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# D1/D5 Moduli in SCFT and Gauge Theory, and Hawking Radiation

Justin R. David[a,*], Gautam Mandal[a,†], and Spenta R. Wadia[a,b,‡]

[a]Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, INDIA.

[b]Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, Trieste 34014, ITALY.



## ABSTRACT

We construct marginal operators of the orbifold SCFT corresponding to all twenty near-horizon moduli in supergravity, including operators involving twist fields which correspond to the blowing up modes. We identify the operators with the supergravity moduli in a 1-1 fashion by inventing a global $SO(4)$ algebra in the SCFT. We analyze the gauge dynamics of the D1/D5 system relevant to the splitting $(Q_1, Q_5) \to (Q_1', Q_5') + (Q_1'', Q_5'')$ with the help of a linear sigma model. We show in supergravity as well as in SCFT that the absorption cross-section for minimal scalars is the same all over the near-horizon moduli space.

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e-mail: justin,mandal,wadia@theory.tifr.res.in
* Address after August 31, 1999: Department of Physics, University of California, Santa Barbara, CA 93106, U.S.A.
† Address after August 31, 1999: Theory Division, CERN, CH 1211, Geneve 23, Switzerland.
‡ Jawaharlal Nehru Center for Advanced Scientific Research. Bangalore 560012, INDIA.

## 1 Introduction

The D1/D5 system has been crucial in understanding black hole physics in string theory. Our current understanding of the microscopic derivation of black hole entropy and Hawking radiation rests mainly on this model [1, 2, 3, 4, 5, 6]. The D1/D5 system also provides a concrete realization of holography in the near horizon limit [7, 8, 9, 10, 11, 12, 13]. Unlike in other examples of $AdS_{d+1}/CFT_d$ duality, this system provides an example where both sides of the duality are well tractable.

The D1/D5 system is constructed as follows. Consider type IIB string theory compactified on $T^4$ (one could also consider $K_3$ ). Let us assume that the coordinates of the compactified directions are $x^{6,7,8,9}$. Let us consider $Q_5$ D5-branes extending along $x^{5,6,7,8,9}$ and wrap four of the directions along the $T^4$. Also let us consider $Q_1$ D1-branes extending along the coordinate $x^5$. This leaves us with a black string in six dimensions carrying electric as well as magnetic charge under the Ramond-Ramond field $C^{(2)} \equiv B'$. The much studied example of the five dimensional black hole solution is obtained by compactifying the $x^5$ coordinate and introducing Kaluza-Klein momentum along this direction [2]. The complete specification of the D1/D5 system includes various moduli. Most of the study of the D1/D5 system so far has been focused on the situation with no moduli. It is known from supergravity that the D1/D5 system with no moduli turned on is marginally stable with respect to decay of a subsystem consisting of $Q_5'$ D5 and $Q_1'$ D1 branes. It has been observed recently [14] that such a decay in fact signals a singularity in the world volume gauge theory associated with the origin of the Higgs branch. The issue of stability in supergravity in the context of the D1/D5 system has also been discussed in [15, 16].

The singularity mentioned above leads to a singular conformal field theory and hence to a breakdown of string perturbation theory. However, generic values of the supergravity moduli which do not involve fragmentation into constituents are described by well-defined conformal field theories and therefore string perturbation theory makes sense. In this paper, we would like to understand the D1/D5 system at a generic point in the moduli space. In particular, we address two issues: (a) What is the boundary SCFT corresponding to the D1/D5 system at generic values of the moduli? (b) To what extent do the various moduli affect the Hawking rate of the minimal scalars computed from the microscopic

SCFT?

The low energy description of the D1/D5 system is given by a $\mathcal{N} = (4,4)$ SCFT on the moduli space of $Q_1$ instantons of a $U(Q_5)$ gauge theory on $T^4$. This moduli space is conjectured to be a resolution of the orbifold $(\widetilde{T}^4)^{Q_1 Q_5}/S(Q_1 Q_5)$ [17] which we shall denote by $\mathcal{M}$. $\widetilde{T}^4$ can be distinct from the compactification torus $T^4$. The evidence for this conjecture is mainly topological and is related to dualities which map the black string corresponding to the D1/D5 system to a perturbative string of Type IIB theory with $Q_1$ units of momentum and $Q_5$ units of winding along the $x^5$ direction [18]. An orbifold theory realized as a free field SCFT with identifications is nonsingular as all correlations functions are finite. The realization of the SCFT of the D1/D5 system as $\mathcal{N} = (4,4)$ theory on $\mathcal{M}$ implies that we are at a generic point in the moduli space of the D1/D5 system and *not at the singularity corresponding to fragmentation*. In other words, the orbifold SCFT corresponds to a bound state of $Q_1$ D1-branes and $Q_5$ D5-branes (henceforth denoted as the $(Q_1, Q_5)$ bound state). Thus we use the free field realization of $\mathcal{N} = (4,4)$ SCFT on the orbifold $\mathcal{M}$ and its resolutions using the marginal operators of this theory to describe the boundary SCFT corresponding to the D1/D5 system at generic values of the moduli. We construct all the 20 marginal operators of this theory including the 4 blow up modes explicitly. We use symmetries, including a new global $SO(4)$ algebra, to identify the marginal operators with their corresponding supergravity moduli.

Once we have found the four marginal operators corresponding to the blow up modes, we address the question how to understand their origin in the gauge theory of the D1/D5 system and also how to describe the splitting of the $(Q_1, Q_5)$ bound state into subsystems $(Q_1', Q_5')$ and $(Q_1'', Q_5'')$ in the gauge theory language. We find that the dynamics of this splitting is described by an effective $(4,4)$ $U(1)$ theory coupled to $Q_1'Q_5'' + Q_1''Q_5'$ hypermultiplets. We show, by an analysis of the D-term equations and the potential, that the splitting is possible only when the Fayet-Iliopoulos terms and the theta term of the effective gauge theory are zero. These, therefore, should correspond to the four SCFT marginal operators corresponding to the blow up modes.

We also address the question whether the Hawking rate of minimal scalars of the five dimensional black hole at a generic point in the moduli space of the D1/D5 system agrees with the SCFT calculation. We find that the absorption cross-section and Hawking rate do not depend on the moduli in supergravity, essentially because the minimal scalars are coupled only to the Einstein metric which remains unchanged under U-duality transformations generating the moduli. In the SCFT the calculation of the absorption cross-section/Hawking radiation depends on the two-point function of the corresponding operators, and we show that turning on the exactly marginal perturbations do not modify the cross-section or the rate.

This paper is organized as follows. In Section 2 we construct all the marginal operators of the $\mathcal{N} = (4,4)$ SCFT on $\mathcal{M}$ and identify their quantum numbers under the symmetries of the SCFT. In Section 3 we make a one-to-one identification of the supergravity moduli with the marginal operators in the SCFT. In Section 4 we analyze the gauge dynamics of the D1/D5 system relevant for splitting into subsystems. In Section 5 we discuss absorption/Hawking radiation of the minimal scalars from supergravity and SCFT in the presence of moduli. Section 6 contains concluding remarks.

## 2 The resolutions of the symmetric product

In this section we will construct marginal operators of the $\mathcal{N} = (4,4)$ SCFT on the symmetric product orbifold $\mathcal{M}$. We will find the four operators which correspond to resolution of the orbifold singularity.

The SCFT is described by the free Lagrangian

$$S = \frac{1}{2} \int d^2 z \; \left[ \partial x^i_A \bar{\partial} x_{i,A} + \psi^i_A(z) \bar{\partial} \psi^i_A(z) + \widetilde{\psi}^i_A(\bar{z}) \partial \widetilde{\psi}^i_A(\bar{z}) \right] \tag{1}$$

Here $i$ runs over the $\widetilde{T}^4$ coordinates 1,2,3,4 and $A = 1, 2, \ldots Q_1 Q_5$ labels various copies of the four-torus.

In order to organize the fermions according to doublets of $SU(2)_R \times \widetilde{SU(2)}_R$, we introduce the following notations:

$\Psi_A(z)$ denotes the row vector of fermions

$$\Psi_A(z) \equiv (\Psi^1_A(z), \Psi^2_A(z)) \equiv \sqrt{1/2}(\psi^1_A(z) + i\psi^2_A(z), \psi^3_A(z) + i\psi^4_A(z)) \tag{2}$$

$\Psi^\dagger_A(z)$ denotes the column vector

$$\Psi_A{}^\dagger(z) = \begin{pmatrix} \Psi_A{}^{\dagger 1}(z) \\ \Psi_A{}^{\dagger 2}(z) \end{pmatrix} = \sqrt{\frac{1}{2}} \begin{pmatrix} \psi^1_A(z) - i\psi^2_A(z) \\ \psi^3_A(z) - i\psi^4_A z) \end{pmatrix} \tag{3}$$

Similarly $\widetilde{\Psi}_A(\bar{z})$ will denote the antiholomorphic counterparts of the above fermions. (See appendix A for more details.)

## 2.1 The untwisted sector

Let us first focus on the operators constructed from the untwisted sector. The operators of lowest conformal weight are

$$\begin{array}{cc} \Psi^1_A(z)\widetilde{\Psi}^1_A(\bar{z}) & \Psi^1_A(z)\widetilde{\Psi}^{2\dagger}_A(\bar{z}) \\ \Psi^{2\dagger}_A(z)\widetilde{\Psi}^1_A(\bar{z}) & \Psi^{2\dagger}_A(z)\widetilde{\Psi}^{2\dagger}_A(\bar{z}) \end{array} \tag{4}$$

where summation over $A$ is implied. These four operators have conformal dimension $(h, \bar{h}) = (1/2, 1/2)$ and $(j^3_R, \tilde{j}^3_R) = (1/2, 1/2)$ under the R-symmetry $SU(2)_R \times \widetilde{SU(2)}_R$ (see Appendix A). Since $(h, \bar{h}) = (j^3_R, \tilde{j}^3_R)$, these operators are chiral primaries and have non-singular operator product expansions (OPE) with the supersymmetry currents $G^1(z), G^{2\dagger}(z), \widetilde{G}^1(\bar{z}), \widetilde{G}^{2\dagger}(\bar{z})$ (defined in Appendix A). These properties indicate that they belong to the bottom component of the short multiplet $(\mathbf{2}, \mathbf{2})_\mathbf{S}$ (See Appendix B for details). Each of the four chiral primaries gives rise to four top components of the short multiplet $(\mathbf{2}, \mathbf{2})_\mathbf{S}$. They are given by the leading pole $((z-w)^{-1}(\bar{z}-\bar{w})^{-1})$ in the OPE's

$$\begin{array}{cc} G^2(z)G^2(\bar{z})\mathcal{P}(w, \bar{w}) & G^2(z)\widetilde{G}^{1\dagger}(\bar{z})\mathcal{P}(w, \bar{w}) \\ G^{1\dagger}(z)\widetilde{G}^2(\bar{z})\mathcal{P}(w, \bar{w}) & G^{1\dagger}(z)\widetilde{G}^{1\dagger}(\bar{z})\mathcal{P}(w, \bar{w}) \end{array} \tag{5}$$

where $\mathcal{P}$ stands for any of the four chiral primaries in (4). From the superconformal algebra it is easily seen that the top components constructed above have weights $(1, 1)$ and transform as $(\mathbf{1}, \mathbf{1})$ under $SU(2)_R \times \widetilde{SU(2)}_R$. The OPE's (5) can be easily evaluated. We find that the 16 top components of the $4(\mathbf{2}, \mathbf{2})_\mathbf{S}$ short multiplets are $\partial x^i_A \bar{\partial} x^j_A$.

