A microscopic theory of black holes in string theory: Thermodynamics and Hawking radiation

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In quantum mechanics, black holes behave like black bodies and they emit thermal radiation which is called Hawking radiation. Black hole evaporation leads to the 'information paradox'. In general relativity these properties follow from the fact that a black hole has a horizon. We explain this deep mystery in terms of the quantum statistical mechanics of an underlying microscopic theory. The microscopic degrees of freedom, for the class of black holes we discuss, are given by the collective excitations of a configuration of *D*-branes which occur in string theory. This article introduces the issues involved and presents a summary of what has been achieved and what more needs to be done.

Black holes and general relativity

THE force of gravity is one of the most important predictions of string theory. In fact string theory explains gravity because its low energy limit is described by Einstein's General Theory of Relativity. Since string theory is consistent with quantum mechanics (in particular it is unitary and free of the usual infinities of quantum field theory) it is widely believed that it is also a consistent theory of quantum gravity. Hence string theory should be able to resolve the conundrums of general relativity. One of these conundrums, to which this article is devoted, has to do with the fact that black holes emit thermal radiation.

Black holes are among the solutions of general relativity that describe gravitational collapse. There is a lot of indirect evidence for their existence and their study occupies a central place in modern high energy astrophysics. Their masses range from a few solar masses for black holes in X-ray binaries to a few hundred million solar masses for black holes that are believed to exist in the centers of galaxies.

At present all theories of astrophysical phenomena associated with black holes are based on classical general relativity. While quantum aspects of black holes seem remote from experimental access they present nagging problems for theoretical physicists. There are various aspects to this.

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Firstly the center of the black hole is a place where the curvature of space–time becomes infinitely large (Figure 1). Presently we do not know how to treat this singularity. However on general grounds it is clear that for large curvatures Einstein's theory needs modification and in a modified framework the singularity would be smoothened out. string theory offers many examples where singularities are understood and resolved by a variety of mechanisms. As of now these mechanisms do not apply to the singularity of a black hole.

Black holes have another property that enables us to ignore the above singularity and discuss their physics in a well-defined way. They have a horizon which is a surface that shields the singularity in a manner of speaking (Figure 1). Penrose has called this property 'cosmic censorship'. The horizon is a null surface in the sense that any two points on the horizon are connected by a ray of light. The horizon of a black hole in 3+1 dims is a 2-dim. surface with a fixed area. Light starting inside the horizon never emerges outside. Physically we can

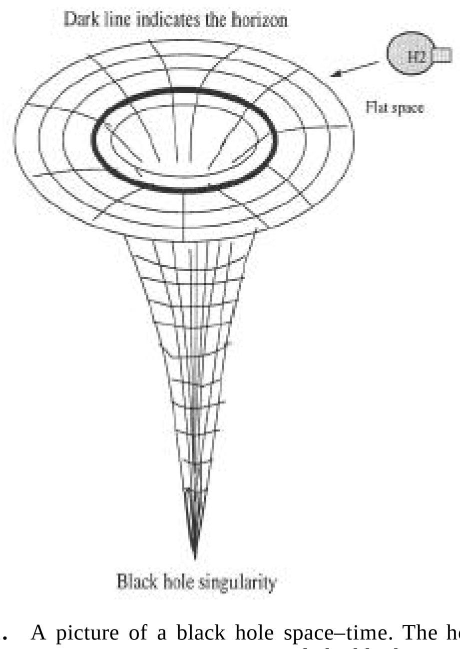

Figure 1. A picture of a black hole space–time. The horizon divides the space–time into two regions and shields the singularity. A jar of hydrogen gas is falling into the black hole.

attribute this to the bending of light by the gravitational field inside the black hole: a kind of a gravitational total internal reflection. The horizon thus partitions the space–time of a black hole into two distinct regions. A freely falling observer crossing the horizon will continue to fall inwards and will never be able to get out. Since the light emitted by a source that is carried by the observer cannot escape, the hole will appear black! That is why an object with a horizon is called a black hole. The name was coined by John Wheeler.

