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Absorption and Hawking Radiation of Minimal and Fixed Scalars, and AdS/CFT correspondence

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ABSTRACT

We present a complete derivation of absorption cross-section and Hawking radiation of minimal and fixed scalars from the Strominger-Vafa model of five-dimensional black hole, starting right from the moduli space of the D1-D5 brane system. We determine the precise coupling of this moduli space to bulk modes by using the AdS/CFT correspondence. Our methods resolve a long-standing problem regarding emission of fixed scalars. We calculate three-point correlators of operators coupling to the minimal scalars from supergravity and from SCFT, and show that both vanish. We make some observations about how the AdS/SYM correspondence implies a close relation between large N equations of motion of d-dimensional gauge theory and supergravity equations on AdS_{d+1}-type backgrounds. We compare with the explicit nonlocal transform relating 1 and 2 dimensions in the context of $c = 1$ matrix model.

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1 Introduction

In the past few years significant progress has been made in our understanding of black hole physics in terms of string theoretic models [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Out of this, the derivation of black hole entropy from string theory, based on a counting of BPS states [3], is an *ab initio* derivation. The discussion of dynamical issues like absorption and Hawking radiation, however, is based on several plausible assumptions, in particular (a) about the degrees of freedom of the D-brane system, and (b) about how these couple to bulk quanta which appear as Hawking radiation [6, 7, 8]. It cannot be overemphasized that without an *ab initio* derivation of Hawking radiation, there will be lingering doubts about any claimed explanation of black hole thermodynamics and information loss within unitary quantum theory. In this paper we will present an *ab initio* derivation of Hawking radiation/absorption starting from the moduli space of low energy degrees of freedom of the gauge theory describing D1-D5 system. We will explicitly determine the gauge-invariant coupling of this moduli space to minimal and fixed scalars and also construct in detail microcanonical ensembles based on the moduli space leading to gauge-invariant S-matrix elements for absorption/emission. Since the microscopic framework here is gauge theory, calculations based on it are obviously unitary.

The low energy degrees of freedom of a large number of D1 and D5 branes in type IIB string theory compactified on B_4 ($B_4 = T^4$ or $K3$) [17, 11, 18] in a nutshell are as follows. The degrees of freedom of the D1-D5 system can be derived in one of two ways. One is by regarding the D1 branes as instantons on the D5 branes, in which case the degrees of freedom are described in terms of an instanton moduli space [19]. This in turn is described in terms of an $\mathcal{N} = (4,4)$ SCFT (superconformal field theory) based on a resolution of the orbifold $(B_4)^{Q_1Q_5}/S(Q_1Q_5)$ [17]. Here $S(p)$ denotes symmetric group of p elements. The second way is to describe the D1-D5 system in terms of a gauge theory arising out of massless modes of various open strings that connect these branes. The important component of this gauge theory are the hypermultiplets which arise out of the open strings connecting D1 and D5 branes [11]. The low energy degrees of freedom of the system have been explicitly solved (for $B_4 = T^4$) and correspond to the hypermultiplet moduli space given by an $\mathcal{N} = (4,4)$ SCFT based on $(B_4)^{Q_1Q_5}/[S(Q_1) \times S(Q_5)]$ [18]. The two representations in terms of instanton moduli space and the hypermultiplet moduli space are conjectured to be equivalent [20]: it would certainly be worthwhile to understand the equivalence in detail, in particular what the nonrenormalization theorem for the hypermultiplet moduli space implies for the instanton moduli space. This question has a bearing on the issue of extrapolation from weak to strong coupling.

In what follows we will consider the SCFT based on (a resolution of) the orbifold $(T^4)^{Q_1Q_5}/S(Q_1Q_5)$. (It is simple to extend our results to the SCFT with the other quotient group.) We will denote the fields of the SCFT as $x^i_A(z,\bar{z})$, $\psi^{a\alpha}_A(z)$ and $\bar{\psi}^{\dot{a}\dot{\alpha}}_A(\bar{z})$. Here i is the vector index of $SO(4)^I$, the local Lorentz group of the 4-torus, and $A = 1,\ldots,Q_1Q_5$ labels $S(Q_1Q_5)$. Also, $a, \dot{a}$ denote spinor labels of $SO(4)^I \equiv SU(2)^I \times SU(2)^I$, and $\alpha, \dot{\alpha}$ denote spinor labels of the R-parity group $SO(4)^E \equiv SU(2)^E \times SU(2)^E$ of $\mathcal{N} = (4,4)$ SCFT. The superscript E anticipates identification of $SO(4)^E$ later on in supergravity with the isometry group of S^3 which is external to the 4-torus. Besides the x's and ψ's we also have spin fields and twist fields.

The plan of the paper is as follows. In Sec. 2 we describe the absorption and emission of minimal scalars, specifically the traceless symmetric deformation of the metric of the 4-torus. We first determine the SCFT operator coupled to this field using the principle of near-horizon symmetry underlying the AdS/CFT correspondence. We present a detailed discussion on how to determine the normalization of the interaction Lagrangian. We construct gauge-invariant density matrices representing the black hole state and use the above coupling to the bulk fields to calculate S-matrix for absorption and emission. In Sec. 3 we present a calculation of 2- and 3-point amplitudes of the SCFT operators from supergravity as dictated by the quantitative version of the AdS/CFT conjecture. We also calculate these amplitudes directly from SCFT. We show that the two-point functions agree precisely for an appropriate choice of normalization of the interaction Lagrangian, and that the three-point functions vanish in either way of computing them. In Sec. 4 we discuss the absorption/emission of fixed scalars and show how the existing discrepancies between semiclassical and D-brane calculations disappear once the correct coupling to SCFT operators is identified. In Sec. 5 we discuss intermediate scalars. In Sec. 6 we make some general remarks about how the large N equations of motion of gauge theories are related to the equations of supergravity on AdS-type backgrounds through the AdS/CFT correspondence. We also discuss how a similar correspondence is effected in $c = 1$ matrix model through a nonlocal transform between 1 and 2 dimensions which we explicitly present. Sec. 7 contains summary and concluding remarks.

2 Minimal Scalars

The massless spectrum of type IIB string theory compactified on T^4 [1] has 25 scalars: the full spectrum is described in Appendix B. Out of these scalars, five pick up masses when D1 and D5 branes are introduced. The remaining twenty satisfy wave equations appropriate for massless scalars minimally coupled to the metric of the D-brane solution. These are called minimal scalars. We will focus our attention on the familiar example of h_{ij}, the traceless symmetric deformations of the 4-torus.

A crucial ingredient in the D-brane method of computation of absorption cross-section for these scalars or the rate of Hawking radiation is the coupling of h_{ij} to the D-branes. This is given by a specific SCFT operator $\mathcal{O}_{ij}(\vec{z})$ ($\vec{z} = (z, \bar{z})$), which *couples* to the bulk mode h_{ij} in the form of an interaction

$$S_{\rm int} = \mu \int d^2\vec{z}[h_{ij}(\vec{z})\mathcal{O}_{ij}(\vec{z})] \tag{1}$$

where $h_{ij}(\vec{z})$ denotes the restriction of h_{ij} to the location of the SCFT, and μ is a number denoting the strength of the coupling. Below we will discuss, given the operator h_{ij}, first how to determine $\mathcal{O}_{ij}$ and later (below equation (4)) how to determine the constant μ.

2.1 Determination of the operator $\mathcal{O}_{ij}$:

Method 1. One way of determining the operator $\mathcal{O}_{ij}$ would be to reanalyze the instanton moduli space or the hypermultiplet moduli space with the metric of the T^4 deformed by h_{ij}. This method is not very easy and we will not dwell on it any further.

