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Gauge Theory Description of D-brane Black Holes: Emergence of the Effective SCFT and Hawking Radiation

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Abstract

We study the hypermultiplet moduli space of an N=4, $U(Q_1) \times U(Q_5)$ gauge theory in 1+1 dimensions to extract the effective SCFT description of near extremal 5-dimensional black holes modelled by a collection of D1- and D5-branes. On the moduli space, excitations with fractional momenta arise due to a residual discrete gauge invariance. It is argued that, in the infra-red, the lowest energy excitations are described by an effective $c = 6$, N=4 SCFT on T^4, also valid in the large black hole regime. The "effective string tension" is obtained using T-duality covariance. While at the microscopic level, minimal scalars do not couple to (1,5) strings, in the effective theory a coupling is induced by (1,1) and (5,5) strings, leading to Hawking radiation. These considerations imply that, at least for such black holes, the calculation of the Hawking decay rate for minimal scalars has a sound foundation in string theory and statistical mechanics and, hence, there is no information loss.

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1 Introduction and Summary

In the recent past there has been encouraging progress in understanding the statistical basis of black hole entropy [1, 2, 3]. This progress is in part due to new developments in superstring theory, both technical and conceptual. The technical part has to do with supersymmetry, which brings some non-perturbative aspects under control, and the conceptual part has to do with D-branes and duality symmetries of string theory. For a review see [4, 5]. The fact that there is a statistical basis for the black hole entropy means that one has understood what is usually dubbed as *intrinsic gravitational entropy* [6], in terms of degrees of freedom hitherto unknown within general relativity. As yet we do not know how these new degrees of freedom are inscribed in the metric and other long range fields and hence, as yet, we do not "understand" the geometric Bekenstein-Hawking entropy formula. However, even with this lacking, we can attempt to answer some of the conceptual issues raised by black hole thermodynamics. The conceptual issue is the so called *information paradox* which says that black hole radiation is exactly thermal [6]. Such an assertion, made within the standard formulation of general relativity, denies a statistical basis of black hole thermodynamics. A statistical basis explains thermodynamics in terms of the statistical averages of unitary amplitudes, in which case, information loss is not intrinsic. Such a view has been advocated by 't Hooft [7]. The success of the statistical derivation of black hole entropy has suggested a derivation of black hole thermodynamics in terms of some constituent degrees of freedom of the black hole [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. In most of these studies, for technical reasons, one focuses on the near extremal black hole of type IIB string theory compactified on a 5-torus in a particular range of parameters. In this range of parameters the black hole has positive specific heat. Even in this specialised situation, the black hole has a large number of degrees of freedom and a study of black hole thermodynamics leads to the "information paradox" as the black hole radiates to its zero temperature ground state. A complete solution of even this simplified problem is not easy and up to now all calculations of black hole thermodynamics have been performed when the effective open string coupling is small, and one is not in the large black hole regime. However the precise agreements of the grey body factors at long wave lengths encourage us to search for a precise microscopic description of the black hole which is valid even when the effective open string coupling is large.

This paper is devoted to a study of the D-brane model of the 5-dimensional black hole of IIB string theory with charges Q_1, Q_5 and N [8]. The model consists of Q_1 D1-branes and Q_5 D5-branes wrapped around $S^1 \times T^4$, carrying excitations of total momentum N along S^1 of radius R. At low energies, this model is described by a $U(Q_1) \times U(Q_5)$ N=(4,4) super Yang-Mills theory on a 2-dimensional cylinder of radius R. When the black hole is macroscopic ($gQ_1, gQ_5 >> 1$), this gauge theory is strongly coupled. A counting argument in [8, 16] indicates that the microscopic degrees of freedom of the black hole correspond to hypermultiplets originating in the (1,5) string sector of the D-brane system. For the D-brane configuration to appear as a black hole in the four directions transverse to $S^1 \times T^4$,

R must be much smaller than the radius of the extremal black hole (the T^4 radii are taken to be of string size). On the other hand, as noted in [9], in order to explain the black hole thermodynamics, the D-brane system must have excitations of energy much lower than $1/R$. To resolve this problem, the picture of "multiply wound" branes was suggested in [9], based on an observation in [19]. This amounts to replacing the radius R by RQ_1Q_5. If the black hole entropy is related to the degeneracy of states in a superconformal field theory then, in order to get the right entropy, the central charge must be set to $c = 6$. One may try to implement the notion of "multiple winding" of D-branes by introducing a Wilson line in the Weyl group of the corresponding gauge theory. This, however, does not seem to lead to a consistent description. An earlier attempt [20] to explain the black hole degrees of freedom in terms of the low-energy excitations of the gauge theory involved a variant of this approach, with Wilson lines in the centre of the gauge group. However, as will be shown here, such constructions are unnecessary and the fractionalization of momentum is a consequence of a residual gauge invariance in the theory which leads to the existence of sectors with twisted-periodic boundary conditions on S^1.

Another approach followed to investigate this D-brane system and its coupling to bulk fields is based on a variant of the Dirac-Born-Infeld action for a D-string on S^1 [11, 13]. While a DBI action for the D1,D5-brane system is not known, one starts with a DBI action for a single D-string with 4 out of its 8 transverse oscillations frozen to simulate the effect of a single D5-brane. To include the effect of multiple D-branes, one resorts to the "multiple winding" picture and enlarges the radius of the circle S^1 from R to RQ_1Q_5. In this way, the black hole is modelled by an effective D-string. Surprisingly, this heuristic construction leads to a rather successful model of the black hole in which many calculations have been performed, although the microscopic origins of this model are not very clear. On expanding the DBI action constructed in this way, one obtains

$$S_{DBI} = T_{eff} \int dt \int_0^{2\pi RQ_1Q_5} d\sigma \partial_\alpha X^m \partial^\alpha X^m \qquad + \quad \text{couplings}\,. \tag{1}$$

This, to lowest order, and after including fermions, is a $c = 6$ superconformal field theory with its target space as the T^4 transverse to the D1-brane. The fields X^m are the transverse oscillations of the D-string in this T^4 which, in the microscopic picture, would be related to the (1,1) strings. This is at variance with the expectation that the black hole degrees of freedom have their origin in the (1,5) string sector of the D-brane system. Furthermore, the effective string tension T_{eff} cannot be derived in this framework. Also, the form of the couplings to bulk fields obtained in this way do not lead to correct results for fixed scalars [14], though this is not the case with coupling to minimal scalars.

In this paper, we study the $U(Q_1) \times U(Q_5)$ gauge theory for the D-brane system in a systematic way and isolate the effective theory for its low-energy excitations that are relevant to the low-energy dynamics of the near extremal black hole in D=5. The content of the paper is organised as follows: In section 2, we describe the relevant features of the hypermultiplet sector of $U(Q_1) \times U(Q_5)$ gauge theory which describes the low-energy dynamics of the D1,D5-brane system. The coupling of the (1,5) hypermultiplets and, partly, the field content of the theory is fixed by imposing covariance under a set of T-dualities that interchanges the D1- and D5-branes.

In section 3, we describe a parametrization of the space of solutions $\mathcal{M}_0$ to the conditions for the vanishing of the D-term potential. After gauge fixing, we show that, generically, this space can be almost entirely parametrized in terms of the (1,5) hypermultiplets, while the (1,1) and (5,5) hypermultiplets induce a metric on it. After gauge fixing, we are still left with a discrete residual gauge group $S(Q_1 - 1) \times S(Q_5 - 1)$ which is a subgroup of the Weyl group of $U(Q_1) \times U(Q_5)$, and maps $\mathcal{M}_0$ to itself. The hypermultiplet moduli space is not renormalized and therefore this description is valid even in the strong coupling limit of the gauge theory which is the regime of macroscopic black hole.

In section 4, we consider oscillations of the moduli fields for which the D-term potential stays zero. These are the massless excitations of the theory on the Higgs branch and are relevant to the black hole degrees of freedom. The residual discrete gauge invariance $S(Q_1 - 1) \times S(Q_5 - 1)$ enables us to impose twisted-periodicity conditions on the moduli fields on S^1. As a result, many components of the moduli fields are sewn into one single field which is periodic on a circle of larger radius, and hence has fractional momentum on the original space. This is how very long wavelength excitations emerge. Our sewing procedure generalizes the one used in [21, 22, 23, 24] for mutually commuting square matirces, to arbitrary rectangular matrices. The non-linear sigma-model on the moduli space, when written in terms of sewn variables, takes a very complicated and non-local form that cannot be analysed directly. We are interested in the infra-red limit of this model, obtained after integrating out all higher momentum modes and retaining only the lowest ones. Assuming that the infra-red theory is local, N=4 supersymmetry along with the compactness of the moduli space and some general considerations leads us to a $c = 6$ superconformal field theory on a target space T^4. This T^4 is different from the one in (1) and is not part of the 10-dimensional space-time. The basic variable is the renormalized form of the sewn (1,5) field with the lowest momentum quantum $\sim 1/RQ_1Q_5$. We regard this field as an order parameter for low-energy excitations of the system in the infra-red. The SCFT has an $SO(4)$ symmetry, instead of an $SU(2)_R$ of the gauge theory. This SCFT is also valid in the strong coupling regime of the gauge theory $gQ_{1,5} > 1$.