We classify the above operators belonging to the top component according to representations of (a) the $SO(4)_I$ rotational symmetry (Appendix A) of the $\widetilde{T}^4$, (The four torus $\tilde{T}^4$ breaks this symmetry but we assume the target space is $R^4$ for the classification of states) (b) $R$ symmetry of the SCFT and (c) the conformal weights. As all of these operators belong to the top component of $(\mathbf{2}, \mathbf{2})_\mathbf{S}$ the only property which distinguishes them is the representation under $SO(4)_I$. The quantum numbers of these operators under the various symmetries are

| Operator | $SU(2)_I \times \widetilde{SU(2)}_I$ | $SU(2)_R \times \widetilde{SU(2)}_R$ | $(h, \bar{h})$ | |
|---|---|---|---|---|
| $\partial x_A^{\{i}(z)\bar{\partial} x_A^{j\}}(\bar{z}) - \frac{1}{4}\delta^{ij}\partial x_A^k(z)\bar{\partial} x_A^k(\bar{z})$ | $(\mathbf{3}, \mathbf{3})$ | $(\mathbf{1}, \mathbf{1})$ | $(1,1)$ | |
| $\partial x_A^{[i}(z)\bar{\partial} x_A^{j]}(\bar{z})$ | $(\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$ | $(\mathbf{1}, \mathbf{1})$ | $(1,1)$ | (6) |
| $\partial x_A^i(z)\bar{\partial} x_A^i(\bar{z})$ | $(\mathbf{1}, \mathbf{1})$ | $(\mathbf{1}, \mathbf{1})$ | $(1,1)$ | |

Therefore we have 16 marginal operators from the untwisted sector. As these are top components they can be added to the free SCFT as perturbations without violating the $\mathcal{N} = (4,4)$ supersymmetry.

## 2.2 $Z_2$ twists.

We now construct the marginal operators from the various twisted sectors of the orbifold SCFT. The twist fields of the SCFT on the orbifold $\mathcal{M}$ are labeled by the conjugacy classes of the symmetric group $S(Q_1Q_5)$ [19, 20]. The conjugacy classes consist of cyclic groups of various lengths. The various conjugacy classes and the multiplicity in which they occur in $S(Q_1Q_5)$ can be found from the solutions of the equation

$$\sum nN_n = Q_1Q_5 \tag{7}$$

where $n$ is the length of the cycle and $N_n$ is the multiplicity of the cycle. Consider the simplest nontrivial conjugacy class which is given by $N_1 = Q_1Q_5 - 2, N_2 = 1$ and the rest of $N_n = 0$. A representative element of this class is

$$(X_1 \to X_2,\ X_2 \to X_1),\ X_3 \to X_3,\ \ldots,\ X_{Q_1Q_5} \to X_{Q_1Q_5} \tag{8}$$

Here the $X_A$'s are related to the $x_A$'s appearing in the action (1) by (64) in Appendix A.

To exhibit the singularity of this group action we go over to the following new coordinates

$$X_{cm} = X_1 + X_2 \ \text{ and } \ \phi = X_1 - X_2 \tag{9}$$

Under the group action (8) $X_{cm}$ is invariant and $\phi \to -\phi$. Thus the singularity is *locally* of the type $R^4/Z_2$. The bosonic twist operators for this orbifold singularity are given by following OPE's [21]

$$\partial\phi^1(z)\sigma^1(w,\bar{w}) = \frac{\tau^1(w,\bar{w})}{(z-w)^{1/2}} \qquad \partial\phi^{1\dagger}(z)\sigma^1(w,\bar{w}) = \frac{\tau'^1(w,\bar{w})}{(z-w)^{1/2}} \tag{10}$$

$$\partial\phi^2(z)\sigma^2(w,\bar{w}) = \frac{\tau^2(w,\bar{w})}{(z-w)^{1/2}} \qquad \partial\phi^{2\dagger}(z)\sigma^2(w,\bar{w}) = \frac{\tau'^2(w,\bar{w})}{(z-w)^{1/2}}$$
$$\bar{\partial}\widetilde{\phi}^1(\bar{z})\sigma^1(w,\bar{w}) = \frac{\widetilde{\tau}'^1(w,\bar{w})}{(\bar{z}-\bar{w})^{1/2}} \qquad \bar{\partial}\widetilde{\phi}^{1\dagger}(\bar{z})\sigma^1(w,\bar{w}) = \frac{\widetilde{\tau}^1(w\bar{w})}{(\bar{z}-\bar{w})^{1/2}}$$
$$\bar{\partial}\widetilde{\phi}^2(\bar{z})\sigma^2(w,\bar{w}) = \frac{\widetilde{\tau}'^2(w,\bar{w})}{(\bar{z}-\bar{w})^{1/2}} \qquad \bar{\partial}\widetilde{\phi}^{2\dagger}(\bar{z})\sigma^2(w,\bar{w}) = \frac{\widetilde{\tau}^2(w,\bar{w})}{(\bar{z}-\bar{w})^{1/2}}$$

The $\tau$'s are excited twist operators. The fermionic twists are constructed from bosonized currents defined by

$$\chi^1(z) = e^{iH^1(z)} \qquad \chi^{1\dagger}(z) = e^{-iH^1(z)} \tag{11}$$
$$\chi^2(z) = e^{iH^2(z)} \qquad \chi^{2\dagger}(z) = e^{-iH^2(z)}$$

Where the $\chi$'s, defined as $\Psi_1 - \Psi_2$, are the superpartners of the bosons $\phi$.

From the above we construct the supersymmetric twist fields which act both on fermions and bosons as follows:

$$\Sigma_{(12)}^{(\frac{1}{2},\frac{1}{2})} = \sigma^1(z,\bar{z})\sigma^2(z,\bar{z})e^{iH^1(z)/2}e^{-iH^2(z)/2}e^{i\widetilde{H}^1(\bar{z})/2}e^{-i\widetilde{H}^2(\bar{z})/2} \tag{12}$$
$$\Sigma_{(12)}^{(\frac{1}{2},-\frac{1}{2})} = \sigma^1(z,\bar{z})\sigma^2(z,\bar{z})e^{iH^1(z)/2}e^{-iH^2(z)/2}e^{-i\widetilde{H}^1(\bar{z})/2}e^{i\widetilde{H}^2(\bar{z})/2}$$
$$\Sigma_{(12)}^{(-\frac{1}{2},\frac{1}{2})} = \sigma^1(z,\bar{z})\sigma^2(z,\bar{z})e^{-iH^1(z)/2}e^{+iH^2(z)/2}e^{i\widetilde{H}^1(\bar{z})/2}e^{-i\widetilde{H}^2(\bar{z})/2}$$
$$\Sigma_{(12)}^{(-\frac{1}{2},-\frac{1}{2})} = \sigma^1(z,\bar{z})\sigma^2(z,\bar{z})e^{-iH^1(z)/2}e^{+iH^2(z)/2}e^{-i\widetilde{H}^1(\bar{z})/2}e^{+i\widetilde{H}^2(\bar{z})/2}$$

The subscript (12) refers to the fact that these twist operators were constructed for the representative group element (8) which exchanges the 1 and 2 labels of the coordinates of $\widetilde{T}^4$. The superscript stands for the $(j_R^3, \widetilde{j}_R^3)$ quantum numbers. The twist operators for the orbifold $\mathcal{M}$ belonging to the conjugacy class under consideration is obtained by summing over these $Z_2$ twist operators for all representative elements of this class.

$$\Sigma^{(\frac{1}{2},\frac{1}{2})} = \sum_{i=1}^{Q_1Q_5}\sum_{j=1,j\neq i}^{Q_1Q_5}\Sigma_{(ij)}^{(\frac{1}{2},\frac{1}{2})} \tag{13}$$

We can define the rest of the twist operators for the orbifold in a similar manner. The conformal dimensions of these operators are $(1/2, 1/2)$. They transform as $(\mathbf{2},\mathbf{2})$ under the $SU(2)_R \times \widetilde{SU(2)}_R$ symmetry of the SCFT. They belong to the bottom component of the short multiplet $(\mathbf{2},\mathbf{2})_{\mathbf{S}}$. The operator $\Sigma^{(\frac{1}{2},\frac{1}{2})}$ is a chiral primary. As before the 4 top components of this short multiplet, which we denote by

$$\begin{array}{c} T^{(\frac{1}{2},\frac{1}{2})},\ \ T^{(\frac{1}{2},-\frac{1}{2})} \\ T^{(-\frac{1}{2},\frac{1}{2})},\ \ T^{(-\frac{1}{2},-\frac{1}{2})} \end{array} \tag{14}$$

are given by the leading pole in the following OPE's respectively

$$\begin{array}{c} G^2(z)\widetilde{G}^2(\bar{z})\Sigma^{(\frac{1}{2},\frac{1}{2})}(w,\bar{w}),\ \ G^2(z)\widetilde{G}^{1\dagger}(\bar{z})\Sigma^{(\frac{1}{2},\frac{1}{2})}(w,\bar{w}), \\ G^{1\dagger}(z)\widetilde{G}^2(\bar{z})\Sigma^{(\frac{1}{2},\frac{1}{2})}(w,\bar{w}),\ \ G^{1\dagger}(z)\widetilde{G}^{1\dagger}(\bar{z})\Sigma^{(\frac{1}{2},\frac{1}{2})}(w,\bar{w}) \end{array} \tag{15}$$

These are the 4 blow up modes of the $R^4/Z_2$ singularity [22] and they have conformal weight $(1,1)$[1]. They transform as $(\mathbf{1},\mathbf{1})$ under the $SU(2)_R \times \widetilde{SU(2)}_R$. As before, since these are top components of the short multiplet $(\mathbf{2},\mathbf{2})_{\mathbf{S}}$ they can be added to the free SCFT as perturbations without violating the $\mathcal{N}=(4,4)$ supersymmetry of the SCFT. The various quantum numbers of these operators are listed below.