Method 2. A simpler but more elegant approach towards finding the operator $\mathcal{O}_{ij}$ that couples to h_{ij} is by appealing to *symmetries*. This method utilizes the dictionary between symmetries of D-brane world-volume and those of spacetime. The steps are: (a) find the symmetries $\mathcal{S}$ of the bulk, (b) find how (all or a part of) these symmetries appear

[1] This 4-torus is not identical to the one appearing in the SCFT described above, but this subtlety [18, 28] need not concern us here.

in D-brane world-volume and consequently how they act on the variables of the SCFT, (c) find how h_{ij} transforms under $\mathcal{S}$, and (d) demand that $\mathcal{O}_{ij}$ should transform under the *same* representation of the symmetry group $\mathcal{S}$ when it acts on the SCFT. The last step arises from the fact that $h_{ij}(\vec{z})$ in (1) is a source for $\mathcal{O}_{ij}$. The hope is that this procedure fixes $\mathcal{O}_{ij}$. It is clear that the normalization of $\mathcal{O}_{ij}$ would not be determined by these arguments. We will discuss the determination of the normalization, or equivalently that of the constant μ of equation (1), below (4).

It has long been recognized that the symmetries $\mathcal{S}' = SO(4)^I \times SO(4)^E$ of the bulk theory (local Lorentz rotation of the 4-torus and rotation in the transverse space) appear naturally in the SCFT of the D-brane world volume as well. The $SO(4)^I$ part is obvious; $SO(4)^E$ appears as the R-parity group (see [20], *e.g.*). Let us now apply steps (c) and (d) above in the context of this symmetry group $\mathcal{S}'$.

The field h_{ij} (symmetric, traceless) transforms as $(\mathbf{3},\mathbf{3})$ under $SO(4)^I \equiv SU(2)^I \times SU(2)^I$ and as $(\mathbf{1},\mathbf{1})$ under $SO(4)^E \equiv SU(2)^E \times SU(2)^E$.

Now there are at least three possible SCFT operators which belong to the above representation of $\mathcal{S}'$:

$$\begin{aligned}
\mathcal{O}_{ij} &= \partial x^i_A \bar{\partial} x^j_A \\
\mathcal{O}'_{ij} &= \psi^\alpha_{aA}(z)\sigma_i^{a\dot{a}}\bar{\psi}^{\dot{\alpha}}_{\dot{a}B}(\bar{z})\psi_{\alpha,bA}(z)\sigma_j^{b\dot{b}}\bar{\psi}_{\dot{\alpha},\dot{b}B}(\bar{z}) \\
\mathcal{O}''_{ij} &= \psi^\alpha_{aA}(z)\sigma_i^{a\dot{a}}\bar{\psi}^{\dot{\alpha}}_{\dot{a}A}(\bar{z})\psi_{\alpha,bB}(z)\sigma_j^{b\dot{b}}\bar{\psi}_{\dot{\alpha},\dot{b}B}(\bar{z})
\end{aligned} \tag{2}$$

The spinor labels are raised/lowered above using the $\epsilon^{\alpha\beta}, \epsilon_{\alpha\beta}$ symbol. The σ_i's denote the matrices : $(1, i\vec{\tau})$. The last two operators differ only in the way the $S(Q_1Q_5)$ labels are contracted. All the three operators should be regarded as symmetric (in (i,j)) and traceless.

The complete list of operators with the same transformation property under $\mathcal{S}'$ contains, in addition, those obtained by multiplying any of the above by singlets. These would necessarily be irrelevant operators, but cannot be ruled out purely by the above symmetries.

It might seem 'obvious' that the operator $\mathcal{O}_{ij}$ should be the right one to couple to the bulk field h_{ij}. However, the simplest guesses can sometimes lead to wrong answers, as we will see later for fixed scalars, where it will turn out that the operator $\partial x^i_A \bar{\partial} x^i_A$ is far from being the right one to couple to h_{ii} (trace). We proceed, therefore, to find out the right operator, by sticking to the principle stated in Method 2 above.

2.2 Incorporation of Near-horizon Symmetry

It has been conjectured recently [21, 22, 23] that if one takes the large gQ ($Q = Q_1, Q_5$) limit, then a powerful correspondence can be built between the physics of the bulk and the physics of the boundary. This has many qualitative and quantitative consequences. For the limited purpose of identifying the SCFT operator, it is enough to use only the most basic part of the conjecture which says that going to the large gQ limit leads to an enhancement of the symmetry. Since the 'proof' of this part is obvious, we will accept results based on this as derived from first principles.

The discussion below has overlap with a number of recent works [24, 25, 26, 27, 28]. These papers, especially the work of de Boer [27], provide the background for many results of the present paper. We have found it useful to carry out a detailed extension of de Boer's method to the low supergravity modes, especially in the case of T^4, which we have included in Appendix B (see also [26]).

In the large gQ for the present system the symmetry group $\mathcal{S}'$ is enhanced to $\mathcal{S} = SO(4)^I \times SO(4)^E \times SU(1,1|2) \times SU(1,1|2)$.

From the spacetime point of view, this happens because in this limit the spacetime geometry is effectively the *near-horizon geometry* of a D1-D5 system (wrapped on $T^4 \times S^1$) which is $AdS_3 \times S^3 \times T^4$ (with x^5 periodic) with appropriate values of the RR two-form field B' (see Appendix A for details)[2]. The factor $SO(4)^I$ acts as before. The group $SO(4)^E$ corresponds in the new picture to the isometry group of S^3. The bosonic part of $SU(1,1|2) \times SU(1,1|2)$ arises as the isometry group of AdS_3 (which is the $SL(2,R)$ group manifold). The $SU(2)$ part which transform the fermions among themselves, and the off-diagonal supersymmetry transformations, are a consequence of $\mathcal{N} = (4,4)$ supersymmetry

[2]The near-horizon geometry of the finite-area black hole actually involves a BTZ black hole ($\times S^3 \times T^4$) which corresponds to the Ramond sector of the SCFT, whereas AdS_3 corresponds to the NS sector. However, the local operators of interest here can be shown to have the same symmetry properties irrespective of whether they belong to the Ramond or the NS sector ([25, 28], see also remarks at the end of Appendix B). The ground state of the Ramond sector is degenerate, as against that of the NS sector; this degeneracy would be reflected in our construction of the black hole state (11) — however, this would not affect the S-matrix relevant for absorption and emission. We consider below the NS sector, corresponding to AdS_3, but for the specific purposes of the paper the discussion is equally applicable to the Ramond sector.

of this compactification. On the SCFT side, the $SO(4)$ groups have actions as before. The $SU(1,1|2)$ is identified with the subgroup of the superconformal algebra generated by $L_{\pm 1,0}, G^{a\alpha}_{\pm 1/2}$ (the other $SU(1,1|2)$ involves $\bar{L}, \bar{G}$).

Let us now apply steps (c) and (d) of Method 2 to this enhanced symmetry group $\mathcal{S}$. How does h_{ij} transform under $SU(1,1|2)$? From the fact that h_{ik} transforms as $(\mathbf{1},\mathbf{1})$ of $SO(4)^E$ we can deduce that its various KK (Kaluza-Klein) modes $h_{ik}^{j,j'}$ will obey the restriction $j = j'$ (see Appendix B for details). If we restrict ourselves for the present to s-waves we have $j = j' = 0$. Now since the h_{ij} is a massless (minimal) scalar, it corresponds to $(L_0, \bar{L}_0) = (1,1)$ where by L's here we mean $SU(1,1)$ generators in the bulk (*cf.*, [25]). Since h_{ij} creates single-particle excitations, let us classify it as a short multiplet (more on this later). Looking at the list (Appendix B) of short multiplets, we find that there is only one short multiplet of $SU(1,1|2)$ which contains the field $j = 0, L_0 = 1$: viz. $(\mathbf{2},\mathbf{2})_S$ of $SU(1,1|2) \times SU(1,1|2)$ (see Appendix B for notation). It is important to note that the $(j = 0, L_0 = 1)$ field occurs as the 'top' component (killed by $G_{-1/2}$, and not by $G_{1/2}$) of that supermultiplet.