In section 5, we discuss the connection between this $c = 6$ SCFT and the black hole in some more detail: The "effective string tension", T_{eff}, that has so far eluded a consistent derivation, is related to the coupling of (1,5) hypermultiplets that is fixed by T-duality and is given by $T_{eff} = \frac{1}{\alpha'^2}\sqrt{\frac{V_4}{g^2Q_1Q_5}}$. This is different from what one would expect if the "effective string" is interpreted as a D-string along the x^5 direction. But it is consistent with the "mean string" picture suggested in [25], based on the requirement that the effective string produces (at least in principle) the correct cross section for higher angular momentum scattering from the black hole. We then describe the identification of the extremal and near extremal black holes in terms of states in the SCFT with degeneracies related to the entropy. For a given extremal black hole, the SCFT states that do not have a near-extremal black hole interpretation are automatically removed by a level matching condition. This condition originates in the residual discrete gauge invariance of the theory on the moduli space and insures the consistency of the description. Coupling to bulk fields are then given by SCFT operators allowed by the level matching condition.

Next, to make contact with Hawking radiation, we discuss the coupling of the SCFT to minimal scalars in the bulk. A generic coupling can be written using the $SO(4)$ invariance of the SCFT emerging in the infra-red limit. However, while the black hole degrees of freedom are contained in the (1,5) hypermultiplets, these, as shown in [26], do not couple to minimal scalars at the microscopic level. In our approach, we can easily see that an effective coupling of the minimal scalars to the black hole degrees of freedom is induced through the coupling of these scalars to (1,1) and (5,5) hypermultiplets. The calculation of emission and absorption rates for these scalars is, in its technical aspects, the same as before [8, 10, 11, 12] and hence one can exactly reproduce the grey body factors as calculated in the semi-classical approach to general relativity. We also comment on the range of validity of comparisons between the D-brane and the thermodynamic descriptions of this black hole. As the black hole radiates and approaches the extermal limit, the thermodynamic description breaks down [27] (since temperature fluctuations blow up) while the SCFT description based on the D-brane model is still valid. Section 6 contains the conclusions.

2 The SUSY Gauge Theory for the D-brane Model of the Black Hole

Type IIB string theory with five coordinates, say $x^5 \cdots x^9$, compactified on $S^1 \times T^4$, admits a black hole solution in the five non-compact directions $x^0 = t, x^1, \cdots, x^4$. This 5-dimensional black hole carries RR charges Q_1 and Q_5, and a Kaluza-Klein charge N corresponding to a momentum along the S^1. In the extremal limit (and in the near extremal region) it is modelled by a collection of low-energy states in a system of Q_1 D1-branes and Q_5 D5-branes [8, 16]. The D1-branes are parallel to the x^5 coordinate compactified to a circle S^1 of radius R, while the D5-branes are parallel to x^5 and $x^6, \cdots, x^9$ compactified on a torus T^4 of volume V_4. The charge N is related to the momenta of very low-energy excitations of this system along S^1. We take the T^4 radii to be of the order of α' and smaller than R which, in turn, is much smaller than the black hole radius. The low-energy dynamics of this D-brane system is described by a $U(Q_1) \times U(Q_5)$ gauge theory in two dimensions with $N = 4$ supersymmetry. In this section, we will describe some aspects of this gauge theory that are relevant to the identification of the black hole degrees of freedom in the D-brane system.

The elementary excitations of the D-brane system correspond to open strings with two ends attached to the branes and there are three classes of such strings: the (1,1), (5,5) (1,5) strings. The associated fields fall into vector multiplets and hypermultiplets, using the terminology of N=2, D=4 supersymmetry. In the following, we will only consider the hypermultiplet sector since this sector contains the low-energy black hole degrees of freedom we are interested in. The part of the spectrum coming from (1,1) strings is simply the dimensional reduction, to $1+1$ dimensions (the $(t, x^5 = \sigma)$-space), of the N=1 $U(Q_1)$ gauge theory in $9+1$ dimensions [4, 28]. The gauge field components $X_m^{(1)}(\sigma, t)$ $(m = 6, 7, 8, 9)$ along the T^4, together with their fermionic superpartners, form a hypermultiplet in the adjoint of $U(Q_1)$, while the remaining components form a vector multiplet. Since x^m are compact, the (1,1) strings can also have winding modes around the T^4. These are, however, massive states in the $(1+1)$-dimensional theory and can be ignored. Similarly, the part of the spectrum coming from (5,5) strings is the dimensional reduction, to $5+1$ dimensions, of the N=1 $U(Q_5)$ gauge theory in $9+1$ dimensions. In this case, the gauge field components $A_m^{(5)}$ $(m = 6, 7, 8, 9)$ also have a dependence on x^m. A set of four T-duality transformations along x^m interchanges D1- and D5-branes and also converts the momentum modes of the (5,5) strings along T^4 into winding modes of (1,1) strings around the dual torus [29]. Since these winding modes have been ignored, a T-duality covariant formulation requires that we should also ignore the associated momentum modes. Thus we can only retain the zero modes of $A_m^{(5)}$ along T^4, denoted by $X_m^{(5)}(\sigma, t)$. These fields fall in a hypermultiplet in the adjoint of $U(Q_5)$, while the remaining fields form a vector multiplet.

The field content obtained so far is that of N=2 $U(Q_1) \times U(Q_5)$ gauge theory in 1+5 dimensions, reduced to 1+1 dimensions on T^4. The $SO(4) \sim SU(2)_L \times SU(2)_R$ rotations of the torus act on the components of the adjoint hypermultiplets $X_m^{(1,5)}$ as an R-symmetry. To this set of fields we have to add the fields from the (1,5) sector that are constrained to live in 1+1 dimensions by the ND boundary conditions. These strings have their ends fixed on different types of D-branes and, therefore, the corresponding fields transform in the fundamental representation of both $U(Q_1)$ and $U(Q_5)$. The (1,5) sector fields also form a hypermultiplet but with only $SU(2)_R$ as the R-symmetry group. We denote these fields by $\chi_i(\sigma, t)$, where i is the $SU(2)_R$ doublet index. The inclusion of these fields breaks the supersymmetry by half, to the equivalent of N=1 in D=6, and the final theory only has an $SU(2)_R$ R-symmetry. To make this manifest, we write the hypermultiplets $X_m^{(1,5)}$ in terms of $SU(2)_R$ doublets $N_i^{(1,5)}$ given by (see for example, [30])

$$\sigma^m X_m = \begin{pmatrix} X_9 + iX_8 & X_7 + iX_6 \\ -X_7 + iX_6 & X_9 - iX_8 \end{pmatrix} = \begin{pmatrix} N_1 & N_2 \\ -N_2^\dagger & N_1^\dagger \end{pmatrix} . \tag{2}$$

Here, $\sigma^m = (i\tau^1, i\tau^2, i\tau^3, \mathbf{1})$, and τ^I are the Pauli matrices.

At low-energies, a lagrangian for the hypermultiplets χ_i, $N_i^{(1)}$ and $N_i^{(5)}$ as well as the two vector multiplets can be easily written by constructing an N=2 $U(Q_1) \times U(Q_5)$ gauge theory in $3+1$ dimensions and then reducing it to $1+1$ dimensions (see for example, [31]). However, note that $N_i^{(1,5)}$ are related to the zero modes of gauge fields an the compact space T^4 either directly, or by T-duality. As a result, these fields are valued on a compact space. This information is not contained in the field theory and we will impose it as an extra condition on the field variables. In the following we will only write down some relevant terms of this lagrangian which are needed for the analysis in the next sections. But first, some notation: The fundamental representation indices for the gauge group $U(Q_5)$ are denoted by $a, b, \ldots$ and those for $U(Q_1)$ are denoted by $a', b', \ldots$. For the adjoint representations, we use the indices s and s', respectively. The indices i, j label the fundamental doublet of $SU(2)_R$ and its generators are denoted by $\tau^I/2$. Thus in components, the scalars in the hypermultiplets take the form $\chi_{ia'a}$, $N^{(1)}_{ia'b'}$ and $N^{(5)}_{iab}$. Under a gauge transformation, χ_i transform as $\chi_i \to U_1 \chi_i U_5^{-1}$ where, $U_5 \in U(Q_5)$ and $U_1 \in U(Q_1)$. Also, $\alpha = 0, 1$ labels the coordinates on the $t, x^5 = \sigma$ space.

