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Aspects of Semiclassical Strings in AdS$_5$

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Abstract: We find an infinite number of conserved currents and charges in the semiclassical limit $\lambda \to \infty$ of string theory in AdS$_5 \times S^5$ and remark on their relevance to conserved charges in the dual gauge theory. We establish a general procedure of exploring the semiclassical limit by viewing the classical motion as collective motion in the relevant part of the configuration space. We illustrate the procedure for semiclassical expansion around solutions of string theory on AdS$_5 \times (S^5/Z_M)$.

Keywords: WKB, String, AdS-CFT.

Contents

1. Introduction

Recently string propagation in pp-wave backgrounds has attracted much interest [12–14]. One of the attractive features of this background is that the tree level string theory is exactly soluble in the Green-Schwarz formalism [15, 16]. In a parallel development Berenstein, Maldacena and Nastase (BMN) [17] presented a derivation of the tree level string theory starting from the dual SYM theory. In fact despite various efforts over the past 25 years [39–49] this is the first time a precise derivation of critical string theory (in a given background) has been given from a gauge theory. We also see how a spacetime background arises from a gauge theory.

Subsequently there has been work along the lines of BMN in generalizing to strings propagating in various spaces related to $AdS_5 \times S^5$ [18] where a similar analysis was done for open strings. For other related developments see [20–38].

In another interesting development Gubser, Klebanov and Polyakov (GKP) [9] proposed to explore time dependent soliton solutions of the sigma model on $AdS_5 \times S^5$ and their relation to specific sectors of the dual gauge theory. For example a collapsed string moving on the great circle of S^5 with angular velocity ω corresponds as expected to a chiral primary state in the gauge theory with a large value of the R-charge: $J = \sqrt{\lambda}\,\omega$, where λ is the 'tHooft coupling: $\lambda = g_{YM}^2 N$. In the semi-classical limit we send $N \to \infty$, holding g_{YM}^2 small and fixed. They also explored various other soliton solutions corresponding to a spinning string in AdS_5. These solutions were generalised in [10, 19, 29]. These solitons do not correspond to chiral primary states and the precise theory needs further exploration.

Our work is an attempt to explore further the GKP proposal. There are two aspects to this paper. First, a formal aspect points out that the sigma models under question have an infinite number of classical conserved currents [3–5]. We point out the relevance of the conserved charges to the classical string theory.

The second aspect of the paper deals with the orbifold $AdS_5 \times S^5/Z_M$. The Penrose limit of this model has been discussed in [24] and the gauge theory correspondence has been studied in [11]. We detail the analysis of the fluctuations around the soliton in $AdS_5 \times S^5/Z_M$ and reproduce the string propagating in a pp-wave geometry with a compact light like circle. In doing this analysis we develop a general procedure of exploring the semiclassical limit by viewing the classical motion as collective motion in the relevant part of the configuration space.

Below, in Section 2, we review the relevant aspects of the string sigma model on $AdS_5 \times S^5$ background [1,2]. In Section 3 we turn to the conserved currents of the non-linear sigma model following [4] and discuss their relevance to string theory for worldsheet solitons. In Section 4, we present a solitonic solution of the sigma model on the orbifold $AdS_5 \times S^5/Z_M$ and describe its quantization in a semiclassical expansion using the method of collective coordinates. The spectrum corresponds to that of a winding string propagating in the pp-wave geometry of [11,24].

2. String theory on $\mathbf{AdS}_5 \times S^5$

Here we start with a brief review of the relevant aspects of string sigma model on $AdS_5 \times S^5$ background. We represent the metric on $AdS_5 \times S^5$ by the line element

$$ds^2 = R^2 \left(dN_\alpha dN^\alpha + dn_a dn^a\right) \tag{2.1}$$

where N^α represents a "timelike" unit vector in $R^{2,4}$:

$$-\eta_{\alpha\beta} N^\alpha N^\beta = N_0^2 + N_5^2 - N_1^2 - N_2^2 - N_3^2 - N_4^2 = 1 \tag{2.2}$$

and n_a is a unit vector in R^6:

$$n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2 + n_6^2 = 1. \tag{2.3}$$

