

# D-brane Black Holes: Large-N Limit and the Effective String Description

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## Abstract

We address the derivation of the effective conformal field theory description of the 5-dimensional black hole, modelled by a collection of  $D1$ - and  $D5$ - branes, from the corresponding low energy  $U(Q_1) \times U(Q_5)$  gauge theory. Finite horizon size at weak coupling requires both  $Q_1$  and  $Q_5$  to be large. We derive the result in the moduli space approximation (say for  $Q_1 > Q_5$ ) and appeal to supersymmetry to argue its validity beyond weak coupling. As a result of a combination of quenched  $Z_{Q_1}$  Wilson lines and a residual Weyl symmetry, the low-lying excitations of the  $U(Q_1) \times U(Q_5)$  gauge theory are described by an effective  $N=4$  superconformal field theory with  $c = 6$  in 1+1 dimensions, where the space is a circle of radius  $RQ_1Q_5$ . We also discuss the appearance of a marginal perturbation of the effective conformal field theory for large but finite values of  $Q_5$ .

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# 1 Introduction

The identification of D-branes as RR charged solitons in [1] has added new substance to string theory. One important physical application is to black hole physics. Various black hole solutions of low energy string theory admit a constituent description in terms of D-branes [2]. For extremal and near extremal black holes the constituent model gives a statistical basis to the notion of black hole entropy and leads to an understanding of the Hawking-Bekenstein formula [2, 3, 5, 4, 6, 7]. In [2] the idea of the effective conformal field theory that accounted for the entropy of the black hole was introduced. In [6] the applications of the D-brane model to black hole thermodynamics were initiated and in [7] the thermodynamics was explained in terms of the effective conformal field theory that lives on a cylinder whose radius is dilated by a factor related to the charges of the black hole. The formulation of [7] was used in [8] to show that the low frequency absorption rates for minimally coupled scalars calculated using D-branes were proportional to the classical calculation. This work also established that the D-brane assembly in the classical limit was indeed “black”. At that time it was not clear whether these rates should be equal because the D-brane calculations were done in a regime of couplings where the event horizon would fall within the string length. Subsequently the equality of these rates was shown in [9] in the weak coupling regime. The absorption calculations were extended to the highly non-trivial case of grey body factors in [10] and also other circumstances in [11, 12, 13]. There are recent works that also discuss disagreements of the D-brane calculations with the classical results [14, 15] and these issues are still to be understood.

It is clear that the remarkable agreements between weak coupling D-brane calculations and the classical results for near extremal black holes need to be explained in terms of the standard theory of the D-brane system [18, 1] which is in principle valid for both weak and strong coupling, when the branes are very near each other. There is also the suggestion of a supersymmetry non-renormalization theorem at work [10] and an attempt to derive, on general grounds, the low energy effective sigma model that describes the collective modes of the black hole [10]. The issue of string loop corrections to the Dirac-Born-Infeld action has also been recently addressed [17].

In this work we attempt to address, in a more systematic way, the issue of the description of the 5-dimensional black hole arising in toroidal compactification of type IIB string theory in terms of an assembly of D-branes. The black hole we consider is the one considered in this connection in [6] and carries charges  $Q_1$ ,  $Q_5$  and  $n$ . The D-brane system corresponding to this black hole is described by an N=4 supersymmetric  $U(Q_1) \times U(Q_5)$  gauge theory in 1+1 dimension with the space dimension as a circle of radius  $R$ . This theory is the dimensional reduction of an N=2 theory in 3+1 dimensions [19, 20]. The low energy collective modes of the gauge theory have an effective description in terms of a superconformal field theory in 1+1 dimensions. The black hole is then identified with a certain state in this conformal field theory and the degeneracy of this state determines the black hole entropy. In this paper, we analyze the supersymmetric gauge theory corresponding to the black hole and obtain the effective theory for its low-lying excitations in an explicit way.

This paper is organized as follows: In section 2, using some observations made in [10] we

recognise, in analogy with QCD, that there is indeed a weak coupling expansion parameter  $1/N$  in the problem, where  $N$  is proportional to  $Q_1$  and  $Q_5$ .  $N$  goes to infinity holding  $Ng$  fixed where  $g$  is the closed string coupling constant. The Newton's constant  $G$  scales as  $1/N^2$ . There is a partial analogy with large- $N$  baryons of QCD. In section 3, we describe some relevant aspects of the  $U(Q_1) \times U(Q_5)$  supersymmetric gauge theory that describes the D-brane system corresponding to the black hole. In section 4 we consider this theory in a phase where, in the ground state, Wilson lines in the centre of the gauge group as well as the hypermultiplets condense and then analyze the ground state structure of the theory. In section 5, we isolate the supersymmetric gauge invariant degrees of freedom of the low-lying excitation spectrum at long wave lengths. We find that a combination of the Wilson lines in the centre of  $SU(Q_1)$ , (say, for  $Q_1 > Q_5$ ) and a residual invariance under Weyl reflections in  $SU(Q_5)$ , leads to the existence of exactly 4 excitations that effectively live on a circle of radius  $RQ_1Q_5$ . We perform a quenched average over the different Wilson lines as is standard in the limit of large  $N$ . The residual Weyl reflections are treated by using the gauge invariant sewing mechanism [21, 22, 23, 24]. This leads to an  $N=4$  supersymmetric  $\sigma$ -model in  $1 + 1$  dimensions with central charge  $c = 6$ . Though this analysis is performed in the weak coupling limit  $g \rightarrow 0$ , one may argue, based on the supersymmetry of the model, that the results are not changed in any drastic way in the black hole regime where  $gN > 1$ . We also discuss the failure of the ‘unitary gauge fixing’ and the appearance of  $Z_2$  vortices. The  $Z_2$  vortex corresponds to a marginal operator in the effective superconformal field theory. Section 6 contains our conclusions and some comments about the possibility of obtaining a description of black holes in the M(atrix) theory [25] which seems poised to unify most of the disparate facts about string theory.

