ON THE IMPLICATIONS OF THE SUBMILLISECOND PULSAR IN SUPERNOVA 1987A

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ABSTRACT

We discuss the implications of the ultra rapid rotation rate of 1968 Hz of the newly discovered pulsar in supernova 1987A, as well as the sinusoidal modulation of the frequency. It is argued that all the available evidence suggests that the pulsar has a very low magnetic field, $\sim 10^9$ G. The significance of this is discussed in detail.

RECENTLY Middleditch *et al*[1] have reported the discovery of an optical pulsar at the site of the supernova 1987A in the Large Magellanic Cloud. Although subsequent observations have not confirmed this, we feel that several extraordinary features of this pulsar merit a discussion of the implications.

The most remarkable characteristic of this pulsar is its ultra short period ($P \sim 0.5$ ms). In this article we wish to discuss the implications of the observed period, the expected slow-down rate, the magnetic field of the pulsar and the observed 8 h modulation of the spin frequency, on the assumption that the observed pulsations are due to the rotation of a neutron star.

THE PERIOD

Newtonian gravitation sets a well defined lower limit to the period of rotation of a self-gravitating body. Instabilities driven by gravitational radiation imply that the actual limiting period of rotation may in fact be $\sim 10\%$ larger than the Newtonian limit. Recent calculations[2] have shown that for a neutron star of 1.4 $M_\odot$ such a relativistic instability limit is around 1.5 ms if the equation of state of the neutron star matter is very stiff. For softer equations of state this limiting period can be smaller. For example, if one adopts the Canuto–Chitre[3] equation of state, one of the softest equations of state available in the literature, then the relativistic stability limit is around ~ 0.7 ms. Thus even in this most favourable case the observed period of 0.5 ms implies a mass $> 1.4\ M_\odot$. There is no consensus on how stiff neutron star matter actually is. However, the fact that the two fastest millisecond pulsars have almost identical periods has led Friedman *et al*[2] to argue that they may, in fact, be spinning very close to their limiting periods. This would imply, according to them, that the equation of state is very stiff. If this is so in the present case also, then the mass of the pulsar under discussion must be significantly greater than 1.4 $M_\odot$. The masses of neutron stars measured so far are all very close to 1.4 $M_\odot$. Under what circumstances can neutron star mass significantly exceed this?

If one accepts the standard scenario that the binary millisecond pulsars such as PSR 1953+29 and PSR 1855+09 were spun up due to mass accretion from a low-mass giant[4], then the evolutionary calculations suggest that these neutron stars must have accreted[5,6] about 0.6 $M_\odot$. This would make the present masses of these two pulsars close to 2 $M_\odot$. If the pulsar in SN 1987A is the new-born pulsar, such arguments are not relevant even if the pre-supernova star was a member of a binary system. A more plausible alternative is that a substantial amount of mass fell back on the newly formed neutron star. Chevalier[7] has in fact suggested such a possibility. He has argued that if the size of the pre-supernova star is rather small, as was the case with the progenitor of SN 1987A, the amount of matter falling back could be ~ 100 times larger than if the star was a red supergiant.

It is interesting to ask if there is any evidence supporting such an infall. The fact that there were *two* bursts of neutrinos separated by about 5 h is perhaps the most direct evidence. Although the reality of the first burst of neutrinos, detected only by the Mont Blanc group, has remained controversial, de Kujula[8] has argued that there is no valid basis for rejecting these observations. In fact he has concluded that $\sim 1\ M_\odot$ of matter falling back onto the neutron star *will* result in a second burst of neutrinos.

Before turning to a discussion of the expected slow-down rate of this pulsar, we wish to remark that the collapse dynamics of such a rapidly rotating object might have resulted in the formation of a jet along the rotation axis[9]. Piran and Nakamura[10] in fact invoked such a jet to explain the "mystery spot". Very recently, Wood and Faulkner[11] from an analysis of the narrow circumstellar emission lines conclude that the stellar wind from the progenitor of the supernova was asymmetric, and possibly bipolar. Further, if bipolar, then the position angle of the mystery spot is close to the direction of the bipolar axis. This strengthens the case for a jet along the rotation axis.