$$\begin{array}{lcccc} \text{Operator} & (j^3,\tilde{j}^3)_I & SU(2)_R\times\widetilde{SU(2)}_R & (h,\bar{h}) & \\ \mathcal{T}^1_{(1)} = T^{(\frac{1}{2},\frac{1}{2})} & (0,1) & (\mathbf{1},\mathbf{1}) & (1,1) & \\ \mathcal{T}^1_{(0)} = T^{(\frac{1}{2},-\frac{1}{2})} + T^{(-\frac{1}{2},\frac{1}{2})} & (0,0) & (\mathbf{1},\mathbf{1}) & (1,1) & \\ \mathcal{T}^1_{(-1)} = T^{(-\frac{1}{2},-\frac{1}{2})} & (0,-1) & (\mathbf{1},\mathbf{1}) & (1,1) & \\ \mathcal{T}^0 = T^{(-\frac{1}{2},-\frac{1}{2})} - T^{(-\frac{1}{2},-\frac{1}{2})} & (0,0) & (\mathbf{1},\mathbf{1}) & (1,1) & \end{array} \tag{16}$$

The first three operators of the above table can be organized as a $(\mathbf{1},\mathbf{3})$ under $SU(2)_I \times \widetilde{SU(2)}_I$. We will denote these 3 operators as $\mathcal{T}^1$. The last operator transforms as a scalar $(\mathbf{1},\mathbf{1})$ under $SU(2)_I \times \widetilde{SU(2)}_I$ and is denoted by $\mathcal{T}^0$. The simplest way of figuring out the $(j^3,\tilde{j}^3)_I$ quantum numbers in the above table is to note that (a) the $\Sigma$-operators of (12) are singlets under $SU(2)_I \times \widetilde{SU(2)}_I$, as can be verified by computing the action on them of the operators $I_1, I_2$ and $\widetilde{I}_1, \widetilde{I}_2$, (b) the $\mathcal{T}$-operators are obtained from $\Sigma$'s by the action of the supersymmetry currents as in (15) and (c) the quantum numbers of the supersymmetry currents under $I_1, I_2$ and $\widetilde{I}_1, \widetilde{I}_2$ are given by (68).

[1]Relevance of $Z_2$ twist operators to the marginal deformations of the SCFT has earlier been discussed in [23, 24]

## 2.3 Higher twists

We now show that the twist operators corresponding to any other conjugacy class of $S(Q_1Q_5)$ are irrelevant. Consider the class with $N_1 = Q_1Q_5 - 3, N_3 = 1$ and the rest of $N_n = 0$. A representative element of this class is

$$(X_1 \to X_2, X_2 \to X_3, X_3 \to X_1),\ X_4 \to X_4, \ldots,\ X_{Q_1Q_5} \to X_{Q_1Q_5}. \tag{17}$$

To make the action of this group element transparent we diagonalize the group action as follows.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \tag{18}$$

where $\omega = \exp(2\pi i/3)$. These new coordinates are identified under the group action (17) $\phi_1 \to \phi_1$, $\phi_2 \to \omega^2\phi_2$ and $\phi_3 \to \omega\phi_3$. These identifications are locally characteristic of the orbifold

$$R^4 \times R^4/\omega \times R^4/\omega^2 \tag{19}$$

The dimension of the supersymmetric twist operator which twists the coordinates by a phase $e^{2\pi ik/N}$ in 2 complex dimensions is $h(k, N) = k/N$[21]. The twist operator which implements the action of the group element (17) combines the supersymmetric twist operators acting on $\phi_2$ and $\phi_3$ and therefore has total dimension

$$h = h(1, 3) + h(2, 3) = 1/3 + 2/3 = 1 \tag{20}$$

It is the superpartners of these which could be candidates for the blow up modes. However, these have weight $3/2$, These operators are therefore irrelevant.

For the class $N_1 = Q_1Q_5 - k$ , $N_k = k$ and the rest of $N_n = 0$, the total dimension of the twist operator is

$$h = \sum_{i=1}^{k-1} h(i, k) = (k - 1)/2 \tag{21}$$

Its superpartner has dimension $k/2$. Now it is easy to see that all conjugacy classes other than the exchange of 2 elements give rise to irrelevant twist operators. Thus the orbifold $\mathcal{M}$ is resolved by the 4 blow up modes corresponding to the conjugacy class represented by (8). We have thus identified the 20 marginal operators of the $\mathcal{N} = (4, 4)$ SCFT on $\widetilde{T}^4$. They are all top components of the $5(\mathbf{2}, \mathbf{2})_\mathbf{S}$ short multiplets.

## 3 The supergravity moduli and the resolutions of the orbifold.

In this section we find the massless scalar fields which couple to the 4 blow up modes of the orbifold $\mathcal{M}$. Type IIB supergravity compactified on $T^4$ has 25 scalars. There are 10 scalars $h_{ij}$ which arise from compactification of the metric. $i, j, k \ldots$ stands for the directions of $T^4$. There are 6 scalars $b_{ij}$ which arise from the Neveu-Schwarz $B$-field and similarly there are 6 scalars $b'_{ij}$ from the Ramond-Ramond $B'$-field. The remaining 3 scalars are the dilaton $\phi$, the Ramond-Ramond scalar $\chi$ and the Ramond-Ramond 4-form $C_{6789}$. These scalars parameterize the coset $SO(5,5)/(SO(5) \times SO(5))$. The near horizon limit of the D1/D5 system is $AdS_3 \times S^3 \times T^4$ [6]. In this geometry 5 of the 25 scalars become massive. They are the $h_{ii}$ (the trace of the metric of $T^4$ which is proportional to the volume of $T^4$), the 3 components of the anti-self dual part of the Neveu-Schwarz $B$-field $b^-_{ij}$ and a linear combination of the Ramond-Ramond scalar and the 4-form [14]. The massless scalars in the near horizon geometry parameterize the coset $SO(5,4)/SO(5) \times SO(4)$ [10].

The near horizon symmetries form the supergroup $SU(1,1|2) \times SU(1,1|2)$. This is the global part of the $\mathcal{N} = (4,4)$ superconformal algebra. We can classify [11, 12] all the massless supergravity fields of Type IIB supergravity on $AdS_3 \times S^3 \times T^4$ ignoring the Kaluza-Klein modes on $T^4$ according to the short multiplets of the supergroup $SU(1,1|2) \times SU(1,1|2)$. The massless fields of the supergravity fall into the top component of the $5(\mathbf{2},\mathbf{2})_{\mathbf{S}}$ short multiplet. We further classify these fields according to the representations of the $SO(4)_I$, the rotations of the $x^{6,7,8,9}$ directions. This is not a symmetry of the supergravity as it is compactified on $T^4$, but it can be used to classify states. The quantum number of the massless supergravity fields are listed below.

| Field | $SU(2)_I \times \widetilde{SU(2)}_I$ | $SU(2)_E \times \widetilde{SU(2)}_E$ | Mass |
|---|---|---|---|
| $h_{ij} - \frac{1}{4}\delta_{ij}h_{kk}$ | $(\mathbf{3},\mathbf{3})$ | $(\mathbf{1},\mathbf{1})$ | 0 |
| $b'_{ij}$ | $(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$ | $(\mathbf{1},\mathbf{1})$ | 0 |
| $\phi$ | $(\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1})$ | 0 |
| $a_1\chi + a_2 C_{6789}$ | $(\mathbf{1},\mathbf{1})$ | $(\mathbf{1},\mathbf{1})$ | 0 |
| $b^+_{ij}$ | $(\mathbf{1},\mathbf{3})$ | $(\mathbf{1},\mathbf{1})$ | 0 |

(22)

The linear combination appearing on the fourth line is the one that remains massless in the near-horizon limit. The $SU(2)_E \times \widetilde{SU(2)}_E$ stands for the $SO(4)$ isometries of the $S^3$. All the above fields are s-waves of scalars in the near horizon geometry.

We would like to match the twenty supergravity moduli appearing in (22) with the twenty marginal operators appearing in (6) and (16) by comparing their symmetry properties under the AdS/CFT correspondence [12].

The symmetries, or equivalently quantum numbers, to be compared under the AdS/CFT correspondence [7, 25, 26, 8] are as follows:

(a) The isometries of the supergravity are identified with the global symmetries of the superconformal field theory. For the $AdS_3$ case the symmetries form the supergroup $SU(1,1|2) \times SU(1,1|2)$. The identification of this supergroup with the global part of the $\mathcal{N} = (4,4)$ superalgebra leads to the following mass-dimension relation

$$h + \bar{h} = 1 + \sqrt{1 + m^2} \tag{23}$$

where $m$ is the mass of the bulk field and $(h, \bar{h})$ are the dimensions of the SCFT operator. Since in our case the SCFT operators are marginal and the supergravity fields are massless, the mass-dimension relation is obviously satisfied.

(b) The $SU(2)_E \times \widetilde{SU(2)}_E$ quantum number of the bulk supergravity field corresponds to the $SU(2)_R \times SU(2)_R$ quantum number of the boundary operator. By an inspection of column three of the tables in (6), (16) and (22), we see that these quantum numbers also match.

(c) The supersymmetry properties of the bulk fields and the boundary operators tell us which component of the short multiplet they belong to. Noting the fact that all the twenty bulk fields as well as all the marginal operators mentioned above correspond to top components of short multiplets, this property also matches.

(d) The above symmetries alone do not distinguish between the twenty operators or the twenty bulk fields. To further distinguish these operators and the fields we identify the $SO(4)_I$ symmetry of the directions $x^{6,7,8,9}$ with the $S0(4)_I$ of the SCFT. At the level of classification of states this identification is reasonable though these are not actual symmetries. Using the quantum numbers under this group we obtain the following matching of the boundary operators and the supergravity moduli.

$$
\begin{array}{lll}
\text{Operator} & \text{Field} & SU(2)_I \times \widetilde{SU(2)}_I \\
\partial x_A^{\{i}(z)\bar{\partial} x_A^{j\}}(\bar{z}) - 1/4\delta^{ij}\partial x_A^k \bar{\partial} x_A^k & h_{ij} - 1/4\delta_{ij}h_{kk} & (\mathbf{3},\mathbf{3}) \\
\partial x_A^{[i}(z)\bar{\partial} x_A^{j]}(\bar{z}) & b'_{ij} & (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) \\
\partial x_A^i(z)\bar{\partial} x_A^i(\bar{z}) & \phi & (\mathbf{1},\mathbf{1}) \\
\mathcal{T}^1 & b_{ij}^+ & (\mathbf{1},\mathbf{3}) \\
\mathcal{T}^0 & a_1\chi + a_2 C_{6789} & (\mathbf{1},\mathbf{1})
\end{array}
\tag{24}
$$

Note that both the representations $(\mathbf{1},\mathbf{3})$ and $(\mathbf{1},\mathbf{1})$ occur twice in the above table. This could give rise to a two-fold ambiguity in identifying either $(\mathbf{1},\mathbf{3})$ or $(\mathbf{1},\mathbf{1})$ operators with their corresponding bulk fields. The way we have resolved it here is as follows. The operators $\mathcal{T}^1$ and $\mathcal{T}^0$ correspond to blow up modes of the orbifold, and since these are related to the Fayet-Iliopoulos terms and the $\theta$-term in the gauge theory (see Section 4), tuning these operators one can reach the singular SCFT [14] that corresponds to fragmentation of the D1/D5 system. In supergravity, similarly, it is only the moduli $b_{ij}^+$ and $a_1\chi + a_2 C_{6789}$ which affect the stability of the D1/D5 system [14, 16, 15]. As a result, it is $b_{ij}^+$ (and not $b_{ij}'^+$) which should correspond to the operator $\mathcal{T}^1$ and similarly $a_1\chi + a_2 C_{6789}$ should correspond to $\mathcal{T}^0$.

