FOUR DIMENSIONAL CURRENT ALGEBRA FROM CHERN-SIMONS THEORY

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Abstract

We study an abelian Chern-Simons theory on a five-dimensional manifold with boundary. We find it to be equivalent to a higher-derivative generalization of the abelian Wess-Zumino-Witten model on the boundary. It contains a $U(1)$ current algebra with an operatorial extension.

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Three-dimensional gauge theories with an action given just by a Chern-Simons (CS) term have been recently analyzed by several authors [1-5] . On a spacetime manifold of the form $\mathbf{R} \times \Sigma$, where $\mathbf{R}$ stands for the time axis and Σ is a Riemann surface without boundary, such a theory has only a finite number of physical degrees of freedom, namely the moduli of flat connections on Σ. When Σ has a boundary, there are additional degrees of freedom corresponding to a Wess-Zumino-Witten (WZW) model on the boundary. This has stirred much interest because it may lead to a classification of all rational conformal field theories [2,3].

These results can be generalized to higher dimensions. For instance, it has been proven that an abelian CS theory on a $2n+1$-dimensional manifold $\mathbf{R} \times \Sigma$ without boundary has again a finite number of physical degrees of freedom; the reduced phase space consists of gauge fields whose curvature has rank $\leq 2(n-1)$ modulo $U(1)$ gauge transformations and diffeomorphisms of Σ [6].

In the present paper we study the case in which Σ has a boundary. For simplicity we will restrict ourselves to the case when $\Sigma = B^4$ (a four dimensional ball) and $\partial\Sigma = S^3$. We show, using canonical methods, that the physical degrees of freedom of the theory are three scalar fields Φ, Ψ and Ω on S^3. We derive a Lagrangian for these fields in analogy to what has been done in the case $\Sigma = B^2$ and, applying again canonical methods, we prove that it gives rise to a current algebra, which in terms of the original field variables A_α, restricted to S^3, can be written:

$$\{\rho(x), \rho(y)\} = \frac{1}{6\alpha} \varepsilon^{\alpha\beta\gamma} \partial_\alpha A_\beta(x) \frac{\partial}{\partial x^\gamma} \delta^{(3)}(x,y) \ , \tag{1}$$

where ρ is the $U(1)$ charge density of the model. One notices that the algebra extension is of the form predicted by cohomological arguments [7]. This is our main result; we shall comment at the end on its generalization to higher dimensions.

As in [6] we start from the action:

$$S = \alpha \int_M d^5x \ \varepsilon^{\lambda\mu\nu\rho\sigma} A_\lambda \partial_\mu A_\nu \partial_\rho A_\sigma \ , \tag{2}$$

where $\varepsilon^{01234} = 1$, α is a dimensionless constant and $M = \mathbf{R} \times \Sigma$ is space-time. We denote with x^0 the coordinate in $\mathbf{R}$ and $\{x^i\}$, with $i = 1,2,3,4$ the coordinates in Σ. In order to ensure that the Gauss law does not get a contribution from the boundary, we will assume that $A_0 = 0$ on the boundary. We then repeat the constraint analysis of [6] keeping track of boundary terms. We have the following set of constraints:

$$0 \approx C^0 = P^0 \ ; \tag{3a}$$

$$0 \approx C^i = P^i - 2\alpha \ \varepsilon^{ijkl} A_j \partial_k A_l \ ; \tag{3b}$$

$$0 \approx G = \frac{3}{4}\alpha \ \varepsilon^{ijkl} F_{ij} F_{kl} \ , \tag{3c}$$

where P^μ are the momenta canonically conjugate to A_μ. Since C^0 commutes with all other constraints, we can choose the gauge condition $A_0 = 0$ and thereby eliminate A_0 and P^0

from the theory. We then remain with the phase space $T^*\mathcal{C}(\Sigma)$, where $\mathcal{C}(\Sigma)$ denotes the space of the gauge potentials A_i, together with the constraints C^i and G. The brackets between the variables A_i and P^i are still the canonical ones, and the Hamiltonian vanishes identically.