The presence of the horizon of a black hole leads to a paradox with the 2nd law of thermodynamics. Imagine a jar of hydrogen gas falling into a black hole (Figure 1). This gas, which is at a certain temperature, would have a certain amount of entropy before falling into the black hole. After falling in this entropy would be entirely unobservable and that would violate the second law of thermodynamics because in the process of the jar falling into the black hole the total entropy of the jar plus the black hole would decrease! The paradox would be resolved if we assume that the black hole has entropy and this entropy increased when the jar fell in.

Bekenstein made the hypothesis that the entropy associated with a black hole is proportional to the area of the horizon of the black hole,

$$S = aA, \tag{1}$$

where a is a universal constant which would be the same for all black holes.

Let us list a few properties of black holes that support his guess.

- The 'no hair theorems', tell us that the state of a classical black hole is completely characterized by a few parameters like its mass, angular momentum and global gauge charges which give rise to long range fields. Electric charge is an example of a gauge charge associated with the long range electromagnetic field. In particular the area of the event horizon depends only on these quantities.
- If we perturb a black hole, say by throwing in some matter, then the perturbation decays in a characteristic time r_h/c (r_h is the horizon radius and c is the speed of light in vacuum), and the new state of the black hole is again characterized by a horizon whose area has increased and is characterized by the changed mass, angular momentum and charge of the final state. In fact in any adiabatic process involving black holes the horizon area never decreases. In particular if we have a fusion of two black holes with horizon areas A_1 and A_2 then the horizon area A_{12} of the final black hole is never less than the sum of the initial areas.

This is as far as we can go using the classical theory where only the change in the entropy of a system makes sense. The absolute entropy of a black hole can only be defined in the framework of quantum mechanics. This crucial step was taken in the work of Hawking who quantized matter in a black hole background.

Quantum field theory and black holes

In the quantum theory since the absorption process is described by the matrix element of a hermitian Hamiltonian, the emission amplitude is necessarily non-zero. Black holes radiate and behave like black bodies. They emit thermal radiation and they are characterized by a temperature that has a purely quantum mechanical origin and depends only on the mass, angular momentum and the global charges of the black hole. The fundamental formula for the temperature, due to Hawking is given by,

$$T = \frac{\hbar \boldsymbol{k}}{2\boldsymbol{p}}, \tag{2}$$

where $\boldsymbol{k}$ is surface gravity (acceleration due to gravity felt by a static observer) at the horizon of the black hole. For a Schwarzschild black hole $\boldsymbol{k} = (1/3G_N M_c)$, where G_N is Newton's constant and M is the mass of the black hole.

The constant of proportionality in eq. (1) is determined by using the first law of thermodynamics, $T\mathrm{d}S = \mathrm{d}M$ and the temperature formula

$$S_{bh} = aA_h, \qquad a = \frac{c^3}{4G_N\hbar}. \tag{3}$$

This is the celebrated Bekenstein–Hawking formula.

A basic question that can be posed at this stage is: Is there a microscopic origin of the black hole entropy? Or equivalently can we understand black hole entropy in terms of Boltzmann's formula?

$$S_{bh} = \ln\Omega, \tag{4}$$

where Ω is the number of micro-states of the black hole. There does not seem to be an answer to this question in the standard framework of quantized general relativity. However if we assume that eq. (3) is a measure of the micro-states of the black hole, its study may teach us something about the hidden degrees of freedom of quantum gravity. string theory does vindicate this expectation for a certain class of black holes.

Note that the universal constant $4a^{-1}$ is the square of the Planck length l_P which is a basic unit of area built out of the 3 fundamental constants: speed of light c, Planck's constant $\hbar$ and Newton's constant G_N. ($l_P^2 = (G_N\hbar/c^3) = 10^{-33}$ cm.) It is remarkable that a counting formula contains the Planck length which is a length scale one would naturally associate with extremely high energy physics. Besides the fact that a black hole behaves like a thermodynamic object, black holes in general spontaneously radiate thermal radiation (called Hawking radiation). In the case of a realistic Schwarzschild black hole, this is reflected in the fact that the specific heat is negative and hence the black hole is expected to evaporate completely.