The only terms needed for the study of dynamics on the Higgs branch of this theory are the kinetic energy terms for the hypermultiplets and the D-term potential in the theory. The kinetic energy terms are given by

$$S_{ke} = k_{11}\mathrm{Tr}\int d^2\sigma \partial_\alpha N^{(1)} \partial^\alpha \bar{N}^{(1)} + k_{15}\mathrm{Tr}\int d^2\sigma \partial_\alpha \chi \partial^\alpha \bar{\chi} + k_{55}\mathrm{Tr}\int d^2\sigma \partial_\alpha N^{(5)} \partial^\alpha \bar{N}^{(5)}\,, \qquad (3)$$

where, $(\bar{\chi})^i = \chi_i^\dagger$ etc., and the traces are normalized to identity. All fields have been scaled using powers of α' such that they have dimensions of length. With this convention, the couplings are given by

$$k_{11} = \frac{1}{\alpha' g Q_1}\,, \quad k_{15} = \frac{1}{\alpha'^2 g}\sqrt{\frac{V_4}{Q_1 Q_5}}\,, \quad k_{55} = \frac{V_4}{\alpha'^3 g Q_5}\,. \qquad (4)$$

where, g is the string coupling constant. As we will see, k_{15} is the only coupling that appears in the effective low-energy theory describing the near extremal dynamics of the black hole and is often referred to as the "effective string tension". Its value has been fixed by a simple T-duality argument: Since the hypermultiplets correspond to D-brane excitations, we know that $k_{11} \sim (\alpha' g)^{-1}$ and $k_{55} \sim V_4(\alpha'^3 g)^{-1}$. The V_4 arises from the reduction, on T^4, of the 5-brane worldvolume theory and α' takes care of the dimensions. In general, k_{15} will be of the form $c(\alpha' g)^{-1}$, where c can only be a function of the dimensionless quantity $V_4\alpha'^{-2}$. Under T-duality along all T^4 directions, V_4 and g transform as

$$V_4' = \frac{\alpha'^4}{V_4}\,, \qquad g' = g\frac{\alpha'^2}{V_4}\,. \qquad (5)$$

This interchanges k_{11} and k_{55} as a consequence of the interchange of D1-branes and D5-branes under such a duality. However, the (1,5) string sector remains unchanged, implying that c/g must go over to itself. This requirement fixes $c = \sqrt{V_4/\alpha'^2}$. Furthermore, the gauge theory is studied in the limit of $g \to 0$ and $Q_1, Q_5 \to \infty$ such that gQ_1 and gQ_5 are finite. To write the theory in a meaningful way, we scale the fields appropriately so that the action depends on the well defined finite couplings. Taking the T-duality covariance into account, this amounts to scaling χ by a factor of $(Q_1Q_5)^{-1/4}$ and $N^{(1,5)}$ by $(Q_{1,5})^{-1/2}$. This leads to the couplings as given in (4).

The lagrangian also contains a D-term potential $k_{11}D^{(1)Is'}D^{(1)Is'} + k_{55}D^{(5)Is}D^{(5)Is}$, with the D-terms given by

$$k_{11}D^{(1)Is'} = \tau^{Ii}_{j}\mathrm{Tr}\left\{T^{s'}\left(k_{15}\chi_i\bar{\chi}^j + k_{11}[N^{(1)}_i, \bar{N}^{(1)j}]\right)\right\} , \quad (6)$$

$$k_{55}D^{(5)Is} = \tau^{Ii}_{j}\mathrm{Tr}\left\{T^{s}\left(k_{15}\bar{\chi}^j\chi_i + k_{55}[N^{(5)}_i, \bar{N}^{(5)j}]\right)\right\} . \quad (7)$$

Here, $T^{s'}_{a'b'}$ and T^s_{ab} are the generators of $U(Q_1)$ and $U(Q_5)$ respectively. In the future, we will suppress the couplings in the D-terms. The remaining terms of the lagrangian, that we have omitted, correspond to two vector multiplets and their couplings to the hypermultiplets and are not needed for our analysis. We will first consider this theory in the perturbative regime where $gQ_{1,5} < 1$ and then argue that our results can safely be extrapolated to the large black hole regime where $gQ_{1,5} > 1$. As stated above, $N^{(1,5)}$ take values on a compact space. Note that, in the limit $V_4 \to \infty$, the (5,5) sector decouples and the gauge group $U(Q_5)$ reduces to a flavour group. Since T^4 goes over to R^4, the fields $N^{(1)}$ are no longer compact. This is the theory analyzed in [32].

3 The Moduli Space

Our aim is to study the low-lying excitations of the gauge theory described in the previous section, in the strong coupling regime $gQ_{1,5} > 1$. This is the macroscopic black hole regime of the D-brane system. However, to begin with, we consider the system in the regime $gQ_{1,5} < 1$ which is the perturbative regime of the gauge theory. To isolate the massless excitations (to be later identified as the black hole degrees of freedom) we restrict ourselves to the Higgs phase where all fields are set to zero except for the hypermultiplets χ_i, $N^{(1)}_i$ and $N^{(5)}_i$, altogether containing $4(Q_1^2 + Q_5^2 + Q_1Q_5)$ components. We then look at configurations of these hypermultiplets for which the D-terms vanish. For constant field configurations, this would normally define the moduli space of vacua on the Higgs branch, though in 1+1 dimensions this notion is not well defined due to strong infra-red fluctuations. However, we are interested in space-time dependent configurations of fields for which the D-terms vanish, and use the term "moduli space" only in this sense. After gauge fixing, these configurations correspond to the independent low-energy degrees of freedom relevant to the black hole problem. In this section, we describe how these degrees of freedom are obtained by setting the D-term potential to zero and fixing the gauge.

The D-terms in (6),(7) are of the form $D^{Is} = \mathrm{Tr}(T^sD^I)$, where D^I are hermitian matrices. The generators T^s also include the identity, corresponding to the overall $U(1)$ factors in $U(Q_{1,5})$. Therefore, $D^{Is} = 0$ implies $D^I = 0$. Thus, the vanishing of the D-terms (6),(7) leads to the following sets of equations (for $I = 1, 2, 3$ and with the couplings suppressed):

$$\tau^{Ii}_{j}\left(\chi_i\bar{\chi}^j + [N^{(1)}_i\, , \bar{N}^{(1)j}]\right)_{a'b'} = 0 \quad , \quad (8)$$

$$\tau^{Ii}_{j}\left(\bar{\chi}^j\chi_i + [N^{(5)}_i\, , \bar{N}^{(5)j}]\right)_{ab} = 0 \quad . \quad (9)$$

Equation (8) has its origin in the $U(Q_1)$ sector of the theory and, using the standard representation of Pauli matrices τ^I, it has the following independent components

$$(\chi_1\chi_1^\dagger - \chi_2\chi_2^\dagger)_{a'b'} + [N_1^{(1)}\,,\, N_1^{(1)\dagger}]_{a'b'} - [N_2^{(1)}\,,\, N_2^{(1)\dagger}]_{a'b'} = 0\,, \tag{10}$$

$$(\chi_1\chi_2^\dagger)_{a'b'} + [N_1^{(1)}\,,\, N_2^{(1)\dagger}]_{a'b'} = 0\,. \tag{11}$$

The first equation is real while the second one is complex, thus there are $3Q_1^2$ constraints coming from this set of equations. Similarly, equation (9) comes from the $U(Q_5)$ sector of the gauge theory and gives rise to the $3Q_5^2$ constraints,

$$(\chi_1^\dagger\chi_1 - \chi_2^\dagger\chi_2)_{ab} + [N_1^{(5)}\,,\, N_1^{(5)\dagger}]_{ab} - [N_2^{(5)}\,,\, N_2^{(5)\dagger}]_{ab} = 0\,, \tag{12}$$

$$(\chi_1^\dagger\chi_2)_{ab} + [N_2^{(5)}\,,\, N_1^{(5)\dagger}]_{ab} = 0\,. \tag{13}$$

Equations (10,11) and (12,13) have the same trace parts corresponding to the vanishing of $U(1)$ D-terms, namely,

$$\chi_{1a'a}\chi^*_{1a'a} - \chi_{2a'a}\chi^*_{2a'a} = 0\,, \qquad \chi_{1a'a}\chi^*_{2a'a} = 0\,, \tag{14}$$

which are three real equations. Therefore, the vanishing of D-terms imposes $3Q_1^2 + 3Q_5^2 - 3$ constrains on the fields. Gauge fixing can remove another $Q_1^2 + Q_5^2 - 1$ scalar field components, thus leaving only $4(Q_1Q_5 + 1)$ components to parametrize the moduli space. The explicit structure of this moduli space is important for our analysis and will be described below.

Let us start with gauge fixing by gauging away components of the adjoint hypermultiplets $N_i^{(1,5)}$. For this, it is convenient to parametrize these complex fields in terms of the hermitian metrices $X_m^{(1,5)}$ as in (2). In this parametrization, we can use the $U(Q_1)$ transformations to diagonalize one of the $X_m^{(1)}$, say, $X_6^{(1)}$. This fixes $U(Q_1)$ down to $U(1)^{Q_1}$. From the remaining $U(1)$'s, $Q_1 - 1$ of them can be used to gauge away that number of phases from any other $X_m^{(1)}$, say $X_7^{(1)}$. Let us gauge away the phases of the components $(X_7^{(1)})_{1\widetilde{b}'}$ where $\widetilde{b}' = 2, \cdots, Q_1$. The remaining $U(1)$ factor is the overall abelian factor of $U(Q_1)$ which leaves fields in the adjoint representation invariant. Similarly, we can fix $U(Q_5)$ down to its overall $U(1)$ subgroup by diagonalizing $X_6^{(5)}$ and removing the phases from $(X_7^{(5)})_{1\widetilde{b}}$, where $\widetilde{b} = 2, \cdots, Q_5$. After fixing the gauge (down to a $U(1)\times U(1)$ subgroup) in this manner, $X_8^{(1,5)}$ and $X_9^{(1,5)}$ remain arbitrary hermitian matrices, while $X_6^{(1,5)}$ and $X_7^{(1,5)}$ reduce to the form

$$X_6 = \begin{pmatrix} v_{11} & 0 & \cdots & 0 \\ 0 & v_{22} & & \\ \vdots & & \ddots & \\ 0 & 0 & & v_{QQ} \end{pmatrix}, \qquad X_7 = \begin{pmatrix} w_{11} & |w_{1\widetilde{a}}| \\ & \\ |w_{1\widetilde{a}}| & \left(w_{\widetilde{a}\widetilde{b}}\right) \end{pmatrix}. \tag{15}$$

Here, $\widetilde{a}, \widetilde{b} = 2, \cdots, Q$ where Q is either Q_1 or Q_5.