We define $\mathcal{P} \in T^*\mathcal{C}(\Sigma)$ to be the subspace where the constraints C^i hold. Since $\mathcal{P}$ is obtained by fixing the values of the momenta P^i as functions of the A_i, $\mathcal{P}$ is diffeomorphic to $\mathcal{C}(\Sigma)$. We also define $\mathcal{D}$ to be the subspace of $\mathcal{P}$ defined by the constraint (3c). A gauge field A_i satisfies (3c) if and only if the matrix F_{ij} is degenerate. Thus $\mathcal{D}$ consists of connections whose curvature form has rank two or zero. The reduced phase space is the quotient of $\mathcal{D}$ by the gauge transformations which are generated by the first class constraints.

It is convenient to smear the constraints (3) with arbitrary test one-forms $\xi = \xi_i dx^i$ and arbitrary functions f on Σ: $C_\xi = \int d^4x\ \xi_i(x) C^i(x)$, and $G_f = \int d^4x\ f(x) G(x)$. Then, we have the following Poisson brackets:

$$\{C_\xi, C_\eta\} = \mathcal{F}(\xi,\eta) + \mathcal{R}(\xi,\eta) \ , \tag{4a}$$
$$\{C_\xi, G_f\} = \mathcal{F}(\xi, df) + \mathcal{S}(\xi, f) \ , \tag{4b}$$
$$\{G_f, G_g\} = 0 \ , \tag{4c}$$

where

$$\mathcal{F}(\xi,\eta) = -3\alpha \int d^4x\ \varepsilon^{ijkl} \xi_i \eta_j F_{kl} \ , \tag{5}$$

$$\mathcal{R}(\xi,\eta) = 2\alpha \int d^4x\ \partial_i(\varepsilon^{ijkl} \xi_j \eta_k A_l) \ , \tag{6}$$

$$\mathcal{S}(\xi, f) = -3\alpha \int d^4x\ \partial_i(\varepsilon^{ijkl} f \xi_j F_{kl}) \ . \tag{7}$$

From these formulae one sees that the following are first class constraints:

$$D_f = C_{df} - G_f \ , \tag{8}$$

when f is zero on the boundary,

$$\Delta_v = C_{i_v F} + D_{i_v A} \ , \tag{9}$$

(with $(i_v F)_i = v^j F_{ji}$) when the vectorfield v is zero on the boundary, and C_η when η satisfies $\eta \wedge F = 0$ everywhere and $\eta \wedge A = 0$ on the boundary. For generic A this implies that η vanishes on the boundary. On $\mathcal{C}(\Sigma) \equiv \mathcal{P}$ these constraints generate the following transformations:

$$\delta_f A_i = \{A_i, D_f\} = \partial_i f \ , \tag{10a}$$
$$\delta_v A_i = \{A_i, \Delta_v\} = v^j \partial_j A_i + (\partial_i v^j) A_j \ , \tag{10b}$$
$$\delta_\eta A_i = \{A_i, C_\eta\} = \eta_i \ . \tag{10c}$$

As in [6] one can show that for every one-form η vanishing on the boundary and satisfying $\eta \wedge F = 0$, there exists a vectorfield v vanishing on the boundary, such that

$$C_\eta = \Delta_v - D_{i_v A} \ . \tag{11}$$

Therefore, every constraint C_η which is first class can be expressed as a combination of a Δ_v and a D_f which are first class. Thus we can take D_f and Δ_v as independent first class constraints. It is seen from (10) that the gauge group generated by these constraints consists of $U(1)$ gauge transformations and diffeomorphisms which are both the identity on the boundary.