The information paradox

The information puzzle is a statement of the conflict that arises between the general principles of quantum mechanics and the fact that black holes evaporate by the emission of thermal radiation. Let us explain this. The in falling matter that forms a black hole is described by a wave function in the standard fashion before it disappears behind the horizon. Given the wave function we know (in principle) all the quantum mechanical correlations between the degrees of freedom of the system. When the black hole evaporates the final state of the system is purely thermal radiation. The final state is a mixed state where the quantum mechanical correlations between the states of the system are averaged over. This evolution of a pure state to a mixed state is in conflict with the standard laws of quantum mechanics because a hermitian Hamiltonian evolves pure states into pure states. The fact that in the presence of black hole a pure state evolves into a mixed thermal state is called the 'information paradox'.

Hawking radiation as calculated in semi-classical general relativity is a mixed state and one can attempt to resolve this paradox in the standard quantum gravity framework by calculating the correlations between the ingoing and outgoing Hawking particles. In this way one may be able to reconstruct the initial state of the black hole. However such a calculation would require a good quantum theory of gravity where controlled approximations are possible. All such attempts have failed in the conventional framework of quantum gravity.

In string theory we propose to resolve the information puzzle (for a certain class of black holes) by discovering the microscopic degrees of freedom of the black hole and their interactions. In string theory a black hole is described like in standard quantum statistical mechanics by a density matrix:

$$\boldsymbol{r} = \frac{1}{\Omega}\sum_i |\, i\rangle\langle i\,|,$$

$$S = \ln\Omega, \tag{5}$$

where $|i\rangle$ is a micro-state. Ω represents the total number of micro-states. Hence in string theory the thermal nature of Hawking radiation is explained just like we explain the thermal spectrum of a piece of hot glowing coal. In principle, if we choose not to use the microcanonical ensemble, we can reconstruct the initial state of the system from the final state. There is no in principle 'information paradox'. It is well worth emphasizing that the existence of black holes in nature compels us to resolve the logical problems of incorporating general relativity into the framework of quantum mechanics. One may take recourse to the fact that for a black hole of a few solar masses (one solar mass is approximately 10^{33} g), the Hawking temperature is very tiny $\sim 10^{-8}$ K, and hence unobservable. However the logical problem that we have described above cannot be wished away and its resolution enhances the case for the string paradigm as a framework for fundamental physics.

D-Branes and micro-states of string theory black holes

It is a fortuitous circumstance that for a certain class of black holes that occur in string theory, we can make precise statements about the derivation of black hole thermodynamics from statistical mechanics including the rates of Hawking radiation. These black holes occur as classical solutions in the low energy limit of type IIB string theory and their space–time is 4+1-dimensional. Their virtue lies in the fact that we can make precise statements about them. They also have the simplifying property that unlike the Schwarzschild black hole they have a positive specific heat.

Let us give a brief description of these black holes. They are characterized by 3 charges Q_5, Q_1 and N which can be chosen to be positive integers. The charges Q_5, Q_1 are generalized gauge charges and N is an electric type charge. To describe the microscopic model we need to first consider 10-dim space–time as a product of $R^{5,1} \times S^1 \times T^4$. So we have 4 non-compact space dimensions, time, a circle of radius R and a 4-torus (a product of 4 circles). The radii of the 4-torus are much smaller than R.

The basic building blocks are the 2 distinct types of solitons of this string theory. They are $D5$-branes and $D1$-branes. A $D5$ brane is a 5-dim. domain wall and a $D1$-brane is a 1-dim. string. These branes are sources of the generalized gauge charges Q_5, Q_1. The fundamental property of these branes is that open strings can be emitted and absorbed by these branes just like photons are emitted and absorbed along the world lines of electrons. Just like a photon emitted by an electron and absorbed by another describes their electromagnetic interactions, open strings emitted and absorbed between branes also mediate interactions between D-branes. The amplitude for the emission of an open string from a D-brane is proportional to the string coupling constant g_s. Figure 2 illustrates this point.