Out of the surviving $U(1)\times U(1)$ gauge group, only a diagonal $U(1)$ subgroup has a non-trivial action on the fields χ_i and, therefore, can be used to gauge away a single phase. We use this to gauge away the phase of the component $(\chi_1)_{1'1}$, leaving us with only three degrees of freedom in $\chi_{i1'1}$. The other diagonal $U(1)$ subgroup does not transform the hypermultiplets and is not broken. This is not unexpected since the scalar component of the vector multiplet associated with this $U(1)$ corresponds to the center of mass position of the combined D-brane system in the physical 4-dimensional space and hence should remain massless. Thus, after gauge fixing, the adjoint hypermultiplets $N_i^{(1)}$ and $N_i^{(5)}$ contain $3Q_1^2+1$ and $3Q_5^2+1$ degrees of freedom, respectively, and the bi-fundamental hypermultiplet χ_i contains $4Q_1Q_5-1$ degrees of freedom.

Now we consider the constraints imposed on these fields by the vanishing of the D-terms. The trace equations (14) can be used to determine $|(\chi_i)_{1'1}|$ and the phase of $(\chi_2)_{1'1}$ in terms of other χ's. This, along with gauge fixing, completely determines $\chi_{i1'1}$, leaving us with $4Q_1Q_5-4$ degrees of freedom in the χ hypermultiplet.

Removing the trace parts from the D-term constraints, we are left with $3Q_1^2+3Q_5^2-6$ equations that can be used to determine that many components of $N_i^{(1,5)}$ in terms of the χ_i. This leaves 8 components (two adjoint hypermultiplets) undetermined. These are easy to identify: The D-term conditions involve only the commutators of $X_m^{(1,5)}$ (or of $N_i^{(1,5)}$) and therefore do not constrain their traces, $x_m^{(1,5)} = \mathrm{Tr}\, X_m^{(1,5)}$, which are the undetermined hypermultiplets. The fields $x_m^{(1)}$ correspond to the center of mass location of the D1-branes inside the D5-branes while $x_m^{(5)}$ are Abelian Wilson lines on T^4 that have a similar interpretation in a dual theory in which the 1-branes and the 5-branes are interchanged. Clearly, these positions are arbitrary which justifies the existence of the two associated massless hypermultiplets. Therefore, with these conventions, the space of independent field components is parametrized by the Q_1Q_5-1 hypermultiplets $\chi_{ia'a}$ (excluding $a'=a=1$) and two adjoint hypermultiplets $x_m^{(1)}$ and $x_m^{(5)}$ [5]. Since $X_m^{(1,5)}$ are compact variables, it is natural to also take χ_i to be compact so that the moduli space can be consistently parametrized in terms of χ_i. Explicitly, this space can be written as

$$\mathcal{M}_0 = (T^4)^{\widetilde{Q}_1} \times (T^4)^{\widetilde{Q}_5} \times (T^4)^{\widetilde{Q}_1\widetilde{Q}_5} \times (T^4)^2\,, \tag{16}$$

where, $\widetilde{Q}_{1,5} = Q_{1,5}-1$.

The parametrization of the $(4Q_1Q_5-4)$-dimensional subspace of fields in terms of the hypermultiplets $\chi_{i\widetilde{a}'1}$, $\chi_{i1'\widetilde{a}}$ and $\chi_{i\widetilde{a}'\widetilde{a}}$ still has a redundancy. This is because our gauge fixing has not properly taken care of a subgroup of the Weyl group of the gauge group. The Weyl group of $U(Q)$ is the symmetric group $S(Q)$, the elements of which act as permutations on the fundamental representation index $a=1,2,\cdots,Q$ of $U(Q)$. Let us denote by $S(Q-1)$ the

[5]This parametrization of the independent degrees of freedom in terms of $\chi_{ia'a}$ (excluding $a'=a=1$) is not globally valid and breaks down when the matrices N_i commute with each other. However, in such a situation one can find a different parametrization. One can choose the diagonal elements of $N_i^{(1,5)}$ as independent degrees of freedom [32] and instead regard $\chi_{i\widetilde{a}'1}$ and $\chi_{i1'\widetilde{a}}$ as being fixed by the D-term constraints. This again leads to spaces of the form $\mathcal{M}_0$ in (16) or $\mathcal{M}$ in (18) below.

subgroup that leaves $a = 1$ unchanged and acts as permutations only on elements labelled by $\widetilde{a} = 2, 3, \cdots, Q$. We are interested in the subgroup $S(Q_1 - 1) \times S(Q_5 - 1)$ of Weyl reflections in $U(Q_1) \times U(Q_5)$. The elements of this subgroup act on the rectangular matrices $\chi_{i\widetilde{a}'\widetilde{a}}$ by permuting their rows and columns amongst themselves. They also act as permutations on $\chi_{i\widetilde{a}'1}$ and $\chi_{i1'\widetilde{a}}$. Denoting the elements of $S(Q_{1,5} - 1)$ by $S_{1,5}$, this action can be written as

$$S_1 \begin{pmatrix} \chi_{1'1} & \chi_{1'\widetilde{a}} \\ \chi_{\widetilde{a}'1} & \chi_{\widetilde{a}'\widetilde{a}} \end{pmatrix} S_5^\dagger = \begin{pmatrix} \chi_{1'1} & \chi_{1'p_5(\widetilde{a})} \\ \chi_{p_1(\widetilde{a}')1} & \chi_{p_1(\widetilde{a}')p_5(\widetilde{a})} \end{pmatrix}, \tag{17}$$

where, $p_{1,5}(a)$ implement the corresponding permutations on the index a. Note that the form (15) of the matrices $X_m^{(1,5)}$, as dictated by our choice of gauge fixing, is manifestly invariant under this subgroup of Weyl reflections. As a result, the points on the space of χ's related by these transformations are gauge equivalent and, therefore, have to be identified. Taking this into account, the moduli space has the structure

$$\mathcal{M} = \frac{(T^4)^{\widetilde{Q}_1}}{S(\widetilde{Q}_1)} \times \frac{(T^4)^{\widetilde{Q}_5}}{S(\widetilde{Q}_5)} \times \frac{(T^4)^{\widetilde{Q}_1\widetilde{Q}_5}}{S(\widetilde{Q}_1) \times S(\widetilde{Q}_5)} \times (T^4)^2 . \tag{18}$$

The dependence of $N^{(1,5)}$ on χ induces a metric on the moduli space $\mathcal{M}$ (or the space $\mathcal{M}_0$) through the kinetic energy terms in (3). For convenience, let us parametrize the complex fields χ_i in terms of real fields $Y^m, (m = 6, 7, 8, 9)$ given by

$$\chi_1 = Y^9 + iY^8 , \qquad \chi_2 = Y^7 + iY^6 , \tag{19}$$

and also parametrize $N_i^{(1,5)}$ in terms of $X_m^{(1,5)}$ as in (2). Then, for example, the kinetic energy term for $N_i^{(5)}$ can be written as

$$\partial_\alpha X_{lab}^{(5)} \partial^\alpha X_{lab}^{(5)} = G_{(5)}^{(mc'd)(ne'f)}(Y) \partial_\alpha Y_{mc'd} \partial^\alpha Y_{ne'f} , \tag{20}$$

with,

$$G_{(5)}^{(mc'd)(ne'f)}(Y) = \frac{\partial X_{lab}^{(5)}}{\partial Y_{mc'd}} \frac{\partial X_{lab}^{(5)}}{\partial Y_{ne'f}} , \tag{21}$$

There are similar contributions from $N^{(1)}$ and $\chi_{i1'1}$. Putting all this together, the metric on the $4(Q_1Q_5 - 1)$-dimensional subspace of the target space spanned by χ_i takes the form

$$G_{(mc'd)(ne'f)} = \delta_{mn}\delta_{c'e'}\delta_{df} + \frac{k_{11}}{k_{15}} G_{(mc'd)(ne'f)}^{(1)} + \frac{k_{55}}{k_{15}} G_{(mc'd)(ne'f)}^{(5)} + G_{(mc'd)(ne'f)}^{(\chi)} \tag{22}$$

However, note that the vanishing of the D-terms (6) and (7) imply that $G^{(1)}$ is proportional to k_{15}/k_{11} and $G^{(5)}$ is proportional to k_{15}/k_{55}. Therefore, the metric G is independent of the couplings. Restricted to the non-trivial $4(Q_1Q_5 - 1)$-dimensional part of the moduli space, the action S_{ke} in (3) becomes

$$S'_{ke} = k_{15} \int d^2\sigma G^{(mc'd)(ne'f)}(Y) \partial_\alpha Y_{mc'd} \partial^\alpha Y_{ne'f} . \tag{23}$$

where, $Y_{ma'a}$ does not include $Y_{m1'1}$.