Thus, we arrive at a one-to-one identification between operators of the SCFT and the supergravity moduli.

## 4 The linear sigma model

In this section we will analyze the gauge theory description of the D1/D5 system. We show that that the gauge theory has four parameters which control the break up of the $(Q_1, Q_5)$ system to subsystems $(Q_1', Q_5')$ and $(Q_1'', Q_5'')$. These are the Fayet-Iliopoulos D-terms and the theta term in the effective $U(1)$ $(4,4)$ linear sigma model of the relative coordinate between the subsystems $(Q_1', Q_5')$ and $(Q_1'', Q_5'')$. To motivate this we will review the linear sigma model corresponding to the of the $R^4/Z_2$ singularity.

## 4.1 The linear sigma model description of $R^4/Z_2$

The linear sigma model is a $1+1$ description of the $R^4/Z_2$ singularity dimensional $U(1)$ gauge theory with $(4,4)$ supersymmetry [27]. It has 2 hypermultiplets charged under the $U(1)$. The scalar fields of the hypermultiplets can be organized as doublets under the $SU(2)_R$ symmetry of the $(4,4)$ theory as

$$\chi_1 = \begin{pmatrix} A_1 \\ B_1^\dagger \end{pmatrix} \text{ and } \chi_2 = \begin{pmatrix} A_2 \\ B_2^\dagger \end{pmatrix} \tag{25}$$

The $A$'s have charge $+1$ and the $B$'s have charge $-1$ under the $U(1)$. The vector multiplet has 4 real scalars $\varphi_i$, $i = 1, \ldots, 4$. They do not transform under the $SU(2)_R$. One can include 4 parameters in this theory consistent with (4,4) supersymmetry. They are the 3 Fayet-Iliopoulos terms and the theta term.

Let us first investigate the hypermultiplet moduli space of this theory with the 3 Fayet-Iliopoulos terms and the theta term set to zero. The Higgs phase of this theory is obtained by setting $\phi_i$ and the D-terms to zero. The D-term equations are

$$\begin{aligned} |A_1|^2 + |A_2|^2 - |B_1|^2 - |B_2|^2 &= 0 \\ A_1B_1 + A_2B_2 &= 0 \end{aligned} \tag{26}$$

The hypermultiplet moduli space is the space of solutions of the above equations modded out by the $U(1)$ gauge symmetry. Counting the number of degrees of freedom indicate that this space is 4 dimensional. To obtain the explicit form of this space it is convenient to introduce the following gauge invariant variables

$$M = A_1B_2 \qquad N = A_2B_1 \tag{27}$$

$$P = A_1B_1 = -A_2B_2 \tag{28}$$

These variables are not independent. Setting the D-terms equal to zero and modding out the resulting space by $U(1)$ is equivalent to the equation

$$P^2 + MN = 0. \tag{29}$$

This homogeneous equation is an equation of the space $R^4/Z_2$. To see this the solution of the above equation can be parametrized by 2 complex numbers $(\zeta, \eta)$ such that

$$P = i\zeta\eta \;\; M = \zeta^2 \;\; N = \eta^2 \tag{30}$$

Thus the point $(\zeta, \eta)$ and $(-\zeta, -\eta)$ are the same point in the space of solutions of (29). We have shown that the hypermultiplet moduli space is $R^4/Z_2$.

The above singularity at the origin of the moduli space is a geometric singularity in the hypermultiplet moduli space. We now argue that this singularity is a genuine singularity of the SCFT that the linear sigma model flows to in the infrared. At the origin of the classical moduli space the Coulomb branch meets the Higgs branch. In addition to the potential due to the D-terms the linear sigma model contains the following term in the superpotential[2]

$$V = (|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2)(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2) \tag{31}$$

Thus at the origin of the hypermultiplet moduli space a flat direction for the Coulomb branch opens up. The ground state at this point is not normalizable due to the non-compactness of the Coulomb branch. This renders the infrared SCFT singular.

This singularity can be avoided in two distinct ways. If one turns on the Fayet-Iliopoulos D-terms, the D-term equations are modified to [27]

$$\begin{aligned} |A_1|^2 + |A_2|^2 - |B_1|^2 - |B_2|^2 &= r_3 \\ A_1 B_1 + A_2 B_2 &= r_1 + i r_2 \end{aligned} \tag{32}$$

Where $r_1, r_2, r_3$ are the 3 Fayet-Iliopoulos D-terms transforming as the adjoint of the $SU(2)_R$. Now the origin is no more a solution of these equations and the non-compactness of the Coulomb branch is avoided. In this case wave-functions will have compact support all over the hypermultiplet moduli space. This ensures that the infrared SCFT is non-singular. Turning on the Fayet-Iliopoulos D-terms thus correspond to the geometric resolution of the singularity. The resolved space is known to be [27, 28] described by an Eguchi-Hanson metric in which $r_{1,2,3}$ parameterize a shrinking two-cycle.

[2] These terms can be understood from the coupling $A_\mu A^\mu \chi^* \chi$ in six dimensions, and recognizing that under dimensional reduction to two dimensions $\varphi_i$'s appear from the components of $A_\mu$ in the compact directions.

The second way to avoid the singularity in the SCFT is to turn on the theta angle $\theta$. This induces a constant electric field in the vacuum. This electric field is screened at any other point than the origin in the hypermultiplet moduli space as the $U(1)$ gauge field is massive with a mass proportional to the vacuum expectation value of the hypers. At the origin the $U(1)$ field is not screened and thus it contributes to the energy density of the vacuum. This energy is proportional to $\theta^2$. Thus turning on the theta term lifts the flat directions of the Coulomb branch. This ensures that the corresponding infrared SCFT is well defined though the hypermultiplet moduli space remains geometrically singular. In terms of the Eguchi-Hanson space, the $\theta$-term corresponds to a flux of the antisymmetric tensor through the two-cycle mentioned above.

The $(4,4)$ SCFT on $R^4/Z_2$ at the orbifold point is well defined. Since the orbifold has a geometric singularity but the SCFT is non-singular it must correspond to the linear sigma model with a finite value of $\theta$ and the Fayet-Iliopoulos D-terms set to zero. Deformations of the $R^4/Z_2$ orbifold by its 4 blow up modes correspond to changes in the Fayet-Iliopoulos D-terms and theta term of the linear sigma model[3] The global description of the moduli of a $\mathcal{N} = (4,4)$ SCFT on a resolved $R^4/Z_2$ orbifold is provided by the linear sigma model. In conclusion let us describe this linear sigma model in terms of the gauge theory of D-branes. The theory described above arises on a single D1-brane in presence of 2 D5-branes. The singularity at the at the point $r_1, r_2, r_3, \theta = 0$ is due to noncompactness of the flat direction of the Coulomb branch. Thus it corresponds to the physical situation of the D1-brane leaving the D5-branes.

## 4.2 The gauge theory description of the moduli of the D1/D5 system

As we have seen in Section 2 the resolutions of the $\mathcal{N} = (4,4)$ SCFT on $\mathcal{M}$ is described by 4 marginal operators which were identified in the last subsection with the Fayet-Iliopoulos D-terms and the theta term of the linear sigma model description of the $R^4/Z_2$ singularity. We want to now indicate how these four parameters would make their appearance in the

[3]If we identify the $SU(2)_R$ of the linear sigma-model with $\widetilde{SU(2)}_I$ of the orbifold SCFT, then the Fayet-Iliopoulos parameters will correspond to $\mathcal{T}^1$ and the $\theta$-term to $\mathcal{T}^0$. This is consistent with Witten's observation [29] that $SO(4)_E$ symmetry of the linear sigma-model (one that rotates the $\phi_i$'s) corresponds to the $SU(2)_R$ of the orbifold SCFT.

gauge theory description of the full D1/D5 system.

The gauge theory relevant for understanding the low energy degrees of freedom of the D1/D5 system is a 1+1 dimensional $(4,4)$ supersymmetric gauge theory with gauge group $U(Q_1)\times U(Q_5)$ [30, 31]. The matter content of this theory consists of hypermultiplets in the adjoint representation of the two gauge groups $U(Q_1)$ and $U(Q_5)$ They can be arranged as doublets under $SU(2)_R$ of the gauge theory as

$$N^{(1)}_{a\bar{a}} = \begin{pmatrix} Y^{(1)}_{4(a\bar{a})} + iY^{(1)}_{3(a\bar{a})} \\ Y^{(1)}_{2(a\bar{a})} - iY^{(1)}_{1(a\bar{a})} \end{pmatrix} \quad \text{and} \quad N^{(5)} = \begin{pmatrix} Y^{(5)}_{4(b\bar{b})} + iY^{(5)}_{3(b\bar{b})} \\ Y^{(5)}_{2(b\bar{b})} - iY^{(5)}_{1(b\bar{b})} \end{pmatrix} \tag{33}$$

where $a,\bar{a}$ runs from $1,\ldots,Q_1$ and $b,\bar{b}$ runs from $1,\ldots,Q_5$. The $N^{(1)}$ transform as adjoints of $U(Q_1)$ and the $N^{(5)}$ transform as adjoints of $U(Q_5)$. $N^{(1)}$ corresponds to massless excitations of open strings joining the D1-branes among themselves and $N^{(5)}$ corresponds to massless excitations of open strings joining D5-branes among themselves. This is clear from the expression for the $N$'s in terms of the $Y$'s. The $Y$'s appear from the components of the gauge field $A_\mu$ along the compact directions of the four torus $T^4$. The gauge theory also has hypermultiplets transforming as bi-fundamentals of $U(Q_1)\times\overline{U(Q_5)}$. These hypermultiplets can be arranged as doublets of the $SU(2)_R$ symmetry of the theory as

$$\chi_{a\bar{b}} = \begin{pmatrix} A_{a\bar{b}} \\ B^\dagger_{a\bar{b}} \end{pmatrix} \tag{34}$$

The hypermultiplets arise from massless excitations of open strings joining the D1-branes and the D5-branes. This gauge theory was analyzed in detail in [31]. Motivated by the D-brane description of the $R^4/Z_2$ singularity we look for the degrees of freedom characterizing the break up of $(Q_1,Q_5)$ system to $(Q_1',Q_5')$ and $(Q_1'',Q_5'')$. Physically the relevant degree of freedom describing this process is the relative coordinate between the centre of mass of the $(Q_1',Q_5')$ system and the $(Q_1'',Q_5'')$. We will describe the effective theory of this degree of freedom below.

