# THE DESIGN OF FALLS WITH REFERENCE TO UPLIFT PRESSURE.

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*Introduction.*

In a series of investigations,[1, 2] carried out in the Institute, it was definitely established that the distribution of uplift pressure under hydraulic works such as weirs and dams follows the same law as the distribution of electrical potential in conductors. Having established this fact, the method is now employed for the investigation of uplift pressure under hydraulic works. The details of the method are described in the papers referred to above. Since then a number of standard cases,[3, 4] and actual weirs[5] have been investigated by this method.

In the present paper, it is proposed to give an account of the investigation of the uplift pressure under hydraulic works on porous foundations, such as are built in canals and rivers.

*Construction of Falls.*

Fig. 1 represents the cross-section of a fall with a length $L_1$ for the upstream apron, $L_2$ for the downstream apron, depth $d_1$ for the fall and $d_2$

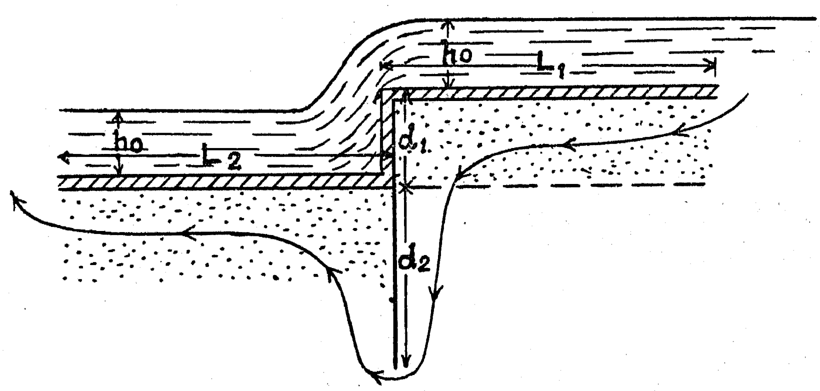


Fig. 1.

for the sheet pile. There is a depth $h_0$ of water upstream and a similar depth of water downstream. As illustrated in the figure, there is a difference of head equal to $d_1$ and due to the seepage flow taking place through the porous subsoil, this head exerts an uplift pressure on the masonry. If there is a lock and there is only water on the upstream side then the head $d_1$ increases to $d_1 + h_0$, since there is no water downstream. It is necessary to know the residual pressures at any point and its distribution for purposes of design.

*Results.*

Two series of experiments were carried out. In the first series, the lengths of downstream apron were varied, the fall $d_1$ and the length of the sheet pile $d_2$ being kept constant. The length, $L_1$ of the upstream apron in the model was maintained at four inches, the fall $d_1$ one inch and the length of the sheet pile $d_2$ 4″ throughout this series. In the second series, the lengths of the downstream and upstream aprons and the depth of the fall were kept constant and the length of the sheet pile varied from 0 to 5″. Nine cases of the former series and eight cases of the latter series were investigated and complete pressure contours were plotted. Fig. 2 shows one such case as an illustration. The pressure contours of the remaining sixteen cases