## 2 Finite Horizon Area and the ‘Large- $N$ ’ Limit

This section has much overlap with some parts of [10, 26], however, given the importance of the observations, it is well worth discussing them with a slightly different emphasis. The black hole solution of IIB string theory (see [19] for a review) compactified on a 5-torus, is characterized by 3 integral charges  $Q_1$ ,  $Q_5$  and  $n$ . This solution saturates the BPS bound.  $Q_1$  and  $Q_5$  are the electric and magnetic type charges associated with the RR 3-form field strength and  $n$  is related to the momentum  $P$  along the 5th circle of radius  $R$  which is quantized as  $P = n/R$ . The entropy of the black hole turns out to be

$$S = 2\pi\sqrt{Q_1Q_5n}, \quad (1)$$

and is related to the horizon area  $A$  by  $S = A/4\pi G$ . Equating these two expressions, we get an expression for the area of the horizon of the black hole,  $A \sim g^2\sqrt{Q_1Q_5n}$ . We have used the fact that, in units of the string length, Newton's coupling is  $G \sim g^2$ , where  $g$  is the coupling constant of closed string theory. Now since the black hole solution has a quantum relevance only if  $g \rightarrow 0$ , we see that for fixed values of the charges the horizon shrinks to zero, and the black hole is not macroscopic. This objection can be averted if we send the charges to infinity in a specific way. Since the D-brane assembly is described by a  $U(Q_1) \times U(Q_5)$  gauge

theory, it is consistent to hold  $gQ_1$  and  $gQ_5$  fixed, because the gauge theory in that case has a systematic expansion in powers of  $1/Q_1$  and  $1/Q_5$ . Hence, a finite horizon area requires holding  $g^2n$  fixed. Further, since the horizon sets the length scale below which there is no black hole, the relevant regime for the gauge theory is  $(gQ_1gQ_5g^2n)^{1/6} \gg 1$ . Since  $Q_1$  and  $Q_5$  both scale as  $1/g$ , they are comparable. It is important to realise that all the 3 charges have to be scaled to infinity in order to describe a finite area of the horizon. Holding any charge at a finite value leads to a zero horizon area in the limit of weak closed string coupling. To facilitate further discussion we introduce the natural notation  $N \sim Q_1 \sim Q_5$ . Needless to say, solving the gauge theory in the regime  $gN > 1$  is a difficult problem, however, it is important to realise that there exists a systematic expansion in the small expansion parameter  $1/N$ . For various aspects of the large  $N$  limit of gauge theories and matrix models which are relevant to us in this section and later on, we refer the reader to [28].

Let us indicate the  $N$  scaling of some relevant quantities. Newton's coupling scales as  $G \sim 1/N^2$ . The mass of the black hole scales as  $M \sim N^2$  and its entropy scales as  $S \sim N^2$ . This is what we expect in the semi-classical limit as  $1/N \rightarrow 0$ . There is an instructive analogy with  $SU(N)$  QCD in the large  $N$  limit. As is well known the low energy effective lagrangian of QCD is the chiral model of mesons whose expansion parameter is given by the inverse of the pion coupling constant  $f_\pi \sim 1/N$ . The baryon is an  $N$  quark bound state interacting via gluons and it is a soliton solution of the chiral Lagrangian with  $M \sim N$ . The baryon is analogous to the black hole that is composed of  $N^2$  open string degrees of freedom, which, as we will subsequently see, arise from the hypermultiplet of  $N^2$  strings in the fundamental representation of  $U(Q_1) \times U(Q_5)$ . Just like in QCD, where meson-baryon couplings are of order  $(1/N)^0$  and of the same order as the pion kinetic energy term, the closed string- black hole couplings are also of order  $(1/N^2)^0$ , of the same order as the graviton kinetic energy term. In both cases the interaction is of order one and that is why there is a non-trivial scattering. Also, the size of the baryon is independent of  $N$  and so is the area of the horizon of the black hole. However the analogy is partial because the lowest lying collective modes of the baryon are described by the collective coordinates of the flavour group, and hence the degeneracy of the ground state does not increase exponentially. An exponential increase in the number of states is of course the main feature of the modern basis of black hole thermodynamics [29, 30], at least for the black hole solutions of string theory. The other significant difference is that the mass of a D-brane is of order  $N$ , and hence it can have strong order  $N$  interactions with the black hole [31].