According to step (d) we now look for a SCFT operator $\mathcal{O}_{ij}$ which is the top component of a $(\mathbf{2},\mathbf{2})_S$ short supermultiplet of $SU(1,1|2) \times SU(1,1|2)$ and also has $(h,\bar{h}) = (1,1)$. We need to find which of the operators $\mathcal{O}_{ij}, \mathcal{O}'_{ij}$ and $\mathcal{O}''_{ij}$ has this property. Note that operators obtained by multiplying with nontrivial $\mathcal{S}'$ singlets does not have $(h,\bar{h}) = (1,1)$. Now, it is easy to see that only $\mathcal{O}_{ij}$ is killed by the 'raising' operators $G_{-1/2}$. Coupled with the fact that it has $j = 0, L_0 = 1$ (and similar equation for the antiholomorphic sector), it matches the transformation property of h_{ij} completely. The other two operators are killed by the lowering operators $G_{1/2}$. So they are 'bottom' components of a supermultiplet. Now, bottom components of all $SU(1,1|2)$ supermultiplets have $j = L_0$. Since both $\mathcal{O}'_{ij}$ and $\mathcal{O}''_{ij}$ have $j = 0 < L_0 = 1$, they cannot be the right operators to couple to h_{ij}.

Hence, we find that $\mathcal{O}_{ij}$ is indeed the right operator to couple to h_{ij} in Equation (1).

This choice was independently arrived at in [18] from their analysis of the hypermultiplet moduli space. The variable x^i_A was denoted there as $y^i_{aa'}$ where a, a' are $S(Q_1)$ and $S(Q_5)$ indices respectively. This operator also appears in [24].

Note that this derivation assumes that h_{ij} (like other fields in the supergravity spectrum, Appendix B) should belong to short multiplets. This assumption is vindicated by the complete accounting of *all* KK modes on S^3 in terms of short multiplets, as we show on Appendix B (see [27] for a more detailed discussion of this issue).

2.3 Absorption and Hawking Radiation:

With the above result in hand, we can now use (1) to perform a D-brane computation of absorption cross-section and Hawking radiation for minimal scalars.

Instead of detailing the entire computation we will emphasize the essential conceptual differences from earlier works [6, 7]. From the above discussion, the interaction Lagrangian is

$$S_{\text{int}} = \mu T_{\text{eff}} \int d^2z \, \left[h_{ij} \partial_z x^i_A \partial_{\bar{z}} x^j_A \right] \tag{3}$$

The effective string tension T_{eff} of the conformal field theory , which also appears in the free part of the action

$$S_0 = T_{\text{eff}} \int d^2z \, \left[\partial_z x^i_A \partial_{\bar{z}} x_{i,A} + \text{fermions} \right] \tag{4}$$

has been discussed in [9, 12, 18]. The specific value of this is not important for the calculation of the S-matrix for absorption or emission, since the factor cancels in the S-matrix between the interaction Lagrangian and the external leg factors. However, the value of μ *is important* to determine since the absorption crosssection and Hawking radiation rates calculated from the SCFT depend on it. We will argue below that $\mu = 1$.

Determination of the normalization constant μ in S_{int}:

A direct string theory computation, as in Method 1 mentioned in the context of determining the operator $\mathcal{O}_{ij}$, would of course provide the constant μ as well (albeit at weak coupling perhaps). This would be analogous to fixing the normalization of the Dirac-Born-Infeld action for a single D-brane by comparing with a one-loop open string diagram [30]. However, for a large number, and more than one type, of D-branes it is a very difficult proposition and we will not attempt to pursue it here. Fortunately, as in Method 2 for determining the operator $\mathcal{O}_{ij}$, the AdS/CFT correspondence helps us determine the value of μ as well. For the latter, however, we need to use the more quantitative version [23, 22] of the conjecture. In Section 3, we will see that for this quantitative conjecture to be true for the two-point function (which can be calculated independently from the $\mathcal{N} = 4$ SCFT and from supergravity) we need $\mu = 1$.

We will see below that the above normalization leads to precise equality between the absorption cross-sections (and consequently Hawking radiation rates) computed from the moduli space of the D1-D5 system and from semiclassical gravity. This method of fixing the normalization can perhaps be criticized on the ground that it borrows from supergravity and does not rely entirely on the SCFT. However, we would like to emphasize two things:

(a) We have fixed $\mu = 1$ by comparing with supergravity around AdS_3 background which does *not* have a black hole. On the other hand, the supergravity calculation of absorption cross-section and Hawking flux is performed around a *black hole background* represented in the near-horizon limit by the BTZ black hole. From the viewpoint of semiclassical gravity these two backgrounds are rather different. The fact that normalizing μ with respect to the former background leads to the correctly normalized absorption cross-section around the black hole background is a rather remarkable prediction.

(b) Similar issues are involved in fixing the coupling constant between the electron and the electromagnetic field in the semiclassical theory of radiation in terms of the physical electric charge, and in similarly fixing the gravitational coupling of extended objects in terms of Newton's constant. These issues too are decided by comparing two-point functions of currents with Coulomb's or Newton's laws respectively. In the present case the quantitative version of the AdS/CFT conjecture [23, 22] provides the counterpart of Newton's law or Coulomb's law at strong coupling. Without this the best result one can achieve is that the Hawking radiation rates computed from D1-D5 branes and from semiclassical gravity are *proportional*.

We should remark that fixing the normalization by the use of Dirac-Born-Infeld action, as has been done previously, is not satisfactory since the DBI action is meant for single D-branes and extending it to a system of multiple D1-D5 branes does not always give the right results as we will see in the section on fixed scalars. The method of equivalence principle to fix the normalization is not very general and cannot be applied to the case of non-minimal scalars, for example.

With these comments, we now go back to the calculation of the S-matrix.

The black hole is represented by a density matrix

$$\rho = \frac{1}{\Omega} \sum_{\{i\}} |i\rangle\langle i| \tag{5}$$

This is the same as Equation (9) of [6]. However, the states $|i\rangle$ now represent gauge-invariant states (invariant under $S(Q_1Q_5)$) from all possible twisted sectors of the orbifold SCFT.

The explicit formula for these states $|i\rangle$ for an arbitrary twisted sector is somewhat involved. Since the maximally twisted sector, defined by the permutation element

$$g : x^i_A \to x^i_{A+1}, \tag{6}$$

has dominant contribution (*cf.* [18]) to the density matrix (5), let us write out the gauge-invariant states $|i\rangle$ for this sector. The variables $x^i_A(z,\bar z)$ belonging to this sector satisfy:

$$x^i_A(\sigma + 2\pi, t) = g(x^i_A)(\sigma, t) \equiv x^i_{A+1}(\sigma, t) \tag{7}$$

In the above we define $A+1 \equiv 1$ when $A = Q_1Q_5$. Similar equations hold for the fermions.