The bosonic part of the world-sheet action of string theory in the above space is [1,2]

$$S = -\frac{R^2}{4\pi\alpha'} \int d^2\xi \ \sqrt{g} g^{\mu\nu} \left[\partial_\mu N^\alpha \partial_\nu N_\alpha + \partial_\mu n^a \partial_\nu n^a\right] \tag{2.4}$$

where we have omitted the fermionic terms (which include the RR couplings). The fermionic terms are of course crucial for a consistent quantum string theory. However, it is clear that for classical solutions of the string world-sheet action, it suffices to set the fermions to zero and consider only the bosonic terms in the action. We have also chosen to work with a Minkowski world sheet. As in [9] we will use the notation

$$\alpha = \alpha'/R^2$$

and understand $\alpha \to 0$ as the classical limit. The 'tHooft coupling of the dual gauge theory is given by

$$\lambda \sim R^4/(\alpha')^2 = 1/\alpha^2.$$

After fixing the conformal gauge on the metric [1] $\sqrt{g}g^{\mu\nu} = \eta^{\mu\nu} = \mathrm{Diag}[-1,1]$, the action becomes a sum of non-linear sigma models on S^5 and AdS_5 in two-dimensional Minkowski space:

$$S = -\frac{R^2}{4\pi\alpha'}\int d^2\xi\ \left[\partial_\mu N^\alpha \partial^\mu N_\alpha + \partial_\mu n^a \partial^\mu n^a\right] \tag{2.5}$$

which comes together with two constraints $T_{++} = T_{--} = 0$. These read

$$\begin{aligned} \partial_+ N^\alpha \partial_+ N_\alpha + \partial_+ n^a \partial_+ n^a &= 0 \\ \partial_- n^a \partial_- n^a + \partial_- N^\alpha \partial_- N_\alpha &= 0 \end{aligned} \tag{2.6}$$

In the above equations, $\partial_\pm = \frac{\partial}{\partial\sigma^\pm}$, $\sigma^\pm = \frac{1}{2}(\tau \pm \sigma)$. The action (2.5) leads to the following equations of motion:

$$\partial^2 n^a = n^a n^b \partial^2 n^b, \tag{2.7}$$

$$\partial^2 N^\alpha = -N^\alpha N_\beta \partial^2 N^\beta \tag{2.8}$$

We should note that at the classical level, the system (2.5) is conformally invariant.

3. Infinite number of conserved currents

Luscher and Pohlmeyer [4] (see also [5]) show that for a classical non-linear sigma model, namely one that satisfies (2.3) and (2.7), there are infinite number of (non-local) conserved currents J^n_μ in the system:

$$\partial_\mu J^\mu_n = 0 \tag{3.1}$$

The discussion in the above papers assumes that the NLSM is defined on a plane while the string theory world-sheet is usually a cylinder. This is not a problem; since the theory classically is conformally invariant, we can make a conformal transformation from a cylinder to a plane to use the results of [4, 5].

These currents are constructed recursively. From a J^μ_n satisfying (3.1), it is always possible to define χ_n so that

$$J_{n,\mu} =: \epsilon_{\mu\nu}\partial^\nu \chi_n \tag{3.2}$$

Define, further,

$$(A_\mu)_{ab} = n_a \partial_\mu n_b - n_b \partial_\mu n_a \tag{3.3}$$

Then, the $n+1$-th current is

$$J_{n+1,\mu} := (\partial_\mu + A_\mu)\chi_n \tag{3.4}$$

Note that equation (2.7) implies $\partial^\mu A_\mu = 0$. Thus we can start with $J^1_\mu = A\mu, (\chi_0 = 1)$ and so on.

A similar remark goes through also for the non-linear sigma-model based on AdS_5 and one can again, independently, construct an infinite number of conserved currents.

At this point, one must note that the equations of motion (2.7) and (2.8) are constrained together by (2.6). Then how does our above construction of infinite number of conserved currents remain valid? After all, we are doing string theory here as opposed to a non-linear sigma model.

This turns out to have an easy answer. The constraints are first class, so they can in principle be gauge-fixed. However, even without gauge fixing, it is easy to see that the equations of motion will be always valid (it only amounts to redundancy in the description of dynamical evolution). Thus, the construction of the conserved currents that we have described above remains valid. Indeed, the argument can be made differently. Suppose we have fixed the gauge; then on the gauge-fixed surface also the equations of motion will remain valid, and hence the construction of the conserved currents. Thus we do have an infinite number of conserved currents in string theory.

3.1 Spacetime significance of these charges

We found above that there are infinite number of classically conserved currents in the $AdS_5 \times S^5$ string theory. How about *charges*? For Minkowski signature of the world-sheet, it is easy to fix a partial gauge $\tau = t$. In this case the infinite number of conserved currents gives rise to an infinite number of conserved charges, which are conserved also in time in the *target space*. We note that typically global charges in the world-sheet theory correspond to a local symmetry in the target space, which, in turn, corresponds to global charges in the dual gauge theory.

