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Research Article

Iterative Receiver Based on SAGE Algorithm for Crosstalk Cancellation in Upstream Vectored VDSL

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We propose the use of an iterative receiver based on the Space Alternating Generalized Expectation maximization (SAGE) algorithm for crosstalk cancellation in upstream vectored VDSL. In the absence of alien crosstalk, we show that when initialized with the frequency-domain equalizer (FEQ) output, the far-end crosstalk (FEXT) can be cancelled with no more real-time complexity than the existing linear receivers. In addition, the suggested approach does not require offline computation of the channel inverse and thus reduces the receiver complexity. In the presence of alien crosstalk, there is a significant gap between the rate performance of the linear receivers as compared with the single-user bound (SUB). The proposed receiver is shown to successfully bridge this gap while requiring only a little extracomplexity. Computer simulations are presented to validate the analysis and confirm the performance of the proposed receiver.

1. Introduction

"Very-high-speed Digital Subscriber Lines (VDSLs)" is a broadband access technology that uses twisted pairs (TPs) as a medium for high-speed data transmission [1]. It is one of the key broadband technologies for solving the "last mile" problem. With the exploitation of a high bandwidth (in tens of a megahertz), it can provide a bidirectional data rate up to 200 Mbps over short loop lengths [2]. Several TPs corresponding to a number of users are contained in a binder, which ultimately connect the central office (CO) or optical network unit (ONU) to the customer premise equipment (CPE). Because of the electromagnetic coupling among closely packed TPs, a "crosstalk" termed as far-end crosstalk (FEXT) is introduced into the far-end signal at each TP. Such crosstalk that arises from subscribers enjoying similar type of services under VDSL systems is referred to as self-crosstalk and degrades the performance significantly, especially for shorter loop lengths [1]. Another major cause of rate degradation is crosstalk that originates from subscribers enjoying other services, and is referred to as alien crosstalk [3]. Such crosstalk exists in practical situations mainly due to the coexistence of broadband over power line (BPL) systems, radio frequency interference (RFI) ingress, and crosstalk from subscribers within the same binder enjoying other DSL services.

With the advent of vectored transmission in [4], which leverages the colocation of receiver modems at the CO, there has been a surge of research interest in receiver designs for crosstalk cancellation [5]. For the design of crosstalk cancelers, computational complexity is an important issue due to the presence of a large number of tones (typically 4096) as well as a number of users per vectored group. Recently, a near-optimal linear zero forcing (ZF) receiver has been proposed in [6] for self-crosstalk cancellation, which requires channel matrix inversion at each tone. Such matrix inversions are frequently required due to changes in user status or variations in crosstalk characteristics [7], and hence cause an increase in overhead on the computational cost of the ZF receiver. To avoid this, a low-order truncated series approximation of the inverse channel matrix was considered for downstream transmissions in [8]. But it was shown that such an approach does not provide performance as good as the ZF receiver when the loop lengths are short (which is often the case with VDSL systems). The authors in [9] have suggested a pilot-based least-mean-square (LMS) tracking algorithm, which requires a large training overhead for self-crosstalk cancellation.

All the above authors assume the absence of alien crosstalk. The presence of an alien noise, originating from an external source, introduces a high spatial correlation among noise at the receivers of different vectored users [10–12]. It was shown in [3] that noise correlation between twisted pairs is more than the correlation between the tones. In theory, the receivers suggested for self-crosstalk can be applied after a whitening procedure. However, the prewhitening operation applied on the spatially correlated noise destroys the columnwise diagonal dominant (CWDD) characteristics of the channel (leading to poor performance of linear receivers for alien crosstalk cancellation). The authors in [10] suggested a nonlinear successive interference canceler to achieve higher data rates than ZF receiver. However, this receiver requires **QR** (here **Q** denotes a unitary matrix, and **R** is an upper triangular matrix) decomposition of the channel matrix and is quite involved computationally because of the search required for **QR** ordering. A computationally expensive turbo receiver based on the minimum mean-square-error (MMSE) criterion for such crosstalk cancellation was suggested in [13]. An alien crosstalk canceler was considered in [14] by assuming perfect symbol estimation after self-crosstalk cancellation. In [3], a joint transmitter-receiver cooperation framework was shown to achieve capacity for alien crosstalk cancellation. However, the proposed algorithm is dependent on knowledge of the channel at the transmitter, and cooperation between the CPEs, which is not feasible in most situations. An algorithm to mitigate a single interference from home local area network using iterative soft cancellation was suggested in [15] for use in downstream DSL. The proposed scheme does not consider vectoring and, therefore, may not be suitable for use in the upstream transmissions.

In practice, there is a need for crosstalk canceler at each tone, which can support both conventional single-user as well as multiuser detection with a limited complexity. Considering a bit loading based on the crosstalk-free signal-to-noise ratio (SNR), the performance after frequency-domain equalization (FEQ) may be degraded in terms of bit error rate (BER). However, it is important to appreciate that the estimated symbols are located within a small radius (of the order of minimum distance between constellation points) of the true symbol value. When the crosstalk is present on the victim line, it can be mitigated by utilizing the estimates of the disturbers (after FEQ) iteratively to yield a relatively smaller variance of the residual crosstalk, ensuring the minimum BER level of 10^{-7} as per the DSL standard. The specific CWDD property of DSL channels facilitates achievement of crosstalk-free performance and hence motivates the deployment of an iterative canceler.

With the above motivation, we propose in this paper an iterative receiver based on a space-alternating generalized expectation maximization (SAGE) algorithm for cancellation of the crosstalk in VDSL systems. Basically, the SAGE algorithm is a variant of the expectation-maximization (EM) algorithm [16] that yields performance close to the maximum likelihood (ML) solution in situations where the ML solution is computationally intractable [17]. We initially consider a situation when alien crosstalk is not present. By employing an ordered SAGE (OSAGE), we derive simple bounds on the achievable signal-to-interference-noise ratio (SINR) to help facilitate the performance analysis. Based on the derived bounds, we show that our proposed canceler provides near crosstalk-free performance while eliminating the need for channel inversion. We also show that the proposed receiver cancels the self-crosstalk by initializing the receiver with the FEQ output and requires only a single iteration to come close to the optimal performance. We next consider the case when alien crosstalk is also present. By deriving a new bound on the CWDD parameter of equivalent channel after noise whitening, we show that the convergence conditions continue to be satisfied in most situations of practical interest and can be exploited to cancel the alien crosstalk. In the presence of alien crosstalk of high power and/or low correlation, the SAGE algorithm is still shown to require only one iteration for approximating the ML solution, though a few additional iterations may be required under the conditions of low power and/or high correlation of alien noise across the TPs. This offers an attractive trade-off between data rate and complexity while canceling the self-crosstalk and mitigating alien crosstalk. Computer simulations are conducted to demonstrate the relevance of the proposed method in practical VDSL deployments.

The organization of this paper is as follows. A description of the system model is given in Section 2. Section 3 presents an iterative receiver based on the SAGE algorithm for self-crosstalk cancellation while Section 4 describes an alien crosstalk cancellation algorithm with noise prewhitening. The performance of the proposed iterative receiver based on the derived bounds as well as computer simulations is assessed in Section 5. Finally, conclusions are drawn in Section 6.

Notation. Vectors (matrices) are denoted by boldface lower (upper) case letters. A_{ij} and $[\mathbf{A}]_{ij}$ denote the ijth element of the matrix $\mathbf{A}$ while X_i denotes the ith element of vector $\mathbf{x}$. All vectors are column vectors. The variance of random variable X is denoted by σ_x^2. The operators $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^\dagger$ denote the conjugate, transpose, and conjugate transpose, respectively. The operators $\mathbb{E}\{\cdot\}$, $\mathrm{tr}(\cdot)$, $|\cdot|$, and $\det(\cdot)$ represent expectation, trace, absolute value, and determinant, respectively. We use $\tilde{z}$ and $\hat{z}$ to differentiate z corresponding to self- and alien crosstalk cancellation, respectively.