For the bound state $(Q_1,Q_5)$ the hypermultiplets the $\chi_{a,\bar{b}}$ are charged under the relative $U(1)$ of $U(Q_1)\times U(Q_5)$, that is under the gauge field $A_\mu = Tr_{U(Q_1)}(A^{a\bar{a}}_\mu) - Tr_{U(Q_5)}(A^{b\bar{b}})$. The gauge multiplet corresponding to the relative $U(1)$ corresponds to the degree of freedom of the relative coordinate between the centre of mass of the collection of $Q_1$ D1-branes and $Q_5$ D5-branes. At a generic point of the Higgs phase, all the $\chi_{a\bar{b}}$'s have expectation values, thus making this degree of freedom becomes massive. This is consistent with the fact that we are looking at the bound state $(Q_1, Q_5)$.

Consider the break up of the $(Q_1, Q_5)$ bound state to the bound states $(Q_1', Q_5')$ and $(Q_1'', Q_5'')$. To find out the charges of the hypermultiplets under the various $U(1)$, we will organize the hypers as

$$\begin{pmatrix} \chi_{a'b'} & \chi_{a'b''} \\ \chi_{a''b'} & \chi_{a''b''} \end{pmatrix}, \quad \begin{pmatrix} Y^{(1)}_{i(a'\bar{a}')} & Y^{(1)}_{i(a'\bar{a}'')} \\ Y^{(1)}_{i(a''\bar{a}')} & Y^{(1)}_{i(a''\bar{a}'')} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} Y^{(5)}_{i(b'\bar{b}')} & Y^{(5)}_{i(b'\bar{b}'')} \\ Y^{(5)}_{i(b''\bar{b}')} & Y^{(5)}_{i(b''\bar{b}'')} \end{pmatrix} \tag{35}$$

where $a', \bar{a}'$ runs from $1, \ldots, Q_1'$, $b', \bar{b}'$ from $1, \ldots, Q_5'$, $a''\bar{a}''$ from $1, \ldots Q_1''$ and $b'', \bar{b}''$ from $1 \ldots, Q_5''$. We organize the scalars of the vector multiplet corresponding to the gauge group $U(Q_1)$ and $U(Q_5)$ as

$$\begin{pmatrix} \phi_i^{(1)a'\bar{a}'} & \phi_i^{(1)a'\bar{a}''} \\ \phi_i^{(1)a''\bar{a}'} & \phi_i^{(2)a'\bar{a}''} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \phi_i^{(5)b'\bar{b}'} & \phi_i^{(5)b'\bar{b}''} \\ \phi_i^{(5)b''\bar{b}'} & \phi_i^{(5)b'\bar{b}''} \end{pmatrix} \tag{36}$$

where $i = 1, 2, 3, 4$.

Let us call the the $U(1)$ gauge fields (traces) of $U(Q_1')$, $U(Q_5'), U(Q_1'')$, $U(Q_5'')$ as $A_1'$, $A_5'$, $A_1''$, $A_5''$ respectively. We will also use the notation $A_\pm' \equiv A_1' \pm A_5'$ and $A_\pm'' \equiv A_1'' \pm A_5''$.

As we are interested in the bound states $(Q_1', Q_5')$ and $(Q_1'', Q_5'')$, in what follows we will work with a specific classical background in which we give *vev*'s to the block-diagonal hypers $\chi_{a'b'}, \chi_{a''b''}, Y^{(1)}_{i(a'\bar{a}')}, Y^{(5)}_{i(b'\bar{b}')}, Y^{(1)}_{i(a''\bar{a}'')}$ and $Y^{(5)}_{i(b''\bar{b}'')}$. These *vev*'s are chosen so that the classical background satisfies the D-term equations [31].

The *vev*'s of the $\chi$'s render the fields $A_-'$ and $A_-''$ massive with a mass proportional to *vev*'s. In the low energy effective Lagrangian these gauge fields can therefore be neglected. In the following we will focus on the $U(1)$ gauge field $A_r = 1/2(A_+' - A_+'')$ which does not get mass from the above *vev*'s. The gauge multiplet corresponding to $A_r$ contains four real scalars denoted below by $\varphi_i$. These represent the relative coordinate between the centre of mass of the $(Q_1', Q_5')$ and the $(Q_1'', Q_5'')$ bound states. We will be interested in the question whether the $\varphi_i$'s remain massless or otherwise. The massless case would correspond to a non-compact Coulomb branch and eventual singularity of the SCFT.

In order to address the above question we need to find the low energy degrees of freedom which couple to the gauge multiplet corresponding to $A_r$.

The fields charged under $A_r$ are the hypermultiplets $\chi_{a'\bar{b}''}$, $\chi_{a''\bar{b}'}$, $Y^{(1)}_{i(a'\bar{a}'')}$, $Y^{(1)}_{i(a''\bar{a}')}$, $Y^{(5)}_{i(b'\bar{b}'')}$, $Y^{(5)}_{i(b''\bar{b}')}$ and the vector multiplets $\phi_i^{(1)a'\bar{a}''}, \phi_i^{(1)a''\bar{a}'}, \phi_i^{(5)b'\bar{b}''}, \phi_i^{(5)b''\bar{b}'}$. In order to find out which of these are massless, we look at the following terms in the Lagrangian of $U(Q_1) \times U(Q_5)$ gauge theory:

$$
\begin{aligned}
L &= L_1 + L_2 + L_3 + L_4 \\
L_1 &= \chi^*_{a_1\bar{b}_1}\phi_i^{(1)a_2\bar{a}_1 *}\phi_i^{(1)a_2\bar{a}_3}\chi_{a_3\bar{b}_1} \\
L_2 &= \chi^*_{a_1\bar{b}_1}\phi_i^{(5)b_1\bar{b}_2 *}\phi_i^{(5)b_3\bar{b}_2}\chi_{a_1\bar{b}_3} \\
L_3 &= Tr([Y_i^{(1)}, Y_j^{(1)}][Y_i^{(1)}, Y_j^{(1)}]) \\
L_4 &= Tr([Y_i^{(5)}, Y_j^{(5)}][Y_i^{(5)}, Y_j^{(5)}])
\end{aligned}
\tag{37}
$$

where the $a_i$'s run from $1, \ldots, Q_1$ and the $b_i$'s run form $1, \ldots, Q_5$. The terms $L_1$ and $L_2$ originate from terms of the type $|A_M\chi|^2$ where $A_M \equiv (A_\mu, \phi_i)$ is the $(4,4)$ vector multiplet in two dimensions. The terms $L_3$ and $L_4$ arise from commutators of gauge fields in compactified directions.

The fields $Y$ are in general massive. The reason is that the traces $y_i'^{(1)} \equiv Y^{(1)}_{i(a'\bar{a}')}$, representing the centre-of-mass position in the $T^4$ of $Q_1'$ D1-branes, and $y_i''^{(1)} \equiv Y^{(1)}_{i(a''\bar{a}'')}$, representing the centre-of-mass position in the $T^4$ of $Q_1''$ D1-branes, are neutral and will have *vev*'s which are generically separated (the centres of mass can be separated in the torus even when they are on top of each other in physical space). The mass of $Y^{(1)}_{i(a'\bar{a}'')}, Y^{(1)}_{i(a''\bar{a}')}$ can be read off from the term $L_3$ in (37), to be proportional to $(y'^{(1)} - y''^{(1)})^2$ Similarly the mass of $Y^{(5)}_{i(b'\bar{b}'')}, Y^{(5)}_{i(b'\bar{b}'')}$ is proportional to $(y'^{(5)} - y''^{(5)})^2$ (as can be read off from the term $L_4$ in (37)) where $y'^{(5)}$ and $y''^{(5)}$ are the centers of mass of the $Q_5'$ D5-branes and $Q_5''$ D5-branes along the direction of the dual four torus $\hat{T}^4$. (At special points when their centres of mass coincide, these fields become massless. The analysis for these cases can also be carried out by incorporating these fields in (40)-(42), with no change in the conclusion) The fields $\phi_i^{(1)a'\bar{a}''}, \phi_i^{(1)a''\bar{a}'}$ are also massive. Their masses can be read off from the $L_1$ in (37). Specifically they arise from the following terms

$$\chi^*_{a''_1\bar{b}''}\phi_i^{(1)a'\bar{a}''_1*}\phi_i^{(1)a'\bar{a}''_2}\chi_{a''_2\bar{b}''}+\chi^*_{a'_1\bar{b}'}\phi_i^{(1)a''\bar{a}'_1*}\phi_i^{(1)a''\bar{a}'_2}\chi_{a'_2\bar{b}'} \tag{38}$$

where $a'_i$ run from $1,\ldots Q'_1$ and $a''_i$ run form $1,\ldots Q''_1$. These terms show that their masses are proportional to the expectation values of the hypers $\chi_{a'b'}$ and $\chi_{a''b''}$. Similarly the terms of $L_2$ in (37)

$$\chi^*_{a''\bar{b}''_1}\phi_i^{(5)b''_1b'*}\phi_i^{(5)b''_2\bar{b}'}\chi_{a''\bar{b}''_2}+\chi^*_{a'\bar{b}'_1}\phi_i^{(5)b'_1\bar{b}''*}\phi_i^{(5)b'_2\bar{b}''}\chi_{a'\bar{b}'_2} \tag{39}$$

show that the fields $\phi_i^{(5)b'\bar{b}''}$ $\phi_i^{(5)b''\bar{b}'}$ are massive with masses proportional to the expectation values of the hypers $\chi_{a'\bar{a}''}$ and $\chi_{a''\bar{b}''}$. In the above equation $b'_i$ take values from $1,\ldots,Q'_5$ and $b''_i$ take values from $1,\ldots,Q''_5$. Note that these masses remain non-zero even in the limit when the $(Q'_1,Q'_5)$ and $(Q''_1,Q''_5)$ are on the verge of separating.