The above discussion would imply that the dual $\mathcal{N} = 4$ super Yang Mills theory has an infinite number of conserved, perhaps nonlocal, charges in the classical limit $\lambda \to \infty$ (note that the time in the string theory is the same as that of the gauge theory). In this paper we will not discuss the rather important question of what happens to these conserved charges at $\lambda < \infty$, in particular whether the conservation laws have anomalies. This issue is somewhat more subtle in string theory compared to usual NLSM because of the inclusion of fermionic terms in the currents in the quantum theory. It is also important to note that the existence of such integrable structures has been hinted at in [6] at the opposite limit $\lambda \to 0$. It would be obviously interesting to unravel any such integrable structure at a general λ (see Figure 1).

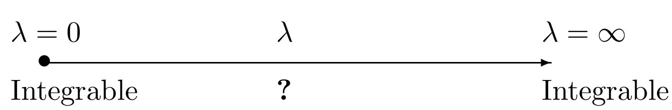

Figure 1

3.2 $\mathbf{AdS}_3 \times S^3$

From the viewpoint of integrability, string theory on $\mathrm{AdS}_3 \times S^3$ appears to be easier. Since S^3 is a group manifold, more is known about its integrable structure and exact solutions. Besides, it is also known that there are additional world-sheet charges which are local [7]. Furthermore, there seems to be a rather direct correspondence between the current algebra on the world-sheet and a corresponding current algebra in spacetime [8]. We hope to return to these issues and its significance for the dual CFT in a future publication.

4. String Solitons on $AdS_5 \times S^5/Z_M$

In the second half of the paper we want to report on semiclassical solutions of string theory on $AdS_5 \times S^5/Z_M$ orbifold. Part of our motivation is to share some insights on the general method of dealing with such semiclassical limits which applies to more general manifolds and of course to the original $\mathrm{AdS}_5 \times S^5$ string theory as well. We seek a solution of the sigma model, in the spirit of [9], which sees the pp-wave geometry of [24]. The dual gauge theory was discussed in [11].

We recall that the metric of $AdS_5 \times S^5$ is given by (2.1),(2.3) and (2.2). We will use below the following explicit representation for the unit vector N^α:

$$\begin{aligned} N^0 &= \cosh\rho \cos t \\ N^5 &= \cosh\rho \sin t \\ N^j &= \sinh\rho\, \Omega^j \quad\quad j = 1,..,4 \quad\quad \text{such that} \quad\quad \Omega^j \Omega^j = 1 \end{aligned} \tag{4.1}$$

With this,

$$ds^2_{AdS_5} = R^2[-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2] \tag{4.2}$$

We will further use the following representation for the unit vector n_a

$$\begin{aligned} n_1 + i\, n_2 &= z_1 = \sin\beta\, e^{i\theta} \\ n_3 + i\, n_4 &= z_2 = \cos\beta \cos\gamma\, e^{i\chi} \\ n_5 + i\, n_6 &= z_3 = \cos\beta \sin\gamma\, e^{i\phi} \end{aligned} \tag{4.3}$$

With this, the metric of S^5 becomes:

$$ds^2_{S^5} = R^2[d\beta^2 + \sin^2\beta\, d\theta^2 + \cos^2\beta\, (d\gamma^2 + \cos^2\gamma\, d\chi^2 + \sin^2\gamma\, d\phi^2)] \tag{4.4}$$

Now we come to $\mathrm{AdS}_5 \times (S^5/Z_M)$. The Z_M orbifold action on S_5 is defined by

$$z_1 \to z_1, z_2 \to e^{2\pi i/M} z_2, z_3 \to e^{-2\pi i/M} z_3 \tag{4.5}$$

As shown in [50], this implies that χ, ϕ in (4.3) can be written in terms of usual 2π-periodic angles as

$$\begin{aligned} \chi &= \eta/M \\ \phi &= -\eta/M + \zeta \\ \eta &\equiv \eta + 2\pi, \zeta \equiv \zeta + 2\pi \end{aligned} \tag{4.6}$$

In other words (4.5) imply the following identifications for ϕ, χ

$$\begin{aligned}\chi &\equiv \chi + \frac{2\pi}{M}.\text{integer} \\ \phi &\equiv \phi + 2\pi.\text{integer} - \frac{2\pi}{M}.\text{integer}\end{aligned} \tag{4.7}$$