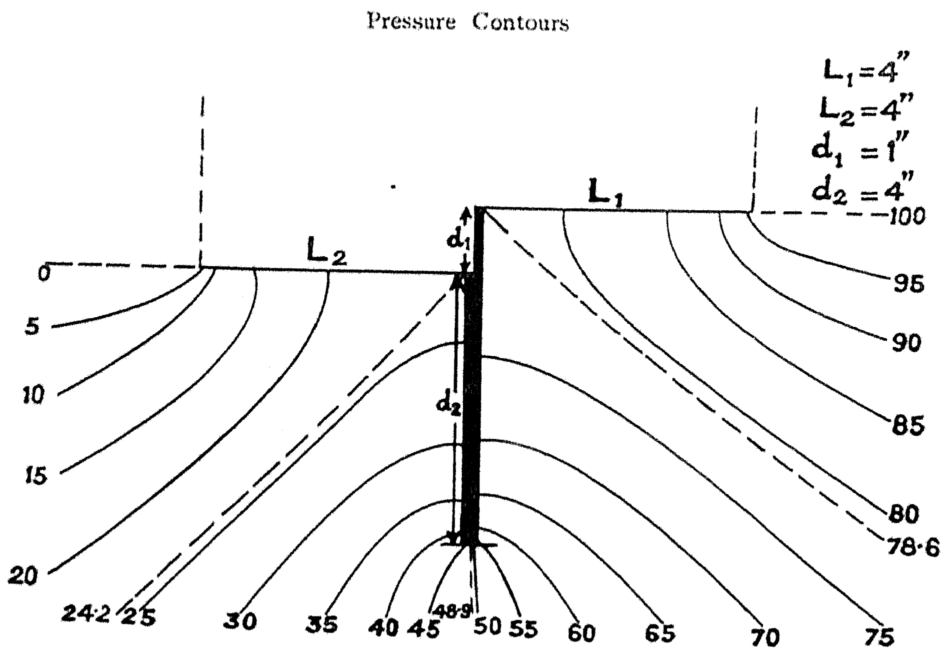


FIG. 2.

are not shown here as the types are similar. From the engineering point of view, the hydraulic gradient along the underside of the masonry is the most important consideration, since it is this gradient that measures the uplift pressure at any point.

Fig. 3 shows the pressure distribution under the masonry for different values of $L_{/2}$. The drop in pressure caused by the sheet pile is shown by the discontinuities in the curves. The percentage drop in pressure is marked for each graph.