Thus the relevant degrees of freedom describing the splitting process is a $1+1$ dimensional $U(1)$ gauge theory of $A_r$ with $(4,4)$ supersymmetry. The matter content of this theory consists of hypermultiplets $\chi_{a'\bar{b}''}$ with charge $+1$ and $\chi_{a''\bar{b}'}$ with charge $-1$.

Let us now describe the dynamics of the splitting process. This is given by analyzing the hypermultiplet moduli space of the effective theory described above with the help of the D-term equations:

$$\begin{aligned} A_{a'\bar{b}''}A^*_{a'\bar{b}''}-A_{a''\bar{b}'}A^*_{a''\bar{b}'}-B_{b''\bar{a}'}B^*_{b''\bar{a}'}+B_{b'\bar{a}''}B^*_{b'\bar{a}''}&=0 \\ A_{a'\bar{b}''}B_{b''\bar{a}'}-A_{a''\bar{b}'}B_{b'\bar{a}''}&=0 \end{aligned} \tag{40}$$

In the above equations the sum over $a',b',a'',b''$ is understood. These equations are generalized version of (26) discussed for the $R^4/Z_2$ singularity in Section 4.1. At the origin of the Higgs branch where the classical moduli space meets the Coulomb branch this linear sigma model would flow to an infrared conformal field theory which is singular. The reason for this is the same as for the $R^4/Z_2$ case. The linear sigma model contains the following term in the superpotential

$$V=(A_{a'\bar{b}''}A^*_{a'\bar{b}''}+A_{a''\bar{b}'}A^*_{a''\bar{b}'}+B_{b''\bar{a}'}B^*_{b''\bar{a}'}+B_{b'\bar{a}''}B^*_{b'\bar{a}''})(\varphi_1^2+\varphi_2^2+\varphi_3^2+\varphi_4^2) \tag{41}$$

As in the discussion of the $R^4/Z_2$ case, at the origin of the hypermultiplet moduli space the flat direction of the Coulomb branch leads to a ground state which is not normalizable.

This singularity can be avoided by deforming the D-term equations by the Fayet-Iliopoulos terms:

$$A_{a'\bar{b}''}A^*_{a'\bar{b}''} - A_{a''\bar{b}'}A^*_{a''\bar{b}'} - B_{b''\bar{a}'}B^*_{b''\bar{a}'} + B_{b'\bar{a}''}B^*_{b'\bar{a}''} = r_3 \tag{42}$$
$$A_{a'\bar{b}''}B_{b''\bar{a}'} - A_{a''\bar{b}'}B_{b'\bar{a}''} = r_1 + ir_3$$

We note here that the Fayet-Iliopoulos terms break the relative U(1) under discussion and the gauge field becomes massive. The reason is that the D-terms with the Fayet-Iliopoulos do not permit all $A, B$'s in the above equation to simultaneously vanish. At least one of them must be non-zero, but it is charged, therefore the U(1) is broken.

The singularity associated with the non-compact Coulomb branch can also be avoided by turning on the $\theta$ term, the mechanism being similar to the one discussed in the previous subsection. If any of the 3 Fayet-Iliopoulos D-terms or the $\theta$ term is turned on, the flat directions of the Coulomb branch are lifted, leading to normalizable ground state is of the Higgs branch. This prevents the breaking up of the $(Q_1, Q_5)$ system to subsystems. Thus we see that the 4 parameters which resolve the singularity of the $\mathcal{N} = (4,4)$ SCFT on $\mathcal{M}$ make their appearance in the gauge theory as the Fayet-Iliopoulos terms and the theta term.

It would be interesting to extract the singularity structure of the the gauge theory of the D1/D5 system through mappings similar to (27)- (30).

### 4.3 The case $(Q_1, Q_5) \to (Q_1 - 1, Q_5) + (1, 0)$: splitting of 1 D1-brane

It is illuminating to consider the special case in which 1 D1-brane splits off from the bound state $(Q_1, Q_5)$. The effective dynamics is again described in terms of a $U(1)$ gauge theory associated with the relative separation between the single D1-brane and the bound state $(Q_1 - 1, Q_5)$. The massless hypermultiplets charged under this $U(1)$ correspond to open strings joining the single D1-brane with the D5-branes and are denoted by

$$\chi_{b'} = \begin{pmatrix} A_{b'} \\ B^\dagger_{b'} \end{pmatrix} \tag{43}$$

The D-term equations, with the Fayet-Iliopoulos terms, become in this case

$$\sum_{b'=1}^{Q_5}\left(|A_{b'}|^2 - |B_{b'}|^2\right) = r_3, \quad \sum_{b'=1}^{Q_5} A_{b'}B_{b'} = r_1 + ir_2 \tag{44}$$

while the potential is

$$V = \left[\sum_{b'=1}^{Q_5}\left(|A_{b'}|^2 + |B_{b'}|^2\right)\right](\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2) \tag{45}$$

The D-term equations above agree with those in [14] which discusses the splitting of a single D1-brane. It is important to emphasize that the potential and the D-term equations describe an *effective dynamics* in the classical background corresponding to the $(Q_1-1, Q_5)$ bound state. This corresponds to the description in [14] of the splitting process in an AdS$_3$ background which represents a mean field of the above bound state.

# 5 Hawking radiation and the resolutions of the orbifold

In this section we will address the question whether dynamical processes like absorption and Hawking radiation from the five-dimensional black hole are affected by the presence of the above moduli, especially the blow-up modes. For our understanding of such processes to be complete, the supergravity calculation and the SCFT calculation of absorption cross-section/Hawking radiation rate should continue to agree even in the presence of these moduli.

## 5.1 Supergravity calculation of absorption/Hawking radiation in presence of moduli

We recall that the five dimensional black hole solution in the absence of moduli is [1, 2] obtained from the D1-D5 system by further compactifying $x^5$ on a circle of radius $R_5$ and adding gravitational waves carrying left-(right-) moving momenta $N_L/R_5$ $(N_R/R_5)$ along $x^5$. The near horizon geometry of this solution is [7, 8] BTZ$\times S^3 \times T^4$ where the BTZ black hole has mass $M = (N_R + N_L)/l$ and angular momentum $J = N_L - N_R$[4].

[4] $l \equiv 2\pi(g_s^2 Q_1 Q_5/V_4)^{1/4}$ is the radius of curvature of the BTZ black hole, $V_4$ being the volume of the four-torus.

The absorption cross-section of minimal scalars in the absence of moduli is given by

$$\sigma_{abs} = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega}{2} \frac{\exp(\omega/T_H) - 1}{(\exp(\omega/2T_R) - 1)(\exp(\omega/2T_L) - 1)} \tag{46}$$

We will now show that the absorption cross-section remains unchanged even when the moduli are turned on.

From the equations of motion of type II supergravity [32], we can explicitly see that the five-dimensional Einstein metric $ds^2_{5,Ein}$ is not changed by turning on the sixteen moduli corresponding to the metric $G_{ij}$ on $T^4$ and the Ramond-Ramond 2-form potential $B'$. As regards the four blowing up moduli, the invariance of $ds^2_{5,Ein}$ can be seen from the fact that turning on these moduli corresponds to $SO(4,5)$ transformation which is part of a U-duality transformation and from the fact that the Einstein metric does not change under U-duality. This statement can be verified by using explicit construction of the corresponding supergravity solution, at least for the $B_{NS}$ moduli [33]. Now we know that the minimal scalars $\phi^i$ all satisfy the wave-equation

$$D_\mu \partial^\mu \phi^i = 0 \tag{47}$$

where the Laplacian is with respect to the Einstein metric in five dimensions. Since it is *only* this wave equation that determines the absorption cross-section completely, we see that $\sigma_{abs}$ is the same as before.

It is straightforward to see that the Hawking rate, given by

$$\Gamma_H = \sigma_{abs}(e^{\omega/T_H} - 1)^{-1} \frac{d^4k}{(2\pi)^4} \tag{48}$$

is also not changed.

### 5.2 SCFT calculation of absorption cross-section/Hawking rate in the presence of moduli

In Section 3 we have listed the twenty (1,1) operators $O_i(z,\bar{z})$ in the SCFT based on the symmetric product orbifold $\mathcal{M}$ which is dual to the D1/D5 system. In order to arrive at the SCFT dual to the *black hole*, we have to first implement the periodic identification $x^5 \equiv x^5 + 2\pi R_5$. It was shown in [8] that this forces the SCFT to be in the Ramond sector. Turning on various moduli $\phi^i$ of supergravity corresponds to perturbing the SCFT

$$S = S_0 + \sum_i \int d^2z \bar{\phi}^i O_i(z,\bar{z}) \tag{49}$$

where $\bar{\phi}^i$ denote the near-horizon limits of the various moduli fields $\phi^i$.

Finally, the Kaluza-Klein momentum of the black hole (equivalently, angular momentum of BTZ) corresponds to excited states of this sector with

$$L_0 = N_L, \bar{L}_0 = N_R \tag{50}$$

We note here that $S_0$ corresponds to the free SCFT based on the symmetric product orbifold $\mathcal{M}$. Since this SCFT is non-singular (all correlation functions are finite), it does not correspond to the marginally stable BPS solution originally found in [1, 2]. Instead, it corresponds to a five-dimensional black hole solution in supergravity with suitable "blow-up" moduli turned on.

Let us now calculate the absorption cross-section of a supergravity fluctuation $\delta\phi_i$ using SCFT. The notation $\delta\phi_i$ implies that we are considering the supergravity field to be of the form

$$\phi^i = \phi^i_0 + \kappa_5 \delta\phi^i \tag{51}$$

where $\phi^i_0$ represents the background value. The factor of $\kappa_5$ above ensures appropriate normalization of the fluctuation $\delta\phi^i$ as explained below in the paragraph above (55). This corresponds to the SCFT action

$$\begin{aligned} S &= S_0 + \int d^2z [\bar{\phi}^i_0 + \kappa_5 \delta\bar{\phi}^i] O_i(z,\bar{z}) \\ &= S_{\phi_0} + S_{int} \end{aligned} \tag{52}$$

where

$$S_{\phi_0} = S_0 + \int d^2z \bar{\phi}^i_0 O_i(z,\bar{z}) \tag{53}$$

$$S_{int} = \kappa_5 \int d^2z \delta\bar{\phi}^i O_i(z,\bar{z}) \tag{54}$$

The absorption cross-section of the supergravity fluctuation $\delta\phi^i$ involves [34, 4, 12] essentially the two-point function of the operator $O_i$ calculated with respect to the SCFT action $S_{\phi_0}$. Since $O_i$ is a marginal operator, its two-point function is determined apart from a normalization constant.