Let us define a periodic variable $\tilde x^i(\sigma, t)$ on a larger circle $\sigma \in [0, 2\pi Q_1Q_5)$ [29, 31] by

$$\tilde x^i(\sigma + 2\pi(A-1), t) \equiv x^i_A(\sigma, t), \sigma \in [0, 2\pi) \tag{8}$$

which will have a normal mode expansion:

$$\tilde x^i(\sigma, t) = (4\pi T_{\text{eff}})^{-1/2} \sum_{n>0} \left[\left(\frac{a^i_n}{\sqrt{n}} e^{in(-t+\sigma)/Q_1Q_5} + \frac{\tilde a^i_n}{\sqrt{n}} e^{in(-t-\sigma)/Q_1Q_5} \right) + \text{h.c.} \right] \tag{9}$$

The twist (6) acts on these oscillators as

$$\begin{aligned} g : a^i_n &\to a^i_n e^{2\pi i n/Q_1Q_5} \\ g : \tilde a^i_n &\to \tilde a^i_n e^{-2\pi i n/Q_1Q_5} \end{aligned} \tag{10}$$

The states $|i\rangle$ are now defined as

$$|i\rangle = \prod_{n=1}^{\infty} \prod_i C(n,i) (a^{i\dagger}_n)^{N^i_{L,n}} (\tilde a^{i\dagger}_n)^{N^i_{R,n}} |0\rangle \tag{11}$$

where $C(n,i)$ are normalization constants ensuring unit norm of the state (*cf.* Equation (3) of [6], which used some given polarization index i). $|0\rangle$ represents the NS ground state. As explained in the footnote on page 6, the present discussion is also valid in the Ramond sector, in which case the ground state will have an additional spinor index but will not affect the S-matrix. We have also suppressed the fermion creation operators which also do not affect the S-matrix.

It is clear that the creation operators create KK momentum along S^1 (parametrized by x^5). The total left (right) moving KK momentum of (11) (in units of $1/\tilde{R}$, $\tilde{R} \equiv Q_1 Q_5 R_5$, R_5 being the radius of the S^1) is N_L (N_R), where

$$N_L = \sum_{n,i} n N^i_{L,n}, \quad N_R = \sum_{n,i} n N^i_{R,n} \tag{12}$$

From (10) and (11), we see that

$$g : |i\rangle \to \exp[\frac{2\pi i}{Q_1 Q_5}(N_L - N_R)|i\rangle \tag{13}$$

Now, the total KK momentum carried by $|i\rangle$ is $p_5 = (N_L - N_R)/(Q_1 Q_5 R_5)$. Quantization of the KK charge requires that $p_5 = \text{integer}/R_5$, which implies that

$$(N_L - N_R)/(Q_1 Q_5) = \text{integer} \tag{14}$$

Thus, using (13) and the above equation, we find the states $|i\rangle$ representing microstates of the black hole to be gauge invariant[3].

The rest of the calculation now follows formally along the lines of [6, 7] and the final results obtained are the same, thus establishing the agreement between the D-brane calculation and the semiclassical calculation.

The present discussion provides, in our perception for the first time, a complete derivation of absorption and Hawking radiation from the five-dimensional black hole.

[3]In (12) we have written down only the bosonic contribution to the KK momentum. If we wrote the total contribution of bosons and fermions, then taking into account an identical gauge transformation property for the fermionic oscillators as in (10) we would arrive at the same conclusion as above, *viz*, that the state $|i\rangle$ is gauge invariant.

3 Two- and Three-point Functions of Minimal Scalars

In this section we will discuss the more quantitative version of the AdS/CFT conjecture [22, 23] to compare 2- and 3-point correlation functions of $\mathcal{O}_{ij}$ from supergravity and from SCFT respectively.

The relation between the correlators are as follows. Let the supergravity Lagrangian be

$$
\begin{aligned}
L &= \int d^3x_1 d^3x_2 b_{ij,i'j'}(x_1,x_2) h_{ij}(x_1) h_{i'j'}(x_2) \\
&+ \int d^3x_1 d^3x_2 d^3x_3 c_{ij,i'j',i''j''}(x_1,x_2,x_3) h_{ij}(x_1) h_{i'j'}(x_2) h_{i''j''}(x_3) + \ldots
\end{aligned} \tag{15}
$$

where we have only exhibited terms quadratic and cubic in the h_{ij}'s. The coefficient b determines the propagator and the coefficient c is the tree-level 3-point vertex in supergravity. b and c can be read out from Appendix A.

The 2-and 3-point functions of the $\mathcal{O}_{ij}$'s (at large gQ) are then given by [22, 23], assuming an $S_{\rm int}$ given by (3)

$$
\begin{aligned}
&\langle \mathcal{O}_{ij}(z_1)\mathcal{O}_{i'j'}(z_2)\rangle \qquad (16)\\
&= 2(\mu T_{\rm eff})^{-2} \int d^3x_1 d^3x_2 \left[b_{ij,i'j'}(x_1,x_2) K(x_1|z_1) K(x_2|z_2) \right],
\end{aligned}
$$

$$
\begin{aligned}
&\langle \mathcal{O}_{ij}(z_1)\mathcal{O}_{i'j'}(z_2)\mathcal{O}_{i''j''}(z_3)\rangle \qquad (17)\\
&= 3!(\mu T_{\rm eff})^{-3} \int d^3x_1 d^3x_2 d^3x_3 \left[c_{ij,i'j',i''j''}(x_1,x_2,x_3) K(x_1|z_1) K(x_2|z_2) K(x_3|z_3) \right]
\end{aligned}
$$

where K is the boundary-to-bulk Green's function for massless scalars [23]

$$
K(x|z) = \frac{1}{\pi} \left[\frac{x_0}{(x_0^2 + (|z_x - z|^2)} \right]^2 \tag{18}
$$

We use complex z for coordinates of the CFT, and $x = (x_0, z_x)$ for the (Poincare coordinates) of bulk theory.

<u>Two-point function:</u>

The right hand side of (16) can be evaluated using equation (17) in [32], with $\eta = Q_1Q_5/(8\pi)$ in our case, where we have been careful to convert to complex coordinates at the boundary. We find that

$$\langle \mathcal{O}_{ij}(z)\mathcal{O}_{i'j'}(w)\rangle = (\mu T_{\rm eff})^{-2}\delta_{ii'}\delta_{jj'}\frac{Q_1Q_5}{16\pi^2}\frac{1}{|z-w|^4} \tag{19}$$

This is exactly the value of the two-point function obtained from the SCFT described by the free Lagrangian (4) provided we put $\mu = 1$. It is remarkable that even at strong coupling the two-point function of $\mathcal{O}_{ij}$ can be computed from the free Lagrangian (4). This is consistent with the nonrenormalization theorems involving the $\mathcal{N} = 4$ SCFT.

The choice $\mu = 1$ ensures that the perturbation (3) of (4) is consistent with the perturbation implied in (31). We have already remarked in Section 2.3 that this choice leads to precise equality between absorption cross-sections (consequently Hawking radiation rates) calculated from semiclassical gravity and from the D1-D5 branes.

Three-point function:

Before proceeding to evaluate (17), let us pause to see what a tree-level CFT calculation of the three-point correlator of the $\mathcal{O}_{ij}$'s gives. It is straightforward to see that the three-point function vanishes:

$$\langle \mathcal{O}_{ij}(\vec{z}_1)\mathcal{O}_{i'j'}(\vec{z}_2)\mathcal{O}_{i''j''}(\vec{z}_3)\rangle = 0 \tag{20}$$

The reason is simply that the correlator splits into a holomorphic and an antiholomorphic factor, and in each of them there are an odd number of x's. Therefore each factor vanishes (ignoring possible contact terms throughout). Interestingly, the three-point function of both $\mathcal{O}'_{ij}$ and $\mathcal{O}''_{ij}$ are nonvanishing.