THE EXPECTED SLOW-DOWN RATE

A rapidly spinning neutron star will slow down due to gravitational radiation and electromagnetic radiation. We first discuss the expected luminosity of gravitational radiation from this pulsar. Since there is no measurement of the slow-down rate, one can at best try to obtain an upper limit for it from the presently available data in the following way: The IAU circular[1] reporting the discovery mentions that the frequency modulation with a period of approximately 8 h can be described as "sinusoidal" with a peak-to-peak amplitude of 3×10^{-3} Hz to within 5%. From this we obtain an upper limit of $\dot{P}$ of 2×10^{-15} s s^{-1}. This corresponds to a limit on the energy loss rate of 6×10^{41} erg s^{-1}. If all of this energy loss is attributed to gravitational radiation, then one obtains a limit on the equatorial ellipticity: $\varepsilon < 2 \times 10^{-7}$. We shall return to this point later.

We turn next to an estimate of the electromagnetic luminosity of the pulsar (we include in this the energy loss in the form of radiation as well as relativistic particles). As was first argued by Ostriker and Gunn[12], if the ejecta is not highly fragmented then one expects the energy radiated by the pulsar into the supernova cavity to be reprocessed and radiated. Thus, after a steady state is reached, one expects the bolometric luminosity of the supernova to be equal to that of the central pulsar. Further, since the pulsar luminosity does not decline significantly during the initial characteristic slow-down time scale, one expects the supernova light curve to be flat for several years. The luminosity of this supernova at light maximum was 10^{42} erg s^{-1}, and then it began to decline. This sets an upper limit to the electromagnetic luminosity of the pulsar. The fact that the luminosity declined exponentially for more than a year with a time constant of 110 days strongly suggests that the main source of energy powering the light curve was, in fact, radioactive decay of ^{56}Co. Thus the pulsar luminosity is likely to be very much less than the limit mentioned above (10^{42} erg s^{-1}). The present bolometric luminosity of the supernova is $\sim 10^{39}$ erg s^{-1} (including the X and γ radiations), and there is still no indication of the light curve "levelling off". Therefore, a more reasonable upper limit to the pulsar luminosity may be obtained by assuming that it is equal to the present supernova luminosity. This implies a period derivative of the pulsar of $\sim 3 \times 10^{-18}$ s s^{-1}, and a surface magnetic field $\sim 10^9$ G.

A second and more direct estimate of the slow-down rate of the pulsar may be obtained from the observed luminosity of the optical radiation from the pulsar. Using the model developed by Pacini and Salvati[13] we estimate that an optical magnitude of 18.5 implies a $\dot{P}$ of the order of 3×10^{-19} s s^{-1}. Using standard values for neutron star mass, radius, etc. the derived magnetic field of this pulsar turns out to be 4×10^8 G. It should be borne in mind that in the model due to Pacini and Salvati the optical luminosity of the pulsar is extremely sensitive to the period of pulsar ($L_{\mathrm{opt}} \propto P^{-10}$) and hence the above estimate should be treated with some caution.

A third estimate for $\dot{P}$ may be obtained by assuming that the efficiency of this pulsar in producing optical radiation is the same as that of the Crab pulsar. Given this assumption, the electromagnetic luminosity of the pulsar will be $\sim 5 \times 10^{39}$ erg s^{-1} yielding a $\dot{P} \sim 10^{-17}$ s s^{-1} and a surface magnetic field $\sim 3 \times 10^9$ G. Interestingly, all these estimates suggest that this submillisecond pulsar has a very small magnetic field comparable to that of the other millisecond pulsars[14].