2. System Model

We consider a system model for upstream transmissions as shown in Figure 1. As is well known, DMT modulation based on Inverse Fast Fourier Transform (IFFT) is used in VDSL. The available frequency band is divided into a number of parallel subcarriers or tones, and IFFT effectively loads symbols onto the multiple tones. The DMT receivers (at each TP in the CO) ignore the cyclic prefix portion of the received signal and use the Fast Fourier Transform (FFT)

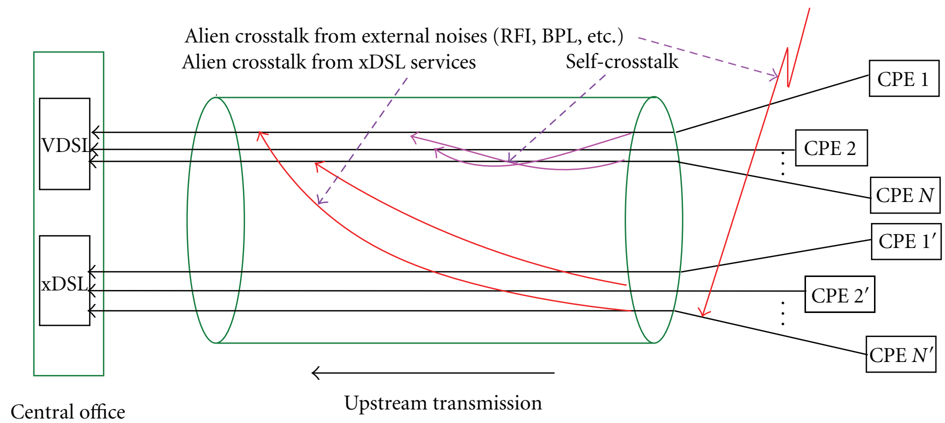

Figure 1: System model of vectored VDSL and crosstalk environment.

for demodulation. Since an adaptive bit loading is used, the QAM constellation size varies according to the receive SNR. We assume that all users are perfectly synchronized and that the impulse response length of channel is no longer than the cyclic prefix length. We now consider a VDSL system with N vectored users as shown in Figure 1. At the receiver of the ith TP, the kth sample of the FFT $Y_{i,k}$ is given by

$$Y_{i,k} = H_{ii,k}X_{i,k} + \sum_{j=1, j\neq i}^{N} H_{ij,k}X_{j,k} + V_{i,k}, \tag{1}$$

where $X_{i,k}$ is the data symbol of the ith user and $V_{i,k}$ is a component of the additive noise at the kth tone that includes the alien crosstalk and thermal noise. The coefficient $H_{ii,k}$ basically arises due to attenuation of the kth tone in the TP, which is usually modeled by the transmission line theory. Crosstalk coupling coefficient $H_{ij,k}$ is the complex channel element of the kth tone from the jth interferer to the ith victim and is modeled as in [18]. The second term on the right-hand side of (1) represents the FEXT. As is well known, this crosstalk is the major factor limiting the performance of DSL systems. We consider signal-level coordination in such a way that the samples $Y_{i,k}$ at the kth tone for all TPs are processed together (referred to as *vectoring* in [4]). Therefore, the received vector on the kth tone can be expressed as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{y}_k = [Y_{1,k}, Y_{2,k}, Y_{3,k}, \ldots, Y_{N,k}]^T$, $\mathbf{x}_k = [X_{1,k}, X_{2,k}, X_{3,k}, \ldots, X_{N,k}]^T$, and $\mathbf{v}_k = [V_{1,k}, V_{2,k}, V_{3,k}, \ldots, V_{N,k}]^T$ represent the vectors of received samples, transmitted symbols, and noise samples, respectively. $\mathbf{H}_k$ is the channel matrix for the kth tone whose diagonal elements correspond to the direct paths between the CPEs and the CO while the off-diagonal elements represent the crosstalk. The maximum ratio of the nondiagonal element to the diagonal element is defined by a parameter α_k^{cwdd} as

$$0 \leq \left|H_{ji,k}\right| \leq \alpha_k^{\text{cwdd}} \left|H_{ii,k}\right|, \quad \forall i \neq j. \tag{3}$$

Equation (3) implies that the magnitude of the crosstalk channel coefficient $|H_{ji,k}|$ from the ith disturbing transmitter into the jth receiver is always weaker than the magnitude of the corresponding direct channel $|H_{ii,k}|$, which indicates the CWDD characteristic of the channel, typically observed in this context [4, 6].

In what follows, we omit the index k for notational simplicity since crosstalk cancellation is carried out tonewise. The analysis is similar for all tones. Henceforth, we refer to $Y_{i,k}$, Y_k, $X_{i,k}$, X_k, $H_{ij,k}$, and so forth, by Y_i, Y, X_i, X, and H_{ij}, respectively. In the next section, we assume the vector of noise samples to be spatially white, while the case of spatially correlated noise (which arises due to the alien crosstalk) is discussed in Section 4.

3. SAGE Algorithm for Self-Crosstalk Cancellation

In this section, we investigate the performance of an iterative receiver based on the SAGE algorithm [16] for the cancellation of self-crosstalk in upstream VDSL. The SAGE algorithm with user ordering is considered in Section 3.1, while its special unitary subset case is dealt with in Section 3.2.

3.1. Iterative Receiver with Ordered SAGE. The magnitude of FEXT on any tone shows a statistical variation from TP to TP because of the variations in the characteristics of twisted pairs, the nature and line length of the disturbers, and so forth. Consequently, some selected TPs are more highly affected than others at a given frequency. By giving priority to these TPs we order the users accordingly in the SAGE algorithm to increase the convergence rate and refer to this as "Ordered SAGE (OSAGE)" algorithm. Here we design our iterative technique such that the crosstalk of an ordered subset of users (grouped according to their decreasing SINR) is cancelled sequentially. As such each step of the OSAGE algorithm updates only one component of each ordered subset at a time, while keeping the estimates of the other components fixed at their previous values. By ordering N users of set $S = \{1, 2, 3, \ldots, N\}$ into M subsets $S_1, S_2, S_3, \ldots, S_M$ containing $N_{S_1}, N_{S_2}, N_{S_3}, \ldots, N_{S_M}$ users, respectively, the algorithm at *each* iteration is described as follows.

(1) Definition step: $L = 1 + S' \bmod M$ where $S' = 0, 1, 2, 3, \ldots, M-1$.

(2) Maximization step:

$$\widetilde{X}_{i\in S_L} = \frac{1}{H_{ii}}\left(Y_i - \sum_{j=1, j\neq i}^{N} H_{ij}\widetilde{X}_{j\in S}\right),$$
$$\widetilde{X}_{j\notin S_L} = \widetilde{X}_{j\in S}, \tag{4}$$

where $\widetilde{X}_{i\in S_L}$ is the symbol update of the ith user of the Lth subset (L is used for indexing subsets) and $\widetilde{X}_{j\in S}$ is the latest available estimate of jth disturber of the set S. Each iteration uses the prior iteration's estimates to generate new estimates of interference and subtracts these recent estimates from the received signal to produce new estimates with lower interference levels. In the rest of this section, we discuss initialization of OSAGE and its performance analysis (convergence behaviour).

3.1.1. Initialization and SINR after FEQ. The initialization process in the considered OSAGE algorithm plays an important role in the receiver performance. The algorithm is initiated by the use of FEQ (a one-tap equalizer) because it has the following advantages. Firstly, it can help to exploit knowledge of the approximate symbol estimates, and secondly, it utilizes easily the accurate knowledge of direct channel coefficients. We divide Y_i in (1) by H_{ii} to find the FEQ estimate $\widetilde{X}^0_{i,\text{feq}}$ of the desired symbol X_i corrupted by $N-1$ disturbers as $\widetilde{X}^0_{i,\text{feq}} = X_i + \widetilde{\xi}^0_{i,\text{feq}}$, where

$$\widetilde{\xi}^0_{i,\text{feq}} = \sum_{j=1, j\neq i}^{N} \frac{H_{ij}}{H_{ii}} X_j + \widetilde{V}_i. \tag{5}$$

In (5) above, $\widetilde{\xi}^0_{i,\text{feq}}$ and $\widetilde{V}_i = V_i/H_{ii}$ represent the FEXT plus noise and noise terms after FEQ, respectively. We define $\text{FEXT}_i = \sum_{j=1, j\neq i}^{N} |H_{ij}|^2 \sigma^2_{x,j}$ as the total crosstalk noise on the ith user which is the sum of individual crosstalks. It is noted that X_j's are independent equiprobable symbols taken typically from a QAM constellation. We assume that the central limit theorem holds for $\widetilde{\xi}^0_{i,\text{feq}}$ [15], so that it can be modeled as a complex Gaussian random variable $\widetilde{X}^0_{i,\text{feq}} \sim \mathcal{CN}(0, \widetilde{\psi}^0_{i,\text{feq}})$, where

$$\widetilde{\psi}^0_{i,\text{feq}} = \sum_{j=1, j\neq i}^{N} \frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\sigma^2_{x,j} + \sigma^2_{\widetilde{v},i} = \frac{\text{FEXT}_i}{|H_{ii}|^2} + \sigma^2_{\widetilde{v},i}. \tag{6}$$

This is reasonable since the FEXT component consists of the weighted sum of independent symbols. The SINR after FEQ can be expressed as $\text{SINR}_i^{\text{feq}} = \sigma^2_{x,i}/\widetilde{\psi}^0_{i,\text{feq}}$. We remark that the value of $\text{SINR}_i^{\text{feq}}$ is low when number of disturbers is large, and specially so at higher frequencies. This is due to the fact that crosstalk power depends on both crosstalk channel gains and the signal strength of the disturbers.