While we do not calculate the metric G, we can make some important general observations: (i) In 1+1 dimensions, a hypermultiplet decomposes into a pair of chiral multiplets of N=2 supersymmetry in D=2, while a vector multiplet decomposes into a chiral and a twisted chiral multiplet [33]. A non-linear σ-model in two dimensions, that admits a superfield representation, can have an antisymmetric tensor filed coupling (the analogue of B_{mn} in string theory) only if it contains twisted chiral multiplets. Therefore, since $\mathcal{M}$ is a hypermultiplet moduli space, an antisymmetric field and hence a torsion is not induced on it. (ii) As a consequence of $N = 4$ supersymmetry and the absence of torsion, the metric on $\mathcal{M}$ is hyperkahler. (iii) Up to now our analysis pertained to the weak coupling regime $gQ_{1,5} < 1$ of the gauge theory. However, the non-renormalization of the hypermultiplet moduli space in gauge theory implies that the classical moduli space discussed in the this section is not affected as we increase the gauge coupling (see for example [34] for a statement of this). This means that the analysis is valid even when $gQ_{1,5} > 1$.

To summarize this section, we have gauge fixed and isolated the independent degrees of freedom which satisfy the D-flatness conditions. These are components $\chi_{ia'a}(\sigma, t)$ (excluding $a' = a = 1$) of the (1,5) hypermultiplets along with $x_m^(1, 5)$. The remaining (1,1) and (5,5) hypermultiplets (on which we fixed the unitary gauge) are determined in terms of these independent components. The slowly varying independent moduli are described by a non-linear sigma-model on a $(4Q_1Q_5+4)$-dimensional target space whose validity is ensured even when $gQ_{1,5} > 1$. In the next section, we describe very low-energy excitations of these moduli fields.

4 Discrete Gauge Symmetry and Low Energy Degrees of Freedom

In this section we describe the mechanism by which very low-energy excitations of the moduli fields, to be identified as the black hole degrees of freedom, arise. One approach to the study of this problem would be to consider the non-linear sigma-model on the orbifold space $\mathcal{M}$ as given in (18), with the induced metric on it. The approach we follow is to consider the sigma-model on $\mathcal{M}_0$, as given by (16), and regard the residual Weyl symmetry $S(Q_1-1) \times S(Q_5-1)$ as a discrete gauge group to be implemented as a Gauss law constraint on the physical states. In the presence of this discrete gauge group, the theory develops sectors with twisted-periodic boundary conditions, leading to excitations with fractional momenta [21, 22, 23, 24]. The effective theory for these modes is argued to be a $c = 6$ superconformal field theory which emerges in the infra-red limit.

Consider $\chi_{ia'a}(\sigma, t)$ (excluding $a' = a = 1$) as taking values in $T^{4(Q_1Q_5-1)}$ with an $S(Q_1 -$

$1) \times S(Q_5 - 1)$ action on it as given by(14). Since configurations related by this residual gauge group are to be identified, the compactness of σ allows us to impose twisted periodicity conditions on the components of the matrix χ, so that

$$\chi_i(\sigma + 2\pi R) = S_1 \chi_i(\sigma) S_5^\dagger \,. \tag{24}$$

S_1 acts as a permutation on the row index $\widetilde{a}' = 2, \cdots, Q_1$ and we denote its action by $p_1(\widetilde{a}')$. Similarly, we denote a permutation of the column index $\widetilde{a} = 2, \cdots, Q_5$, under the action of S_5, by $p_5(\widetilde{a})$. Then, in terms of components, the twisted boundary condition above breaks up into

$$\chi_{i\widetilde{a}'1}(\sigma + 2\pi R) = \chi_{ip_1(\widetilde{a}')1}(\sigma) \,, \tag{25}$$

$$\chi_{i1'\widetilde{a}}(\sigma + 2\pi R) = \chi_{i1'p_5(\widetilde{a})}(\sigma) \,, \tag{26}$$

$$\chi_{i\widetilde{a}'\widetilde{a}}(\sigma + 2\pi R) = \chi_{ip_1(\widetilde{a}')p_5(\widetilde{a})}(\sigma) \,. \tag{27}$$

Note that $\chi_{i1'1}$, which is not an independent parameter, is invariant. The theory develops different sectors depending on the elements $S_{1,5}$ chosen to implement the twisted boundary conditions. In each sector, defined by a choice of S_1 and S_5, all elements of χ_i that are related to each other by the twisted boundary condition, can be sewn into a single function defined on a circle of larger radius, as explained below. These sewn functions are the natural variables corresponding to very low-energy excitations of the D-brnane system.

The elements of the symmetric group $S(Q)$ fall into conjugacy classes that are characterized by the *order* and *number* of cyclic subgroups that an element can generate. Suppose that one of the cyclic subgroups generated by $S_1 \in S(Q_1 - 1)$ is of order l_1. This cyclic group will permute l_1 rows of the matrices χ_i amongst themselves (keeping the first row invariant). Let us consider one of these rows labelled by the fundamental representation index $\widetilde{a}'_0$. After going once around the compact direction, this row index is transformed into $\widetilde{a}'_1 = p_1(\widetilde{a}'_0)$, and after going n times around this S^1, we get $\widetilde{a}'_n = p_1(\widetilde{a}'_{n-1}) = p_1^n(\widetilde{a}'_0)$. Clearly, $\widetilde{a}'_{l_1} = p_1^{l_1}(\widetilde{a}'_0) = a'_0$. A similar discussion applies to a column index $\widetilde{a}_0$ transforming under a permutation group of order l_5 which is one of the cyclic groups generated by S_5. To describe the sewing procedure for the elements of χ, let us first consider the simpler case involving the elements $\chi_{i\widetilde{a}'1}$ that transform only under S_1, and are invariant under S_5. Consider a specific matrix element labelled by $(\widetilde{a}'_0, 1)$ that transforms under a cyclic subgroup of order l_1. As we go around the circle l_1 times, the twisted boundary condition (25) forces the function $\chi_{i\widetilde{a}'_0 1}$ to go through the l_1 functions $\chi_{i\widetilde{a}'_n 1}$, for $n = 1, \cdots, l_1$, before coming back to itself. To sew these l_1 functions, each defined on a circle of radius R, into one single function $\widetilde{\chi}_{i(l_1)}$ defined on a circle of radius Rl_1, let $\widetilde{\sigma}$ be a parameter along the larger circle ($0 \leq \widetilde{\sigma} \leq 2\pi R l_1$). Then, $\widetilde{\chi}_{i(l_1)}(\widetilde{\sigma})$ can be defined such that

$$\widetilde{\chi}_{i(l_1)}(2\pi R n \leq \widetilde{\sigma} \leq 2\pi R(n+1)) = \chi_{ip_1^n(\widetilde{a}'_0)1}(\sigma) \equiv \chi_{i\widetilde{a}'_n 1}(\sigma) \,, \tag{28}$$

where $\widetilde{\sigma} = 2\pi R n + \sigma$.

The kinetic energy terms for the above components of χ_i, that have been sewn into a single function, can be easily expressed in terms of the sewn field. Using (28), it is easy to see that (suppressing the $SU(2)_R$ index i)

$$\begin{aligned}\int_0^{2\pi Rl_1} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{(l_1)}(\widetilde{\sigma})\partial^\alpha\widetilde{\chi}^*_{(l_1)}(\widetilde{\sigma}) &\equiv \sum_{n=0}^{n=l_1-1}\int_{2\pi Rn}^{2\pi R(n+1)} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{(l_1)}(\widetilde{\sigma})\partial^\alpha\widetilde{\chi}^*_{(l_1)}(\widetilde{\sigma}) \\ &= \sum_{n=0}^{n=l_1-1}\int_0^{2\pi R} d\sigma\partial_\alpha\chi_{a'_n1}(\sigma)\partial^\alpha\chi^*_{a'_n1}(\sigma)\,. \end{aligned}\tag{29}$$

In general, S_1 can generate many cyclic groups of various orders l_1 such that $\sum_{\{l_1\}} l_1 = Q_1-1$. Then, all the Q_1-1 elements $\chi_{\widetilde{a}'1}$ can be sewn into a smaller number of functions $\widetilde{\chi}_{(l_1)}$, each living on circle of larger radius Rl_1. Hence, the part of the kinetic energy term involving these elements can be written entirely in terms of the sewn fields as

$$\sum_{\widetilde{a}'=2}^{Q_1}\int dt\int_0^{2\pi R} d\sigma\partial_\alpha\chi_{i\widetilde{a}'1}\partial^\alpha\chi^*_{i\widetilde{a}'1} = \sum_{\{l_1\}}\int dt\int_0^{2\pi Rl_1} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{i(l_1)}\partial^\alpha\widetilde{\chi}^*_{i(l_1)}\,. \tag{30}$$

The most important outcome of the sewing procedure is that, while the compact dimension has a physical radius R, the momenta of the sewn fields $\widetilde{\chi}_{(l_1)}$ are quantized in units of $1/Rl_1$. Furthermore, the number of independent fields reduces from $4(Q_1-1)$ for $\chi_{i\widetilde{a}'1}$ to the number of sewn fields $\widetilde{\chi}_{i(l)}$. This number is given by the number of cyclic groups generated by S_1 and could be much smaller than $4(Q_1-1)$. Obviously, the above procedure carries over, without any change, to the elements $\chi_{1'\widetilde{a}}$ that are permuted by $S_5^\dagger$, but are invariant under S_1.