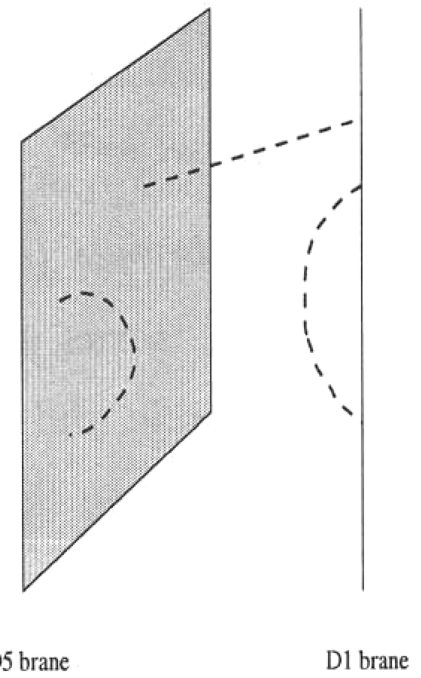

Figure 2. Open strings (dotted lines) emitted and absorbed by $D5$ and $D1$-branes.

The model of the black hole is built by wrapping the $D5$-branes on the 4-torus and the circle of radius R, and wrapping the $D1$-branes on the same circle. In this way these coincident branes share exactly one spatial dimension, namely the circle of radius R.

Now that we know the placement in space–time of the basic building blocks, let us enumerate the various open strings that mediate the interaction amongst the assembly of D-branes. There will be various types of open strings attached to the branes depending on their end points. For example there is an open string that connects the ith Q_1 brane with the jth Q_5 brane. We can assemble these open strings into $Q_1 \times Q_5$ matrices. Similarly we can assemble open strings that join the ith and jth Q_1 branes into $Q_1 \times Q_1$ matrices and similarly for the 5 branes we have $Q_5 \times Q_5$ matrices. Since the D-branes are charged the (j, i) open strings are complex conjugates of the (i, j) open strings. Hence the matrices which assemble the open strings are hermitian matrices. See Figure 3.

The low energy dynamics of these open strings are described by a supersymmetric non-abelian gauge theory on a cylinder of radius R with time running along its length. This is just the cylinder that is common to both the $D1$ and $D5$-branes. The gauge group is $U(Q_1) \times U(Q_5)$. Since there are a large number of branes involved, the effective coupling constant of this gauge theory is $g_s \sqrt{Q_1 Q_5}$. We see that even though the coupling g_s can be small the effective coupling can be large since a macroscopic black hole is composed of a large number of branes. Hence black hole physics is described by the strong coupling limit of the gauge theory.

In order to have a workable solution to our problem we are aided (immensely) by the fact that the gauge theory has a scale invariant description with a high degree of supersymmetry. Supersymmetry enables us to circumvent the would-be hard strong coupling problem. However we leave this unexplained for the purposes of this article. Using this fact we can explain several features of black hole thermodynamics without a detailed knowledge of the dynamics. The scale invariant dynamics has a Hamiltonian H that evolves the system along the cylinder, a momentum P that generates rotations around the cylinder and a central charge c which is a measure of the number of degrees of freedom that enter the Hamiltonian. In the case at hand $c = 6Q_1Q_5$ and the ground state of our black hole is modeled by the eigenvalue condition $H = P = N/R$. The fact that such a state is a stable ground state is a consequence of supersymmetry.

Entropy of the extremal black hole

We now indicate the fundamental result that the entropy of the above state exactly matches the Bekenstein–Hawking entropy of the black hole. Henceforth our convention will be to work in units which set $c = \hbar = 1$.

For large values of charges the $H = P = N/R$ state, is highly degenerate, and in fact the degeneracy grows exponentially. A well-known asymptotic formula counts this number and the entropy is the logarithm of the degeneracy

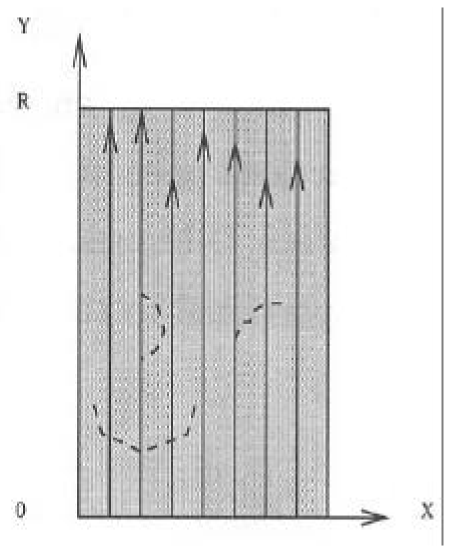

Figure 3. A drawing of the assembly of $D5$-branes (shaded region) and $D1$-branes. The X axis stands for the directions of the 4-dim. torus and the Y axis stands for a circle of radius R. The $D1$-branes wind around the circle and they are indicated by the lines with arrows. The various open strings are illustrated by the dotted lines.