The vertex factor c, as seen from Appendix A, is clearly non-vanishing. So (20) seems to be at variance with the fact that the vertex factor c in (17) is nonzero. Before going to ascribe the difference to strong and weak coupling, let us evaluate the r.h.s. of (17).

Using the list of integrals in [32] (eqs. 19,20,25,29) we find, rather remarkably, that (17) gives a zero answer too! Note that there are two surprises leading to this answer: (a) the vanishing of the integral in (17), as just mentioned, and (b) the vanishing of the coefficient of the cubic term coupled to RR backgrounds; if this coefficient were not to vanish, the corresponding integral in (17) would have been different from the one coming from terms coupled to the metric, and would have been nonzero.

Thus, the three-point function calculated from the CFT (ostensibly at weak coupling) and that calculated from the supergravity agree, and both vanish!

4 Fixed Scalars

Out of the 25 scalars mentioned earlier which form part of the spectrum of IIB supergravity on T^4, five become massive when further compactified on $AdS_3 \times S^3$. There is an important additional scalar field which appears after this compactification: h_{55}. Our notation for coordinates here is as follows: $AdS_3 : (x^0, x^5, r), S^3 : (\chi, \theta, \phi); T^4 : (x^6, x^7, x^8, x^9)$. r, χ, θ, ϕ are spherical polar coordinates for the directions x^1, x^2, x^3, x^4. In terms of the D-brane wrappings, the D5 branes are wrapped along the directions 5-9 and D1 branes are wrapped along direction 5. The field h_{55} is scalar in the sense that it is a scalar under the local Lorentz group $SO(3)$ of S^3.

In what follows we will specifically consider the three scalars ϕ_{10}, h_{ii} and h_{55}. The equations of motion of these fields in supergravity are coupled and have been discussed in detail in the literature [9, 10]. It turns out that a linear combination of h_{ii} and ϕ_{10} remains massless; it is part of the twenty massless (minimal) scalars previously discussed. Two other linear combinations λ and ν satisfy coupled differential equations. These are examples of what are called 'fixed scalars'.

Understanding the absorption and emission properties of fixed scalars is an important problem, because the D-brane computation and semiclassical black hole calculation of these properties appear to be at variance [10]. The discrepancy essentially originates from the 'expected' couplings of λ and ν to SCFT operators with $(h, \bar{h}) = (1, 3)$ and $(3, 1)$. These SCFT operators lead to qualitatively different greybody factors from what the fixed scalars exhibit semiclassically[4]. The semiclassical greybody factors are in agreement with D-brane computations if the couplings were only to (2,2) operators. The precise agreement depends on the normalization of this coupling which is given in terms of the effective string tension, discussed at length in [9, 18][5].

The coupling to $(1, 3)$ and $(3, 1)$ operators is guessed from qualitative reasoning based on the Dirac-Born-Infeld action. Since we now have a method of *deducing* the couplings $\int \phi \mathcal{O}$ based on near-horizon symmetries, let us use it to the case of the fixed scalars.

The steps are similar to the case of the minimal scalars, so we will be brief. It is obvious

[4]For earlier attempts at understanding this difference see [34].

[5]The normalization could also be 'determined' by demanding [35] that the two-point functions in the bulk and at the boundary agree in the sense of [23, 22].

that the fixed scalars transform as $(\mathbf{1},\mathbf{1})$ of $SO(4)^E$ (and of $SO(4)^I$ as well, although that will not play any role). Furthermore, the equations of motion for λ and ν decouple in the near-horizon limit (this point has not been emphasized much in the literature), and each corresponds to a massive scalar field with mass $m^2 = 8/R^2$ (where R is the radius of curvature of AdS_3 defined in Appendix A). By scale invariance of the interaction term $\int \lambda \mathcal{O}$ we deduce that $h + \bar{h} = 4$ (similarly for ν)[6]. This is of course compatible with $(h, \bar{h}) = (2, 2), (1, 3), (3, 1), (0, 4)$ and $(4, 0)$ (note that for $h + \bar{h} = 0$ we automatically have $h = \bar{h} = 0$). As before, we now see which of these values, together with $(j, \bar{j}) = (0, 0)$, occur, in short multiplets of $SU(1, 1|2)$. We find that $(h, \bar{h}) = (2, 2)$ is the only choice.

We also find that the fixed scalars belong to the short multiplet $(\mathbf{3},\mathbf{3})_S$ of $SU(1, 1|2) \times SU(1, 1|2)$. This, together with the fact that $(h, \bar{h}) = (2, 2)$ occurs as the 'top' component of the supermultiplet, leads to only two SCFT operators

$$\begin{aligned} \mathcal{O}_1 &= \partial x^i_A \partial x^i_A \bar{\partial} x^j_B \bar{\partial} x^j_B \\ \mathcal{O}_2 &= \partial x^i_A \partial x^j_A \bar{\partial} x^i_B \bar{\partial} x^j_B \end{aligned} \tag{21}$$

corresponding to the two bulk fields λ and ν. Which specific linear combinations of these couple to the two fields respectively, remains undetermined at this stage, but the D-brane calculation for absorption/emission using either leads to the same result[7]. This accords with the semiclassical calculations since λ and ν satisfy identical differential equation, leading to the same absorption/emission properties. We emphasize that in this analysis too, we have assumed that the fixed scalars should form short multiplets. This assumption is amply justified in Appendix B, where all the KK modes on S^3 are correctly classified as a result of this assumption (*cf.* [27]).

In summary, since the $(1, 3)$ *and* $(3, 1)$ *operators are ruled out by our analysis, the discrepancy between the D-brane calculation and the semiclassical calculation of absorption*

[6] If we had assumed, as in [25], that these fields are Virasoro primaries, then we could have read off the spins as well; however, we can assume this generally only for 'bottom' components.

[7] The appearance of the $S(Q_1Q_5)$ indices A, B introduces considerable subtlety into the D-brane calculations. As in the case of minimal scalars, the dominant contribution comes from the maximally twisted sector; so one can again introduce the extended variable $\tilde{x}^i(\sigma)$. However, both $\mathcal{O}_1$ and $\mathcal{O}_2$ contain products of operators which are generally at two different points σ, σ' of the circle. The calculation of the S-matrix can still be carried out and it can be shown that the absorption cross-section or rate of Hawking radiation is not changed by this to leading order in frequency ω of the bulk quantum.

and emission rates disappears. It is important to note here that couplings guessed from reasoning based on Dirac-Born-Infeld action turn out to be incorrect.

5 Intermediate Scalars

We just make the remark that the classification presented in Appendix B correctly account for all sixteen intermediate scalars, and predict that they should couple to SCFT operators with $(h, \bar{h}) = (1, 2)$ belonging to the short multiplet $(\mathbf{2}, \mathbf{3})_S$ or operators with $(h, \bar{h}) = (2, 1)$ belonging to the short multiplet $(\mathbf{3}, \mathbf{2})_S$. This agrees with the 'phenomenological' prediction made earlier in the literature [36].

6 Large N classical equations of motion of gauge theories

In previous sections, the superconformal field theory arose from a large N gauge theory (in either description of the moduli space). The aspect of large N that was used there was that in the large N (more precisely large gQ) limit, the symmetries of the gauge theory and those of the supergravity solution could be identified. The precise role of large N in the gauge theory as such was not used. In this section we make some remarks concerning this issue. In particular, we discuss elements of large N classical dynamics of gauge theories are encoded in AdS spacetimes through the AdS/SYM correspondence and also discuss how a similar correspondence appears in $c = 1$ matrix model.