### 3 The Supersymmetric Gauge Theory for the D-brane System

The five dimensional black hole in type IIB string theory compactified on  $T^5$  and carrying charges  $Q_1$  and  $Q_5$ , is modelled by a system of  $Q_1$  D1-branes and  $Q_5$  D5-branes [6]. This system is, in turn, described by a  $U(Q_1) \times U(Q_5)$  gauge theory in two dimensions which is the dimensional reduction of an N=1 supersymmetric  $U(Q_1) \times U(Q_5)$  gauge theory in six dimensions, or equivalently, that of an N=2 supersymmetric theory in four dimensions.

In this section, we briefly describe the massless spectrum of this D-brane system and write down the relevant terms of the corresponding 2-dimensional gauge theory Lagrangian.

Consider a system of  $Q_5$  D5-branes parallel to  $x^1, x^2, x^3, x^4, x^5$  and  $Q_1$  D1-branes parallel to  $x^1$ . The excitations of this system correspond to open strings with the two ends attached to the branes and there are four classes of such strings: the (1,1), (5,5) (1,5) and (5,1) strings. In the absence of D1-branes, the part of the spectrum corresponding to (5,5) strings is the dimensional reduction, to  $5 + 1$  dimensions, of the  $N=1$   $U(Q_5)$  gauge theory in  $9 + 1$  dimensions. In six dimensions, this consists of a vector multiplet and a hypermultiplet in the adjoint representation of the gauge group. When we introduce D1-branes, say along  $x^1$ , then this gauge theory has to be further restricted to two dimensions, *i.e.*, to the  $x^1 - t$  space. Now, we also have (1,1) strings and the states coming from these correspond to the dimensional reduction, to two dimensions, of  $N=1$   $U(Q_1)$  gauge theory in ten dimensions. The field content obtained so far is the same as that of  $N=2$   $U(Q_1) \times U(Q_5)$  gauge theory in 6 dimensions, reduced to 2 dimensions. However, there are also other fields coming from the (1,5) and (5,1) strings. These strings have ND boundary conditions along the coordinates  $x^2, x^3, x^4, x^5$ , that are transverse to the D1-branes, but parallel to the D5-branes. The excitations coming from these strings fall in a hypermultiplet and their inclusion reduces the  $N=2$  supersymmetry in 6 dimensions to  $N=1$ . Since these strings have their ends fixed on different types of D-branes, the corresponding fields transform in the fundamental representation of both  $U(Q_1)$  and  $U(Q_5)$ .

A theory describing these excitations at low energies can be easily written down by first constructing an  $N = 2$   $D = 4$  gauge theory with gauge group  $U(Q_1) \times U(Q_5)$  containing a pair of vector multiplets and a pair of hypermultiplets in the adjoint representations of the gauge groups, along with a hypermultiplet in the fundamental representation of each factor of the gauge group (see, for example, [32, 33]). The dimensional reduction of this theory to 2 dimensions then gives the gauge theory describing the low-energy dynamics of the D-brane system. In the following we will only write down the few relevant terms of this Lagrangian which are needed for the analysis of the next section. But first, some notation: The fundamental representation indices are denoted by  $a, b, \dots$  for  $U(Q_5)$ , and  $a', b', \dots$  for  $U(Q_1)$ . The indices  $i, j$  label the fundamental doublet of  $SU(2)_R$  and its generators are denoted by  $\tau^I/2$ . The scalar components of the hypermultiplet in the fundamental representation of the gauge groups are denoted by  $\chi_{ia'a}$  and its spinor components are denoted by  $\psi_{a'a}$  and  $\tilde{\psi}_{a'a}$ . For the gauge fields of  $U(Q_1)$  and  $U(Q_5)$  we use the notations  $A_\alpha^{(1)s'}$  and  $A_\alpha^{(5)s}$  respectively, where  $\alpha = 0, 1$  and  $s, s'$  label the adjoint representations. The scalar components of the adjoint hypermultiplets are denoted by  $N_i^{(1)}$  and  $N_i^{(5)}$ , and their fermionic superpartners are denoted by  $\Sigma^{(1)}, \tilde{\Sigma}^{(1)}$  and  $\Sigma^{(5)}, \tilde{\Sigma}^{(5)}$ , respectively. Under a gauge transformation,  $\chi_i$  transform as  $\chi_i \rightarrow U_1 \chi_i U_5^{-1}$  where,  $U_5 \in U(Q_5)$  and  $U_1 \in U(Q_1)$ . The relevant terms in the Lagrangian are the ones involving the hypermultiplets and are given by

$$L = \int d^2x \left[ - (D_\alpha \chi_i)^\dagger_{aa'} (D^\alpha \chi_i)_{a'a} - \frac{1}{2} \bar{\psi}^{aa'} \gamma^\alpha (D_\alpha \psi)_{a'a} - \frac{1}{2} \bar{\tilde{\psi}}^{aa'} \gamma^\alpha (D_\alpha \tilde{\psi})_{a'a} \right. \\ \left. - \text{Tr} \left( D_\alpha N^{i(1)\dagger} D^\alpha N_i^{(1)} + \frac{1}{2} \bar{\Sigma}^{(1)} \gamma^\alpha D_\alpha \Sigma^{(1)} + \frac{1}{2} \bar{\tilde{\Sigma}}^{(1)} \gamma^\alpha D_\alpha \tilde{\Sigma}^{(1)} \right) \right]$$