Having demonstrated the sewing procedure in a simpler situation, let us now consider a matrix element labelled by $(\widetilde{a}'_0, \widetilde{a}_0)$, where the first index transforms under a cyclic group of order l_1 and the second, under one of order l_5. Due to the twisted boundary condition (27), after going around the circle n times, this element is transformed into one labelled by $(\widetilde{a}'_n, \widetilde{a}_n) = (p_1^n(\widetilde{a}'_0), p_5^n(\widetilde{a}_0))$. Clearly, the element $(\widetilde{a}'_0, \widetilde{a}_0)$ comes back to itself after going around the circle l times, where l is the *least common multiple* of l_1 and l_5. This means that the function $\chi_{i\widetilde{a}'_0\widetilde{a}_0}$ goes through l functions $\chi_{i\widetilde{a}'_n\widetilde{a}_n}$, for $n = 1, \cdots, l$, before coming back to itself. These l functions, each defined (but not periodic) on a circle of radius R, can be sewn into one single function $\widetilde{\chi}_{i(l)}$ defined on a circle of radius Rl as before: Let $\widetilde{\sigma}$ be a parameter along the larger circle ($0 \le \widetilde{\sigma} \le 2\pi Rl$). Then, $\widetilde{\chi}_{i(l)}(\widetilde{\sigma})$ can be defined such that

$$\widetilde{\chi}_{i(l)}(2\pi Rn \le \widetilde{\sigma} \le 2\pi R(n+1)) = \chi_{ip_1^n(\widetilde{a}'_0)p_5^n(\widetilde{a}_0)}(\sigma) \equiv \chi_{i\widetilde{a}'_n\widetilde{a}_n}(\sigma)\,, \tag{31}$$

where $\widetilde{\sigma} = 2\pi Rn + \sigma$. Using this, the kinetic energy terms for the sewn components of $\chi_{\widetilde{a}'\widetilde{a}}$ can be written in terms of the single $\widetilde{\chi}_{(l)}$ as (again, suppressing the $SU(2)_R$ index)

$$\begin{aligned}\int dt\int_0^{2\pi Rl} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{(l)}(\widetilde{\sigma})\partial^\alpha\widetilde{\chi}^*_{(l)}(\widetilde{\sigma}) &\equiv \sum_{n=0}^{n=l-1}\int dt\int_{2\pi Rn}^{2\pi R(n+1)} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{(l)}(\widetilde{\sigma})\partial^\alpha\widetilde{\chi}^*_{(l)}(\widetilde{\sigma}) \\ &= \sum_{n=0}^{n=l-1}\int dt\int_0^{2\pi R} d\sigma\partial_\alpha\chi_{a'_na_n}(\sigma)\partial^\alpha\chi^*_{a'_na_n}(\sigma)\,. \end{aligned}\tag{32}$$

Depending on the structure of S_1 and S_5, all the $(Q_1-1)(Q_5-1)$ complex elements $\chi_{i\widetilde{a}'\widetilde{a}}$, defined on a circle of radius R, can be sewn into a smaller number of functions $\widetilde{\chi}_{i(l)}$ defined on circles of radii Rl, such that $\sum_{\{l\}} l = (Q_1-1)(Q_5-1)$. The kinetic energy terms for these components can then be written as

$$\sum_{\widetilde{a}'=2}^{Q_1}\sum_{\widetilde{a}=2}^{Q_5}\int dt\int_0^{2\pi R} d\sigma\partial_\alpha\chi_{i\widetilde{a}'\widetilde{a}}\partial^\alpha\chi^*_{i\widetilde{a}'\widetilde{a}} = \sum_{\{l\}}\int dt\int_0^{2\pi Rl} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{i(l)}\partial^\alpha\widetilde{\chi}^*_{i(l)}\,. \tag{33}$$

The field $\widetilde{\chi}_{(l)}$ has momentum quantized in units of $1/Rl$. Of course, if we are interested in long wavelength effects, then only sectors with the largest l will contribute (these sectors are also more favourable entropically).

A sewn field $\widetilde{\chi}$ with the lowest quantum of momentum can be obtained if $\widetilde{Q}_1 = Q_1 - 1$ and $\widetilde{Q}_5 = Q_5 - 1$ are coprime and $S_{1,5}$ in (24) are chosen such that they generate cyclic groups of order $\widetilde{Q}_{1,5}$. This can be easily achieved by choosing $S_{1,5}$ to cyclically permute the indices $(\widetilde{a}', \widetilde{a})$: $(p_1(\widetilde{a}'), p_5(\widetilde{a})) = (\widetilde{a}'+1, \widetilde{a}+1)$. In this case, using (31), all the $4\widetilde{Q}_1\widetilde{Q}_5$ real components of $\chi_{i\widetilde{a}'\widetilde{a}}$ can be sewn into a 4 real functions $\widetilde{\chi}_i$ that are periodic on a circle of radius $R\widetilde{Q}_1\widetilde{Q}_5$, and thus have their momenta quantized in units of $1/R\widetilde{Q}_1\widetilde{Q}_5$. The kinetic energy term for these components of χ takes a very simple form:

$$\sum_{\widetilde{a}'=2}^{Q_1}\sum_{\widetilde{a}=2}^{Q_5}\int dt\int_0^{2\pi R} d\sigma\partial_\alpha\chi_{i\widetilde{a}'\widetilde{a}}\partial^\alpha\chi^*_{i\widetilde{a}'\widetilde{a}} = \int dt\int_0^{2\pi R\widetilde{Q}_1\widetilde{Q}_5} d\widetilde{\sigma}\partial_\alpha\widetilde{\chi}_{i}\partial^\alpha\widetilde{\chi}^*_{i}\,. \tag{34}$$

With this twisted boundary condition, the elements $\chi_{\widetilde{a}'1}$ ($\chi_{1'\widetilde{a}}$) can be sewn into a function $\widetilde{\chi}_{(\widetilde{Q}_1)}$ ($\widetilde{\chi}_{(\widetilde{Q}_5)}$) with momentum quantized in units of $1/R\widetilde{Q}_1$ ($1/R\widetilde{Q}_5$). The remaining two moduli fields $x^{(1)}_m$ and $x^{(5)}_m$ are periodic on the circle of radius R. Ignoring the contribution of the fields $N^{(1,5)}$ for the time being, it is reasonable to expect that for large Q_1 and Q_5 only the sewn field appearing in (34), with 4 real components, would be relevant to the physics of the black hole at low energies. In case $\widetilde{Q}_1$ and $\widetilde{Q}_5$ are not coprime, one can choose S_1 and S_5 such that they contain cyclic subgroups of maximum possible order l_1 and l_5 which are coprime. The sewn field $\widetilde{\chi}_{(l_1 l_5)}$ coming from this sector will then have the lowest momentum quantum and will dominate the low-energy physics. For sectors corresponding to other conjugacy classes of the Weyl group, one can construct tunnelling configurations in the gauge theory that could induce transitions between these sectors. As a result, smallest cycles may go over to longer cycles for reasons of phase space, because the entropy associated with long cycle is not less than the sum of entropies of smaller cycles.

So far, we have only considered the moduli fields $\chi_{ia'a}$ (excluding $\chi_{i1'1}$) and their kinetic energy terms. The full kinetic energy action (3) also contains the fields $N^{(1,5)}_i$ and $\chi_{i1'1}$ which give rise to a metric on the moduli space through their dependence on the χ-moduli as in (23). On imposing twisted boundary conditions, the presence of $G_{(mc'd)(ne'f)}$ leads to *non-local* interactions for the sewn fields $\widetilde{\chi}_{(l)}$ (or equivalently, for $\widetilde{Y}_{(l)}$) in the theory with a

4-dimensional target space defined on the circle of radius Rl. This is not difficult to see: The index $(a'a)$ of an element of χ_i determines a small interval on the larger circle on which $\chi_{ia'a} = \widetilde{\chi}_{i(l)}$. Therefore, $\chi_{ic'd}$ and $\chi_{ie'f}$ (provided they are in the same sector of the twisted boundary condition) may correspond to the values of some $\widetilde{\chi}_{i(l)}$ at different points on the circle of radius Rl. If $\chi_{ic'd}$ and $\chi_{ie'f}$ are not in the same sector, then G may introduce couplings between different sectors which may have their fractional momenta quantized in different units. The metric will also introduce couplings between the components $\chi_{i\widetilde{a}'1}$,$\chi_{i1'\widetilde{a}}$ and $\chi_{i\widetilde{a}'\widetilde{a}}$.