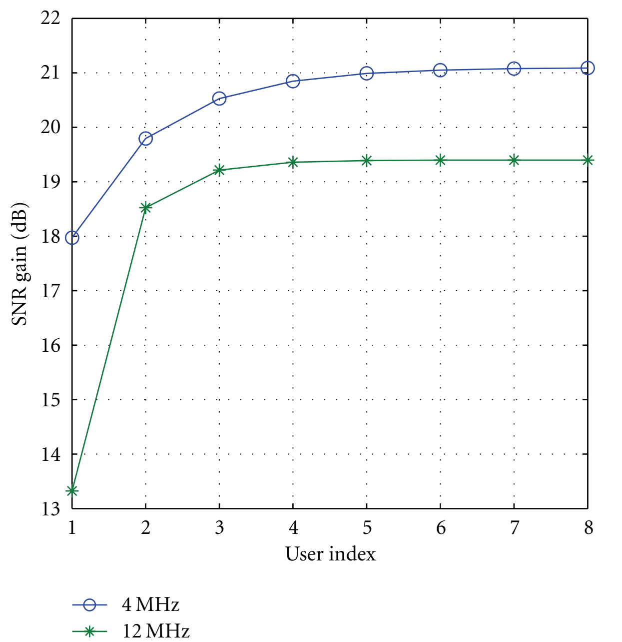

Figure 2: SNR gain per tone for each user after self-crosstalk cancellation. The line length of the 8 users ranges from 300 m (user 1) to 1000 m (user 8).

The $\text{SINR}_i^{\text{feq}}$ needs to be maximized by employing FEXT cancellation techniques. In the ideal scenario, all the crosstalk gets removed to get a parameter $\text{SNR}_i^{\text{awgn}} = \sigma^2_{x,i}/\sigma^2_{\widetilde{v},i}$ as the SNR with only additive white Gaussian noise (AWGN). Higher rate performance is of course indicated by the single-user bound (SUB). SUB is the capacity achieved when a single-user is assumed to be transmitting and whose signal can be detected by all the receivers, either from direct channel or coupling paths [6]. However, it has been established in [6] that the SUB is only marginally higher than the data rate obtained with $\text{SNR}_i^{\text{awgn}}$. Therefore, since the proposed OSAGE algorithm is essentially a crosstalk cancellation algorithm, it aims to achieve the performance close to crosstalk-free value, that is, $\text{SNR}_i^{\text{awgn}}$. To quantify the gain of our designed crosstalk canceler, we define $\text{SNR}_i^{\text{gain}}$ as

$$\text{SNR}_i^{\text{gain}} = \frac{\text{SNR}_i^{\text{awgn}}}{\text{SINR}_i^{\text{feq}}} = \frac{\sigma^2_{v,i} + \text{FEXT}_i}{\sigma^2_{v,i}}. \tag{7}$$

For insights into the SNR gain, we provide computer simulations for 8 users of different line lengths (300 m to 1000 m) within a binder, and at different tones (4 and 12 MHz) in Figure 2. It can be seen that the SNR gain per tone for most of the users is significantly high (about 21 and 19 dB at 4 and 12 MHz, resp.). To understand the impact on data rate, using a thumb rule of 3 dB per bit for every tone, we can see that a substantial improvement in data rate is effected by FEXT cancellation, for the typically large number of tones used in the upstream VDSL system scenario.

3.1.2. Performance of OSAGE Algorithm. We now consider the performance of OSAGE by obtaining an expression for the SINR after the qth iteration, for users in each of the M subsets. This is done by first looking at the variance $\tilde{\psi}_i^1$ of the estimation error $\tilde{\xi}_i^1$ after the first iteration for the ith user and then generalizing these to the corresponding values $\tilde{\psi}_i^q$ (variance of $\tilde{\xi}_i^q$) after the qth iteration, for each of the subsets S_L, $L = 1, 2, 3, \ldots, M$.

We first observe that $\mathbb{E}\{\tilde{\xi}^0_{i,\text{feq}}\tilde{\xi}^{0*}_{j,\text{feq}}\}$ is small as compared to $\mathbb{E}\{|\tilde{\xi}^0_{i,\text{feq}}|^2\}$. To see this, we can use (5) by assuming equal transmit signal power to write

$$\mathbb{E}\left\{\tilde{\xi}^0_{i,\text{feq}}\tilde{\xi}^{0*}_{j,\text{feq}}\right\} = \sum_{j \neq i}^{N}\sum_{l \neq j}^{N}\frac{H_{ij}H_{jl}^*}{H_{ii}H_{jj}^*}\sigma_x^2. \tag{8}$$

Since H_{ij}/H_{ii} and H_{jl}/H_{jj} have small amplitudes (in view of the CWDD nature of the channel) and arbitrary phase values, $\mathbb{E}\{\tilde{\xi}^0_{i,\text{feq}}\tilde{\xi}^{0*}_{j,\text{feq}}\}$ for a large N is small enough to be assumed to be negligible.

As stated earlier, the OSAGE receiver performs the crosstalk cancellation on a victim user by initializing the iteration with FEQ output. We use (4) to get an estimation error $\tilde{\xi}^1_{i\in S_1}$ for the ith user of the first subset ($S' = 0$) after the first iteration (superscript denotes the iteration step) as

$$\tilde{\xi}^1_{i\in S_1} = \tilde{X}^1_{i\in S_1} - X_i = \sum_{j=1, j\neq i}^{N}\frac{H_{ij}}{H_{ii}}\left\{X_j - \tilde{X}^0_{j,\text{feq}}\right\} + \tilde{V}_i. \tag{9}$$

The summation term in (9) corresponds to the residual crosstalk after cancellation while the second term represents AWGN after frequency equalization. It is assumed that residual crosstalk is Gaussian distributed, which is reasonable when the number of users in the binder is large due to the central limit theorem. With such an assumption on $\tilde{\xi}^1_{i\in S_1}$, the residual crosstalk $\tilde{\psi}^1_{i\in S_1}$ for the ith user can be expressed as follows:

$$\tilde{\psi}^1_{i\in S_1} = \sum_{j=1, j\neq i}^{N}\frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\mathbb{E}\left\{\left|X_j - \tilde{X}^0_{j,\text{feq}}\right|^2\right\} + \sigma^2_{\tilde{v},i}. \tag{10}$$

Substituting (6) for the jth disturber in the above expression, we get

$$\tilde{\psi}^1_{i\in S_1} = \sum_{j=1, j\neq i}^{N}\frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\left[\sum_{p=1, p\neq j}^{N}\frac{\left|H_{jp}\right|^2}{\left|H_{jj}\right|^2}\sigma^2_{x,p} + \sigma^2_{\tilde{v},j}\right] + \sigma^2_{\tilde{v},i}. \tag{11}$$

Interchanging $|H_{ii}|^2$ and $|H_{jj}|^2$ in the denominator of (11) with $\sigma^2_{v,i} = \sigma^2_{v,j}$ (note that we assume thermal noise variance of each TP to be equal but not so for the noise power after FEQ), the above equation can be expressed as

$$\tilde{\psi}^1_{i\in S_1} = \sum_{j=1, j\neq i}^{N}\frac{\left|H_{ij}\right|^2}{|H_{jj}|^2}\left[\frac{\text{FEXT}_j}{|H_{ii}|^2} + \sigma^2_{\tilde{v},i}\right] + \sigma^2_{\tilde{v},i}. \tag{12}$$

To simplify further, we define two channel parameters as follows:

$$\alpha_{ij} = \frac{\left|H_{ij}\right|}{\left|H_{jj}\right|}, \quad \alpha \triangleq \max_{i,j}\alpha_{ij} \quad \forall i \neq j, \tag{13}$$

where α is the CWDD parameter corresponding to the longest coupling length of the binder, which can be easily estimated without binder configuration. It is important to emphasise that the value of α is independent of the binder provided the maximum reach of VDSL is fixed. By considering (6) and FEXT on the longest TP (maximum) together with (13) in (12), we get an upper bound on the residual crosstalk as

$$\tilde{\psi}^1_{i\in S_1} \leq (N-1)\alpha^2\tilde{\psi}^0_{i,\text{feq}} + \sigma^2_{\tilde{v},i}. \tag{14}$$