$$\begin{aligned}
& -\text{Tr} \left( D_\alpha N^{i(5)\dagger} D^\alpha N_i^{(5)} + \frac{1}{2} \bar{\Sigma}^{(5)} \gamma^\alpha D_\alpha \Sigma^{(5)} + \frac{1}{2} \tilde{\Sigma}^{(5)} \gamma^\alpha D_\alpha \tilde{\Sigma}^{(5)} \right) \\
& - 8g^2 D_I^{(1)s'} D_I^{(1)s'} - 8g^2 D_I^{(5)s} D_I^{(5)s} + \dots \Big], \tag{2}
\end{aligned}$$

where the covariant derivatives are

$$\begin{aligned}
(D_\alpha \chi_i)_{a'a} &= \partial_\alpha \chi_{ia'a} - i A_{\alpha a'}^{(1)b'} \chi_{ib'a} + i \chi_{ia'b} A_{\alpha a}^{(5)b}, \\
D_\alpha N_i^{(1,5)} &= \partial_\alpha N_i^{(1,5)} + i [N_i^{(1,5)}, A_\alpha^{(1,5)}], \tag{3}
\end{aligned}$$

and the D-terms are given by

$$\begin{aligned}
D_I^{(1)s'} &= \chi_{iaa'} T_{a'b'}^{s'} \tau_{Ij}^i \chi_{b'a}^j - \tau_{Ij}^i T_{a'b'}^{s'} [N_i^{(1)\dagger}, N^{(1)j}]_{b'a'} \\
&\equiv \text{Tr} \left\{ T^{s'} (\chi^j \tau_{Ij}^i \chi_i^\dagger - \tau_{Ij}^i [N_i^{(1)\dagger}, N^{(1)j}]) \right\}, \tag{4}
\end{aligned}$$

$$\begin{aligned}
D_I^{(5)s} &= \chi_{ia'a} T_{ab}^s \tau_{Ij}^i \chi_{ba'}^{*j} - \tau_{Ij}^i T_{ab}^s [N_i^{(5)\dagger}, N^{(5)j}]_{ba} \\
&\equiv \text{Tr} \left\{ T^s (\chi^{\dagger j} \tau_{Ij}^i \chi_i - \tau_{Ij}^i [N_i^{(5)\dagger}, N^{(5)j}]) \right\}. \tag{5}
\end{aligned}$$

In the above, we have chosen the signature of the 2-dimensional metric as  $\{-, +\}$  and the  $\gamma$  matrices are given by  $\gamma^0 = i\sigma_1$ ,  $\gamma^1 = -\sigma_2$ , and  $\gamma_5 = \gamma^0 \gamma^1 = \sigma_3$ . The conjugate of a spinor is defined by  $\bar{\psi} = -i\psi^\dagger \gamma^0$ . The remaining terms of the Lagrangian that we have omitted, correspond to two vector multiplets and their couplings to the hypermultiplets. As we will argue, these omitted fields are not needed for our analysis in the next section.

The gauge theory we have described above corresponds to a system of D1-branes and D5-branes. However, under T-duality transformations along directions transverse to both the 1-branes and the 5-branes, this system is equivalent to a collection of  $Q_1$  D5-branes inside  $Q_5$  D9-branes which is described by a gauge theory in (5+1) dimensions. Our (1+1)-dimensional gauge theory is related to this (5+1)-dimensional theory by dimensional reduction. On the other hand, under a T-duality along the  $x^1$  direction, which is inside both the D1-branes and the D5-branes, this system is equivalent to a collection of  $Q_1$  D0-branes and  $Q_5$  D4-branes. The corresponding gauge theory is obtained from the theory we will analyze here by dimensional reduction to (0+1) dimensions. This later theory is relevant in connection with the matrix theory which contains 0- and 1-branes. In all these cases the structure of the D-terms remain unchanged.

## 4 Analysis of the Gauge Theory Vacuum

In this section we analyze the classical vacuum of the 2-dimensional  $U(Q_1) \times U(Q_5)$  gauge theory on a circle of radius  $R$ , in the limit  $g \rightarrow 0$ , where  $g$  now denotes the gauge coupling constant. Our intention, in this section and the next, is to see if this theory has a phase in which the low-lying excitations admit an effective description in terms of a conformally invariant  $\sigma$ -model on a 4-dimensional moduli space of vacua (and hence with central charge  $c=6$ ) living on a circle of radius  $Q_1 Q_5 R$ .