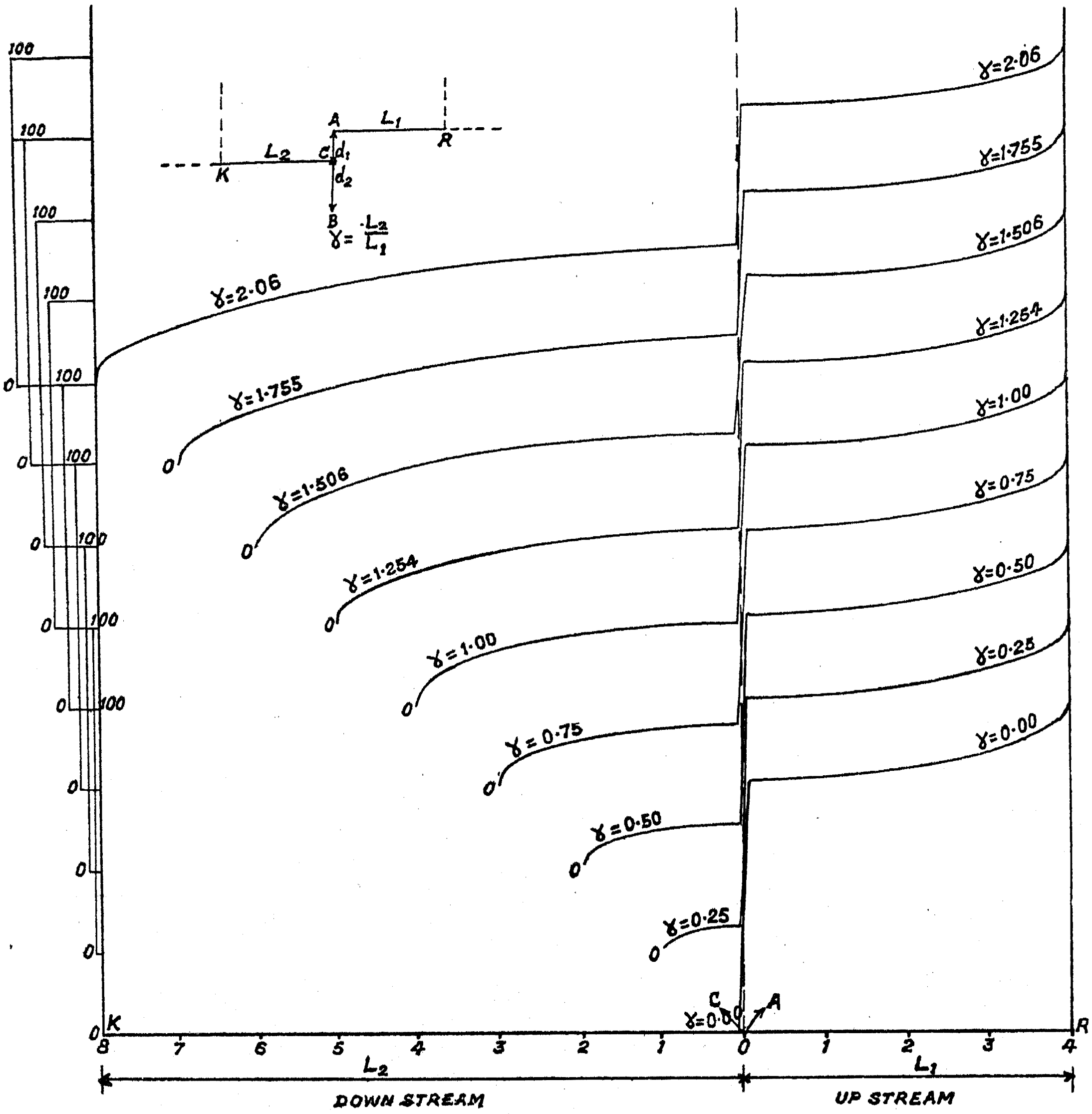


FIG. 3.

While the total drop of pressure caused by the sheet pile is shown by the discontinuity of lines in Fig. 3, the actual distribution along the sheet pile is shown in Fig. 4. It may be seen from this figure that while $L_{/2}$

changes from 0 to 8″ in the model, the changes of pressure at the upstream sides of the sheet piles are not very considerable. This point is of importance in connection with the application of the results to design.