THE MODULATION OF FREQUENCY

Another interesting feature of this pulsar is the sinusoidal modulation of its frequency with a period of 8 h. There are three possible ways in which this could arise: (i) Doppler shift due to the motion of the neutron star in a binary orbit, (ii) Periodic change in the rotation rate of the neutron star, and (iii) Phase modulation of the pulsar signal due to precession or nutation of the neutron star. We shall discuss each of these three possibilities separately below:

MOTION IN A BINARY SYSTEM

If the observed modulation of the frequency arises due to the motion of the neutron star in a binary system, then the nearly sinusoidal nature of the frequency modulation implies that the orbit of the neutron star in the binary system must be nearly circular (i.e. eccentricity $e < 0.05$). The observed amplitude of frequency modulation ($\sim 1.5 \times 10^{-3}$ Hz) yields a mass function of the binary system $\sim 4 \times 10^{-10}\ M_{\odot}$. This will imply a companion of the neutron star with a mass $\sim 10^{-3}\ M_{\odot}$ ($\approx$mass of Jupiter) at a distance $\sim 2\ R_{\odot}$, for the most probable range of angles of inclination of the orbit to the line of sight. Such a tight binary system could have resulted from the spiral-in of a Jupiter-sized companion of the supernova progenitor. As the star evolved into a giant and engulfed its companion, the latter's orbit could have shrunk to the present size due to friction. It is, however, difficult to see how such a binary could remain bound after the explosion. The amount of matter lost from within the orbital radius would have been much more than the total mass of the present binary system. Such a large mass loss would certainly have unbound the system. On the other hand, one might argue that the immediate pre-supernova size of the companion's orbit was much less than that at present, and the mass lost from within that size was not sufficient to unbind the orbit, but increased the orbital separation considerably. However, in such a case, one would be left with a highly eccentric orbit after the explosion. Even if one circumvents this there is an added difficulty. The momentum transferred to the companion in the impact with the expanding material would, by itself, be enough to unbind the system unless most of the blast energy resides in the form of thermal energy and not kinetic energy when the impact occurs. The only conceivable situation in which such a pre-supernova binary system could survive in a near-circular orbit is if the newly born neutron star received a "kick" due to an asymmetric ejection of mass in the supernova, the effect of which almost exactly cancelled that due to mass loss and impact. This requires an extreme fine tuning of several parameters and appears rather unlikely.

It is worth mentioning that the above-mentioned orbital characteristics would be consistent if the neutron star was formed due to an accretion-induced collapse in a low-mass binary system. Such a scenario was, in fact, suggested for SN 1987A by Srinivasan[15], with the prediction that one might expect to find an ultrafast, low magnetic field pulsar in a near-circular orbit with a very low mass companion. Although the observed properties of this pulsar, if interpreted as due to a binary motion, are in good agreement with these predictions, it is very hard to reconcile the observed light curve of the supernova with such a scenario.

The above discussion leaves us with the intriguing possibility that the Jupiter-sized companion of the pulsar could in fact have formed after the supernova explosion. While discussing the ultra short period of the pulsar we invoked the possibility of a substantial fall-back of material after the core collapse. It has been suggested by Michel[16] that the matter falling back onto the neutron star is likely to collapse into a disk. Let us suppose that instabilities in such a disk resulted in its fragmentation and the formation of planets. If such a thing is conceivable, then it is reasonable to suppose that several such planets formed, and indeed part of the disk still remains. The observed near-sinusoidal frequency modulation would then imply the presence of a single large planet among them.