The general solution of the Gauss law (3c) is

$$A = \Phi d\Psi + d\Omega \ , \tag{12}$$

where Φ, Ψ are real-valued functions and $e^{i\Omega}$ is a $U(1)$-valued function (so if Σ had not been simply connected, Ω could be polydromic). Thus we can parametrize the constrained surface $\mathcal{D}$ by the three functions Φ, Ψ, Ω. Note that the decomposition (12) is not unique. For instance, constant (space-independent) shifts in Ψ and Ω leave A unchanged, as do the transformations $\Phi \rightarrow \Phi + c$, $\Psi \rightarrow \Psi$, $\Omega \rightarrow \Omega - c\Psi$ and

$$\begin{aligned}
&\Phi \longrightarrow \Phi \cos\theta + \Psi \sin\theta \\
&\Psi \longrightarrow -\,\Phi \sin\theta + \Psi \cos\theta \\
&\Omega \rightarrow \Omega + \Phi\Psi \sin^2\theta + \frac{1}{2}(\Phi^2 - \Psi^2) \sin\theta \cos\theta \ ,
\end{aligned}$$

where c and θ are space-independent parameters. We shall discuss below the consequences of this arbitrariness.

We can now count the degrees of freedom: we have eight canonical variables $A_i(x)$, $P^i(x)$ and five independent constraints, *e.g.* $C_i(x)$ and $G(x)$. In the interior of B^4 one has to add three gauge fixing conditions, one for each independent first-class constraint; therefore in the interior there can be at most a finite number of degrees of freedom. On the boundary all constraints are second class and therefore there remain three physical fields.

All this can be made more precise using global geometric arguments. If one considers the action of the transformations (10) on Φ, Ψ, Ω, one sees that their values in the interior of Σ are gauge degrees of freedom. More precisely, by means of a $U(1)$ gauge transformation which is the identity on the boundary one can transform every Ω into any other Ω' which has the same boundary values as Ω, and by means of a diffeomorphism which is the identity on the boundary one can transform every Φ, Ψ into any other Φ', Ψ' which have the same boundary values as Φ, Ψ. Therefore, the physical degrees of freedom of the theory are the boundary values of the scalar fields Φ, Ψ, Ω. Note that if we had worked with fixed boundary values for the fields, the reduced phase space would be a point; this is a special case of the results in [6], since $\Sigma = B^4$ is simply connected. If Σ is a more complicated four-manifold with $\partial\Sigma = S^3$ one finds in addition to the fields Φ, Ψ, Ω on the boundary also finitely many degrees of freedom in the interior, as in [6].

To further explore the physical content of the theory one could proceed canonically by fixing the gauge for the first class constraints, compute Dirac brackets etc. This is technically complicated. Instead, we shall derive an action for the physical degrees of freedom by inserting the solution of the Gauss law (3c) into the original action (2). This produces a surface term which, using Stokes' theorem, can be written:

$$S = 2\alpha \int_{\mathbf{R}\times S^3} d^4x \ \varepsilon^{\alpha\beta\gamma}\partial_\alpha\Phi\partial_\beta\Psi\partial_\gamma\Omega \ (\Phi\dot{\Psi} + \dot{\Omega}) \ , \tag{13}$$

where x^α with $\alpha = 1,2,3$ are coordinates on S^3 and a dot signifies derivative with respect to time. The action (13) is invariant under diffeomorphisms of S^3 and also under the transformations of Φ, Ψ, Ω which leave A invariant. In particular, among these are the global (space-independent) shifts of Ω. The Noether charge corresponding to these transformations is $j^0 = 2\alpha\rho$, where

$$\rho = \varepsilon^{\alpha\beta\gamma}\partial_\alpha\Phi\partial_\beta\Psi\partial_\gamma\Omega \ . \tag{14}$$