First, we select a branch of the vacuum moduli space on which all fermions as well as the scalars in the vector multiplets are set to zero but the scalar components of the hypermultiplets, that is,  $\chi_i$ ,  $N_i^{(1)}$  and  $N_i^{(5)}$  are non-zero. Furthermore, we work in a gauge  $A_0 = 0$ . We can also have Wilson lines in the two gauge groups condensing in the vacuum. These will in general break the gauge symmetry, except when the Wilson lines are elements of the centers  $Z_{Q_1}$  and  $Z_{Q_5}$  of the  $SU(Q_1) \times SU(Q_5)$  subgroups of the gauge group. We do not consider Wilson lines in the overall  $U(1)$  factors of the gauge groups as this will give a mass to the fundamental representation fields. For definiteness, let us consider the case  $Q_1 > Q_5$ . Then it turns out, as we will argue below, that only Wilson lines in  $SU(Q_1)$  are important. Therefore, we only consider a Wilson lines in the center  $Z_{Q_1}$  of  $SU(Q_1)$ , which, in a simple parametrization, can be written as  $w(px, 0)$ , where  $p = 1, 2, \dots, Q_1 - 1$ , and

$$w(x, 0) = e^{i \int_0^x A_x^{(1)0} dx} = \begin{pmatrix} e^{ix/Q_1 R} & 0 & \dots & 0 \\ 0 & e^{ix/Q_1 R} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & e^{-i(Q_1-1)x/Q_1 R} \end{pmatrix}. \quad (6)$$

The vacuum is determined by setting the covariant derivatives (3) in the presence of the above Wilson lines, as well as the D-terms (4) and (5) equal to zero. This leads to the solution

$$\chi_{ip}^0(x) = w(px, 0)q_i^0(0), \quad N_{ip}^{(1)0}(x) = w(px, 0)n_i^{(1)0}(0)w(0, xp), \quad N_i^{(5)0}(x) = n_i^{(5)0}(0), \quad (7)$$

where,  $w(2\pi R, 0) \in Z_{Q_1}$  and  $q_i^0(0)$ ,  $n_i^{(1,5)0}(0)$  satisfy the D-term vanishing conditions. This solution is consistent with supersymmetry transformations and leads to zero variation for the fermionic fields. Note that, to write the above solution, we have picked the point  $x = 0$  as the reference point and  $\chi_{ip}^0(x)$  is the parallel transport of  $q_i^0(0)$  in the presence of the Wilson line  $w(px, 0)$ .

We will now analyze the D-term vanishing conditions: The D-terms in (4) and (5) are of the form  $D_I^s = \text{Tr}(T^s A_I)$ , where,  $A_I$  is a hermitian matrix and the generators  $T^s$  also include the identity matrix. As a result,  $D_I^s = 0$  implies  $A_I = 0$ . Therefore, the vanishing of the D-terms leads to

$$(\chi^j \tau_{Ij}^i \chi_i^\dagger - \tau_{Ij}^i [N_i^{(1)\dagger}, N^{(1)j}])_{a'b'} = 0 \quad , \quad (8)$$

$$(\chi^{\dagger j} \tau_{Ij}^i \chi_i - \tau_{Ij}^i [N_i^{(5)\dagger}, N^{(5)j}])_{ab} = 0 \quad . \quad (9)$$

We want to obtain an explicit solution for the  $\chi_i$  by analyzing these equations for  $I = 3, 2, 1$ . First, let us make an ansatz for the adjoint hypermultiplets as

$$N_1^{(1)} = N_2^{(1)}, \quad N_1^{(5)} = N_2^{(5)}. \quad (10)$$

With this ansatz, the  $I = 3$  components of (8) and (9) reduce to

$$\chi_1 \chi_1^\dagger - \chi_2 \chi_2^\dagger = 0 \quad (11)$$

$$\chi_1^\dagger \chi_1 - \chi_2^\dagger \chi_2 = 0 \quad (12)$$

The  $Q_1$  dimensional matrix  $\chi_1\chi_1^\dagger$  can be diagonalized using the gauge group  $U(Q_1)$ . Since this matrix has rank  $Q_5 < Q_1$ , it can always be put in the form  $diag[v_1^2, \dots, v_{Q_5}^2, 0, \dots, 0]$  with real  $v$ 's. This breaks the gauge group  $U(Q_1)$  down to  $SU(Q_1 - Q_5)$ . Note that the Weyl group of  $U(Q_1)$  has a broken subgroup  $S(Q_5)$  which permutes the non-zero eigenvalues of  $\chi_1\chi_1^\dagger$ , keeping its diagonal form unchanged. As we shall see, this has an important implication for the theory. Furthermore, equation (11) implies that  $\chi_2\chi_2^\dagger$  has the same form as  $\chi_1\chi_1^\dagger$ .

Let us denote the matrix  $diag[v_1, \dots, v_{Q_5}]$  by  $V$  and the  $Q_5 \times Q_5$  dimensional non-zero blocks of  $\chi_i$  by  $X_i$ . Now, any generic  $X_i$  can be decomposed as  $X_i = H_i U_i$ , where  $H_i$  is a hermitian matrix and  $U_i$  is a unitary matrix. Then, the discussion so far implies that  $H_1 = V$  and  $H_2 = V'$  where,  $V'$  is the same as  $V$  modulo arbitrary negative signs for the diagonal elements. As we shall see later, the existence of this relative negative signs is crucial for the existence of the solution. Furthermore, equation (12) reduces to  $U_1^\dagger V^2 U_1 = U_2^\dagger V'^2 U_2$ . The matrix  $U_2$  can be set to identity by a  $U(Q_5)$  gauge transformation. This fixes the  $U(Q_5)$  gauge completely, because  $U_2$  is an arbitrary unitary matrix. This also means that the moduli we obtain are  $U(Q_5)$  invariant. The vanishing of (12) then determines  $U_1$  to be of the form  $U_1^0 = diag[e^{i\theta_1}, \dots, e^{i\theta_{Q_5}}]$ , and we get  $\chi_1 = V U_1^0$ ,  $\chi_2 = V'$ .