Writing $\text{SNR}_i^{\text{gain}}$ of (7) as $\text{SNR}_i^{\text{gain}} = (\sigma^2_{\tilde{v},i} + \text{FEXT}_i/|H_{ii}|^2)/\sigma^2_{\tilde{v},i}$, the crosstalk power after FEQ of (6) can be represented in terms of SNR gain as

$$\tilde{\psi}^0_{i,\text{feq}} = \sigma^2_{\tilde{v},i}\text{SNR}_i^{\text{gain}}. \tag{15}$$

On substituting (15) into (14), we have

$$\tilde{\psi}^1_{i\in S_1} \leq (N-1)\alpha^2\sigma^2_{\tilde{v},i}\text{SNR}_i^{\text{gain}} + \sigma^2_{\tilde{v},i}. \tag{16}$$

Consequently, a lower bound of SINR for the ith user of the subset S_1 ($S' = 0$) is given by

$$\widetilde{\text{SINR}}^1_{i\in S_1} \geq \frac{\text{SNR}_i^{\text{awgn}}}{(N-1)\alpha^2\text{SNR}_i^{\text{gain}} + 1}. \tag{17}$$

Similarly, the residual crosstalk for the ith user of the second subset S_2 can be expressed as

$$\begin{aligned}\tilde{\psi}^1_{i\in S_2} = &\sum_{j=1}^{N_{S_1}}\frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\mathbb{E}\left\{\left|X_j - \tilde{X}^1_{j\in S_1}\right|^2\right\} \\ &+ \sum_{j=1, j\neq i}^{N-N_{S_1}}\frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\mathbb{E}\left\{\left|X_j - \tilde{X}^0_{j,\text{feq}}\right|^2\right\} + \sigma^2_{\tilde{v},i}.\end{aligned} \tag{18}$$

By applying the similar approach as for the subset S_1, the residual crosstalk for $i \in S_2$ ($S' = 1$) is upper bounded as

$$\tilde{\psi}^1_{i\in S_2} \leq (N_{S_1})\alpha^2\tilde{\psi}^1_{i\in S_1} + (N-1-N_{S_1})\alpha^2\sigma^2_{\tilde{v},i}\text{SNR}_i^{\text{gain}} + \sigma^2_{\tilde{v},i}. \tag{19}$$

Doing so recursively for subsets, we can represent the residual crosstalk after the first iteration ($q = 1$) for $S' \geq 1$ as

$$\begin{aligned}\tilde{\psi}^1_{i\in S_L} \leq &\sum_{j=1}^{S'-1}\left(N_{S_j}\right)\alpha^2\tilde{\psi}^1_{i\in S_j} \\ &+ \left(N - 1 - \sum_{j=1}^{S'-1}N_{S_j}\right)\alpha^2\sigma^2_{\tilde{v},i}\text{SNR}_i^{\text{gain}} + \sigma^2_{\tilde{v},i}\end{aligned} \tag{20}$$

and the lower bound on SINR for the ith user of the subset S_L as

$$\begin{aligned}&\widetilde{\text{SINR}}^1_{i\in S_L} \\ &\geq \frac{\text{SNR}_i^{\text{awgn}}}{\sum_{j=1}^{S'-1}\left(N_{S_j}\right)\alpha^2\sigma^{-2}_{\tilde{v},i}\tilde{\psi}^1_{i\in S_L} + \left(N-1-\sum_{j=1}^{S'-1}N_{S_j}\right)\alpha^2\text{SNR}_i^{\text{gain}} + 1}.\end{aligned} \tag{21}$$

For a general iteration ($q \geq 2$), we can find the residual crosstalk of the ith user of the first subset as

$$\tilde{\psi}^{q}_{i\in S_1} \leq \sum_{j=2}^{M}\left(N_{S_j}\right)\alpha^2\left(\tilde{\psi}_{i\in S_j}\right)^{q-1} + \left(N_{S_1}-1\right)\alpha^2\left(\tilde{\psi}_{i\in S_1}\right)^{q-1} + \sigma^2_{\tilde{v},i}, \tag{22}$$

where M denotes the total number of subsets. Similarly, the residual crosstalk of the ith user of the last subset (Mth) can be expressed as

$$\begin{aligned}\tilde{\psi}^{q}_{i\in S_M} \leq &\sum_{j=1, j\neq M}^{M}\left(N_{S_j}\right)\alpha^2\left(\tilde{\psi}_{i\in S_j}\right)^{q} \\ &+ \left(N_{S_M}-1\right)\alpha^2\left(\tilde{\psi}_{i\in S_M}\right)^{q-1} + \sigma^2_{\tilde{v},i}.\end{aligned} \tag{23}$$

It can be seen from (22) and (23) that users of the last subset enjoy the maximum benefit of convergence while those of the first subset the least. This, of course, is acceptable in practical scenarios since the first subset is associated with users with good SINR. For a general subset ($1 \leq L \leq M$), residual crosstalk can be obtained as

$$\begin{aligned}\tilde{\psi}^{q}_{i\in S_L} \leq &\sum_{j=1, j\neq L, j<L}^{M}\left(N_{S_j}\right)\alpha^2\left(\tilde{\psi}_{i\in S_j}\right)^{q} + \left(N_{S_L}-1\right)\alpha^2\left(\tilde{\psi}_{i\in S_L}\right)^{q-1} \\ &+ \sum_{j=1, j\neq L, j>L}^{M}\left(N_{S_j}\right)\alpha^2\left(\tilde{\psi}_{i\in S_j}\right)^{q-1} + \sigma^2_{\tilde{v},i}.\end{aligned} \tag{24}$$

Using (24), a lower bound on SINR after the qth iteration ($q \geq 2$) can be obtained as

$$\widetilde{\mathrm{SINR}}^{q}_{i\in S_L} \geq \frac{\mathrm{SNR}^{\mathrm{awgn}}_i}{\sum_{j=1, j\neq L, j<L}^{M}\left(N_{S_j}\right)\alpha^2\sigma^{-2}_{\tilde{v},i}\tilde{\psi}^{q}_{i\in S_j} + \left(N_{S_L}-1\right)\alpha^2\sigma^{-2}_{\tilde{v},i}\tilde{\psi}^{q-1}_{i\in S_L} + \sum_{j=1, j\neq L, j>L}^{M}\left(N_{S_j}\right)\alpha^2\sigma^{-2}_{\tilde{v},i}\tilde{\psi}^{q-1}_{i\in S_j} + 1}. \tag{25}$$

Combining (17), (21), and (25) together generalizes the analysis of the considered iterative receiver for each user of every subset after any given iteration step.

Some of the important implications and conclusions of this analysis are taken up in the following remarks.

Remark 1. The bound on SINR serves as a performance predictor because our focus is on achieving high data rate for a given quality of service (BER is usually fixed at 10^{-7}). It can be seen that the data rate for a practical DSL system with an SNR gap of Γ and K tones having spacing of Δ_f is

$$\text{Data Rate} = \sum_{k=1}^{K}\Delta_f\log_2\left(1+\Gamma^{-1}\widetilde{\mathrm{SINR}}^{q}_{i\in S_L,k}\right). \tag{26}$$

It follows that the performance of OSAGE can be assessed by the manner in which $\widetilde{\mathrm{SINR}}^{q}_{i\in S_L}$ approaches $\mathrm{SNR}^{\mathrm{awgn}}_i$ at each tone. It is useful here to consider this behaviour through a practical example. Using typical values of $\alpha \approx 10^{-2}$, $\mathrm{SNR}^{\mathrm{gain}}_i \approx 20$ dB, and $N = 25$ in (17), it is found that the SNR loss (with respect to $\mathrm{SNR}^{\mathrm{awgn}}_i$) for user of the first subset after the first iteration is $(N-1)\alpha^2\mathrm{SNR}^{\mathrm{gain}}_i \approx 0.24$ (approx. 0.93 dB) and eventually approaches zero for the users of subsequent subsets. The resulting effect on the data rate is very small, as computed from (26). Thus, our proposed algorithm effectively cancels the crosstalk with a single iteration, with an associated computational complexity of $\mathcal{O}(N^2)$ per tone.

In contrast to the ML receiver which requires a computational complexity of $\mathcal{O}(K\mathcal{C}^N)$ (large constellation size $\mathcal{C}$ is often used in VDSL), the complexity of the OSAGE algorithm of the qth iteration is $\mathcal{O}(qKN^2)$. With a single iteration, online complexity of the considered receiver is similar to that of the ZF receiver, which incurs a computational cost of $\mathcal{O}(KN^2)$. Since the ZF receiver requires a computation of inverse of the channel matrix, the proposed iterative receiver promises to be computationally efficient due to its ability to avoid this offline computation.