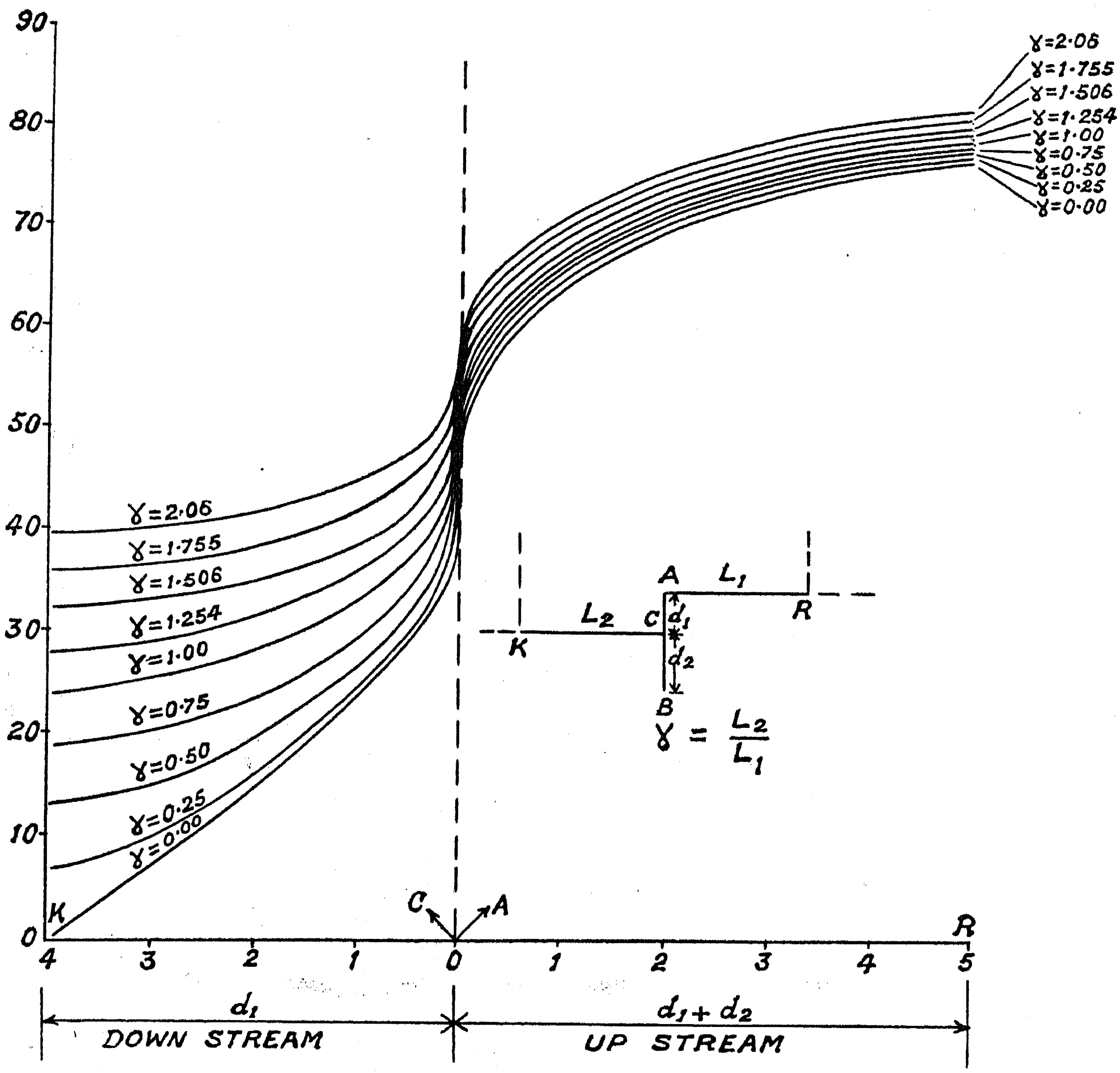


FIG. 4.

Fig. 5 shows the distribution of pressure when the lengths $d_2$ of the sheet pile change from 0 to 5″. $L_1$ and $L_2$ were maintained at 7″ in all the experiments referred to in this figure. Again while the total drop of pressure along the sheet pile is given by the discontinuity in the curves in Fig. 5, the nature of the distribution of pressure along the sheet pile is given by Fig. 6. As is seen in Fig. 6, the dotted line joining the pressures

