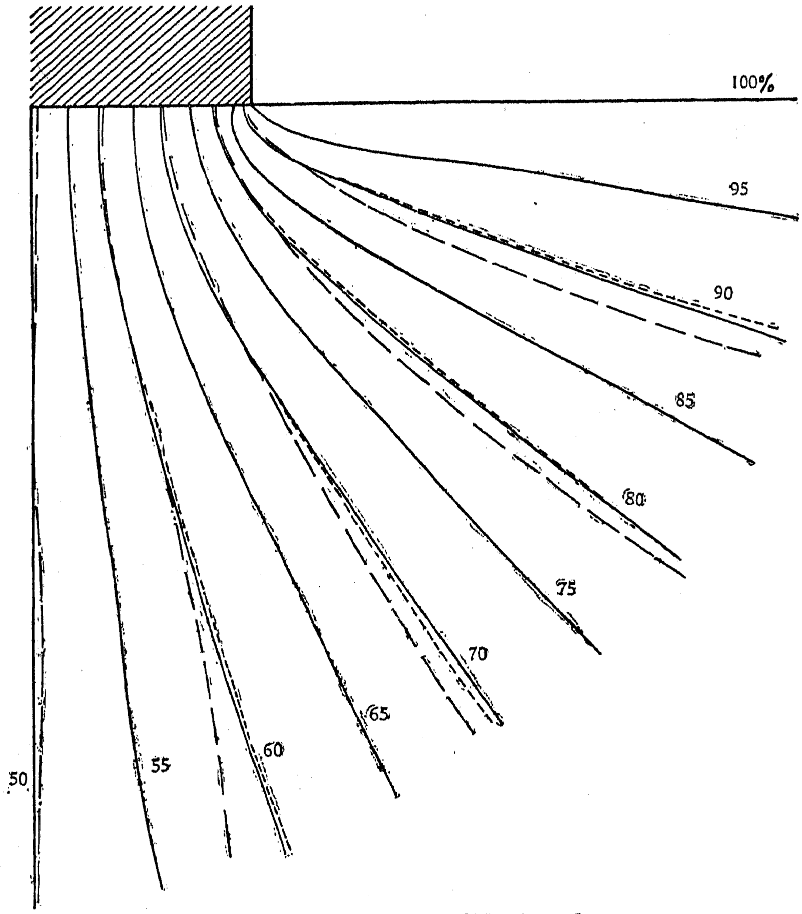

FIG. 3.
Flush Case of the Floor.

calculations fall on the curve showing that the results of the electrical method are identical with those of direct measurements and of theoretical calculations.

In the curves α represents the ratio of the length of the horizontal masonry to that of the sheet-pile. $\alpha = \infty$ therefore means that there is no sheet-pile.

CASE II.

(*a*) *Same as Case* (*I*) *with a sheet-pile at an end.*—When a non-conductor of 2·4 inches is introduced at one end, at right angles to the non-conductor which is twelve inches long, the ratio α becomes 5. The results of the electrical method, of direct measurements and of the theoretical calculations for such a case are compared in Curve 2, Fig. 4. The pressure distribution along the horizontal part is alone shown in the diagram. The results again agree.