MODULATION OF ROTATION RATE

Precession of a triaxial ellipsoid

The observed modulation of the frequency could also be due to a change in the intrinsic rotation period of the neutron star. The spin rate of a neutron star for a given angular momentum can undergo modulation if the moment of inertia of the star around the rotation axis undergoes a periodic change. This can happen, for example, if the neutron star is a triaxial ellipsoid and the spin axis undergoes a precession around a principal axis of the star[17]. Let us suppose that this axis is the shortest principal axis (this corresponds to the lowest energy configuration). The maximum amplitude of frequency modulation that can be produced this way is $\delta f/f \lesssim \delta I/I$ where f is the spin frequency, I the moment of inertia around the shortest axis, and δI the difference between the moments of inertia about the other two axes (in evaluating δI one should not include that part of the oblateness which follows the instantaneous axis of rotation). The observed value of $\delta f/f \sim 10^{-6}$ requires $\delta I/I$ to be larger than this. The fact that the modulation is nearly sinusoidal imposes a further constraint namely that the precession amplitude must be small. This would require $\delta I/I \gg 10^{-6}$. As discussed earlier, such a spinning triaxial body will be a strong source of gravitational radiation and one would expect a corresponding slow-down rate $\dot{P} \gg 2 \times 10^{-13}\,\mathrm{s\,s^{-1}}$, inconsistent with the limits obtained from present observations. Thus the precession of a triaxial ellipsoid does not seem to offer a viable explanation for the observed frequency modulation.

Torsional oscillations

Another way of producing a modulation in rotation frequency of the neutron star is by torsional oscillations. At present we do not have a way to reliably estimate the time-scale or amplitude of such oscillations. However, it is interesting to note that a differential rotation between the crust and the core, modulated by magnetic stresses, may be able to produce the observed 8 h period as well as the amplitude of the frequency modulation if there exists a submerged toroidal field. The observed frequency modulation then implies ~10 complete differential rotations between the crust and the core during a ~2 h period (quarter cycle of the modulation). The resultant rate of change of the angular momentum of the crust requires a field strength $\sim 10^9 (I_{45}/R_6^3)^{1/2}$ G, where I_{45} is the moment of inertia of the crust in units of $10^{45}\,\mathrm{g\,cm^2}$, and R_6 the stellar radius in units of 10 km.

PHASE MODULATION

A third possible source for the observed frequency modulation is free precession (wobble) of the neutron star. An ellipticity of magnitude $\delta I/I \sim 10^{-8}$ (which does not follow the instantaneous rotation axis) will result in a precession period ~8 h for this neutron star[17]. As the precession will change the "longitude" of an emitting point, this will result in a "phase modulation" of the observed signal, which will produce an apparent change in frequency. Ruderman[18] proposed this as the mechanism for the sinusoidal modulation in the pulse arrival times reported for the Crab pulsar[19]. It is worth examining whether this could be the cause of the modulations seen in the present case. Because the phase modulation in this scenario is produced by shifting the apparent longitude of emission points, the maximum amplitude of the modulation obtained this way could only be $\sim 2\pi$. This will give a corresponding maximum modulation in frequency $\delta f \sim f_p$, where f_p is the precession frequency. However, in the present case, the precession frequency $f_p \sim 3 \times 10^{-5}$ Hz (i.e. 1/8 h), while the amplitude of frequency modulation $\delta f \sim 1.5 \times 10^{-3}$ Hz. Thus the observed modulations cannot be explained by a wobble of the neutron star.

THE MAGNETIC FIELD

We now turn to a discussion of the implications of the magnetic field of this pulsar. Various considerations put forward earlier lead one to suspect that this pulsar may have a very low surface magnetic field. If this is confirmed by observations, then it raises several questions. We discuss some of these here.

Is this a typical pulsar or a very unusual one? If indeed most pulsars are born with such low fields, then there are some interesting conclusions which can be drawn. We discuss two possibilities: (i) Such low field pulsars eventually blend with the population of high field pulsars, and (ii) They are a very different population.

(i) Most of the ~400 or so pulsars discovered so far have fields in the range $\sim 10^{11}$ to 10^{13} gauss[14]. It may be that their fields are built up after their birth from small initial values. Indeed such suggestions already exist in the literature[20, 21]. One of the mechanisms suggested is a thermoelectric process operating in the crust or in the liquid layer above it[21]. Unfortunately, there are no reliable predictions about the growth rate. Let us discuss two extreme cases: (a) the field builds up to $\sim 10^{12}$ G in a few years, and (b) the time-scale is much longer: $\gtrsim 10^6$ years.