4.1 Semiclassical limit

The string theory action (2.5) on $\text{AdS}_5 \times (S^5/Z_M)$, comprising the bosonic terms, is given by

$$\begin{aligned}S &= -\frac{R^2}{4\pi\alpha'}\int d^2\sigma \Big[-\cosh^2\rho(\partial t)^2 + (\partial\rho)^2 + \sinh^2\rho(\partial\Omega)^2 + \\ &\sin^2\beta(\partial\theta)^2 + (\partial\beta)^2 + \cos^2\beta\left((\partial\gamma)^2 + \cos^2\gamma(\partial\chi)^2 + \sin^2\gamma(\partial\phi)^2\right)\Big]\end{aligned} \tag{4.8}$$

where the identifications (4.7) are understood. $(\partial\rho)^2$ means $-(\partial_\tau\rho)^2 + (\partial_\sigma\rho)^2$ etc. The range of σ is $[0, 2\pi]$ and of τ is $(-\infty, \infty)$.

It is worth writing out the constraints (2.6) explicitly:

$$\begin{aligned}T_{++} = & -\cosh^2\rho(\partial_+ t)^2 + (\partial_+\rho)^2 + \sinh^2\rho(\partial_+\Omega)^2 + \sin^2\beta(\partial_+\theta)^2 + \\ & (\partial_+\beta)^2 + \cos^2\beta\left((\partial_+\gamma)^2 + \cos^2\gamma(\partial_+\chi)^2 + \sin^2\gamma(\partial_+\phi)^2\right) = 0 \\ T_{--} = & -\cosh^2\rho(\partial_- t)^2 + (\partial_-\rho)^2 + \sinh^2\rho(\partial_-\Omega)^2 + \sin^2\beta(\partial_-\theta)^2 + \\ & (\partial_-\beta)^2 + \cos^2\beta\left((\partial_-\gamma)^2 + \cos^2\gamma(\partial_-\chi)^2 + \sin^2\gamma(\partial_-\phi)^2\right) = 0\end{aligned} \tag{4.9}$$

Now we seek a solution of the equations of motion (2.7),(2.8) and constraint equations (4.9) above.

4.1.1 Collective coordinates

We note that since $\frac{\partial}{\partial t}, \frac{\partial}{\partial \chi}$ are isometries of $\text{AdS}_5 \times S^5$ (and of the orbifold), any solution $\Big(n_a(t(\tau,\sigma), \rho(\tau,\sigma), \ldots),\ N^\alpha(\chi(\tau,\sigma), \beta(\tau,\sigma), \ldots)\Big)$ of the above equations of motion and constraints will have the following shift symmetry

$$\begin{aligned}&\Big(n_a(t(\tau,\sigma), \rho(\tau,\sigma), \ldots),\ \ N^\alpha(\chi(\tau,\sigma, \beta(\tau,\sigma), \ldots)\Big) \to \\ &\qquad \Big(n_a(t(\tau,\sigma) + a, \rho(\tau,\sigma), \ldots), N^\alpha(\chi(\tau,\sigma) + b, \beta(\tau,\sigma), \ldots)\Big)\end{aligned} \tag{4.10}$$

We will further assume that we are considering solutions satisfying $\rho = 0, \beta = 0$ (solutions localized near the centre of AdS_5, and close to an equator of S^5).

As is well-known [51], to take care of the dynamics along the shift directions we can elevate the parameters a, b to functions of τ; these are called collective coordinates. Instead of calling these functions $a(\tau), b(\tau)$ we will permit ourselves an abuse of notation, calling them $t(\tau), \chi(\tau)$.

The collective coordinate action is easily seen to be

$$\frac{S_{coll}}{\alpha} = \frac{1}{\alpha}\int d\tau\ \left[(\dot\chi)^2 - (\dot t)^2\right],\ \alpha \equiv \alpha'/R^2 \tag{4.11}$$

The constraints (4.9) become

$$(\dot\chi)^2 - (\dot t)^2 = 4\dot\chi^+\dot\chi^- = 0 \tag{4.12}$$

where we have introduced

$$\chi^{\pm} = \frac{1}{2}(\chi \pm t) \tag{4.13}$$

The canonical Hamiltonian

$$H = \frac{1}{4} P_+ P_- \tag{4.14}$$

vanishes because of the constraint. Here

$$P_+ = \frac{4}{\dot{\chi}^+}, P_- = \frac{4}{\alpha}\dot{\chi}^- \tag{4.15}$$

are the canonical momenta. We have treated α as an effective $\hbar$.