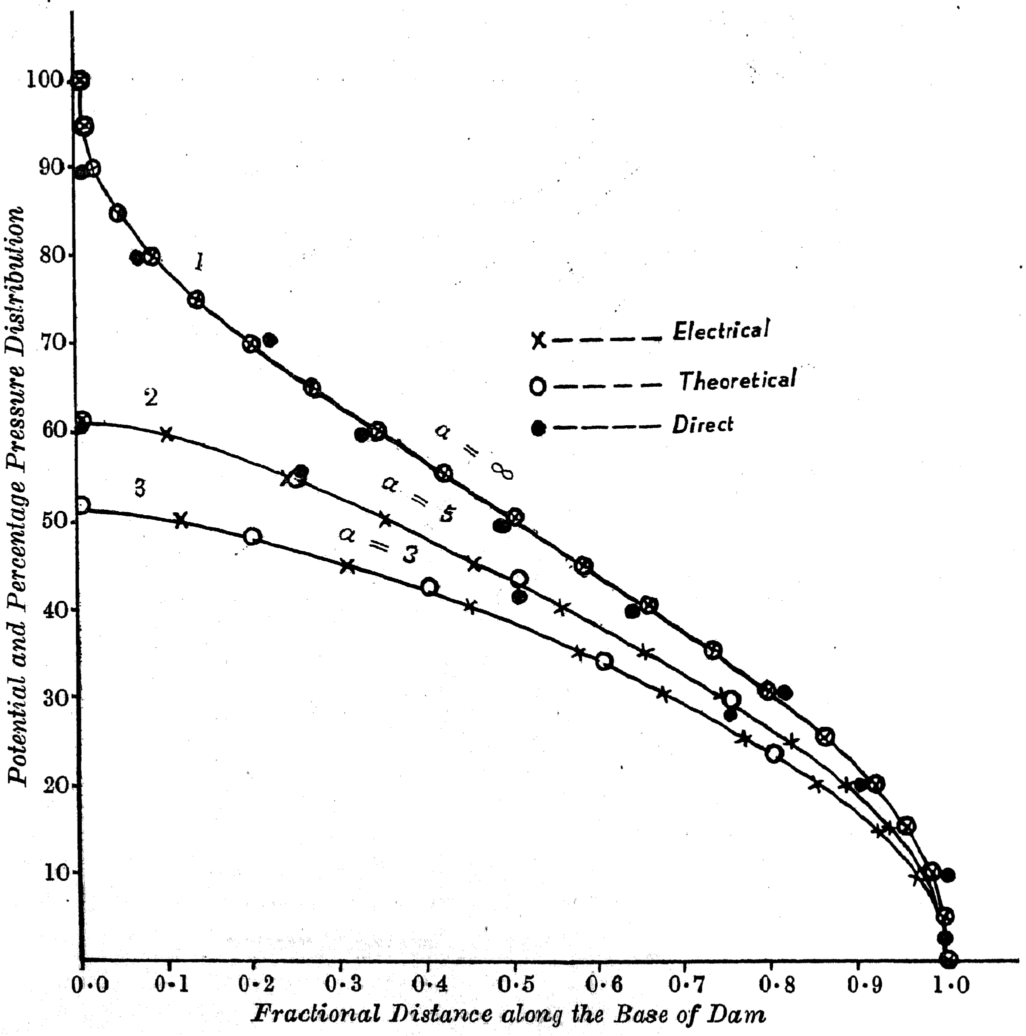

FIG. 4. Distribution of uplift pressure across Base of Dam in per cent. of headwater pressure for various Ratios and of width of Base of Dam to depth of Sheet-Piling at the Heel.

(*b*) *A simple floor with a sheet-pile the ratio* a *being* 3.—In this case no direct measurements were available. The results obtained by the electrical method again agree with those of the theoretical calculations as shown in Curve 3, Fig. 4.

OTHER CASES.

Some other cases of complicated models were also examined by the electrical method and the results were compared with those from direct measurements. The results in those cases agreed, but they are of more practical importance to the engineers, and no mathematical calculations have so far been made for them. They are not therefore shown here.

7. *Discussion.*

It is obvious from the results obtained that *the law of flow of water in the sub-soil under dams is the same as the law of potentials in conductors.* That previous experimenters failed to obtain identical results from direct measurements and from the electrical method, must be attributed to faulty technique in their methods. The fact that the potential distribution in conductors and the pressure distribution in the sub-soil under dams are of the same form, establishes the mathematical foundation for designs of such works and as such should be considered as a great advance on our existing knowledge. The experiments also incidentally verify the theory worked out by Weaver. From the practical standpoint, it becomes very easy now by the electrical method to determine the pressure distribution for any complicated design of a dam. It is however worth while to remark that the geological conditions such as stratification of the layers modify the pressure distribution, but this is a local condition and does not alter the fundamental basis of the laws of design. Attempts are being made in the Institute to try to simulate the geological conditions as well.

Summary.

It has been shown by experiments that the distribution of pressure in the sub-soil under dams is the same as potential distribution in conductors. The theoretical basis of sub-soil pressures under dams is thus established.

REFERENCES.

1. Reference to Pavlovsky's work is found in the *Proceedings of the International Congress of Dams,* 1933.

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4. Warren Weaver.—*Journal of Mathematics and Physics,* 1932, 11, No. 2, 114. Refer also to Lamb's *Hydrodynamics,* page 70, 4th Edition.