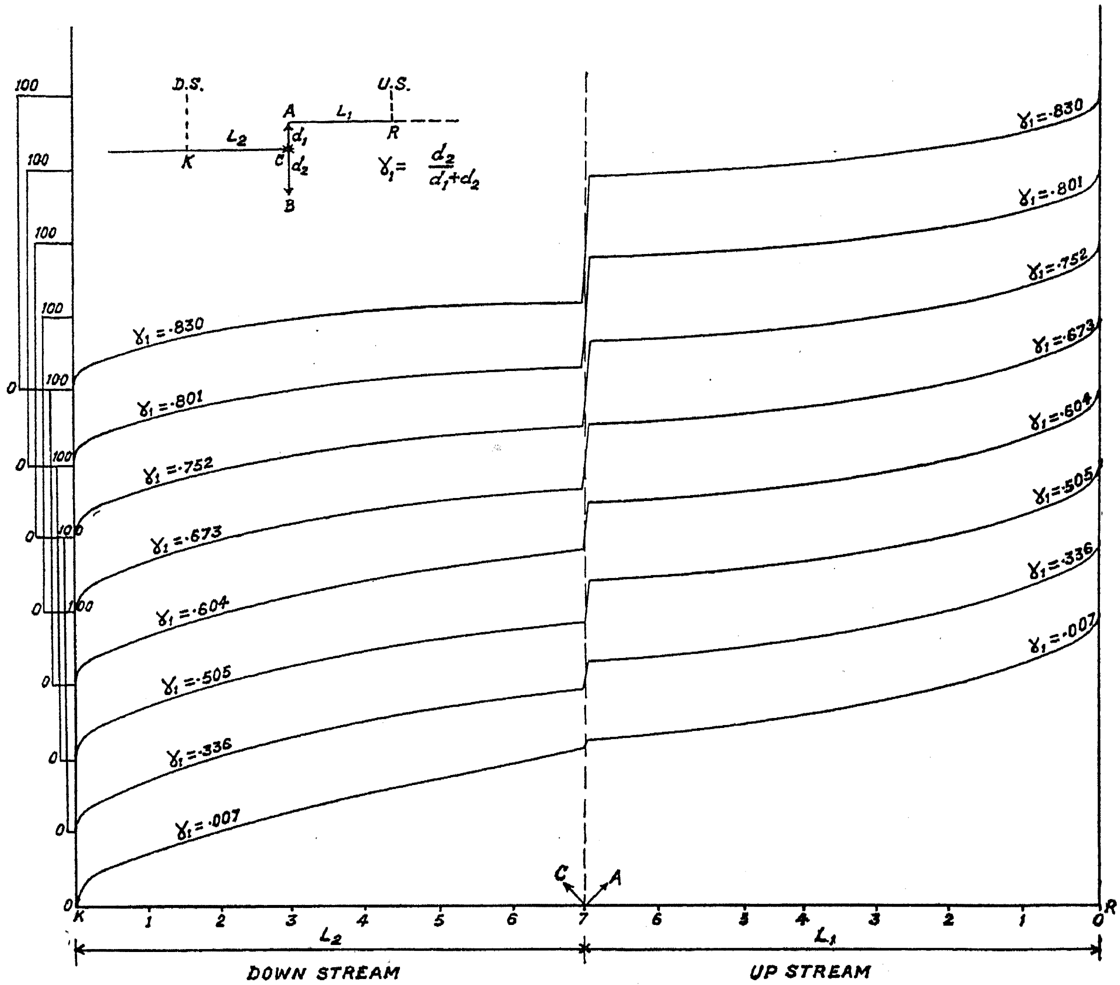


FIG. 5.

at the corners A and C (*i.e.*) the ends of the pressure distribution curves, has only a very small curvature. The value of the pressures at A and C for any particular combinations of $d_2/d_1 + d_2$ can thus be obtained from this curve, by interpolation.

*On the Design of Falls.*

When a fall is to be designed for a particular combination of floor and sheet pile, the best method is to subject the design to an experimental examination. In the Punjab, this is always done.

The results of the present investigation can be used for the ordinary combinations of sheet pile and floor, but it is not possible to apply the results so far obtained to all possible designs. There is no accurate theoretical expression by means of which the uplift pressure can be derived for every such case of a fall. Comparison of the present data shows that