Remark 2. As convergence is an important concern for any iterative technique, we now discuss the condition for convergence for our proposed OSAGE-based iterative algorithm. For ease of presentation, we consider a case of single subset ($M = 1$) in (22) and apply $\tilde{\psi}^{q}_{i\in S_1} < \tilde{\psi}^{q-1}_{i\in S_1}$ to get $(N-1)\alpha^2 < (\mathrm{SNR}^{\mathrm{gain}}_i - 1)/\mathrm{SNR}^{\mathrm{gain}}_i$ as a necessary and sufficient condition for the convergence of the OSAGE algorithm. In a crosstalk-limited DSL system ($\mathrm{SNR}^{\mathrm{gain}}_i \gg 1$), the condition reduces to $(N-1)\alpha^2 < 1$ for the convergence of the considered iterative receiver. It is worth mentioning that this convergence criterion holds good even for the case of a large number of users since α usually takes a small value (typically of the order of $\approx 10^{-2}$). In Figure 3, we plot $(N-1)\alpha^2$ for different values of N and tones for various loop lengths. It can be seen that the convergence condition is readily satisfied.

Remark 3. The rapid convergence shown in Remark 2 can be also explained intuitively. Considering an initial bit loading (assuming an AWGN channel) in the presence of crosstalk, the estimated symbols after FEQ may be in error. However, the true symbols lie within a small radius, of the order of minimum distance $d_{\min}$ between the constellation points. If the crosstalk due to the jth TP is removed using the FEQ estimates (by subtracting $H_{ij}\tilde{X}_{j,\mathrm{feq}}$), the variance of the residual crosstalk is a small multiple of $|H_{ij}|^2 d^2_{\min}$. This is much smaller than the variance of the original crosstalk ($|H_{ij}|^2\sigma^2_{x,j}$) since $d^2_{\min}$ is a small fraction of $\sigma^2_{x,j}$ due to

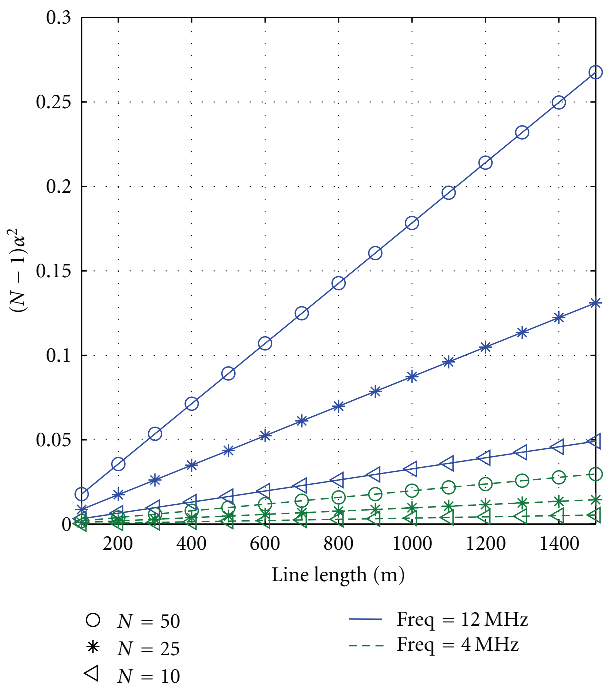

Figure 3: Convergence condition of SAGE algorithm for self-crosstalk cancellation versus line length.

the high bit loading (bit loading of 13-14 bits per tone is not uncommon in VDSL). This coupled with the fact that the crosstalk channel coefficients are small due to the CWDD property accounts for the surprisingly fast convergence for the OSAGE algorithm.

Remark 4. Since the bounds in (17), (21), and (25) are derived by considering the maximum possible values of the CWDD parameter and FEXT, these are tight in most of the situations. This can be verified from the fact that the CWDD parameter as well as the FEXT (hence SNR gain) is not very sensitive to the variation of line length (except at shorter line length as shown in **Figure 2**). The tightness of bound is obvious in the case of a typical VDSL equal length lines scenario as both α and FEXT remain almost same for all users in this case. Furthermore, in the near-far problem (that occurs when the binder contains loops with widely varying lines), equal FEXT for different users can be achieved by employing upstream power back-off (UPBO) under which the upstream transmitters vary their power spectral densities (PSDs) in accordance with the line lengths.

Remark 5. From a system design viewpoint, our derived bounds are of considerable importance. The main design parameters considered here are SNR gain and q. The SNR gain can be changed at the level of designer discretion as per the requirement of user data rates. The iterating parameter q indicates the complexity. The trade-off between data rate and complexity can be realized by varying these parameters. Thus, the bounds in (17), (21), and (25) are useful tools for theoretical analysis of the performance of the considered canceler. Moreover, the bounds depend on local parameters such as binder size, CWDD parameter, and $\mathrm{SNR}_i^{\mathrm{awgn}}$, and not on the binder configuration. This together with independent single-user water filling algorithms on each TP greatly simplifies the adaptive bit loading for the SRA operation. Here, SRA stands for seamless rate adaptation whereby the receiver monitors the SNR of the channel, and according to channel conditions, sends a message to the transmitter to initiate a change in bit loading.

3.2. The USAGE Algorithm and Its Performance. From the discussion in the previous section, we observe that a near crosstalk-free performance can be achieved even by considering only the first subset in the OSAGE algorithm. Motivated by this observation, we consider a special case of OSAGE, termed as "Unitary-subset SAGE (USAGE)," algorithm in this section in which all users are grouped into a single subset. With only one subset, each user is updated at the same time to realize a simultaneous cancellation of crosstalk terms. It offers the advantage of parallel implementation with savings in computation time but at the price of some performance loss. For its performance analysis, we can express the symbol estimates for the ith user at the qth iteration using (4) as

$$\widetilde{X}_i^q = \left[\frac{1}{H_{ii}}\left(Y_i - \sum_{j=1, j\neq i}^{N} H_{ij}\widetilde{X}_j^{q-1}\right)\right], \tag{27}$$

where $\widetilde{X}_j^{q-1}$ is the symbol estimate of a disturber after the $(q-1)$th iteration. It should be noted that $\widetilde{X}_j^0 = \widetilde{X}_{j,\mathrm{feq}}^0$ represents the post-FEQ estimate of the jth crosstalker. Using (27), we obtain the power $\widetilde{\psi}_i^q$ of residual crosstalk as

$$\widetilde{\psi}_i^q = \sum_{j=1, j\neq i}^{N} \frac{\left|H_{ij}\right|^2}{\left|H_{ii}\right|^2} \mathbb{E}\left\{\left|X_j - \widetilde{X}_j^{q-1}\right|^2\right\} + \sigma_{\widetilde{v},i}^2. \tag{28}$$

Since $\mathbb{E}\{|X_j - \widetilde{X}_j^{q-1}|^2\}$ is the residual crosstalk at the $(q-1)$th iteration, we can represent (28) in the form of a recursive relation via

$$\widetilde{\psi}_i^q = \sum_{j=1, j\neq i}^{N} \frac{\left|H_{ij}\right|^2}{\left|H_{ii}\right|^2} \widetilde{\psi}_j^{q-1} + \sigma_{\widetilde{v},i}^2. \tag{29}$$

By representing all the terms of $\widetilde{\psi}_j^{q-1}$, we can reexpress (29) as

$$\widetilde{\psi}_i^q = \underbrace{\sum_{j=1, j\neq i}^{N} \sum_{p=1, p\neq j}^{N} \cdots \sum_{m=1, m\neq p'}^{N} \frac{\left|H_{ij}\right|^2}{\left|H_{ii}\right|^2} \frac{\left|H_{jp}\right|^2}{\left|H_{jj}\right|^2} \cdots \frac{\left|H_{p'm}\right|^2}{\left|H_{p'p'}\right|^2}}_{q \text{ summation terms}} \times \left[\frac{\mathrm{FEXT}_m}{\left|H_{mm}\right|^2} + \sigma_{\widetilde{v},m}^2\right] + \mathcal{S}_N^q + \sigma_{\widetilde{v},i}^2, \tag{30}$$

where

$$\mathcal{S}_N^q = \underbrace{\sum_{j=1,j\neq i}^{N} \frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\sigma_{\tilde{v},j}^2 + \sum_{j=1,j\neq i}^{N}\sum_{p=1,p\neq j}^{N} \frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\frac{|H_{jp}|^2}{\left|H_{jj}\right|^2}\sigma_{\tilde{v},p}^2 + \cdots + \underbrace{\sum_{j=1,j\neq i}^{N}\sum_{p=1,p\neq j}^{N}\cdots\sum_{m=1,m\neq p'}^{N}\frac{\left|H_{ij}\right|^2}{|H_{ii}|^2}\frac{\left|H_{jp}\right|^2}{\left|H_{jj}\right|^2}\cdots\frac{\left|H_{p'm}\right|^2}{\left|H_{p'p'}\right|^2}\sigma_{\tilde{v},m}^2}_{q \text{ summation terms}}}_{q-1 \text{ summation terms}}. \tag{31}$$