The lagrangian in (13) can be written in the form $\mathcal{L} = \dot{\varphi}^a\mathcal{A}_a(\varphi)$, where the index $a = 1,2,3$ labels the fields $\varphi^a = (\Phi, \Psi, \Omega)$ and $\mathcal{A}$ is a one-form on the space of fields given by $\mathcal{A}_a = (0, 2\alpha\rho\Phi, 2\alpha\rho)$. Then, as discussed in [8], the symplectic form on the space of the fields is given by the functional curl of $\mathcal{A}$:

$$\begin{aligned}\mathcal{F}_{ab}(x,y) &= \frac{\delta\mathcal{A}_b(y)}{\delta\varphi^a(x)} - \frac{\delta\mathcal{A}_a(x)}{\delta\varphi^b(y)} \\ &= -2\alpha\begin{bmatrix} 0 & \Phi X_\Phi & X_\Phi \\ \rho + \Phi X_\Phi & 2\ \Phi X_\Psi & X_\Psi + \Phi X_\Omega \\ X_\Phi & X_\Psi + \Phi X_\Omega & 2X_\Omega \end{bmatrix} \delta^{(3)}(x,y) \ ,\end{aligned} \tag{15}$$

where

$$X_\Phi = X^\gamma_\Phi \ \partial_\gamma = \varepsilon^{\alpha\beta\gamma}\partial_\alpha\Psi\partial_\beta\Omega \ \partial_\gamma \ , \tag{16a}$$

$$X_\Psi = X^\gamma_\Psi \ \partial_\gamma = \varepsilon^{\alpha\beta\gamma}\partial_\alpha\Omega\partial_\beta\Phi \ \partial_\gamma \ , \tag{16b}$$

$$X_\Omega = X^\gamma_\Omega \ \partial_\gamma = \varepsilon^{\alpha\beta\gamma}\partial_\alpha\Phi\partial_\beta\Psi \ \partial_\gamma \ , \tag{16c}$$

and the operators appearing in the matrix act on the variable x. The canonical brackets between the fields Φ, Ψ and Ω are given by the inverse of $\mathcal{F}$. However, $\mathcal{F}$ has a nontrivial kernel. These are associated entirely with the arbitrariness in the decomposition (12), and therefore this does not contradict our earlier result that in the variables A_i the CS theory has no first class constraints on the boundary. To see this, we note that the equation

$$\int d^3y \ \mathcal{F}_{ab}(x,y) \ v^b(y) = \ 0 \ , \tag{17}$$

is covariant under coordinate transformations and thus can be studied in any coordinate system. For almost every point in phase space the functions Φ, Ψ and Ω define a local coordinate system on S^3: $\bar{x}^1 = \Phi$, $\bar{x}^2 = \Psi$, $\bar{x}^3 = \Omega$. (These are adapted coordinates in which the only non-vanishing components of F_{ij} are $F_{12} = -F_{21} = 1$.) In these coordinates, $\frac{1}{\rho}X_\Phi = \overline{\partial}_1$, $\frac{1}{\rho}X_\Psi = \overline{\partial}_2$, $\frac{1}{\rho}X_\Omega = \overline{\partial}_3$, $\rho = 1$, and (17) reduces to (suppressing the bars):

$$x^1\partial_1 v^2 + \partial_1 v^3 = 0 \,, \tag{18a}$$
$$v^1 + x^1\partial_1 v^1 + 2\, x^1\partial_2 v^2 + \partial_2 v^3 + x^1\partial_3 v^3 = 0 \,, \tag{18b}$$
$$\partial_1 v^1 + \partial_2 v^2 + x^1\partial_3 v^2 + 2\,\partial_3 v^3 = 0 \,. \tag{18c}$$

After some algebraic manipulations one finds that $\partial_i v^i = 0$, and the previous system is equivalent to:

$$x^1\partial_1 v^2 + \partial_1 v^3 = 0 \,, \tag{19a}$$
$$v^1 + x^1\partial_2 v^2 + \partial_2 v^3 = 0 \,, \tag{19b}$$
$$x^1\partial_3 v^2 + \partial_3 v^3 = 0 \,. \tag{19c}$$

But the quantities on the left hand side of these equations are exactly the variations of A_α under the transformation $\varphi^a \to \varphi^a + v^a$, written in adapted coordinates. Therefore, the gauge degrees of freedom corresponding to the null eigenvectors of $\mathcal{F}$ give exactly the arbitrariness in the decomposition (12).