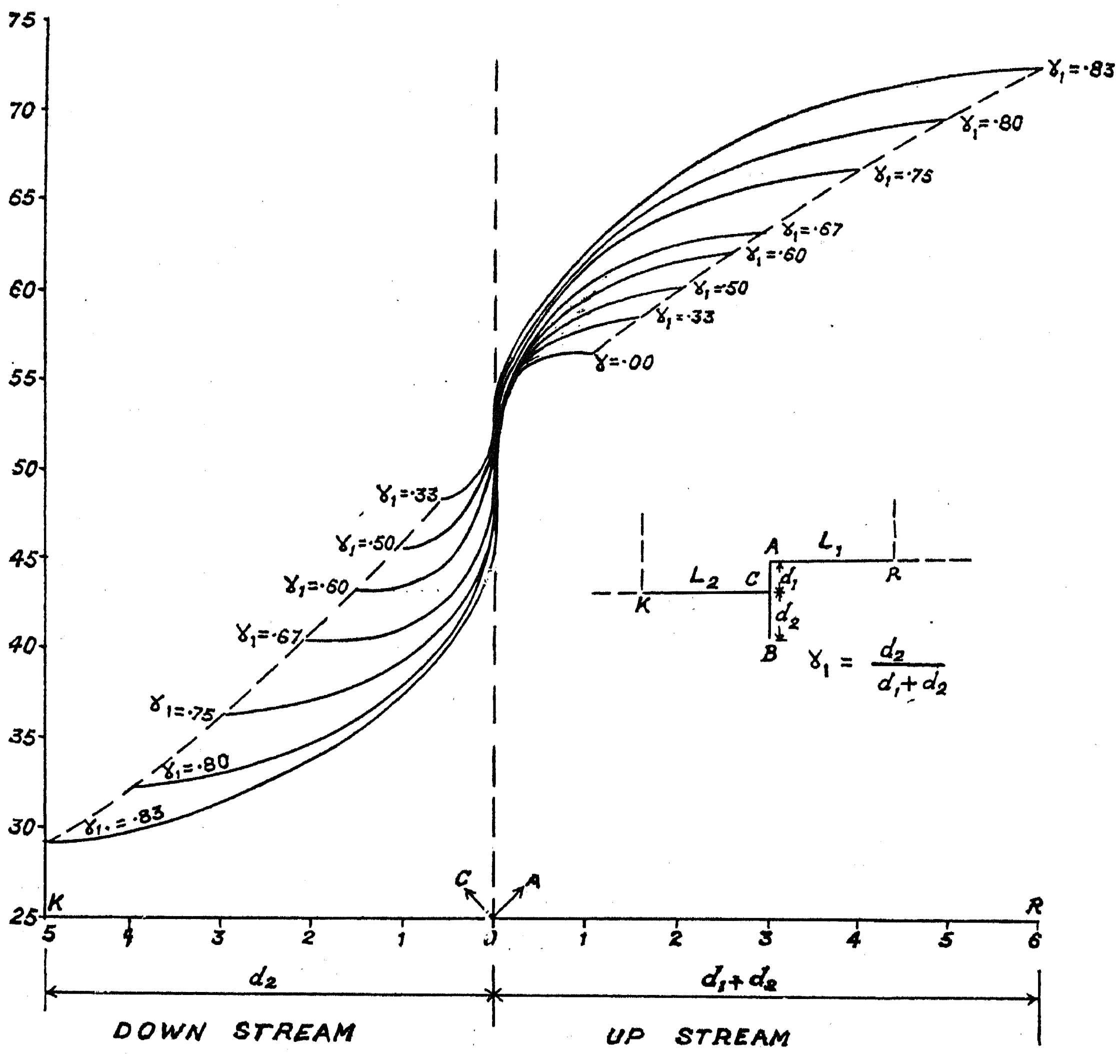


FIG. 6.

the pressure distribution for the type of works indicated by the model in Fig. 1, corresponds very nearly with those of a single sheet pile case under a flush floor, when the fall $d_1$ is small compared with $L_1$ and $L_2$ and sheet-pile not at the heel or toe. For such a case, namely, a sheet pile under a simple floor, as is shown in Fig. 7, a complete theory has been worked out by Weaver.[6] The expression to find the pressure on the base of the dam is, according to him,

$$p\ (x,\ y) = \frac{p_0}{\pi}\ c\ (x,\ y) = \frac{p_0}{\pi} \cos^{-1} \left[ \frac{d\ (\chi - \lambda) \pm \sqrt{x^2 + d^2}}{d\ \lambda} \right]$$

the origin $x = 0$ corresponding to the position of the pile and the value of $x$ increasing from the upstream to the downstream side. In this expression

$$\chi = \sqrt{1+\alpha_1^2}$$

$$\lambda = \frac{\sqrt{1+\alpha_1^2} + \sqrt{1+(\alpha-\alpha_1)^2}}{2}$$

$$\alpha_1 = \frac{L_1}{d} \quad \text{and} \quad \alpha = \frac{L}{d}$$

$d$ = depth of the sheet pile and

L = length of the floor. (See Fig. 7.)