Assuming equal thermal noise at each TP, we can represent (31) in terms of $\sigma_{\tilde{v},i}^2$ (the noise after FEQ for ith user) and α (the CWDD parameter of the binder) by making use of $\sigma_{\tilde{v},j}^2 = \sigma_{v,j}^2/|H_{jj}|$, $\sigma_{\tilde{v},p}^2 = \sigma_{v,p}^2/|H_{pp}|, \ldots, \sigma_{\tilde{v},m}^2 = \sigma_{v,m}^2/|H_{mm}|$ and interchanging the diagonal terms to obtain

$$\mathcal{S}_N^q = \mathcal{S}_N^q(\alpha)\sigma_{\tilde{v},i}^2, \tag{32}$$

where

$$\mathcal{S}_N^q(\alpha) \leq \sum_{r=0}^{q-1}\left((N-1)\alpha^2\right)^r - 1 = \frac{\left((N-1)\alpha^2\right)^q - (N-1)\alpha^2}{(N-1)\alpha^2 - 1}. \tag{33}$$

Using (30), we can obtain residual crosstalk of ith user after the qth iteration in terms of initial crosstalk (post-FEQ) iteration by interchanging $|H_{ii}|^2$ and $|H_{mm}|^2$ and replacing FEXT_m with the FEXT on the longest wire (maximum) together with appropriate interchanges of other diagonal terms, as

$$\tilde{\psi}_i^q \leq \left((N-1)\alpha^2\right)^q \tilde{\psi}_i^0 + \mathcal{S}_N^q + \sigma_{\tilde{v},i}^2. \tag{34}$$

By substituting $\tilde{\psi}_i^0 = \text{SNR}_i^{\text{gain}}\sigma_{\tilde{v},i}^2$ from (15) in (34) and using the result along with (32), we obtain a lower bound of SINR after the qth iteration as

$$\widetilde{\text{SINR}}_i^q \geq \frac{\text{SNR}_i^{\text{awgn}}}{\left((N-1)\alpha^2\right)^q \text{SNR}_i^{\text{gain}} + \mathcal{S}_N^q(\alpha) + 1}. \tag{35}$$

Although the USAGE algorithm can achieve a near crosstalk-free performance with a single iteration in most situations, some performance loss for the case of shorter line lengths may occur, which is quantified as

$$\delta\text{SNR}_{\text{dB}} \leq 10\log_{10}\left(\left((N-1)\alpha^2\right)^q \text{SNR}_i^{\text{gain}} + \mathcal{S}_N^q(\alpha) + 1\right), \tag{36}$$

where

$$\delta\text{SNR}_{\text{dB}} = \left(\text{SNR}_i^{\text{awgn}}\right)_{\text{dB}} - \left(\widetilde{\text{SINR}}_i^q\right)_{\text{dB}}. \tag{37}$$

At shorter loop lengths, the loss can be as much as 3 dB for a typical number of users. Improvement in performance however can be realized by increasing the iterations since $(N-1)\alpha^2 < 1$ ensures convergence. This convergence condition (obtained by applying $\tilde{\psi}_i^q < \tilde{\psi}_i^{q-1}$ from (34) and using $\text{SNR}_i^{\text{gain}} \gg 1$) is similar to that obtained in Remark 2. Under the condition of convergence, USAGE algorithm achieves rate performance close to crosstalk-free rate since when $\alpha \to 0$ and $q \to \infty$, $\delta\text{SNR}_{\text{dB}} \to 0$, and

$$\widetilde{\text{SINR}}_i^q \approx \text{SNR}_i^{\text{awgn}}. \tag{38}$$

The lower bound of (35) is simple and can be used to obtain the data rate without the explicit knowledge of the crosstalk coupling coefficients.

4. Iterative Receiver for Alien Crosstalk Cancellation

So far we have studied the cancellation of self-crosstalk assuming the presence of only white thermal noise. However, as discussed earlier noise from other external sources (alien crosstalk) may significantly affect the functionality of the vectored VDSL systems. In practical scenarios, such a crosstalk may arise due to the use of TPs and power lines over the same spectrum for broadband access. Further, the TPs near the customer premises are generally unshielded so that RFI from nearby radio transmitter couples with these wires to induce alien crosstalk. There is also a crosstalk due to the presence of nonvectored DSL lines (usually unsynchronized) within the same binder that is carrying the vectored lines. This alien crosstalk manifests itself as spatially correlated (i.e., correlated across TPs) additive noise at each tone at the receiver of the considered VDSL system and hence degrades its performance even with the effective cancellation of self-crosstalk. Modeling of alien crosstalk is discussed in [3, 10]. Alien crosstalk is easily amenable in the vectoring framework (due to the spatial correlation between different noise samples across the TPs). In order to facilitate this, we study the use of iterative technique of the previous section for the self-crosstalk cancellation but preceded with a prewhitening filter. Our aim is to develop an effective crosstalk canceler for VDSL systems in order to achieve considerable improvement in performance in the presence of alien crosstalk. The issue here is whether the prewhitening operation affects the CWDD property of the effective channel to enable the SAGE algorithms to converge fast on the postwhitened signals. We, therefore, investigate analytically the behavior of the modified channel (channel together with the noise whitening filter) in the next section and the receiver performance based on the modified channel.

4.1. CWDD Characteristic of Equivalent Channel after Whitening. To proceed, we denote the magnitude and phase of the correlation coefficient between the noise samples on various TPs by ρ and ϕ, respectively and the alien noise power by σ_a^2. We define a noise covariance matrix $\mathbf{R}[r_{ij}]$ whose ijth element is given as

$$\begin{aligned} r_{ii} &= \sigma_{v,i}^2 + \sigma_{a,i}^2, \\ r_{ij} &= \rho_{ij}\sigma_{a,i}\sigma_{a,j}e^{J\phi_{ij}} \quad \forall i \neq j, \\ r_{ji} &= r_{ij}^* \quad \forall j \neq i, \end{aligned} \tag{39}$$

where $\jmath = \sqrt{-1}$. We use a noise whitening filter at the receiver which multiplies (2) with $\mathbf{R}^{-1/2}$ to obtain

$$\hat{\mathbf{y}} = \left(\mathbf{R}^{-1/2}\mathbf{H}\right)\mathbf{x} + \mathbf{R}^{-1/2}\mathbf{v} = \widehat{\mathbf{H}}\mathbf{x} + \hat{\mathbf{v}}, \tag{40}$$

where $\widehat{\mathbf{H}} = \mathbf{R}^{-1/2}\mathbf{H}$ is the postwhitening, equivalent channel matrix and $\hat{\mathbf{v}} = \mathbf{R}^{-1/2}\mathbf{v}$ represents the whitened noise with correlation matrix $\mathbb{E}\{\hat{\mathbf{v}}\hat{\mathbf{v}}^H\} = \mathbf{I}_N$. If the ijth element of a matrix $\mathbf{A}$ is A_{ij}, then define $|\mathbf{A}|$ as a matrix with the ijth element $|A_{ij}|$. We also define $\mathbf{A}(i,:)$ and $\mathbf{A}(:,j)$ as the ith row and the jth column of the matrix $\mathbf{A}$. By taking the absolute value and applying the triangular inequality on each element of the equivalent channel $\widehat{\mathbf{H}} = \mathbf{R}^{-1/2}\mathbf{H}$, that is, $|\widehat{H}_{ij}| = |\mathbf{R}^{-1/2}(i,:)\mathbf{H}(:,j)| \leq |\mathbf{R}^{-1/2}(i,:)||\mathbf{H}(:,j)|$ for all i, j, we define the absolute value of the modified channel matrix $\widehat{\mathbf{H}}$ as an inequality, given as

$$\left|\widehat{\mathbf{H}}\right| \leq \left|\mathbf{R}^{-1/2}\right||\mathbf{H}|. \tag{41}$$