Thus, the full theory in terms of the sewn variables is a very complicated, non-local sigma-model. However, we are only interested in the lowest energy excitations on the moduli space. To isolate this sector, we have to integrate out all the higher momentum sectors, retaining the lowest one, say $\widetilde{\chi}_{i(\widetilde{Q}_1\widetilde{Q}_5)}$ (where, for simplicity, we assume that $\widetilde{Q}_1$ and $\widetilde{Q}_5$ are coprime). Although this cannot be done explicitly, this procedure leads to an effective theory for the renormalized form of $\widetilde{\chi}_{i(\widetilde{Q}_1\widetilde{Q}_5)}$ which we denote by $\widetilde{\chi}_i$, and which lives on a circle of radius $R\widetilde{Q}_1\widetilde{Q}_5$. Essentially, $\widetilde{\chi}_i$ is an order parameter in terms of which the infra-red physics can be described. While we cannot obtain the effective low-energy theory for $\widetilde{\chi}_i$ by explicitly integrating out the higher momentum sectors, we make the assumption that this theory is local to lowest order in energy. This implies that we are dealing with a non-linear sigma-model on a compact 4-dimensional manifold,

$$k_{15}\int dt\int_0^{2\pi R\widetilde{Q}_1\widetilde{Q}_5} d\widetilde{\sigma} G_{mn}(\widetilde{Y})\partial_\alpha\widetilde{Y}^m\partial^\alpha\widetilde{Y}^n\,, \tag{35}$$

where, $\widetilde{Y}^m, (m = 6,7,8,9)$ are the real components of $\widetilde{\chi}_i$ given by $\widetilde{\chi}_1 = \widetilde{Y}^9 + i\widetilde{Y}^8$ and $\widetilde{\chi}_2 = \widetilde{Y}^7 + i\widetilde{Y}^6$. $N = 4$ supersymmetry, which is expected to survive in the infra-red limit, implies that this manifold is hyperkahler [35] and therefore, it is either T^4 or a $K3$ surface. This great simplification, over the original complicated non-local theory, is a consequence of N=4 supersymmetry and compactness of the moduli space $\mathcal{M}_0$, together with our assumption of the locality of the infra-red theory. To make a choice between T^4 and $K3$, let us consider $K3$ in the orbifold limit T^4/Z_2. The modding by Z_2 identifies $\widetilde{Y}^m$ with $-\widetilde{Y}^m$, whereas, such an identification is not required in the original gauge theory, nor it is imposed by the D-term constraints or the infra-red limit. While, strictly speaking, this argument only rules out T^4/Z_2, it may also be an indication of the existence of more general obstructions to $K3$. There is also an important *a posteriory* argument in favour of T^4: Locally, the metric on T^4 has an $SO(4)$ invariance which is crucial in determining the couplings of the minimal scalars to the low-energy excitations of the D-brane system, leading to the correct calculation of Hawking emission and absorption rates. Therefore, it is reasonable to regard (35) as a sigma model on T^4. The emergence of the $SO(4)$ invariance is then a consequence of the infra-red limit.

Clearly, the compactness of the moduli space is responsible for restricting our choice of the hyperkahler metrics to T^4 and $K3$. If the moduli space $\mathcal{M}_0$ in (16) is not compact (as would be the case for a generic gauge theory), that is, if the factors of T^4 in (16) are replaced by R^4, then the dynamics of the order parameter is governed by a sigma-model on a 4-dimensional non-compact hyperkahler, and hence Ricci flat, manifold. In order to deduce further implications of the vanishing of the Ricci tensor, we need to specify the boundary conditions on the metric on this space. If we make the simplest assumption that, for large $\widetilde{Y}^m$, this space has the structure of the 4-dimensional Euclidean space, then a theorem by Witten [36] states that this space is a flat 4-dimensional Euclidean space. Furthermore, an argument similar to the one ruling out T^4/Z_2 may be used to rule out spaces with non-trivial topology at infinity. In spite of this, there are many more choices than in the compact case.

To summarize, in this section we have shown that the residual discrete gauge invariance of the theory leads to fractional excitations of the moduli fields. We then identified an order parameter $\widetilde{\chi}_i$ (or $\widetilde{Y}^m$), periodic on a circle of radius $R\widetilde{Q}_1\widetilde{Q}_5$, in terms of which the low-energy dynamics of the system in the infra-red limit can be described. The corresponding effective theory, including the fermions, is a $c = 6$ superconformal field theory on T^4 with extended $N = 4$ supersymmetry. The $SU(2) \times SU(2)$ R-symmetry of this superconformal algebra acts on the fermionic partners of $\widetilde{Y}_m$ and is identified as the $SO(4)$ group of rotations in the physical 4-dimensional space transverse to $S^1 \times T^4$ [18]. Therefore, fermionic excitations in the superconformal field theory carry angular momentum corresponding to rotations in the physical space transverse to the D-brane system. In the next section, we will focus on the relation between this effective theory and the 5-dimensional black hole.

5 The Black Hole, Coupling to Bulk Fields and Hawking Radiation

We have shown that the D1,D5-brane system has low-energy degrees of freedom with fractional momenta, which in the infra-red limit, are effectively described by a $c = 6$ superconformal field theory. Writing only the the bosonic terms,

$$S_{eff} = T_{eff} \int dt \int_0^{2\pi R\widetilde{Q}_1\widetilde{Q}_5} d\widetilde{\sigma} \partial_\alpha \widetilde{Y}^m \partial^\alpha \widetilde{Y}^m \,, \tag{36}$$

where, up to a normalization, T_{eff} is given by

$$T_{eff} = k_{15} = \frac{1}{\alpha'^2 g} \sqrt{\frac{V_4}{Q_1 Q_5}} \,. \tag{37}$$

As discussed in the introduction, a theory of this form, usually called the "effective string picture", has been used to model the 5-dimensional black hole with rather good success. However, in our case, the T^4 on which the sigma-model is defined does not have a space-time interpretation and, in particular, is not the T^4 on which the D5-brane is compactified. Thus, strictly speaking, our T^4 is not the target space of some effective string, to be regarded as some kind of D-string, though in some cases this picture can prove very useful. The obtaining of the model (36) from the D-brane gauge theory not only explains its success but, more importantly, sheds light on some of its less understood features as will be discussed below. Moreover, this approach puts the calculation of the Hawking radiation and the resolution of the information paradox for these black holes on a more solid foundation. It may also lead to an understanding of higher angular momentum [37, 25] processes, though we will not dwell on this further.

Our derivation relates the coupling T_{eff}, often called the "effective string tension", to the (1,5) hypermultiplet coupling k_{15} (37), which was determined in section 2. As a check, this can be compared with the behaviour of T_{eff} that one expects in order to get agreement between various cross sections calculated in the SCFT framework and the corresponding ones obtained by semi-classical black hole calculations. T_{eff} does not appear in the cross sections for minimally coupled scalars calculated in the $c = 6$ superconformal field theory, though the ones for fixed scalars do depend on it [13, 14]. It is also argued to appear in the cross sections for higher angular momentum modes of the minimal scalars, to the extent that their structure can be surmised within this SCFT approach [25]. In fact, in [25], the dependence of T_{eff} on various parameters of the theory, notably its dependence on V_4, was suggested so that the SCFT has the ability to reproduce the cross sections for higher angular momentum modes in agreement with semi-classical black hole calculations. The form of T_{eff} we have obtained is in agreement with these calculations, as well as with the fixed scalar calculations, where they can be performed.

In the SCFT (36), the extremal 5-dimensional black hole can be identified with left moving states $|N_L>$ at certain level N_L in the standard way [2, 3, 8, 9]. From the expansion

$$\widetilde{Y}^m(\widetilde{\sigma}, t) = \sum_p \alpha_p^m e^{i(p/R\widetilde{Q}_1\widetilde{Q}_5)(\widetilde{\sigma}+t)} + (\text{right moving part})\,, \tag{38}$$

it is clear that a state at level N_L is associated with total left-moving momentum $N_L/R\widetilde{Q}_1\widetilde{Q}_5$. Since the black hole charge N is related to the left-moving momentum N/R along x^5, the states corresponding to it in the CFT should be at level $N_L = N\widetilde{Q}_1\widetilde{Q}_5$. These states preserve the same amount of supersymmetry as the classical black hole solution. The entropy is then related to the degeneracy of states at this level and for large N_L is $S = 2\pi\sqrt{N_L c/6} = 2\pi\sqrt{N\widetilde{Q}_1\widetilde{Q}_5}$. For large $Q_{1,5}$ this agrees with the Hawking-Bekenstein entropy $2\pi\sqrt{NQ_1Q_5}$ for this black hole. It is important to recall that, as argued in section 3, the moduli space analysis there could be extrapolated from the perturbative gauge theory regime of $gQ_{1,5} < 1$ to the large black hole regime of $gQ_{1,5} > 1$. The sigma-model (36) is therefore valid in the large black hole regime (otherwise, $T_{eff} \to \infty$ as $gQ_{1,5} \to 0$ and it becomes almost impossible to excite the black hole away from extremality).

The black hole can be perturbed away from extremality, keeping its macroscopic charges unchanged, by adding small and equal amounts of right and left moving fractional momenta along x^5 [8, 9] (we do not consider going off extremality by adding anti-branes). In the conformal field theory, the corresponding states are $|N_L + \delta N_R > \otimes |\delta N_R >$. These states are no longer BPS saturated and are not protected by supersymmetry. However, based on the non-renormalization of the hypermultiplet moduli space, we expect that the basic SCFT description is valid even at strong coupling and therefore, the correspondence between these non-BPS states and non-extremal black holes holds even in the macroscopic black hole limit. This expectation is supported by the accuracy with which Hawking radiation calculations can be performed. In general, in the vicinity of the extremal black hole states $|N_L > \otimes |0 >$ with $N_L = N\widetilde{Q}_1\widetilde{Q}_5$, the CFT could also contain states like $|N_L > \otimes |\delta N_R >$ that do not have an interpretation in terms of the original black hole. If the SCFT is to provide a faithful description of the extremal black hole and its near extremal deformations, then it should automatically prevent such states from appearing. This is achieved by a level matching condition that, in our approach, is inbuilt in the conformal field theory: The residual discrete gauge transformations (14) of the super Yang-Mills theory correspond to discrete translations along $\widetilde{\sigma}$ in (36). Imposing invariance under these translations as a Gauss law constraint implies that the allowed physical states are the ones for which the momentum operator along $\widetilde{\sigma}$ has integer eigenvalues,

$$\frac{L_0 - \bar{L}_0}{\widetilde{Q}_1\widetilde{Q}_5} = n \text{ (integer)} . \tag{39}$$

For the extremal black hole $n = N$, and any change in n corresponds to going away from extremality by a large amount. Thus the only allowed states in the vicinity of the extremal state are the ones for which $\Delta(L_0 - \bar{L}_0) = 0$. This condition eliminates the conformal field theory states that do not have a counter part on the black hole side.