The constraint can be made to vanish either by choosing $P_- = 0$ or $P_+ = 0$. We will choose the branch $P_- = 0$. As usual, the constraint generates redundant motion. This can be fixed by an appropriate gauge choice. We will choose the gauge

$$\chi_- = 0. \tag{4.16}$$

This, together with the constraint $P_- = 0$, become a pair of second-class constraints.

The quantization of this system is simple. The wavefunctions of the system which satisfies the constraint are of the form

$$\psi(t, \chi) = \exp[iP_+ \chi^+ / \alpha] \tag{4.17}$$

Eqns. (4.7) imply the following identification for $\chi^{\pm}$:

$$\chi^+ \equiv \chi^+ + 2\pi/M, \; \chi^- \equiv \chi^- + 2\pi/M, \tag{4.18}$$

which in turn quantizes P_+:

$$P_+ = \alpha.k.M, k = \text{integer} \tag{4.19}$$

Quantized geodesic

The quantum mechanical wavefunction (4.17), together with the gauge (4.16) is equivalent to the following classical trajectory in the sigma-model path integral (quantized as above)

$$\begin{aligned} \chi = t = w\tau, \\ \rho = 0, \beta = 0 \end{aligned} \tag{4.20}$$

This represents a spinning particle in the χ direction in S^5. The quantization (4.19), through (4.15) and (4.20) implies

$$w = \alpha.k.M = k.\alpha' M/R^2 \tag{4.21}$$

It can be easily checked that the constraints (4.9) are satisfied, as we anyway ensured in the discussion of the collective coordinate dynamics.

Thus we see that the classical solution can be consistently discussed only in the framework of collective coordinates since it involves $1/\alpha = 1/\hbar$ effects.

In view of Eqn. (4.21) we note already that the semiclassical limit $\alpha = \alpha'/R^2 \to 0$ is meaningful only if M is also sent to infinity in such a way that

$$\alpha \to 0, M \to \infty, R^2/(\alpha' M) \equiv R_- = \text{fixed} \tag{4.22}$$

4.2 Fluctuations

We now consider the field-theory fluctuations orthogonal to the collective coordinate motion (4.20).

We will make the gauge choice that the χ^+ fluctuation vanishes:

$$\delta\chi^+ = 0$$

In other words the gauge is

$$\chi^+ = w\tau \tag{4.23}$$

With this, the constraints (4.9) enable us to solve for χ^- in terms of the other eight variables which we will treat as independent. One can easily check that the gauge (4.23) can be fixed. The solvability of the dependent variable χ^- in terms of the independent variables gives an independent justification for the gauge choice (4.23).

In order to proceed, we will make a semiclassical expansion of the fluctuations in powers of $\sqrt{\alpha}$. It is easy to see that in order to have canonically normalized kinetic terms for the independent variables, we need (since these variables were zero for the collective motion, we will not explicitly write $\delta\rho$ etc. and write ρ instead)

$$\begin{aligned}
\rho &= \sqrt{\alpha}\bar{\rho} \\
\Omega &= \bar{\Omega} \\
\beta &= \sqrt{\alpha}\bar{\beta} \\
\phi &= \bar{\phi} \\
\theta &= \bar{\theta} \\
\gamma &= \sqrt{\alpha}\bar{\gamma}
\end{aligned} \tag{4.24}$$

In view of the classical value (4.16) and the difference of the two constraints in (4.9), it is clear that that the dependent variable χ_- starts with

$$\chi_- = \alpha\bar{\chi}^-, \ \bar{\chi}^- = F(\bar{\rho}, \bar{\Omega}, \ldots) \tag{4.25}$$

This equation resolves an important subtlety. Note that (4.18) appears to be problematic in the limit (4.22). However, the fact that (see (4.25)) χ^- turns out of order $\alpha = \alpha'/R^2$ resolves this problem. In other words, (4.18) means that the periodicity of $\bar{\chi}^-$ is

$$\bar{\chi}^- \equiv \bar{\chi}^- + 2\pi.m/(M\alpha) = 2\pi.mR_-, \ m = \text{integer} \tag{4.26}$$

so that the period is of order one. We have used above the scaling (4.22). This is consistent with the second equation of (4.25) where $\bar{\chi}^-$ is determined in terms of scaled quantities and hence of order one.