Using the definition of matrix inverse, we represent the elements of $|\mathbf{R}^{-1/2}|$ as

$$\left|\left[\mathbf{R}^{-1/2}\right]_{ij}\right| = \frac{\left|\det\left(\overline{\mathbf{R}_{ji}^{1/2}}\right)\right|}{|\det(\mathbf{R}^{1/2})|}, \tag{42}$$

where $\overline{\mathbf{R}_{ji}^{1/2}}$ is the $(N-1)\times(N-1)$ submatrix by removing the jth row and the ith column from the square root of the covariance matrix $\mathbf{R}^{1/2}$. We define

$$\gamma_r = \max_i \left|\det\left(\overline{\mathbf{R}_{ii}^{1/2}}\right)\right|, \qquad \gamma_t = \max_{ij}\left|\det\left(\overline{\mathbf{R}_{ji,j\neq i}^{1/2}}\right)\right| \tag{43}$$

as the maximum absolute value of determinant of principal and nonprincipal submatrices of $\mathbf{R}^{1/2}$, respectively. Now it can be shown using (39) that $\gamma_r > \gamma_t$ for the given covariance matrix $\mathbf{R}$. By using (42) in (41) along with the results of (43), we get a bound on the absolute value of elements of $\widehat{\mathbf{H}}$ as

$$\left|\widehat{H}_{ji}\right| \leq \begin{cases} G[\gamma_r + (N-1)\alpha\gamma_t] & \text{if } j = i, \\ G[\alpha\gamma_r + \gamma_t + (N-2)\alpha\gamma_t] & \text{if } j \neq i, \end{cases} \tag{44}$$

where $G = |H_{ii}|/|\det(\mathbf{R}^{1/2})|$ and α as defined in (13). We use (44) to find an upper bound on $\hat{\alpha}$ for the equivalent channel as

$$\hat{\alpha} \leq \frac{\alpha\gamma_r + \gamma_t + (N-2)\alpha\gamma_t}{\gamma_r + (N-1)\alpha\gamma_t} = \frac{\alpha + \eta + (N-2)\alpha\eta}{1 + (N-1)\alpha\eta}, \tag{45}$$

where $\eta = \gamma_t/\gamma_r$. From (45), one can verify that $\hat{\alpha} = \alpha$ for the case of uncorrelated noise. Since $0 \leq \eta < 1$, it is found that $\hat{\alpha} \geq \alpha$ for all the values of α. It can be however shown that $\hat{\alpha} < 1$ as long as $\alpha + \eta(1-\alpha) < 1$ holds for $\alpha < 1$ and $0 \leq \eta < 1$. Although this does not clearly establish the CWDD property $\hat{\alpha} \ll 1$, it is seen that for typical practical values of N, alien noise powers and correlation parameters, the required convergence conditions for the convergence of OSAGE and USAGE are satisfied. This is confirmed later in the simulation results presented in Section 5.

4.2. Iterative Cancellation of Alien Crosstalk. Alien crosstalk cancellation after noise whitening follows analysis similar to that for self-crosstalk. To highlight the difference, we provide the distinct parameters in terms of the characteristics of alien noise. The CWDD characteristic of the equivalent channel has been already considered in the previous section. The noise variance is given by $\hat{\sigma}^2_{v,i} = 1$ after whitening, while after FEQ it is given as $\hat{\sigma}^2_{\tilde{v},i} = |\widehat{H}_{ii}|^{-2}$. Making use of (44), we find an upper bound on SNR after noise whitening (denoted by $\mathrm{SNR}_i^{\mathrm{white}}$) as

$$\mathrm{SNR}_i^{\mathrm{white}} = \left|\widehat{H}_{ii}\right|^2\sigma^2_{x,i} \leq \frac{|H_{ii}|^2\sigma^2_{x,i}}{|\det(\mathbf{R})|}(\gamma_r + (N-1)\alpha\gamma_t)^2. \tag{46}$$

We provide the bounds on $|\det(\mathbf{R})|$, γ_r, and γ_t in (A.1) and (A.6), (A.2), and (A.10), respectively, with proofs given in the appendix. The FEXT on the ith TP after whitening filter can be represented as

$$\widehat{\mathrm{FEXT}}_i = \sum_{j=1, j\neq i}^{N}\left|\widehat{H}_{ij}\right|^2\sigma^2_{x,j} = \sum_{j=1, j\neq i}^{N}\hat{\alpha}_j^2\mathrm{SNR}_j^{\mathrm{white}}. \tag{47}$$

Substituting $\hat{\sigma}^2_{v,i} = 1$ in (7) along with the result of (47), we can express SNR gain of the ith user after noise whitening as

$$\widehat{\mathrm{SNR}}_i^{\mathrm{gain}} = 1 + \sum_{j=1, j\neq i}^{N}\widehat{\mathrm{FEXT}}_i = 1 + \sum_{j=1, j\neq i}^{N}\hat{\alpha}_j^2\mathrm{SNR}_j^{\mathrm{white}}. \tag{48}$$

The SINR bounds for the present case can be obtained by simply invoking the above terms in (17), (21), (25), and (35). To show the effectiveness of the proposed iterative receiver for alien crosstalk cancellation, we study the case of equidistant (equal line length) VDSL system. By representing (48) for equidistant TPs as $\widehat{\mathrm{SNR}}_i^{\mathrm{gain}} \leq 1 + (N-1)\hat{\alpha}^2\mathrm{SNR}_i^{\mathrm{white}}$ and substituting this in (17), we find the SINR of the OSAGE algorithm for the ith user of the first subset after first iteration as

$$\widehat{\mathrm{SINR}}^1_{i\in S_1} \geq \frac{\mathrm{SNR}_i^{\mathrm{white}}}{(N-1)^2\hat{\alpha}^4\widehat{\mathrm{SNR}}_i^{\mathrm{white}} + (N-1)\hat{\alpha}^2 + 1}. \tag{49}$$

Similarly, using (48) along with the result of (32) in (35), a simple lower bound on the performance of the USAGE algorithm for the case of alien crosstalk can be obtained as

$$\widehat{\mathrm{SINR}}_i^q \geq \frac{\mathrm{SNR}_i^{\mathrm{white}}}{((N-1)\hat{\alpha}^2)^{q+1}\mathrm{SNR}_i^{\mathrm{white}} + \sum_{r=0}^{q-1}((N-1)\hat{\alpha}^2)^r + 1}. \tag{50}$$

It can be seen from the above expressions that performance depends on noise correlation and power of alien disturbers, since $\mathrm{SNR}_i^{\mathrm{white}}$ and $\hat{\alpha}$ are functions of these quantities. Specifically, $\mathrm{SNR}_i^{\mathrm{white}}$ and $\hat{\alpha}$ increase with correlation while the former decreases significantly and the latter increases marginally with alien power. With the help of these facts, the proposed iterative receiver can be designed to achieve the near crosstalk-free performance. It may be expected from (49) and (50) that fast convergence (i.e., within one iteration) may be exhibited under low correlation and/or high alien power. However, additional iterations may be required under conditions of high correlation and/or low alien noise power scenarios.

5. Numerical and Simulation Results

In this section, we present numerical results based on the analytical expressions derived here, along with MATLAB simulation experiments to investigate the performance of the proposed iterative receiver. For performing the simulations, we adopt the stochastic channel model [19] and consider the binder to consist of eight lines such that there are seven disturbers per line. We have simulated the proposed algorithm for various VDSL scenarios in the context of real VDSL deployments. Scenario 1 deals with the distributed case consisting of eight VDSL users with lines varying from 300 m to 1000 m in 100 m increments. Scenario 2 includes the case of equidistant lines. In crosstalk limited DSL systems, a near-far problem may occur with the coexistence of long lines with short lines in the same binder. This case is considered under scenario 3 which includes 4 near-end users (at 400 m) and 4 far-end users (located at 800 m) from the CO. For the analysis of OSAGE algorithm, we assume four subsets with sizes 2, 2, 3, 1 with appropriate ordering, for equidistant scenario and with 2 elements per subset for other scenarios at each tone by giving priority to the users with shorter line lengths. A bandplan 998 ADE is incorporated with 3 upstream bands US0 (25–138 kHz), US1 (3.75–5.2 MHz), and US2 (8.5–12 MHz). Other simulation parameters are used in accordance with DSL standards [2] as listed in Table 1.

5.1. Self-Crosstalk Cancellation. In this section, the performance of the proposed (OSAGE and USAGE) receiver is investigated for self-crosstalk cancellation. We consider mainly the performance after a *single* iteration, in order to keep the real-time complexity of $\mathcal{O}(N^2)$ per tone. We consider SINR and the corresponding data rate characteristics for the numerical studies. The analytical lower bounds, derived in (17) and (21) for OSAGE algorithm and in (35) for USAGE algorithm (with $q = 1$), are also validated through the simulations.

The main observations that can be drawn from the numerical investigations and simulations are summarized below.