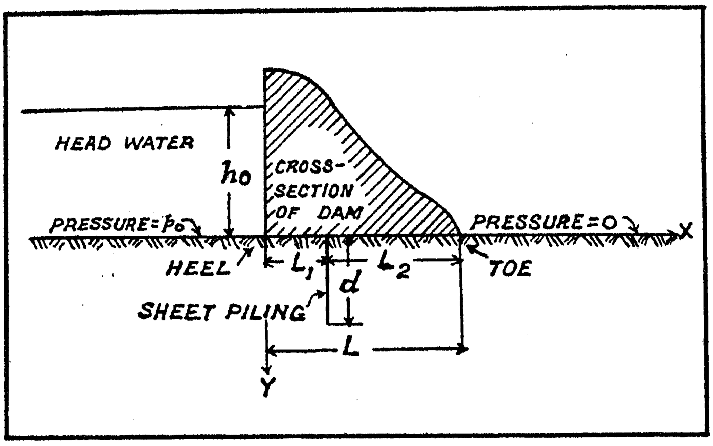


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FIG. 7.

If it is desired to obtain an approximate value of the uplift pressure without recourse to an experiment, in the absence of a general solution Weaver's expression may be applied to the case of falls as has already been stated. In applying this equation to the case of a fall, $d_1 + d_2$ may be regarded as the length $d$ of the pile for purposes of calculation on the upstream side and $d_2$ for the length of the pile on the downstream side. The justification for so doing lies in the fact that, as seen from Fig. 4, the changes in $L_2$ between limits shown in the figure do not affect the nature of the distribution of pressure on the upstream side by any appreciable amount.

Adopting such a method of calculation as is indicated here, the results of a comparison between the theory and experiment are shown in the table. The extreme variation is about 8%. It is generally much less as can be seen by comparing columns 3 and 4, and 7 and 8 in the table. This degree of accuracy is sufficient for practical purposes, since a factor of safety is usually introduced in the design.

*Comparison between the Theoretical and Experimental Values.*

(For Positions of A and C, see Inset in Fig. 3.)

| Upstream Side $P_A$ denotes pressure at A | | | | Downstream Side $P_C$ denotes pressure at C | | | | |
|---|---|---|---|---|---|---|---|---|
| $a$ | $a_1$ | $P_A$ Theoretical | $P_A$ Experimental | $a$ | $a_1$ | $P_C$ Theoretical | $P_C$ Experimental | |
| 0·800 | 0·80 | 77·2 | 77·0 | 1·00 | 1·00 | 0·0 | 0·0 | $L_2$ varying; $L_1$, $d_1$ and $d_2$ constant. |
| 1·000 | 0·80 | 77·3 | 77·1 | 1·25 | 1·00 | 7·3 | 6·8 | |
| 1·200 | 0·80 | 77·6 | 77·4 | 1·50 | 1·00 | 13·8 | 12·9 | |
| 1·400 | 0·80 | 78·0 | 78·2 | 1·75 | 1·00 | 19·8 | 18·7 | |
| 1·600 | 0·80 | 78·5 | 78·6 | 2·00 | 1·00 | 25·0 | 24·2 | |
| 1·800 | 0·80 | 79·1 | 79·3 | 2·25 | 1·00 | 29·5 | 27·9 | |
| 2·000 | 0·80 | 79·7 | 79·9 | 2·50 | 1·00 | 33·3 | 31·9 | |
| 2·200 | 0·80 | 80·2 | 80·7 | 2·75 | 1·00 | 36·6 | 37·1 | |
| 2·400 | 0·80 | 80·7 | 81·7 | 3·00 | 1·00 | 39·5 | 39·4 | |
| 9·33 | 4·67 | 56·7 | 58·4 | 28·00 | 14·00 | 47·7 | 48·2 | $d_2$ varying; $L_1$, $L_2$ and $d_1$ constant. |
| 7·00 | 3·50 | 58·8 | 60·2 | 14·00 | 7·00 | 45·5 | 45·6 | |
| 5·60 | 2·80 | 60·9 | 62·1 | 9·33 | 4·67 | 43·3 | 43·3 | |
| 4·67 | 2·33 | 62·9 | 63·2 | 7·00 | 3·50 | 41·1 | 40·4 | |
| 3·50 | 1·75 | 66·5 | 66·9 | 4·67 | 2·33 | 37·1 | 36·2 | |
| 2·80 | 1·40 | 69·7 | 69·7 | 3·50 | 1·75 | 33·5 | 32·3 | |
| 2·33 | 1·17 | 72·6 | 72·6 | 2·80 | 1·40 | 30·2 | 29·2 | |