$$S_{bh} = 2\pi\sqrt{Q_1Q_5N}. \qquad (6)$$

This result of Strominger and Vafa coincides with the Bekenstein–Hawking result for the corresponding extremal black hole! The temperature of this black hole is zero. However there is no conflict with the Nernst theorem in classical thermodynamics since here we are referring to the absolute entropy of a highly degenerate quantum ground state.

Entropy of near extremal thermal black holes

Let us now describe the more interesting case when the black hole is excited above its ground state. As we shall see in this case it has a small Hawking temperature and one can address the question of Hawking radiation to a state of lower mass. The quantum state corresponding to the near extremal black hole is modeled by the eigenvalue conditions $H = N/R + 2n/R$ and $P = N/R$, $n \ll N$.

The entropy of the excited black hole in the microcanonical ensemble can be once more calculated using asymptotic formulas

$$S_{\mathrm{BH}} = \ln\Omega = 2\pi\sqrt{Q_1Q_5}\,(\sqrt{N+n}+\sqrt{n}). \qquad (7)$$

The small Hawking temperature of this near extremal black hole is also readily obtained from the above formula since we know that the change in energy from the ground state is $2n/R$.

$$T_{\mathrm{H}} = \frac{\pi R}{2}\sqrt{\frac{n}{Q_1Q_5}}. \qquad (8)$$

The above thermodynamic formulas again match with the calculations done in general relativity for large values of the charges Q_1, Q_5, N. It is important to note that the entropy and temperature of this black hole are independent of the 5-dimensional Newtons's constant. It is also easy to see from the above formulas that the specific heat is positive since the change in mass is proportional to the square of the temperature: $n \sim T_{\mathrm{H}}^2$. This circumstance is unlike the (more difficult) case of the Schwarzschild black hole. Another interesting feature of the above thermodynamic formulas is the fact that the entropy above the ground state is proportional to the temperature. This is characteristic of a gas of particles in 1-dimension and has a natural explanation in the microscopic theory.

Hawking radiation of near extremal thermal black holes

The next issue of importance is the calculation of the Hawking rates of emission of the various particles by the near extremal black hole as it decays towards its ground state. The emission is most copious in the case of massless particles and fortunately these can be exactly calculated. The derivation of these rates turns out to be a difficult problem because it involves an understanding of the interaction of the absorbed and emitted modes with the black hole micro-states. In order to set up the interaction Hamiltonian to first order, we need a correspondence between the absorbed and emitted modes and operators in the microscopic theory that couple to them. The interaction Hamiltonian is then uniquely fixed up to a normalizing constant by the high degree of symmetry in the microscopic theory at hand. The basic idea is similar to the case of pion–nucleon couplings that are fixed to first order by the use of $SU(2)$ isospin symmetry of the strong interactions. In the case at hand the symmetry turns out to be a combination of conformal symmetry and the $SO(4)$ group of rotations in 4-space dimensions. The solution of this problem that applies to all the possible Hawking particles is provided by using a duality discovered by Maldacena. This principle was originally formulated for the problem of $D3$ branes. It is indeed fortunate that it can also be adapted to the black hole modeled by the $D1$–$D5$ system.

Let us explain this. In the limit $G_5 \to 0$ the interaction of the Hawking particles (absorbed and radiated from the black hole) and the black hole micro-states is vanishingly small and hence we can isolate the black hole micro-states from the Hawking radiation. Also, since the absorption cross section of very long wavelength particles into the black hole is proportional to the area of the horizon, $A_h = 4G_5\sqrt{Q_1Q_5N}$, this too is vanishingly small in this limit. Note that since the entropy and temperature of the black hole in consideration are independent of G_5 they are unaffected by this decoupling limit and it makes sense to study the microscopic theory in isolation.