One of the most important realizations that came out of the study of the large N limit of field theories (including gauge theories) is the fact that the large N limit can be formulated as a systematic semiclassical expansion in 1/N. The theory is formulated in terms of appropriate operators which satisfy the factorization condition at large N: $\langle X^2 \rangle = \langle X \rangle^2 + o(1/N)$ [37]. For example in gauge theories such operators are $(1/N)\mathrm{tr}\, W(C)^n$ ($W(C)$ is the Wilson loop operator). In the c=1 matrix model we have the collective fields $(1/N)\mathrm{tr}\, \exp(ikM)$. These quantities (or appropriate variables constructed out of them, such as the density variable $\rho(\lambda, t)$ or fermion bilinear $\psi(x, t)\psi^\dagger(y, t)$ of $c = 1$ models [38]) satisfy a set of classical equations of motion valid at $N = \infty$. Let us denote such equations, generically as

$$\mathcal{F}(\Phi) = 0 \tag{22}$$

The classical solution Φ_0 of these equations of motion is called the 'master field'. Fluctuations around this solution, defined by

$$\Phi = \Phi_0 + \frac{1}{\sqrt{N}}\delta\Phi \tag{23}$$

satisfy linear equations (to $o(1)$)

$$\left.\frac{\partial\mathcal{F}}{\partial\Phi}\right|_0 \delta\Phi = 0 \tag{24}$$

giving the spectrum of the theory at large N. The $o(1/N)$ and higher terms involve $\partial^2\mathcal{F}/(\partial\Phi)^2|_0$ etc. represent interaction. A simple example of such a procedure can be found in [40] where 2-dimensional QCD is solved in terms of fermion bilinears whose 'master field' is presented explicitly.

We will illustrate how the AdS/SYM correspondence at large $\lambda = gN$ essentially determines the various coefficients in the Taylor series expansion of $\mathcal{F}$ around Φ_0 except the solution Φ_0 itself. Let us consider the example [41] of the confining phase of the d=3 YM theory at large N and large λ. This is *dual* to the AdS Schwarzschild black hole (X_2 in the notation of [41]). An analysis of the solution of the scalar wave equation indicates an asymptotic solution given by

$$\phi(\rho, x) \sim e^{ik.x}/\rho^4, \ \rho \to \infty \tag{25}$$

where x denotes coordinates on the boundary ($\rho \to \infty$), assumed Euclidean. An analysis of the full solutions $\phi(\rho, x)$ shows that normalizable solutions occur only at discrete values $k^2 = -m_n^2 < 0$. Any of these solutions $\phi_n(\rho, x)$, therefore, leads to a wave on the boundary satisfying the equation

$$(-\partial_t^2 + \partial_{\mathbf{x}}^2 + m_n^2)\psi_n(x) = 0 \tag{26}$$

where we have Wick rotated the equation to Lorentzian signature to emphasize that this corresponds to a physical particle. Indeed, this represents a scalar glueball of mass m_n.

Equation (26) can be regarded as the equation for the expectation value of a small Wilson loop in 2-dimensions. In the light of the remarks made in the beginning of this section, this equation is the linear equation (24) for fluctuations around the large-N classical solution. In terms of a Lagrangian formulation, this tells us about the quadratic fluctuation operator around the classical solution, for instance that the eigenvalues of this operator are discrete and calculable from supergravity [42, 43]. More detailed information about this operator can be obtained by computing the two-point function of various tr F^{2n} operators from the AdS supergravity and looking at their spectral distributions.

If we carry on to compute the various n-point correlations from AdS supergravity, we can reconstruct the various orders of the large N equation around the classical solution. It is interesting to note that the classical solution itself cannot be obtained by this method. It is tempting to think that the knowledge of this solution must be tied to the choice of the specific solution of supergravity.

$c = 1$ matrix model:[8]

It is clear that in the above example the knowledge of the spectrum in the gauge theory is tied to the differential equation in the AdS background. Alternatively it is related to the bulk-to-boundary Green's function (*cf.* (18)) in this particular background. Such a Green's function relating a d-dimensional physics to a $d+1$ dimensional physics is already familiar from our study of [39]. We refer the reader for detail to these references, but the essential point is this:

The $c = 1$ matrix model is a theory of one-dimensional $N \times N$ matrices $M_{ij}(t)$ (much like a 'gauge theory' in $d = 1$ would be). Variables of this theory at large N can be related by nonlinear nonlocal transforms to 'spacetime' fields living in 1+1 dimensions (related to the 'tachyon' of two-dimensional string theory). The specific transform is given in the form

$$\begin{aligned} \mathcal{T}(x,t) &= \sum_{p,q} G_{1,p,q}\delta u_{p,q}(t) + \ldots \\ \delta u_{p,q}(t) &= u_{p,q}(t) - u_{0,p,q}(t) \\ u_{p,q}(t) &= \int d\lambda e^{-ip\lambda}\psi^\dagger(q-\lambda/2,t)\psi(q+\lambda/2,t) \end{aligned} \tag{27}$$

Here $\psi(\lambda,t)$ denotes the fermion field of the $c = 1$ matrix model [39]. The ellipsis denotes non-linear terms in the transform (*cf.* Eqn. (3.4) of [44]). The exact details of these equations are given in [44, 45]. Just like in the gauge theory example discussed above, the equations of motion for the master field $u_{p,q}(t)$ (called $u(p,q,t)$ in the references just mentioned) in the 1-dimensional theory get related to the equations for the 1+1

[8]This section has been developed in collaboration with Avinash Dhar.

dimensional fields. Also like above, the interactions of the one-dimensional fields are related to those of the two-dimensional fields $\mathcal{T}(x,t)$ through this transform.

This indeed represents a holographic realization of $c = 1$ matrix model[9], except that a geometric interpretation of the nonlocal transform in (27) is not available. Hopefully we will be able to report on this on another occasion.

7 Conclusion

(a) We presented a complete derivation of absorption cross-section and Hawking radiation of minimal and fixed scalars in the D-brane picture from the five-dimensional black holes starting from the moduli space of the D1-D5 brane system. In this we have deduced specific CFT operators coupling to these bulk modes by demanding that the coupling should respect the symmetries of the theory. These symmetries in the limit of large gQ should include the symmetries of the near-horizon geometry, as emphasized by the AdS/CFT correspondence. In addition to finding the right CFT operator, we construct a gauge-invariant microcanonical ensemble in the Hilbert space of the orbifold CFT and calculate gauge-invariant S-matrix elements which agree with the semiclassical result.

(b) We determine the normalization of the interaction Lagrangian which couples CFT operators to bulk modes by using the quantitative version of the AdS/CFT correspondence, where we compare two-point functions computed from CFT and from supergravity around AdS_3 background. The normalization fixed this way remarkably leads to precise equality of absorption cross-sections (consequently Hawking radiation rates) computed from CFT and from supergravity around the black hole background.

(c) We have computed 2- and 3-point functions of CFT operators corresponding to minimal scalars (metric fluctuations of the torus) from tree-level supergravity and by a direct CFT calculation. We found that with appropriate choice of normalization of the interaction Larangian (mentioned above) the two-point functions agree precisely. The 3-point functions also agree and they both vanish.

(d) We have settled a long-standing problem in the context of fixed scalars by showing that consistency with near-horizon symmetry demands that they cannot couple to (1,3) or (3,1) operators. They can only couple to (2,2) operators. This removes earlier dis-

[9] While this paper was being completed, we received [46] which mentions related issues.

crepancies between D-brane calculations and semiclassical calculations of absorption and emission.