(a) Fast growth: Such a possibility would imply in the present case a dramatic increase in the electromagnetic luminosity of the pulsar, with many predictable consequences. For example, the luminosity of the pulsed optical radiation should change. As long as the magnetosphere remains optically thin, the luminosity will increase. Eventually it may become optically thick and the radiation may be self-absorbed, resulting in a dramatic decrease in the optical luminosity. A second consequence of the increased energy output of the pulsar is the following. If the field growth is sufficiently rapid, the period of the pulsar will not lengthen significantly during the growth. The luminosity of the pulsar with a period of the order of a millisecond and magnetic field $\sim 10^{12}$ G will be $\sim 10^{44}\,\mathrm{erg\,s^{-1}}$. Thus the energy pumped into the supernova cavity in the form of electromagnetic radiation and relativistic particles will over a few years become comparable to the initial kinetic energy of the ejecta. This will result in an acceleration of the ejecta like in the case of the Crab nebula. This process is likely to be accompanied by further filamentation due to Rayleigh–Taylor instability.

Even if such a rapid build-up of the field occurs, this cannot happen in the majority of cases. For if it did, most pulsars after a few years will have strong fields and be spinning rapidly. The statistics of bright and long-lived nebulae that such pulsars will produce, and the paucity of short-period high-field pulsars[22] argue against such a rapid field build-up as a common occurrence[23].

(b) Slow growth: Thus if fast pulsars with low initial fields are to eventually blend with the observed population due to field growth, then to be consistent with the constraints mentioned above, such an evolution should occur over a time-scale $\gtrsim 10^6$ years. A similar scenario has in fact been invoked earlier by Blandford *et al*[21], to reconcile the poor association between pulsars and supernova remnants, the idea being that by the time the field builds up and the pulsar blends with the observed population, the supernova remnant would have dispersed in the interstellar medium. An immediate implication of this scenario is that there should be a deficit of high-field pulsars in the galactic disk, where they are presumably born.

(ii) The alternative possibility is that pulsars with very small fields comprise a very different population. The solitary millisecond pulsar PSR 1937+21 may be a member of this class[24]. What is the birth rate of such low field pulsars? If the *majority* are born into this distinct population then the birth rate of normal pulsars must be much less than the present estimate of ~ 1 in 40 years[25] in order for the *total* pulsar birth rate not to exceed the supernova rate. It is more likely that the birth rate of low field pulsars is rather small. The first phase of the recent Princeton–Arecibo Survey[22] of an area of the sky ~ 300 square degrees detected about 30 "normal" pulsars and 3 low field millisecond pulsars. (In our opinion the two millisecond pulsars in binaries are members of yet another population of old pulsars spun up by mass accretion.) If the lifetime of the solitary millisecond pulsar is comparable to that of the normal pulsars, then their birth rate is ~ 30 times smaller than that of the high field pulsars. This estimate is uncertain since one does not have a good idea of the luminosity function of millisecond pulsars. On the other hand if the magnetic field of such low field pulsars do not decay[6, 26, 27] then their lifetime will be $\sim 10^{10}$ years and their birth rate ~ 1 in 10^6 years and one has witnessed the birth of a very rare pulsar!

SUBMERGED FIELD?

Before concluding this section it is worth exploring the possibility that this pulsar was in fact born with a canonical field ($\sim 10^{12}$ G), but that this field has been "submerged" by matter that fell back on the neutron star. While the physics of how this would happen is not clear, the fact that the low field pulsars are predominantly in binary systems in which they are expected to have accreted a great deal of matter[4–6] makes it worthwhile to explore this possibility. Although the strong field in the interior may not have any consequence for the electromagnetic luminosity of the pulsar it could have other interesting and predictable consequences. For example, a strong interior field will produce a magnetic distortion in the shape of the neutron star, and unless the symmetry axis of this distortion coincides with the rotation axis there will be gravitational radiation. To be specific, a magnetic field $\sim 10^{13}$ G will lead[28] to a moment difference $\delta I/I \sim 10^{-9}$. The luminosity of gravitational radiation corresponding to this would be $\sim 6\times 10^{37}\,\mathrm{erg\,s^{-1}}$, yielding a $\dot{P}\sim 2\times 10^{-19}\,\mathrm{s\,s^{-1}}$. Although this value is comparable to that produced by electromagnetic torques on a $\sim 10^8$ G field pulsar, it can be discerned by measuring the braking index n as defined below:

$$n=\Omega\ddot{\Omega}/\dot{\Omega}^2$$

where Ω = angular frequency of rotation; $n=3$ for a dipole electromagnetic radiation, and $n=5$ for gravitational radiation.

CONCLUSIONS

We summarize below our main conclusions based on the presently available information.

1. If the observed periodicity in the optical radiation is interpreted as due to the rotation of a neutron star, then we conclude that the mass of the neutron star is in excess of $1.4\,M_\odot$. The stiffer the equation of state, the more massive the neutron star has to be. The most plausible reason for this in the present case might be the "fall back" of a substantial amount of mass on the newly formed neutron star.

Interestingly, this might itself be responsible for spinning up the neutron star from a slightly longer period at birth. This will require the neutron star to be close to the limiting mass regardless of the equation of state.

2. Unless most of the energy output of the pulsar is in the form of low-frequency magnetic dipole radiation with no observable consequences, which is unlikely, the present bolometric luminosity of the supernova implies that the surface magnetic field of this pulsar must be low: $\lesssim 10^9$ G. The observed optical luminosity of the pulsar also suggests a similar limit on the magnetic field. If this is a common feature of the majority of newly born pulsars, then there are two possibilities:

(a) These might eventually blend with the observed population of pulsars due to a build up of the magnetic field. Various constraints elaborated in the previous section require that in the majority of cases the *e*-folding time-scale should be $\gtrsim 10^5$ yr. This would predict a deficit of high field pulsars in the galactic disk, and a large population of low field ($\sim 10^8$–10^{10} G) millisecond pulsars.

(b) Such pulsars with very small initial fields comprise a very different population. If the birth rate of such pulsars is very large then in order for the total pulsar birth rate not to exceed the supernova rate, the birth rate of the "normal" population must be much less than ~ 1 in 40 years.

3. The sinusoidal modulation of the pulse frequency:

(a) The observed peak-to-peak modulation is very hard to reconcile with a free precession of the neutron star.

(b) The upper limit on $\dot{P}$ already implied by the present observations seems to rule out the precession of a triaxial ellipsoid as the origin of the modulation.

(c) Torsional oscillations between the core and the crust of the neutron star driven by elastic or magnetic stresses could produce the observed effect.

(d) An equally plausible scenario is that the frequency modulation is due to the motion in a binary. In this case the "Jupiter-sized" companion must have formed after the core collapse (perhaps from the matter falling back onto the neutron star).

As was mentioned in the beginning, we gather that this pulsar has not been detected again. How is this to be understood? One possibility is that there is variable extinction due to transverse motion of the ejected material. Such a model has in fact been invoked earlier to explain the observed variation of the soft X-ray flux from the supernova[29]. There is another possibility. Earlier we have suggested that there might still be a disk of material surrounding the pulsar. If there is sporadic accretion of matter from such a disk onto the neutron star, then it might smother pulsar action from time to time. This will result in X-ray emission when the pulsar activity is off. However, one may not find such an anti-correlation between X-ray emission and the pulsed optical emission if the pulsar is also a strong X-ray source. If this pulsar is eventually detected at radio wavelengths, then there is a chance of indirectly inferring the presence of a binary companion, as well as a disk, by looking for a modulation of the dispersion measure (such as that observed in the case of PSR 1957+20). This might result due to the ablation of the companion as well as the disk by the hard radiation from the pulsar. Of course, one expects this effect to be pronounced only if the line of sight makes a large angle with the orbit normal.

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