Regarding the marginality of the operators $O_i$, it is easy to establish it upto one-loop order by direct computation ($c_{ijk} = 0$). The fact that these operators are exactly marginal can be argued as follows. The twenty operators $O_i$ arise as top components of five chiral primaries. It is known that the number of chiral primaries with $(j_R, \tilde{j}_R) = (m, n)$ is the Hodge number $h_{2m,2n}$ of the target space $\mathcal{M}$ of the SCFT. Since this number is a topological invariant, it should be the same at all points of the moduli space of deformations.

In the case studied in [12] it was found that if the operator $O_i$ is canonically normalized (OPE has residue 1) and if $\delta\phi_i$ is canonically normalized in supergravity, then the normalization of $S_{int}$ as in (54) ensures that $\sigma_{abs}$ from SCFT agrees with the supergravity result. The crucial point now is the following: once we fix the normalization of $S_{int}$ at a given point in moduli space, at some other point it may acquire a constant ($\neq 1$) in front of the integral when $O_i$ and $\delta\phi_i$ are canonically normalized at the new point. This would imply that $\sigma_{abs}$ will get multiplied by this constant, in turn implying disagreement with supergravity. We need to show that that does not happen.

To start with a simple example, let us first restrict to the moduli $g_{ij}$ of the torus $\widetilde{T^4}$. We have

$$S = \int d^2z \partial x^i \bar{\partial} x^j g_{ij} \tag{55}$$

The factor of string tension has been absorbed in the definition of $x^i$.

In [12] we had $g_{ij} = \delta_{ij} + \kappa_5 h_{ij}$, leading to

$$\begin{aligned} S &= S_0 + S_{int} \\ S_0 &= \int d^2z \partial x^i \bar{\partial} x^j \delta_{ij} \\ S_{int} &= \kappa_5 \int d^2z \partial x^i \bar{\partial} x^j h_{ij} \end{aligned} \tag{56}$$

As we remarked above, this $S_{int}$ gives rise to the correctly normalized $\sigma_{abs}$.

Now, if we expand around some other metric

$$g_{ij} = g_{0ij} + \kappa_5 h_{ij} \tag{57}$$

the above action (55) implies

$$\begin{aligned} S &= S_{g_0} + S_{int} \\ S_{g_0} &= \int d^2z \partial x^i \bar{\partial} x^j g_{0ij} \end{aligned}$$

$$S_{int} \quad = \quad \kappa_5 \int d^2z \partial x^i \bar{\partial} x^j h_{ij} \tag{58}$$

Now the point is, neither $h_{ij}$ nor the operator $O^{ij} = \partial X^i \bar{\partial} X^j$ in $S_{int}$ is canonically normalized at $g_{ij} = g_{0,ij}$. When we do use the canonically normalized operators, do we pick up an additional constant in front?

Note that

$$\langle O^{ij} O^{kl} \rangle_{g_0} = g_0^{ik} g_0^{jl} |z-w|^{-4} \tag{59}$$

and

$$\langle h_{ij}(x) h_{kl}(y) \rangle_{g_0} = g_{0,ik} g_{0,jl} \mathcal{D}(x,y) \tag{60}$$

$\mathcal{D}(x,y)$ is the massless scalar propagator.

This shows that

Statement (1):*The two-point functions of $O^{ij}$ and $h_{ij}$ pick up inverse factors.*

As a result, $S_{int}$ remains correctly normalized when re-written in terms of the canonically normalized $h$ and $O$, and no additional constant is picked up.

The above result is in fact valid in the full twenty dimensional moduli space $\widetilde{\mathcal{M}}$ because Statement (1) above remains true generally.

To see this, let us first rephrase our result for the special case of the metric moduli (55) in a more geometric way. The $g_{ij}$'s can be regarded as some of the coordinates of the moduli space $\widetilde{\mathcal{M}}$ (known to be a coset $SO(4,5)/(SO(4) \times SO(5))$). The infinitesimal perturbations $h_{ij}, h_{kl}$ can be thought of as defining tangent vectors at the point $g_{0,ij}$ (namely the vectors $\partial/\partial g_{ij}, \partial/\partial g_{kl}$). The (residue of the) two-point function given by (59) defines the inner product between these two tangent vectors according to the Zamolodchikov metric [35].

Consider, on the other hand, the propagator (inverse two-point function) of $h_{ij}, h_{kl}$ in supergravity. The moduli space action of low energy fluctuations is nothing but the supergravity action evaluated around the classical solutions $g_{0,ij}$. The kinetic term of such a moduli space action defines the metric of moduli space. The italicized statement above is a simple reflection of the fact that the Zamolodchikov metric defines the metric on moduli space, and hence

Statement (2): *the propagator of supergravity fluctuations, viewed as a matrix, is the inverse of the two-point functions in the SCFT.*

The last statement is of course not specific to the moduli $g_{ij}$ and is true of all the moduli. We find, therefore, that fixing the normalization of $S_{int}$ (54) at any one point $\phi_0$ ensures that the normalization remains correct at any other point $\phi_0'$ by virtue of Statement (2). We should note in passing that Statement (2) is consistent with, and could have been derived from, AdS/CFT correspondence as applied to the two-point function.

Thus, we find that $\sigma_{abs}$ is independent of the moduli, in agreement with the result from supergravity.

### 5.3 Entropy and area

We make a brief mention here of the fact that the correspondence between Bekenstein-Hawking entropy and the SCFT entropy remains true in the presence of all the twenty moduli. The reason is that in supergravity the Einstein metric remains unchanged (see Section 5.1) and therefore the area of the event horizon remains the same (this can be explicitly verified using the supergravity solution in [33]). In the SCFT, since the operators corresponding to the above moduli are all exactly marginal (Section 5.2) therefore the central charge remains unchanged and hence, by Cardy's formula, the entropy does not change, in agreement with the Bekenstein-Hawking formula.

## 6 Conclusions

(a) We presented an explicit construction of all the marginal operators in the SCFT of the D1/D5 system based on the orbifold $\mathcal{M}$. These are twenty in number, four of which are constructed using $Z_2$ twist operators and correspond to blowing up modes of the orbifold.
(b) We classified the the twenty near-horizon moduli of supergravity on $\mathrm{AdS}_3 \times S^3 \times T^4$ according to representations of $SU(1,1|2) \times SU(1,1|2) \times SO(4)_I$.
(c) We established one-to-one correspondence between the supergravity moduli and the marginal operators by inventing a new $SO(4)$ symmetry in the SCFT which we identified with the $SO(4)_I$ of supergravity.

(d) We analyzed gauge theory dynamics of the D1/D5 system relevant for the splitting of the bound state $(Q_1, Q_5) \to (Q_1', Q_5') + (Q_1'', Q_5'')$.
(e) We showed in supergravity as well as in SCFT that the absorption cross-section for minimal scalars is the same for all values of the moduli, therefore establishing the agreement between SCFT and supergravity all over the moduli space.

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## A The $\mathcal{N} = 4$ superconformal algebra

To set up our notations and conventions we review the $\mathcal{N} = 4$ superconformal algebra. The algebra is generated by the stress energy tensor, four supersymmetry currents, and a local $SU(2)$ $R$ symmetry current. The operator product expansions of the algebra with central charge $c$ are given by (See for example [36].)

$$\begin{aligned}
T(z)T(w) &= \frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c}{2(z-w)^4}, \\
G^a(z)G^{b\dagger}(w) &= \frac{2T(w)\delta_{ab}}{z-w} + \frac{2\bar{\sigma}^i_{ab}\partial J^i}{z-w} + \frac{4\bar{\sigma}^i_{ab}J^i}{(z-w)^2} + \frac{2c\delta_{ab}}{3(z-w)^3}, \\
J^i(z)J^j(w) &= \frac{i\epsilon^{ijk}J^k}{z-w} + \frac{c}{12(z-w)^2}, \\
T(z)G^a(w) &= \frac{\partial G^a(w)}{z-w} + \frac{3G^a(z)}{2(z-w)^2}, \\
T(z)G^{a\dagger}(w) &= \frac{\partial G^{a\dagger}(w)}{z-w} + \frac{3G^{a\dagger}(z)}{2(z-w)^2}, \\
T(z)J^i(w) &= \frac{\partial J^i(w)}{z-w} + \frac{J^i}{(z-w)^2}, \\
J^i(z)G^a(w) &= \frac{G^b(z)(\sigma^i)^{ba}}{2(z-w)}, \\
J^i(z)G^{a\dagger}(w) &= -\frac{(\sigma^i)^{ab}G^{b\dagger}(w)}{2(z-w)}
\end{aligned} \tag{61}$$

Here $T(z)$ is the stress energy tensor, $G^a(z), G^{b\dagger}(z)$ the $SU(2)$ doublet of supersymmetry generators and $J^i(z)$ the $SU(2)$ $R$ symmetry current. The $\sigma$'s stand for Pauli matrices. In the free field realization desribed below, the above holomorphic currents occur together with their antiholomorphic counterparts, which we will denote by $\widetilde{J}(\bar{z}), \widetilde{G}(\bar{z})$ and $\widetilde{T}(\bar{z})$. In particular, the R-parity group will be denoted by $SU(2)_R \times \widetilde{SU(2)}_R$.

A free field realization of the $\mathcal{N} = 4$ superconformal algebra with $c = 6Q_1Q_5$ can be constructed out of $Q_1Q_5$ copies of four real fermions and bosons. The generators are given by

$$\begin{aligned} T(z) &= \partial X_A(z)\partial X_A^\dagger(z) + \frac{1}{2}\Psi_A(z)\partial\Psi_A^\dagger(z) - \frac{1}{2}\partial\Psi_A(z)\Psi_A^\dagger(z) \\ G^a(z) &= \begin{pmatrix} G^1(z) \\ G^2(z) \end{pmatrix} = \sqrt{2}\begin{pmatrix} \Psi_A^1(z) \\ \Psi_A^2(z) \end{pmatrix} \partial X_A^2(z) + \sqrt{2}\begin{pmatrix} -\Psi_A^{2\dagger}(z) \\ \Psi_A^{1\dagger}(z) \end{pmatrix} \partial X_A^1(z) \\ J_R^i(z) &= \frac{1}{2}\Psi_A(z)\sigma^i\Psi_A^\dagger(z) \end{aligned} \tag{62}$$

We will use the following notation for the zero mode of the R-parity current:

$$J_R^i = \frac{1}{2}\int \frac{dz}{2\pi i}\Psi_A(z)\sigma^i\Psi_A^\dagger(z) \tag{63}$$