Maldacena's duality states that in the decoupling limit the gauge theory of the micro-states, which we have discussed, is dual to string theory in a 5+1 dimensional space–time which is $M = AdS_3 \times S^3$. AdS_3 stands for Anti de Sitter space in 3-dimensions and S^3 stands for a 3-dimensional sphere (note 1). In fact the gauge theory lives on the boundary of this space, which is the 2-dimensional cylinder that we encountered before. The low energy limit of this string theory describes a gravity theory in M. It has a black hole solution which captures the near-horizon features of the original black hole whose properties we are interested in. In a manner of speaking it is obtained by scaling (stretching) the region near the horizon of the black hole to infinity. The scale invariance of the microscopic model matches well with the fact that as we move away from the horizon of a black hole energies of particles undergo a gravitational red shift. Another way of putting this is that if one examines the potential energy of a particle in the black hole geometry then in the near horizon limit, the wall of the pit (characteristic of a black hole) becomes infinitely high so that the modes near the horizon cannot escape to infinity. In more technical parlance as Q_1 and Q_5 go to infinity the horizon degrees of freedom become exactly massless and decouple from the bulk degrees of freedom.

Once the exact correspondence of the various operators of the theory on the boundary, and the Hawking particles that live in the bulk, is known, we can use the first order interaction Hamiltonian, valid for small but non-zero G_5, to calculate the absorption and emission rates. The interaction is described by the Hamiltonian

$$H_{\text{int}} = \int \boldsymbol{j}|_{\text{B}} O, \tag{9}$$

where $\boldsymbol{j}$ is a wave field corresponding to an absorbed or emitted particle and O is the corresponding operator in the microscopic theory. $\boldsymbol{j}|_{\text{B}}$ means the value of the wave field on the boundary of Anti de Sitter space. The absorption of a particle described by the wave field $\boldsymbol{j}$ causes a transition from a micro-state $|i\rangle$ to a microstate $|f\rangle$ with amplitude

$$S_{\text{if}} = \langle f \,|H_{\text{int}}\, (|i\rangle |\boldsymbol{j}\rangle). \tag{10}$$

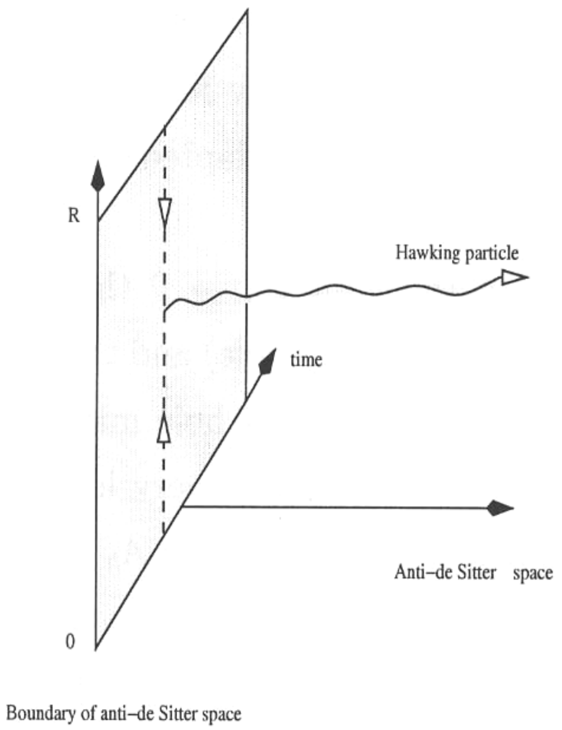

Figure 4. The effective theory that describes the dynamics of the $D1$–$D5$ system lives on the boundary of Anti de Sitter space. The dotted lines indicate excitations of this effective theory giving rise to a Hawking particle (represented by the wave field $\boldsymbol{j}$) that is emitted by the black hole.

The emission and absorption amplitudes are equal by time reversal invariance. Since the large degeneracy of the black hole states enable a density matrix description, the probability of the process is given by the 'unpolarized' expression:

$$\text{Prob}_{\text{abs}} = \frac{1}{\Omega_{\text{I}}} \sum_i \sum_f |\, S_{\text{if}}\,|^2 . \tag{11}$$

Note that the 'unpolarized' transition probability corresponds to *averaging* over initial states and *summing* over final states. Ω_{I} is the total number of initial micro-states corresponding to the macroscopic charges of the black hole. A similar expression for the emission process is given by

$$\text{Prob}_{\text{em}} = \frac{1}{\Omega_{\text{F}}} \sum_i \sum_f |\, S_{\text{if}}\,|^2 , \tag{12}$$

where Ω_{F} is the total number of final micro-states corresponding to the macroscopic charges of the black hole.