(e) We present a detailed and explicit classification in Appendix B of KK modes of IIB supergravity multiplets on $AdS_3 \times S^3 \times T^4$ in terms of short multiplets of $SU(1,1|2) \times SU(1,1|2)$ (see also [26]).

(f) We have given an interpretation of the AdS/CFT (more generally AdS/SYM) correspondence which relates supergravity equations on AdS-type backgrounds to large N equations of gauge theory.

(g) We have discussed the explicit nonlocal transform between the one-dimensional matrix model ($c = 1$) and two-dimensional field theory in the language of holography.

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A The Supergravity Equations

We begin with the bosonic sector of Type IIB supergravity. The Lagrangian is (we follow the conventions of [47])

$$
\begin{aligned}
I &= I_{\rm NS} + I_{\rm RR} \\
I_{\rm NS} &= -\frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G}\left[e^{-2\phi}\left(R - 4(d\phi)^2 + \frac{1}{12}(dB)^2\right)\right] \\
I_{\rm RR} &= -\frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G}\left(\sum_{n=3,7}\frac{1}{2n!}(H^n)^2\right)
\end{aligned}
\tag{28}
$$

We use $\hat{M}, \hat{N} \dots$ to denote 10 dimensional indices, $i, j, \dots$ to denote coordinates on the torus T^4, $M, N \dots$ to denote the remaining 6 dimensions and $\mu, \nu, \dots$ to denote coordinates on the AdS$_3$. $k_{10}^2 = 64\pi^7 g^2$ (we use $\alpha' = 1$). We have separately indicated the terms depending on NS-NS and RR backgrounds.

Our aim will be to obtain the Lagrangian of the minimally coupled scalars in the D1-D5-brane system. We will find the Lagrangian up to cubic order in the near horizon limit. Let us first focus on $I_{\rm NS}$.

The solution of the supergravity equations for the D1-D5 system in the string metric is the following (see, e.g., [12] whose notations are used below)

$$\begin{aligned} ds^2 &= f_1^{-\frac{1}{2}} f_5^{-\frac{1}{2}}(-dt^2 + dx_5^2) + f_1^{\frac{1}{2}} f_5^{\frac{1}{2}}(dx_1^2 + \cdots + dx_4^2) \\ &\quad + f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}}(dx_6^2 + \cdots + dx_9^2), \\ e^{-2(\phi_{10}-\phi_\infty)} &= f_5 f_1^{-1}, \\ B'_{05} &= \frac{1}{2}(f_1^{-1} - 1), \\ H'_{abc} &= (dB')_{abc} = \frac{1}{2}\epsilon_{abcd}\partial_d f_5, \quad a,b,c,d = 1,2,3,4 \end{aligned} \tag{29}$$

Where we have substituted $N = 0$ in the solutions given in [12] and f_1 and f_2 are given by

$$f_1 = 1 + \frac{c_1 Q_1}{r^2}, \quad f_5 = 1 + \frac{c_5 Q_5}{r^2} \tag{30}$$

Here $r^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2, c_1 = 16\pi^4 g/V_4, c_5 = g$. We now substitute the above values of the fields in the the Type IIB Lagrangian with the following change in the metric

$$f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}}\delta_{ij} \to f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}}(\delta_{ij} + h_{ij}). \tag{31}$$

Where h_{ij} are the minimally coupled scalars. Their trace is zero. These scalars are functions of the 6 dimensional coordinates. The Lagrangian unto cubic order in h ignoring the traces is

$$I_{\rm NS} = -\frac{V_4}{2k_{10}^2}\int d^6x\sqrt{-G}\frac{G^{MN}}{4}\left[\partial_M h_{ij}\partial_N h_{ij} + \partial_M(h_{ik}h_{kj})\partial_N h_{ij}\right] \tag{32}$$

In the above equation we have used the near horizon limit and V_4 is the volume of the T^4. The metric G_{MN} near the horizon is

$$ds^2 = \frac{r^2}{R^2}(-dx_0^2 + dx_5^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_3^2 \tag{33}$$

We make a change of variables to the Poincare coordinates by substituting

$$z_0 = \frac{R}{r} \tag{34}$$

$$
\begin{aligned}
z_1 &= \frac{x_0}{R} \\
z_2 &= \frac{x_5}{R}
\end{aligned}
$$

The metric becomes

$$ds^2 = R^2 \frac{1}{z_0^2}(dz_0^2 - dz_1^2 + dz_2^2) + R^2 d\Omega_3^2. \tag{35}$$

Here $R = (c_1 Q_1 c_5 Q_5)^{1/4}$ is the radius of curvature of AdS_3 (also of the S^3). For s-waves the minimal scalars do not depend on the coordinates of the S^3. Combining all this, $I_{\rm NS}$ accurate till the cubic order in the h's, is given by

$$I_{\rm NS} = -\frac{V_4}{8k_{10}^2} R^3 V_{S^3} \int d^3 z \sqrt{-g} g^{\mu\nu} \left[\partial_\mu h_{ij} \partial_\nu h_{ij} + \partial_\mu (h_{ik} h_{kj}) \partial_\nu h_{ij}\right], \tag{36}$$

We would now like to show that up to cubic order $I_{RR} = 0$. The relevant terms in our case are

$$I_{RR} = -\frac{1}{4 \times 3! k_{10}^2} \int d^{10} x \sqrt{-G} H_{\hat{M}\hat{N}\hat{O}} H^{\hat{M}\hat{N}\hat{O}}. \tag{37}$$

We substitute the values of B' due to the magnetic and electric components of the RR charges and the value of G. The contribution from the electric part of B', after going to the near-horizon limit and performing the integral over, is

$$\frac{V_4}{4k_{10}^2} R V_{S^3} \int d^3 z \sqrt{-g} \sqrt{\det(\delta_{ij} + h_{ij})} \tag{38}$$

The contribution of the magnetic part of B' in the same limit is

$$-\frac{V_4}{4k_{10}^2} R V_{S^3} \int d^3 z \sqrt{-g} \sqrt{\det(\delta_{ij} + h_{ij})} \tag{39}$$

We note that the contribution of the electric and the magnetic parts cancel giving no couplings for the minimal scalars to the RR background.

B The Supergravity Spectrum

In this section we analyze the spectrum of Type IIB string theory compactified on $AdS_3 \times S^3 \times T^4$. We ignore the KK modes on the T^4. We show that the KK spectrum of the six dimensional theory on $AdS_3 \times S^3$ can be completely organized as short multiplets of the supergroup $SU(1,1|2) \times SU(1,1|2)$. We will follow the method developed by [27].

The massless spectrum of Type IIB on $T^4 \times R^{(5,1)}$ consists of:

a graviton; 8 gravitinos; 5 two forms; 16 gauge fields; 40 fermions; and 25 scalars.

Since these are massless, they fall into various representations R_4 of the little group $SO(4)$ of $R^{(5,1)}$. On further compactifying $R^{(5,1)}$ into $AdS_3 \times S^3$, each representation R_4 decomposes into various representations R_3 of $SO(3)$, the local Lorentz group of the S^3. The dependence of each of these fields on the angles of S^3 leads to decomposition in terms of KK modes on the S^3 which transform according to some representation of the isometry group $SO(4)$ of S^3. Only those representations of $SO(4)$ occur in these decompositions which contain the representation R_3 of S^3. Once the complete set of KK modes are obtained we organize them into short multiplets of the supergroup $SU(1,1|2)\times SU(1,1|2)$.