In the above the summation over $A$ which runs from 1 to $Q_1Q_5$ is implied. The bosons $X$ and the fermions $\Psi$ are

$$\begin{aligned} X_A(z) &= (X_A^1(z), X_A^2(z)) = \sqrt{1/2}(x_A^1(z) + ix_A^2(z), x_A^3(z) + ix_A^4(z)), \\ \Psi_A(z) &= (\Psi_A^1(z), \Psi_A^2(z)) = \sqrt{1/2}(\psi_A^1(z) + i\psi_A^2(z), \psi_A^3(z) + i\psi_A^4(z)) \\ X_A^\dagger(z) &= \begin{pmatrix} X_A^{1\dagger}(z) \\ X_A^{2\dagger}(z) \end{pmatrix} = \sqrt{\frac{1}{2}}\begin{pmatrix} x_A^1(z) - ix_A^2(z) \\ x_A^2(z) - ix_A^2(z) \end{pmatrix} \\ \Psi_A^\dagger(z) &= \begin{pmatrix} \Psi_A^{1\dagger}(z) \\ \Psi^{2_A\dagger}(z) \end{pmatrix} = \sqrt{\frac{1}{2}}\begin{pmatrix} \psi_A^1(z) - i\psi_A^2(z) \\ \psi_A^3(z) - i\psi_A^4(z) \end{pmatrix} \end{aligned} \tag{64}$$

In addition to the local $R$ symmetry the free field realization of the $\mathcal{N} = 4$ superconformal algebra has additional global symmetries which can be used to classify the states. There are 2 global $SU(2)$ symmetries which correspond to the $SO(4)$ rotations of the 4 bosons $x^i$. The corresponding charges are given by

$$\begin{aligned} I_1^i &= \frac{1}{4}\int \frac{dz}{2\pi i} X_A \sigma^i \partial X_A^\dagger - \frac{1}{4}\int \frac{dz}{2\pi i} \partial X_A \sigma^i X_A^\dagger + \frac{1}{2}\int \frac{dz}{2\pi i} \Phi_A \sigma^i \Phi_A^\dagger \\ I_2^i &= \frac{1}{4}\int \frac{dz}{2\pi i} \mathcal{X}_A \sigma^i \partial \mathcal{X}_A^\dagger - \frac{1}{4}\int \frac{dz}{2\pi i} \partial \mathcal{X}_A \sigma^i \mathcal{X}_A^\dagger \end{aligned} \tag{65}$$

Here

$$\begin{aligned} \mathcal{X}_A = (X_A^1, -X_A^{2\dagger}) \qquad & \mathcal{X}^\dagger = \begin{pmatrix} X_A^{1\dagger} \\ -X_A^2 \end{pmatrix} \\ \Phi_A = (\Psi_A^1, \Psi_A^{2\dagger}) \qquad & \Phi_A^\dagger = \begin{pmatrix} \Psi_A^{1\dagger} \\ \Psi_A^2 \end{pmatrix}. \end{aligned} \tag{66}$$

These charges are generators of $SU(2) \times SU(2)$ algebra:

$$\begin{aligned} [I_1^i, I_1^j] = i\epsilon^{ijk} I_1^k \qquad & [I_2^i, I_2^j] = i\epsilon^{ijk} I_2^k \\ [I_1^i, J_2^j] &= 0 \end{aligned} \tag{67}$$

The commutation relation of these new global charges with the various local charges are given below

$$\begin{aligned} [I_1^i, G^a(z)] = 0 \qquad & [I_1^i, G^{a\dagger}(z)] = 0 \\ [I_1^i, T(z)] = 0 \qquad & [I_1^i, J(z)] = 0 \\ [I_2^i, \mathcal{G}^a(z)] = \frac{1}{2}\mathcal{G}^b(z)\sigma_{ba}^i \qquad & [I_2^i, \mathcal{G}^{a\dagger}(z)] = -\frac{1}{2}\sigma_{ab}^i \mathcal{G}^{b\dagger}(z) \\ [I_2^i, T(z)] = 0 \qquad & [I_2^i, J(z)] = 0 \end{aligned} \tag{68}$$

where

$$\mathcal{G} = (G^1, G^{2\dagger}) \qquad \mathcal{G}^\dagger = \begin{pmatrix} G^{1\dagger} \\ G^2 \end{pmatrix} \tag{69}$$

The following commutations relation show that the bosons transform as $(\mathbf{2}, \mathbf{2})$ under $SU(2)_{I_1} \times SU(2)_{I_2}$

$$\begin{aligned} [I_1^i, X_A^a] = \frac{1}{2} X_A^b \sigma_{ba}^i \qquad & [I_1^i, X_A^{a\dagger}] = -\frac{1}{2}\sigma_{ab}^i X_A^{b\dagger} \\ [I_2^i, \mathcal{X}_A^a] = \frac{1}{2} \mathcal{X}_A^b \sigma_{ba}^i \qquad & [I_2^i, \mathcal{X}_A^{a\dagger}] = -\frac{1}{2}\sigma_{ab}^i \mathcal{X}_A^{b\dagger} \end{aligned} \tag{70}$$

The fermions transform as $(\mathbf{2},\mathbf{1})$ under $SU(2)_{I_1}\times SU(2)_{I_2}$ as can be seen from the commutations relations given below.

$$[I_1^i,\Phi_A^a]=\frac{1}{2}\Phi_A^b\sigma_{ba}^i \qquad [I_1^i,\Phi_A^{a\dagger}]=-\frac{1}{2}\sigma_{ab}^i\Phi_A^{b\dagger} \tag{71}$$
$$[I_2^i,\Psi^a]=0 \qquad [I_2^i,\bar{\Psi}^a]=0$$

We are interested in studying the states of the $\mathcal{N}=(4,4)$ SCFT on $\mathcal{M}$. The classification of the states and their symmetry properties can be analyzed by studying the states of a free field realization of a $\mathcal{N}=(4,4)$ SCFT on $R^{4Q_1Q_5}/S(Q_1Q_5)$. This is realized by considering the holomorphic and the anti-holomorphic $\mathcal{N}=4$ superconformal algebra with $c=\bar{c}=6Q_1Q_5$ constructed out of $Q_1Q_5$ copies of four real fermions and bosons. So we have an anti-holomorphic component for each field, generator and charges discussed above. These are labelled by the same symbols used for the holomorphic components but distinguished by a tilde.

The charges $I_1,I_2$ constructed above generate $SO(4)$ transformations only on the *holomorphic* bosons $X_A(z)$. Similarly, we can construct charges $\widetilde{I}_1,\widetilde{I}_2$ which generate $SO(4)$ transformations only on the *antiholomorphic* bosons $\widetilde{X_A}(\bar{z})$. Normally one would expect these charges to give rise to a global $SO(4)_{hol}\times SO(4)_{antihol}$ symmetry. However, the kinetic term of the bosons in the free field realization is not invariant under independent holomorphic and antiholomorphic $SO(4)$ rotations. It is easy to see, for example by using the Noether procedure, that there is a residual $SO(4)$ symmetry generated by the charges

$$J_I=I_1+\widetilde{I}_1 \qquad \widetilde{J}_I=I_2+\widetilde{I}_2 \tag{72}$$

We will denote this symmetry as $SO(4)_I=SU(2)_I\times \widetilde{SU(2)}_I$, where the $SU(2)$ factors are generated by $J_I,\widetilde{J}_I$. These charges satisfy the property that (a) they correspond to $SO(4)$ transformations of the bosons $X_A(z,\bar{z})=X_A(z)+\widetilde{X_A}(\bar{z})$ and (b) they fall into representations of the $\mathcal{N}=(4,4)$ algebra (as can be proved by using the commutation relations (70) of the $I$'s). The bosons $X(z,\bar{z})$ transform as $(\mathbf{2},\mathbf{2})$ under $SU(2)_I\times\widetilde{SU(2)}_I$.

# B Short multiplets of $SU(1,1|2)$

The supergroup $SU(1,1|2)$ is the global part of the $\mathcal{N}=4$ superconformal algebra. The representations of this supergroup are classified according to the conformal weight and

$SU(2)_R$ quantum number. The highest weight states $|\mathrm{hw}\rangle = |h, \mathbf{j}_R, j_R^3 = j_R\rangle$ satisfy the following properties

$$
\begin{aligned}
L_1|\mathrm{hw}\rangle = 0 \quad L_0|\mathrm{hw}\rangle = h|\mathrm{hw}\rangle \\
J_R^{(+)}|\mathrm{hw}\rangle = 0 \quad J_R^{(3)}|\mathrm{hw}\rangle = j_R|\mathrm{hw}\rangle \\
G_{1/2}^a|\mathrm{hw}\rangle = 0 \quad G_{1/2}^{a\dagger}|\mathrm{hw}\rangle = 0
\end{aligned} \tag{73}
$$

Where $L_{\pm,0}, J_R^{(\pm),(3)}$ are the global charges of the currents $T(z)$ and $J_R^{(i)}(z)$. The charges $G^a_{1/2,-1/2}$ are the global charges of the supersymmetry currents $G^a(z)$ in the Neveu-Schwarz sector. Highest weight states which satisfy $G^{2\dagger}_{-1/2}|\mathrm{hw}\rangle = 0, \quad G^1_{-1/2}|\mathrm{hw}\rangle = 0$ are chiral primaries. They satisfy $h = j$. We will denote these states as $|\mathrm{hw}\rangle_S$. Short multiplets are generated from the chiral primaries through the action of the raising operators $J_-, G^{1\dagger}_{-1/2}$ and $G^2_{-1/2}$. The structure of the short multiplet is given below

$$
\begin{array}{cccc}
\text{States} & j & L_0 & \text{Degeneracy} \\
|\mathrm{hw}\rangle_S & h & h & 2h+1 \\
G^{1\dagger}_{-1/2}|\mathrm{hw}\rangle_S, G^2_{-1/2}|\mathrm{hw}\rangle_S & h-1/2 & h+1/2 & 2h+2h=4h \\
G^{1\dagger}_{-1/2}G^2_{-1/2}|\mathrm{hw}\rangle_S & h-1 & h+1 & 2h-1
\end{array} \tag{74}
$$

The short multiplets of the supergroup $SU(1,1|2)\times SU(1,1|2)$ are obtained by the tensor product of the above multiplet. We denote the short multiplet of $SU(1,1|2)\times SU(1,1|2)$ as $(\mathbf{2h+1}, \mathbf{2h'+1})_{\mathbf{S}}$. These stand for the degeneracy of the bottom component, the top row in (74). The top component of the short multiplet are the states belonging to the last row in (74). The short multiplet $(\mathbf{2},\mathbf{2})_{\mathbf{S}}$ is special, it terminates at the middle row of (74). For this case, the top component is the middle row. These states have $h = \bar{h} = 1$ and transform as $(\mathbf{1},\mathbf{1})$ of $SU(2)_R \times \widetilde{SU(2)}_R$. There are 4 such states for each $(\mathbf{2},\mathbf{2})_{\mathbf{S}}$.

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