The  $I = 2$  components of (8) and (9) reduce to

$$\chi_2\chi_1^\dagger - \chi_1\chi_2^\dagger = 0, \quad \chi_2^\dagger\chi_1 - \chi_1^\dagger\chi_2 = 0, \quad (13)$$

This implies that  $U_1^0 = 1$ , so that in terms  $q_i^0$  of (7), the vacuum can be parametrized by

$$q_1^0(0) = \begin{pmatrix} v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & v_{Q_5} \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}, \quad q_2^0(0) = \begin{pmatrix} v'_1 & 0 & \cdots & 0 \\ 0 & v'_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & v'_{Q_5} \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \quad (14)$$

where  $v_a$  and  $v'_a$  can differ by a negative sign. The  $I = 1$  components of (8) and (9) give

$$\chi_2\chi_1^\dagger + \chi_1\chi_2^\dagger = 2[N_1^{(1)\dagger}, N_1^{(1)}], \quad \chi_2^\dagger\chi_1 + \chi_1^\dagger\chi_2 = 2[N_1^{(5)\dagger}, N_1^{(5)}], \quad (15)$$

These constrain  $N_i^{(1,5)}$  in terms of  $\chi_i$ . Note that the right hand side of the above equation is traceless. This in turn constrains the  $q_i^0(0)$  in (14) so that

$$\sum_{a=1}^{Q_5} v_a v'_a = 0 \quad (16)$$

Thus, the vacuum is parametrized by (14) subject to the above constraint. As we will see in the next section, the precise structure of  $N_i^{(1,5)}$  is not relevant to our problem.

Our starting theory also had a global  $SU(2)_R$  symmetry which is broken by the above solution. As a result, the moduli space also has three more parameters coming from the



$SU(2)_R$  rotations of the doublet  $q_{ia}^0$ , where  $a = 1, \dots, Q_5$  labels the diagonal elements in (14). These give rise to Goldstone bosons in the effective theory. If we denote this  $SU(2)_R$  rotated doublet by  $q_a^0(0)$ , then in the standard parametrization of  $SU(2)$ , we can write

$$q_a^0(0) = \begin{pmatrix} q_{1a}^0(0) \\ q_{2a}^0(0) \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \begin{pmatrix} v \\ v' \end{pmatrix} \quad (17)$$

where,  $|a|^2 + |b|^2 = 1$ . Below,  $q_a^0(0)$  always refers to the above rotated doublet.

The parametrization (14) does not fix the action of a Weyl group diagonally embedded in the two gauge groups: The Weyl group of  $SU(N)$  acts as a permutation group on the entries of a diagonal matrix transforming in the adjoint representation of the group. Therefore, a diagonal combination of the Weyl group  $S(Q_5)$  of  $U(Q_5)$  acting from the right, and the subgroup  $S(Q_5)$  of the Weyl group  $S(Q_1)$  of  $U(Q_1)$  acting from the left will preserve the diagonal form of  $q^0$ , while permuting its eigenvalues. Therefore, the actual moduli space is parametrized by (14) with proper identifications under the action of this diagonal Weyl group. We will discuss this in more detail in the next section.

## 5 The Effective String Theory

Having parametrized the vacuum, we now study the effective theory for the low-lying excitations on the moduli space of vacua. This has to be done carefully because of the presence of the Wilson line in the vacuum which affects the periodicity of the fields on  $S^1$ . For the fields  $\chi_i$ , the excitations around the ground state can be written as

$$\chi_i(x) = w(px, 0)q_{ip}(x), \quad \text{where,} \quad q_{ip}(x) = w(0, px)q_i^0(x) \quad (18)$$

These are still flat directions for the D-terms. Since the ground state was defined with respect to a reference point  $x = 0$ , the Wilson line  $w(0, px)$  in  $q_{ip}(x)$  is needed to transport back  $q_i^0(x)$  to this reference point. The  $q_{ip}(x)$  now also contains the Goldstone modes coming from the  $SU(2)_R$  rotations. Other components of  $\chi_{iaa'}$  do not enter the low-energy description since they are either gauge degrees of freedom that have been gauged away (we fix the same ‘‘unitary type’’ local gauge on the fluctuations at the point  $x$  as we did in the previous section at the point  $x = 0$ ), or they do not correspond to flat directions and hence are massive. The field  $\chi(x)$  appears in the original theory and is periodic on a circle of radius  $R$ . The decomposition in (18) implies that  $q^0(x)$  is also periodic on the same circle while  $q_p(x)$  is periodic on a circle of radius  $RQ_1$ . Since the gauge is fixed with reference to the point  $x = 0$ ,  $q_{ip}(x)$  is gauge invariant under a local gauge transformation and hence it is the true physical degree of freedom. Substituting (18) in the kinetic energy term in (2), one can easily see that while  $q_i^0(x)$  has a mass term coming from the Wilson line,  $q_{ip}(x)$  is massless.