The graviton transforms as $(\mathbf{3},\mathbf{3})$ of the little group in 6 dimensions. The KK harmonics of this field are

$$\begin{aligned}&(\mathbf{1},\mathbf{1})+2(\mathbf{2},\mathbf{2})+(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})\\ &+3\oplus_{\mathbf{m}\geq\mathbf{3}}(\mathbf{m},\mathbf{m})+2\oplus_{\mathbf{m}\geq\mathbf{2}}[\,(\mathbf{m}+\mathbf{2},\mathbf{m})+(\mathbf{m},\mathbf{m}+\mathbf{2})\,]\\ &+\oplus_{\mathbf{m}\geq\mathbf{1}}[\,(\mathbf{m}+\mathbf{4},\mathbf{m})+(\mathbf{m},\mathbf{m}+\mathbf{4})\,]\end{aligned} \tag{40}$$

The little group representations of the 8 gravitinos is $4(\mathbf{2},\mathbf{3})+4(\mathbf{3},\mathbf{2})$. Their KK harmonics are

$$\begin{aligned}&8[\,(\mathbf{1},\mathbf{2})+(\mathbf{2},\mathbf{1})\,]+16\oplus_{\mathbf{m}\geq\mathbf{2}}[\,(\mathbf{m}+\mathbf{1},\mathbf{m})+(\mathbf{m},\mathbf{m}+\mathbf{1})\,]\\ &+8\oplus_{\mathbf{m}\geq\mathbf{1}}[\,(\mathbf{m}+\mathbf{3},\mathbf{m})+(\mathbf{m},\mathbf{m}+\mathbf{3})\,]\end{aligned} \tag{41}$$

The KK harmonics of the 5 two-forms transforming in $(\mathbf{1},\mathbf{3})+(\mathbf{3},\mathbf{1})$ of the little group are

$$10\oplus_{\mathbf{m}\geq\mathbf{2}}(\mathbf{m},\mathbf{m})+10\oplus_{\mathbf{m}\geq\mathbf{1}}[\,(\mathbf{m}+\mathbf{2},\mathbf{m})+(\mathbf{m},\mathbf{m}+\mathbf{2})\,] \tag{42}$$

The KK harmonics of the 16 gauge fields, $(\mathbf{2},\mathbf{2})$ are

$$16(\mathbf{1},\mathbf{1})+32\oplus_{\mathbf{m}\geq\mathbf{2}}(\mathbf{m},\mathbf{m})+16\oplus_{\mathbf{m}\geq\mathbf{1}}[\,(\mathbf{m},\mathbf{m}+\mathbf{2})+(\mathbf{m}+\mathbf{2},\mathbf{m})\,] \tag{43}$$

The 40 fermions $20(\mathbf{2},\mathbf{1})+20(\mathbf{1},\mathbf{2})$ give rise to the following harmonics

$$40\oplus_{\mathbf{m}\geq\mathbf{1}}[\,(\mathbf{m},\mathbf{m}+\mathbf{1})+(\mathbf{m}+\mathbf{1},\mathbf{m})\,] \tag{44}$$

The 25 scalars $(\mathbf{1},\mathbf{1})$ give rise to the harmonics

$$25\oplus_{\mathbf{m}\geq\mathbf{1}}(\mathbf{m},\mathbf{m}) \tag{45}$$

Putting all this together the complete KK specturm of Type IIB on $AdS_3 \times S^3 \times T^4$ yields

$$
\begin{aligned}
&42(\mathbf{1},\mathbf{1}) + 69(\mathbf{2},\mathbf{2}) + 48[\,(\mathbf{1},\mathbf{2}) + (\mathbf{2},\mathbf{1})\,] + 27[\,(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})\,] \\
&70 \oplus_{\mathbf{m}\geq\mathbf{3}} (\mathbf{m},\mathbf{m}) + 56 \oplus_{\mathbf{m}\geq\mathbf{2}} [\,(\mathbf{m},\mathbf{m}+\mathbf{1}) + (\mathbf{m}+\mathbf{1},\mathbf{m})\,] \\
&+28 \oplus_{\mathbf{m}\geq\mathbf{2}} [\,(\mathbf{m},\mathbf{m}+\mathbf{2}) + (\mathbf{m}+\mathbf{2},\mathbf{m})\,] + 8 \oplus_{\mathbf{m}\geq\mathbf{1}} [\,(\mathbf{m},\mathbf{m}+\mathbf{3}) + (\mathbf{m}+\mathbf{3},\mathbf{m})\,] \\
&+ \oplus_{\mathbf{m}\geq\mathbf{1}} [\,(\mathbf{m},\mathbf{m}+\mathbf{4}) + (\mathbf{m}+\mathbf{4},\mathbf{m})\,]
\end{aligned} \tag{46}
$$

We now organize the above KK modes into short representations of $SU(1,1|2) \times SU(1,1|2)$ [27]. The short multiplet of $SU(1,1|2)$ consists of the following states

j	L_0	Degeneracy
h	h	$2h+1$
$h-1/2$	$h+1/2$	$2(2h)$
$h-1$	$h+1$	$2h-1$

(47)

In the above table j labels the representation of $SU(2)$ which is identified as one of the $SU(2)$'s of the isometry group of S^3. L_0 denotes the conformal weight of the state. We denote the short multiplet of $SU(1,1|2) \times SU(1,1|2)$ as $(\mathbf{2h}+\mathbf{1}, \mathbf{2h}'+\mathbf{1})_S$. On carefully organizing the KK spectrum into short multiplets we get the following set

$$
\begin{aligned}
&5(\mathbf{2},\mathbf{2})_S + 6 \oplus_{\mathbf{m}\geq\mathbf{3}} (\mathbf{m},\mathbf{m})_S \\
&\oplus_{\mathbf{m}\geq\mathbf{2}}[\,(\mathbf{m},\mathbf{m}+\mathbf{2})_S + (\mathbf{m}+\mathbf{2},\mathbf{m})_S + 4(\mathbf{m},\mathbf{m}+\mathbf{1})_S + 4(\mathbf{m}+\mathbf{1},\mathbf{m})_S\,]
\end{aligned} \tag{48}
$$

Equation (46) shows that there are $42(\mathbf{1},\mathbf{1})$ $SO(4)$ representations in the supergravity KK spectrum. We know that one of these arises from the s-wave of g_{55} from equation (40). This is one of the fixed scalars. $16(\mathbf{1},\mathbf{1})$ comes from the s-waves of the 16 gauge fields (the components along x^5) as seen in equation (43). The remaining 25 comes from the 25 scalars of the six dimensional theory. We would like to see where these $42(\mathbf{1},\mathbf{1})$ fit in the short multiplets of $SU(1,1|2) \times SU(1,1|2)$. From equation (48) one can read that 20 of them are in the $5(\mathbf{2},\mathbf{2})_S$ with $(j=0, L_0=1;\, j=0, L_0=1)$. 6 of them are in in $6(\mathbf{3},\mathbf{3})_S$ with $(j=0, L_0=2;\, j=0, L_0=2)$. These correspond to the fixed scalars. Finally, the remaining 16 of them belong to $4(\mathbf{2},\mathbf{3})_S + 4(\mathbf{3},\mathbf{2})_S$. 8 of them have $(j=0, L_0=1;\, j=0, L_0=2)$ and 8 of them have $(j=0, L_0=2;\, j=0, L_0=1)$. These scalars can be recognized as the intermediate scalars. We remark that so far as the scalars are concerned, the symmetry properties do not depend on whether we are using periodic or antiperiodic boundary conditions on the fermions. The reason is that the j-values and the L_0 values remain the same under spectral flow upto a normal ordering constant [28].

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