As far as the relation to black hole physics is concerned, the most important issue is the coupling of this effective $c = 6$ SCFT to the bulk fields. These couplings are responsible for the processes of Hawking emission and absorption of the bulk fields [8, 10, 11, 12, 13, 14, 17, 18]. Having identified the extremal and near extremal black holes in terms of states in the SCFT, it is natural to describe couplings to bulk fields by operators in this SCFT, subject to the level matching condition. However, a first principle derivation of the interaction between the effective SCFT and bulk matter (closed strings states) is somewhat difficult and therefore, as yet, there is no known mechanism of systematically identifying these operators (though the Dirac-Born-Infeld action approach is useful and may give some hint [13]). In the remaining part of this section, we will discuss coupling to the minimal scalars in our setup.

The simplest example of coupling to bulk fields is the coupling to the scalar fields $h_{mn}(t, x^1, \cdots x^9), (m \neq n)$ that correspond to the components of the 10-dimensional graviton along the compactification T^4 in the directions x^6, x^7, x^8, x^9. Since we are interested in low-energy processes, we restrict h_{mn} to its zero modes along T^4. Furthermore, the momentum of h_{mn} along x^5 is quantized in units of $1/R$ as opposed to $1/R\widetilde{Q}_1\widetilde{Q}_5$ for the modes of the effective SCFT. Therefore, at low energies, only the zero momentum modes of h_{mn} along x^5 couple to the SCFT. Since, the SCFT lives at the origin of uncompactified space, we restrict the interaction to take place at $x^1, \cdots, x^4 = 0$. Hence, we only have to consider $h_{mn}(t, x^\mu = 0)$. At long wavelengths, and to lowest order, a coupling of these minimal scalars to (36) can be written simply based on symmetry principles. We first write down this "phenomenological" action and then see how it could arise from the microscopic point of view, in the process resolving a puzzle noted in [26]. As already noted, in the long wavelength limit, the $SU(2)_R$ symmetry of the super Yang-Mills theory is enlarged to the $SO(4)$ symmetry of S_{eff} in (36). The simplest form of coupling to h_{mn} is basically dictated by this $SO(4)$ and the relevant phenomenological action can be written as

$$T_{eff} \int dt \int_0^{2\pi R \widetilde{Q}_1 \widetilde{Q}_5} d\widetilde{\sigma} (\delta_{mn} + h_{mn}(t)) \partial_\alpha \widetilde{Y}^m \partial^\alpha \widetilde{Y}^n \,. \tag{40}$$

The origin of this coupling deserves some attention. Although, even a simple counting argument indicates that the (1,5) hypermultiplets χ_i are responsible for the black hole degrees of freedom [8, 16], the calculation in [26] shows that these hypermultiplets do not couple to minimal scalars at the microscopic level. This is contrary to the expectation that these scalars should contribute to Hawking emission and absorption by the black hole, as suggested by semi-classical black hole calculations. We can see that the coupling (40) in the effective low-energy theory arises from the coupling, at the microscopic level, of gravitons to the (1,1) and (5,5) hypermultiplets $N_i^{(1,5)}$ (or, in terms of real components, $X_m^{(1,5)}$). Such a coupling modifies the kinetic energy terms (3) of these hypermultiplets (now written in terms of real components) to $(\delta^{mn} + h^{mn})\partial_\alpha X_m^{(1,5)} \partial^\alpha X_n^{(1,5)}$. This in turn, modifies equations (20)-(22) so that, in the infra-red theory, the G_{mn} in (35) is replaced by $(\delta_{pq} + h_{pq})G_{mn}^{pq}$. In the absence of h_{pq}, we argued that G_{mn}^{pq} is of the form $\delta_m^p \delta_n^q$. Regarding h_{pq} as a small perturbation and hence retaining the same form for G_{mn}^{pq}, leads us to the effective coupling (40).

Equation (40), which is valid in the regime $gQ_{1,5} > 1$, is one of our main results. From here one can compute the grey body factors and match them with the General Relativity calculations. These calculations, which for the scalar emission considered above, do not depend on T_{eff}, have already been performed based on a model due to Maldacena and Susskind [9]. The interaction lagrangian that was used in [10] has a form almost identical to (40). From the DBI action, used in [11, 13], the same form of interaction emerges. This agreement, in spite of the difference in the underlying pictures, is not surprising since this coupling is more or less dictated by $SO(4)$ symmetry.

Regarding the coupling of fixed scalars to the effective SCFT, from [14] it is clear that this coupling involves SCFT operators with more than two derivatives and hence it is presently beyond the scope of the moduli space approximation and the long wavelength limit to which the non-renormalization theorem applies. Incorporation of the fixed scalars will probably require a better understanding of the model.

One of the most important issues that any derivation of black hole thermodynamics from string theory must face is the issue of thermalization which, even in standard systems, is often only argued for rather than proven. We note that the non-local interactions that arise as a result of applying the sewing procedure to (23) can help the system thermalize relatively easily because every bit of the "long string" is in contact with every other bit. This would justify the use of the canonical ensemble for the SCFT.

We end this section by discussing the range of validity of comparisons between the D-brane and semi-classical results: Unlike Schwarzschild black holes, the 5-dimensional black hole we have considered has a positive specific heat $C = \partial M/\partial T_H \sim T_H$, where T_H goes to zero with the non-extremality parameter r_0 [12]. The temperature fluctuations ΔT are related to the specific heat C by [27]

$$\frac{< (\Delta T)^2 >}{T^2} = \frac{1}{C}\,, \tag{41}$$

and therefore, blow up as $T_H \to 0$. Thus, as the black hole approaches the extermal limit, the thermodynamic picture breaks down. For the black hole we have considered, $C \simeq V_4 R^2 r_0^2 \sqrt{Q_1 Q_5/N}$. Therefore, the comparison with semi-classical picture is valid only when

$$r_0^2 \sqrt{\frac{Q_1 Q_5}{N}} >> 1\,. \tag{42}$$

Outside this range, the thermodynamic picture breaks down while the SCFT calculation in the D-brane framework is still valid. This problem also manifests itself in the calculation of the entropy of the near extremal black holes [8]: If the right-moving oscillator number δN_R is small (which it is, very close to extremality), the density of states does not grow exponentially with δN_R and hence does not lead to the thermodynamic entropy. This is not surprising as the thermodynamic picture is not valid in this regime.

6 Conclusions

In summary, we have studied the D-brane constituent model of the near extremal black hole of IIB string theory in 5-dimensions. We have extracted the low-energy degrees of freedom for $gQ_{1,5} > 1$ and the corresponding effective theory which describes black hole thermodynamics. This turns out to be an $N = 4$ free superconformal field theory with $c = 6$ and an enhanced $SO(4)$ symmetry. The coupling T_{eff} is given by (37). The level matching condition (39) guarantees a faithful description of the near extremal excitation of the black hole in terms of SCFT states. We have also discussed the phenomenological lagrangian that describes the interaction of bulk minimal scalars with this SCFT. While the minimal scalars do not couple to the (1,5) hypermultiplets at the microscopic level, a coupling to the black hole degrees of freedom is induced in the effective theory. This provides a string theoretic basis for the Hawking decay of the excited black hole to its ground state. Now that the derivation of the Hawking decay rate has a sound basis in string theory we can argue that, at least in this model, there is no information loss when the black hole radiates to its ground state by the emission of long wavelength quanta. The black hole thermodynamics is born of the usual statistical averaging procedure of quantum statistical mechanics. The so called "information loss", which would seem to occur in the de-excitation of a near extremal black hole in General Relativity, is now explained as arising from the averaging procedure of an S-matrix which knows about all the phase correlations. This resolution is of course based on an S-matrix approach. However it would be of great interest to make contact with the geometry of the black hole that arises from the D-brane model. Only then we would have understood the Hawking-Bekenstein formula for the black hole entropy and resolved some of the mysteries that underly Hawking's original derivation of black hole thermodynamics [38].

There is also another approach to the study of these black holes [39, 40, 41] and it is interesting to explore the relationship between the two associated superconformal field theories. The idea is that by an appropriate U-duality transformation, the D-brane black hole can be mapped into another classical solution carrying only NS-NS charges with the structure $BTZ \times S^3 \times T^4$. This also happens to be the near-horizon geometry of the $D1, D5$-brane black hole. When the momentum along x^5 is set to zero, this reduces to $AdS_3 \times S^3 \times T^4$ and is associated with a $N = (4, 4)$ WZNW model on $SL(2) \times SU(2) \times T^4$. The black hole is then associated with states in this SCFT. Since entropy is U-duality invariant, it can be computed in this SCFT. In this approach one can avoid extrapolation in the string coupling as well as the issue of coupling bulk fields to SCFT for R-R excitations. However, the microscopic origin of the system is not as evident as in the D-brane black hole case. Understanding the connection between these two approaches will certainly lead to a better understanding of not only the black hole problem, but also some aspects of string theory.

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