## *Other Considerations.*

One important point in the design of such works is the pressure gradient at the toe. When the water escapes from beneath the work in an upward direction, the incoherent material from below the masonry is removed by the water. This causes cavities to form under the works and frequently such a work will fail at the toe. If a sheet pile is driven at the toe, this pressure gradient flattens as can be shown both experimentally and theoretically. The toe has, therefore, to be protected by a suitable length of piling or depressed masonry. The correct length of this piling or depression has to be estimated taking various considerations such as stratification head, etc. into account. When once the length of this piling is fixed, there is a specific length for the floor, which will give the maximum safety. Any extra length does not add to the safety as might be expected.

An important factor is the geological stratification with its consequent modification of the pressure distribution. When stratification is known to occur, the pressure distribution can only be determined by an experiment in which the stratification is reproduced. The effect of stratification will be appreciable only if it occurs near the work. As has been pointed out by Weaver, " the distortion which a sheet piling produces in the lines of flow does not extend as far from the piling as one might expect. The distortion due to the piling extends a distance in all directions equal, roughly to the piling depth. The interest in this observation lies in the strong suggestion that non-homogeneities in the foundation material very likely are effective in disturbing seepage flow only in a neighbourhood, whose dimensions are roughly twice those of the non-homogeneity. One has thus a crude means of judging, when and to what extent is applicable an analysis based on homogeneity."

*Summary.*

In this investigation the uplift pressures under hydraulic works on porous foundations, such as are built at the falls in canals and rivers, have been determined.

Two series of experiments have been carried out and the pressure distributions have been obtained for ordinary combinations of sheet pile and floor. A method of obtaining the pressure distribution approximately by the application of theory has also been indicated.

Our thanks are due to Dr. E. McKenzie Taylor, the Director, for his keen interest in the problem.

REFERENCES.

1. Gurdas Ram, V. I. Vaidhianathan and E. McKenzie Taylor, *Proceedings of the Indian Academy of Sciences,* 1935, **2**, 22.
2. V. I. Vaidhianathan and Gurdas Ram, *Research Publication* of the Irrigation Research Institute, Punjab, Lahore, 1935, **5**, 4.
3. V. I. Vaidhianathan, Gurdas Ram and E. McKenzie Taylor, *Proceedings of the Indian Academy of Sciences,* 1935, **2**, 646.
4. V. I. Vaidhianathan, Gurdas Ram and E. McKenzie Taylor, *Research Publication* of the Irrigation Research Institute, Punjab, Lahore, 1936, **5**, 6.
5. V. I. Vaidhianathan and Gurdas Ram, *Punjab Engineering Congress,* 1936, No. 190.
6. W. Weaver, *Journal of Mathematics and Physics,* 1932, **11**, 114.