Recall that in the corresponding analysis of the 2 + 1-dimensional CS theory with boundary, the component A_φ of the gauge field along the boundary satisfies an abelian Kac-Moody algebra. The field A_φ could be interpreted as the current density of the WZW model because, thanks to Gauss' law, A_φ can be written as $\partial_\varphi\Omega$, where Ω is the WZW field and $\partial_\varphi\Omega$ is the Noether charge density for the transformations $\Omega \to \Omega +$ constant. Note also that Gauss' law smeared with the test function $f = 1$ could be written as:

$$G_1 = \int_{B^2} d^2x\ \varepsilon^{ij} F_{ij} = 2\int_{S^1} d\varphi A_\varphi \,. \tag{20}$$

The corresponding current in our 4 + 1-dimensional theory is $\varepsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma$. In fact the analog of (20) is

$$G_1 = \frac{3}{4}\alpha\int_{B^4} d^4x\ \varepsilon^{ijkl}F_{ij}F_{kl} = 3\alpha\int_{S^3} d^3x\ \varepsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma \,. \tag{21}$$

When translated into variables Φ, Ψ and Ω, $\varepsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma = \rho$, as defined in (14), and this is also the Noether charge associated with the global shifts of Ω.

Let us compute therefore the bracket $\{\rho(x), \rho(y)\}$. Although it is possible to actually invert $\mathcal{F}$ in adapted coordinates, this is not particularly instructive. One can compute the algebra of quantities of interest without explicit inversion of $\mathcal{F}$. It is convenient to perform a linear transformation in field space:

$$\mathcal{T}_a{}^c(x,z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\Phi(x) \\ 0 & 0 & 1 \end{bmatrix} \delta^{(3)}(x,z) \,. \tag{22}$$

Then we get

$$\begin{aligned}\tilde{\mathcal{F}}_{ab}(x,y) &= \int d^3z \int d^3t\ \mathcal{T}_a{}^c(x,z)\ \mathcal{T}_b{}^d(y,t)\ \mathcal{F}_{cd}(z,t) \\ &= -2\alpha \begin{bmatrix} 0 & -\rho & X_\Phi \\ \rho & 0 & X_\Psi - \Phi X_\Omega \\ X_\Phi & X_\Psi - \Phi X_\Omega & 2X_\Omega \end{bmatrix} \delta^{(3)}(x,y)\ . \end{aligned} \tag{23}$$

If we write

$$\left(\tilde{\mathcal{F}}^{-1}\right)^{ab}(x,y) = -\frac{1}{2\alpha}\begin{bmatrix} A(x,y) & B(x,y) & C(x,y) \\ -B(y,x) & D(x,y) & E(x,y) \\ -C(y,x) & -E(y,x) & F(x,y) \end{bmatrix}\ , \tag{24}$$

then the condition $\int d^3z\ \tilde{\mathcal{F}}_{ab}(x,z)\left(\tilde{\mathcal{F}}^{-1}\right)^{bc}(z,y) = \delta_a{}^c\delta^{(3)}(x,y)$ gives differential equations for the kernels entering in (24). Using these equations one can express all kernels in terms of $F(x,y)$; furthermore, one must have

$$\left([\frac{1}{\rho}X_\Psi, \frac{1}{\rho}X_\Phi] - \Phi[\frac{1}{\rho}X_\Omega, \frac{1}{\rho}X_\Phi] + 3\ \frac{1}{\rho}X_\Omega\right) F(x,y) = \frac{1}{\rho}\delta^{(3)}(x,y)\ , \tag{25}$$