We now have the situation of a field defined with twisted boundary conditions on a circle of radius  $R$ . In the limit of large  $Q_1$  the gauge field average can be replaced by a quenched average over the twisted boundary conditions which is familiar from the Eguchi-Kawai reduction at large  $N$  [28]. This fact leads to a single massless field  $q_i(x)$  which is

periodic on the circle of radius  $Q_1 R$ . Hence we have an effective theory of  $Q_5 + 3$  massless fields on a circle of radius  $RQ_1$ . The action (2) also shows that all these fields are accompanied by their fermionic superpartners. A similar discussion applies to the field  $N^{(1)}$ . However, since  $N^{(1)}$  is in the adjoint representation and the Wilson line is in the center of the group, these excitations live on circle of radius  $R$ . Therefore, their momenta are quantized in units of  $1/R$  (as opposed to  $1/RQ_1$  for the fundamental representation hypermultiplet) and hence, at low energies, they decouple for the theory. The same is the case with all the other fields we have not considered above.

Let us now consider some further periodicity properties of this solution. Since  $q_i(x)$  are defined modulo the action of the diagonal Weyl group  $S(Q_5)$ , we have

$$q_i^0(x + 2\pi R) = S q_i^0(x) S^\dagger, \quad S \in S(Q_5) \quad (19)$$

If we label the diagonal elements of  $q_i^0(x)$  by the index  $a = 1, \dots, Q_5$ , then this means  $q_{ia}^0(x + 2\pi R) = q_{ib}^0(x)$ . This property can be easily taken into account if we sew [21, 22] the functions  $q_{ia}^0(x)$  into a single function  $f_i^0(x)$  with period  $2\pi RQ_5$ , defined over a circle of radius  $RQ_5$ . In fact,  $q_{ia}^0(x) = f_i^0(x + 2\pi aR)$  so that for any gauge invariant function  $F(q_{ia}^0(x))$  of the moduli, we have

$$\sum_{a=1}^{Q_5} \int_0^{2\pi R} dx F(q_{ia}^0(x)) = \int_0^{2\pi RQ_5} dx F(f_i^0(x)) \quad (20)$$

Now, instead of the functions  $q_{ia}^0(x)$ , we can sew the  $Q_5$  functions  $q_{ia}(x)$  in (18) into a single function  $f_i(x)$ . Taking into account the three  $SU(2)_R$  Goldstones,  $f_i(x)$  describes precisely 4 massless modes on a circle of radius  $RQ_1Q_5$ .

Upto now we have ignored the presence of the Wilson lines in the centre of  $SU(Q_5)$ . In the previous section we have already indicated that the gauge fixed moduli are invariant under  $U(Q_5)$  transformations. Hence they simply do not see these Wilson lines. It is important to realize that this is true only within the moduli space approximation one is working with.

In the above discussion we have only included hypermultiplets in the fundamental representation since it is for these fields that the radius of the circle  $S^1$  is dailated and momentum is quantized in units of  $1/RQ_1Q_5$ . In the limit of large  $Q_1$  and  $Q_5$ , all other fields are much heavier than these and decouple in the long wavelength limit. Therefore they do not contribute to the spectrum of the low-lying excitations of the theory. Now that we have 4 massless bosons and their superpartners with a flat metric on the moduli space, we have an  $N = 4$  SCFT with central charge  $c = 6$ . The black hole in its ground state is described by states at level  $Q_1Q_5n$  in this conformal field theory. This leads to the correct entropy formula (1) for the black hole [7]. Excited states of the black hole correspond to the presence of both left and right moving oscillations of the effective string.

Our analysis has been performed in the weak coupling limit of the gauge theory while the black hole corresponds to the strong coupling limit. However, we note that 2-dimensional gauge theory is ultraviolet finite upto normal ordering of the Hamiltonian. Moreover, the moduli space we obtain is a very special hyperKahler manifold (related to  $N=4$  supersymmetry of the theory) in 4 dimensions: the flat space. If there are corrections to this metric,

they will become more and more important as the coupling grows. However, the hyper-Kähler geometry is very restrictive [36] and it appears that there are no such corrections to interpolate between the flat space and some other non-trivial hyperKähler geometry. Thus we can conclude that our moduli space survives in the strong coupling limit.

We now discuss issues related to the Weyl symmetry in some detail. In the above discussion we considered elements of  $S(Q_5)$  with the longest cycle. These lead to the largest dilation of the radius:  $RQ_1 \rightarrow RQ_1Q_5$ . For large  $Q_5$ , these configurations are more favourable since their entropy grows much faster with  $Q_5$  than the entropy of configurations corresponding to smaller cycles of the permutation group. For example, compare the entropy for the largest cycle of length  $Q_5$  with that of two cycles of length  $Q_5/2$ . The entropy of the largest cycle is greater than that of the shorter cycles, clearly indicating that when one averages over all cycles of the Weyl group, the leading contribution comes from the largest cycle. We also note that the relative contribution of a cycle of length  $Q_5$  and cycles of length  $Q_5 - 1$  and 1 is suppressed by a factor of  $1/Q_5$ . This situation is relevant to the case when the cycle of length 1 shrinks to zero, *i.e.*, when two adjacent eigenvalues cross each other.