The above expressions for the absorption and emission probabilities express the most basic message of this article that black hole thermodynamics is a consequence of the quantum statistical mechanics of the microscopic degrees of freedom associated with the horizon of a black hole. These formulas can be used to calculate the decay rate of the black hole in a standard fashion. The string calculation, for the decay into all the dominant massless modes, is valid in the domain where string theory is well approximated by supergravity, due to the high degree of supersymmetry of the effective theory of the micro-states. The formulas derived in this way agree with Hawking's semi-classical calculation in general relativity,

$$\Gamma(k) = \boldsymbol{s}(k)_{\text{abs}}\, \boldsymbol{r}(k) \frac{\text{d}^4 k}{(2\boldsymbol{p})^4}, \tag{13}$$

where $\boldsymbol{r}(k) = (\exp k/T_{\text{H}} - 1)^{-1}$ and $\boldsymbol{s}(k)_{\text{abs}}$ is the absorption cross section of the incident wave, of wave number k (in a particular channel), on the black hole (note 2).

Concluding remarks and future problems

- The successful description of black hole thermodynamics that we have given, hinged on discovering a microscopic description of the near horizon degrees of freedom of a class of black holes in string theory. The technical methods (which we did not describe in any detail) that made possible calculations of the Hawking process, which is basically a strong coupling problem, rely on (i) the ability to isolate the microscopic dynamics (Maldacena's decoupling limit) from the other degrees of freedom of the theory and (ii) the high degree of supersymmetry of the *D*1–*D*5 system. The question is whether a new conceptual breakthrough is needed to tackle the more physical Schwarzschild or Kerr black holes where neither of the above 2 points are obviously valid.

- We need to explain the Bekenstein–Hawking formula from the microscopic theory: why is the entropy proportional to the area of the horizon? An answer to this question will throw light on the question of how the microscopic theory creates (explains?) the space–time of the black hole.
- Our discussion focused entirely on the region of space–time outside the horizon. What about the region inside the horizon which is singular. A fundamental theory must address this question.
- String theory must address the difficult question of the actual formation and evaporation of a black hole.
- In this article we have focused on one of the important conundrums of general relativity on which string theory has made progress. We end this article by stating another conundrum that string theory faces, in light of the recent astronomical observations that indicate an accelerated expansion of the universe. Presently it is not known how to accommodate such a cosmological solution in string theory. Besides, this string theory must also address itself to the deep conceptual issues related to the 'big bang' and the origin of the universe.

Notes

1. Anti-de Sitter space is a solution of Einstein's equations in 2+1 dims, with negative cosmological constant just like the 3-dimensional sphere is a solution with positive cosmological constant. In the present case the magnitude of the cosmological constant is the same for both the spaces.
2. The theory of Hawking radiation for the class of black holes considered here has now been worked out after much effort. For details we refer the reader to the technical reviews quoted at the end of this article.

Suggested further reading

1. Bunn, T., 'Black Holes FAQ List', http: //cfpa.berkeley.edu/BHfaq.html.
2. Hawking, S. W. and Penrose, R., 'The Nature of Space Time', *Sci. Am.*, July 1996.
3. Susskind, L., 'Black Holes and the Information Paradox', *Sci. Am.*, April 1997.
4. Articles by Mukhi, S., Sen, A., Das, S. and Minwalla, S., *Curr. Sci.*, 1999, **77**, 1624–1668.
5. For technical reviews and guide to the literature see, David, J. R., 'String Theory and Black Holes', TIFR Ph D thesis, hep-th/9911003; Mandal, G., 'A review of the *D*1–*D*5 system and five dimensional black hole from supergravity and brane viewpoint', hep-th/0002184; Wadia, S. R., 'A review of the microscopic modelling of the 5-dim. black hole of IIB string theory', *Pramana*, 2001, **56**, 1–46, hep-th/0006190.