where [,] denotes the Lie bracket of vectorfields on S^3 and the differential operator on the left hand side acts on the variable x. Note that ρ is a scalar density and that X_a are vector densities, so $\frac{1}{\rho}X_a$ are true vectorfields. As observed before, in adapted coordinates $\frac{1}{\rho}X_a$ form natural bases on S^3 and hence commute. Thus the equation for $F(x,y)$ actually reduces to

$$X_\Omega^\alpha(x)\frac{\partial}{\partial x^\alpha}F(x,y) = \frac{1}{3}\delta^{(3)}(x,y)\ . \tag{26}$$

A direct calculation using (23), (24) and (26) then shows that

$$\begin{aligned}\{\rho(x),\rho(y)\} &= \int d^3z \int d^3t\ \left(\mathcal{F}^{-1}\right)^{ab}(z,t)\ \frac{\delta\rho(x)}{\delta\varphi^a(z)}\frac{\delta\rho(y)}{\delta\varphi^b(t)} \\ &= -\frac{1}{2\alpha}X_\Omega^\alpha(x)X_\Omega^\beta(y)\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}F(x,y) \\ &= \frac{1}{6\alpha}X_\Omega^\alpha(x)\frac{\partial}{\partial x^\alpha}\delta^{(3)}(x,y)\ . \end{aligned} \tag{27}$$

If we use the relation (12), this is exactly the announced result (1).

We conclude with some remarks on the action (13). It is a generalization to four dimensions of the action $\int_{\mathbf{R}\times S^1} dt d\varphi\ \dot{\Omega}\partial_\varphi\Omega$ which describes an abelian WZW model in light-cone coordinates. It is also similar to the purely WZW actions discussed in [9] and to higher-derivative sigma models. However in these models the contraction of the derivative indices is effected by means of a totally antisymmetric four-dimensional tensor or by means of two metric tensors, respectively. Instead, when (13) is formally covariantized, it involves a tensor $t^{\mu\nu\rho\sigma}$ which is antisymmetric in three indices.

As in the purely WZW models, dynamics is trivial: the Hamiltonian vanishes and all fields are time-independent. This is due to the invariance of the action (13) under diffeomorphisms of S^3, which in this theory are true symmetries and not gauge invariances. These provide infinitely many conserved charges $Q_v = 2\alpha \int d^3x\ \rho\ v^\beta(\Phi\partial_\beta\Psi + \partial_\beta\Omega)$, where v^β is any vectorfield on S^3.

In $2n+1$ dimensions, the Gauss law is $0 = F \wedge \ldots \wedge F$ (n times) and its general solution is $A = \Phi_1 d\Psi_1 + \ldots + \Phi_{n-1}d\Psi_{n-1} + d\Omega$. The analog of (13) is

$$S = n\alpha \int_{\mathbf{R}\times S^{2n-1}} d^{2n}x\ (\Phi_1\dot{\Psi}_1 + \ldots + \Phi_{n-1}\dot{\Psi}_{n-1} + \dot{\Omega})\ \rho\ , \tag{28}$$

where $\rho = \varepsilon^{\alpha_1\ldots\alpha_{2n-1}}\partial_{\alpha_1}\Phi_1\partial_{\alpha_2}\Psi_1\ldots\partial_{\alpha_{2n-2}}\Psi_{n-1}\partial_{\alpha_{2n-1}}\Omega$. It is natural to expect that the algebra (1) generalizes to

$$\{\rho(x),\rho(y)\} \approx \varepsilon^{\alpha_1\ldots\alpha_{2n-1}}\partial_{\alpha_1}A_{\alpha_2}\ldots\partial_{\alpha_{2n-3}}A_{\alpha_{2n-2}}\frac{\partial}{\partial x^{\alpha_{2n-1}}}\delta^{(2n-1)}(x,y)\ . \tag{29}$$

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