Let us now discuss the important issue of coincident eigenvalues. As is well known the Weyl measure in the functional integral over the moduli space [34] vanishes when two eigenvalues coincide. Coincident eigenvalues are a subleading  $1/N$  effect and hence their effects are proportional to the string coupling  $g$ . When the eigenvalues coincide, the unitary gauge condition on the fluctuations cannot be fixed and it signals the appearance of topological objects in the space-time under consideration [35]. In our problem, since the spacetime is 2-dimensional (here, we assume an analytic continuation to euclidean space), the topological object at a point on this surface where the eigenvalues coincide is a vortex with a string singularity. It is like a vortex in a  $SO(3)$  theory and the vortex charges are given by the elements of the homotopy group  $\pi_1(SU(2)/Z_2) = Z_2$ . Hence the string singularity is a square root branch cut in the world sheet emanating from the position of the vortex. It is natural to introduce a local operator on the world sheet describing the  $Z_2$  vortex. Such a construction has been described in [24] using twist operators and their corresponding spin fields [37]. However, in our case since the SCFT has  $c=6$ , we get a marginal operator [38] that creates the  $Z_2$  vortex. In general, when  $n$  eigenvalues coincide, one is naturally lead to  $Z_n$  vortices characterized by  $\pi_1(SU(n)/Z_n) = Z_n$ . However, the higher twist operators ( $n \geq 3$ ) are relevant and hence tachyonic. From supersymmetry, one expects that such operators are not allowed in the theory and are projected out.

The appearance of the marginal operator for the  $Z_2$  vortex may cause some concern. In the strict  $N = \infty$  limit, it is clear that the SCFT has a target space  $R^4$  which is not renormalized at strong coupling. In this case, we expect the calculations in [8, 9] to be valid in the strongly coupled black hole regime. However, even for a very large value of  $N$ , not strictly equal to infinity, the target space at weak coupling ( $g \rightarrow 0$ ) is the orbifold  $R^4/Z_2$  which is a direct consequence of the existence of this operator. At strong coupling, we expect the full space to be described by the Eguchi-Hanson metric. In light of the excellent agreement obtained in [9], it is unlikely that such a SCFT describes the Hilbert space of a single black hole. One resolution is to interpret the marginal operator as an interaction in a second quantized theory of black holes [41].

## 6 Discussion and Concluding Remarks

In this paper we have discussed the derivation of the low energy effective string picture for the 5-dimensional black hole in the framework of the effective gauge theory that describes the assembly of a large number of D1-branes and D5-branes that form the black hole. It turns out that in the limit of large  $N \sim Q_1 \sim Q_5$ , the effective string corresponds to the gauge invariant collective modes of the condensed  $(1, 5)$  and  $(5, 1)$  open strings in the fundamental representation of the gauge group which mediate the interactions of the branes. Given the fact that the effective theory is a flat four dimensional sigma model with N=4 supersymmetry, and that hyperKahler geometry is very restrictive, we expect the weak coupling answers to persist beyond weak coupling. Our work is one further step in the direction of modelling of black holes by D-brane constituents.

It should be mentioned that though the picture presented here resembles the one suggested in [7], the two differ in some essential ways. According to [7], D-branes are joined to form a long brane which is multiply wound around  $S^1$ . This picture amounts to condensing Wilson lines in the two gauge groups such that their eigenvalues are the  $Q_1$ th and  $Q_5$ th roots of unity [27]. Such Wilson lines are seen by all fields and therefore, both the fundamental and the adjoint representation fields live on a circle of larger radius. However, in our case, the Wilson line is in the centre of the group and only the collective modes of the hypermultiplets in the fundamental representation are described by a theory on the circle of radius  $RQ_1Q_5$ . All other fields have higher momenta and their excitations decouple at sufficiently low energies.

The system of  $Q_1$  D1-branes and  $Q_5$  D5-branes is equivalent to a system of  $Q_5$  D1-branes and  $Q_1$  D5-branes under T-duality in the internal dimensions, and hence the two types of branes can be treated symmetrically.

We would like to mention that our treatment of the D-flatness conditions used an explicit ansatz for the hypermultiplet fields in the adjoint representation of the gauge groups. An improved treatment which uses the full set of fields will be presented in a future communication [42].

We conclude with a comment on matrix theory. As was indicated in section 3, we can use T-duality transformations to transform the  $D1, D5$ -brane system to a  $D0, D4$ -brane system, which is described by an N=8 SUSY Yang-Mills theory in  $0 + 1$  dimensions based on the gauge group  $U(Q_1) \times U(Q_5)$ . The structure of the hypermultiplet is of course the same as before. We know that the  $D_0 - D_4$  system exists in M(atric) theory [39], and one would like to ask whether the matrix model that we have mentioned can be derived as an effective description from M(atric) theory. This is an important issue because any fundamental theory of quantum gravity should explain black holes.

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## Note Added

The appearance of the marginal operator corresponding to the  $R^4/Z_2$  orbifold has also been observed in [40]. We would like to thank T. Banks and P. Horava for pointing this out to us.

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