4.2.1 Explicit details of fluctuation calculation

With the expansions given in (4.24), the constraints (4.9) become, upto $o(\alpha)$,

$$\begin{aligned} T_{++} &= 4\partial_+\chi^+\partial_+\chi^- + \alpha\left[(\partial_+\vec{r})^2 + (\partial_+\vec{y})^2 - w^2(\vec{r}^2 + \vec{y}^2)\right] = 0 \\ T_{--} &= 4\partial_-\chi^+\partial_-\chi^- + \alpha\left[(\partial_-\vec{r})^2 + (\partial_-\vec{y})^2 - w^2(\vec{r}^2 + \vec{y}^2)\right] = 0 \end{aligned} \tag{4.27}$$

where $\vec{r} = (r_1, r_2, r_3, r_4), \vec{y} = (y_1, y_2, y_3, y_4)$ are defined as follows

$$\begin{aligned} &r_i = \bar{\rho}\Omega_i, \\ &(y_1, y_2) = \bar{\beta}(\cos\bar{\theta}, \sin\bar{\theta}), (y_3, y_4) = \bar{\gamma}(\cos\bar{\phi}, \sin\bar{\phi}) \end{aligned} \tag{4.28}$$

We note that the periodicity of $\phi \equiv \bar{\phi}$ becomes standard in the $M \to \infty$ limit (see Eqn. (4.7)). This ensures that (y_3, y_4) represent a plane just like (y_1, y_2).

By using the gauge condition (4.23) we clearly see from the above constraints that χ_- must be $O(\alpha)$, as written earlier in (4.25). Using the gauge condition (4.23) and taking the difference of the two constraints we get

$$\partial_\tau\bar{\chi}^- = -\frac{1}{4w}\left[(\partial_\tau\vec{r})^2 + (\partial_\sigma\vec{r})^2 + (\partial_\tau\vec{y})^2 + (\partial_\sigma\vec{y})^2 - w^2(\vec{r}^2 + \vec{y}^2)\right] \tag{4.29}$$

$$\partial_\sigma\bar{\chi}^- = -\frac{1}{4w}\left[\partial_\tau\vec{r}.\partial_\sigma\vec{r} + \partial_\tau\vec{y}.\partial_\sigma\vec{y}\right] \tag{4.30}$$

Integrating (4.30), using $\bar{\chi}^-(2\pi) - \chi^-(0) = 2\pi.m.R_-$ according to (4.26), and the value of w from (4.21), we get

$$4km + \frac{1}{2\pi}\int_0^{2\pi} d\sigma\left[\partial_\tau\vec{r}.\partial_\sigma\vec{r} + \partial_\tau\vec{y}.\partial_\sigma\vec{y}\right] = 0 \tag{4.31}$$

This gives the level matching condition of the string theory.

The Eqn. (4.29) can be used to determine both the charges corresponding to the isometries $\partial/\partial t, \partial/\partial\chi$, namely the energy E and the angular momentum J (and consequently $E - J$), as follows. It is easy to see, e.g. by using the Noether prescription, that E and J are given by

$$\begin{aligned} E &= \frac{1}{2\pi\alpha}\int_0^{2\pi} d\sigma\cosh^2\rho\;\partial_\tau t \\ J &= \frac{1}{2\pi\alpha}\int_0^{2\pi} d\sigma\cos^2\beta\cos^2\gamma\;\partial_\tau\chi \end{aligned} \tag{4.32}$$

By using the expansions (4.24) and (4.29), we get for $E - J$

$$E - J = \frac{1}{2w}\int_0^{2\pi} d\sigma\left[(\partial_\tau\vec{r})^2 + (\partial_\sigma\vec{r})^2 + (\partial_\tau\vec{y})^2 + (\partial_\sigma\vec{y})^2 + w^2(\vec{r}^2 + \vec{y}^2)\right] \tag{4.33}$$

This gives the spectrum of anomalous dimensions (Δ, which equals E) in the dual gauge theory [17].

Fermions

We should remark that at this order of fluctuations we should include fermions. It follows from considerations of supersymmetry that the eigenvalues of the energy and angular momentum described above remain the same, however the spectrum becomes supersymmetric.

Other examples

It is clear from the preceding discussion that our methods are rather general and can be easily applied to other orbifolds of $\mathrm{AdS}_5 \times S^5$, in particular the ones involving $S^5/(Z_M \times Z_{M'})$ which preserve $\mathcal{N} = 1$ supersymmetry. Note that the continuous isometries of such spaces are the same as those of $\mathrm{AdS}_5 \times S^5$.

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