(i) First, we consider a practical VDSL scenario such that the binder consists of distributed users with varying line lengths from the CO. For this case, we have simulated the SINR and the corresponding data rate characteristics for the considered algorithms, and the results are illustrated in Figures 4 and 5, respectively. From these figures, we see that both algorithms achieve performance close to crosstalk-free performance for a broad range of line lengths and bandwidths. It is further observed that the proposed algorithms have excellent performance for lower- and medium-frequency tones. Although there is a small loss in SNR with the USAGE algorithm (Figure 4) at the higher-frequency tones, the overall effect on data rate is negligible over the entire bandwidth as shown in Figure 5.

(ii) Next, we consider a simplified scenario of equidistant lines, which can be feasible in some scenarios. Figures 6 and 7 incorporate this case and show that the USAGE algorithm works well and achieves close to crosstalk-free performance for longer loop lengths. Even for shorter loop lengths, it gives satisfactory performance for lower-frequency tones. However, the OSAGE algorithm always outperforms USAGE and achieves date rate close to crosstalk-free performance as shown in Figure 7.

(iii) Finally, we address the performance of our proposed algorithms in a mixed scenario, to consider the impact of the near-far problem. This problem occurs when the binder contains loops with widely varying line lengths, which can be alleviated in practice by effective transmit power controls of nearby users. However, as shown in Figure 8, our algorithms offer far-end users to achieve crosstalk-free performance without compromising the data rates for the near-end users, thus potentially avoiding the need for such power controls.

(iv) The ZF receiver exploits the CWDD property of DSL channels and achieves close to self-crosstalk-free performance. As stated earlier, it requires additional computation of channel matrix inversions. Further, it has been shown in many publications [6, 20] that linear receivers (ZF and MMSE) experience significant performance degradation in the presence of alien crosstalk. The SAGE algorithm can effectively mitigate the effect of alien crosstalk as discussed in next section.

Table 1: Simulation parameters.

Twisted pair type	24 AWG (0.5 mm)
Band plan	998 ADE FDD bandplan [2]
Profile	12a VDSL2 [2]
Tone width	4.3125 KHz
Transmit signal PSD mask	−60 dBm/Hz
Noise PSD	−130 dBm/Hz [21]
Error probability	10^{-7}
Coding gain	3 dB
Noise margin	6 dB
SNR gap Γ	12.8 dB

5.2. Alien Crosstalk Cancellation. For this scenario, we investigate the performance of the proposed OSAGE receiver for various alien crosstalk powers and noise correlations by assuming equal length TPs. We consider correlation coefficient values for the alien noise components in the

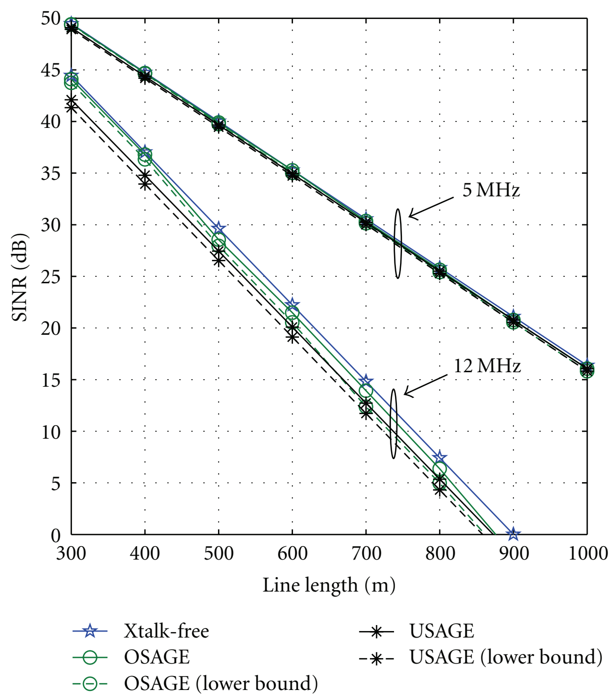

Figure 4: SINR performance of each user with OSAGE and USAGE algorithms for self-crosstalk cancellation under distributed scenario (each user located at a length varying from 300 m to 1000 m).

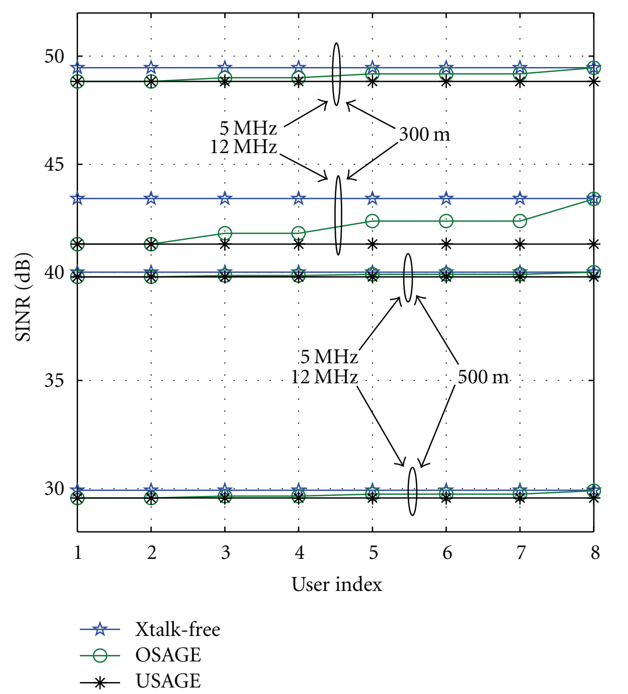

Figure 6: SINR performance of each user with OSAGE and USAGE algorithms for self-crosstalk cancellation under equidistant scenario (all users are assumed to be at the same line length).

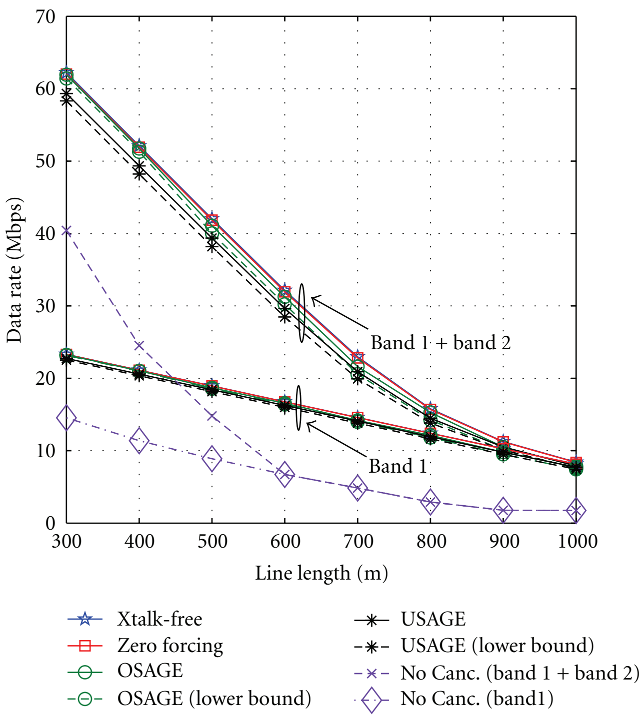

Figure 5: Upstream data rate of each user with OSAGE and USAGE algorithms for distributed scenario (each user located at a length varying from 300 m to 1000 m). Band 1 comprises lower upstream bands (US0 and US1) whereas Band 2 is the higher frequency band (US2). "No Canc." stands for the case of without crosstalk cancellation.

various TPs, in the range of 0.50–0.99. We have also taken an alien power of −100 dBm/Hz as it lies between the levels of −120 dBm/Hz (alien-free environment) and −80 dBm/Hz (indicating a very strong alien crosstalk) [21]. We assume known covariance matrix $\mathbf{R}$ and obtain numerically the parameters γ_r and γ_t in (43) and $|\det(\mathbf{R})|$.

We demonstrate the effect of whitening on the original channel through the CWDD behaviour of modified channel (after noise whitening) in Figures 9 and 10. Simulation results and upper bound (derived in (45)) of the CWDD parameter are also shown in Figure 9. It can be observed that the CWDD parameter increases with the increase in both alien power and correlation, but the effect of the former is insignificant. It can be seen that the CWDD characteristic of the modified channel is weaker as compared to that of the original channel. However, even the weaker CWDD property is seen to satisfy the convergence condition (for typical values of N) and hence can be utilized for alien crosstalk cancellation, via the OSAGE receiver.

Figures 11 and 12 highlight the receiver performance in the presence of alien crosstalk. It is observed from the figures that high spatial correlation can be efficiently utilized in alien crosstalk cancellation. The number of iterations required for such a cancellation follows the similar features as for self-crosstalk cancellation, depending on noise correlation and alien power. For achieving a higher data rate improvement at shorter loop length, the proposed iterative receiver can take 2-3 iterations when the alien crosstalk has a low power and high noise correlation. However, a single iteration is sufficient to mitigate crosstalk of high alien power and/or

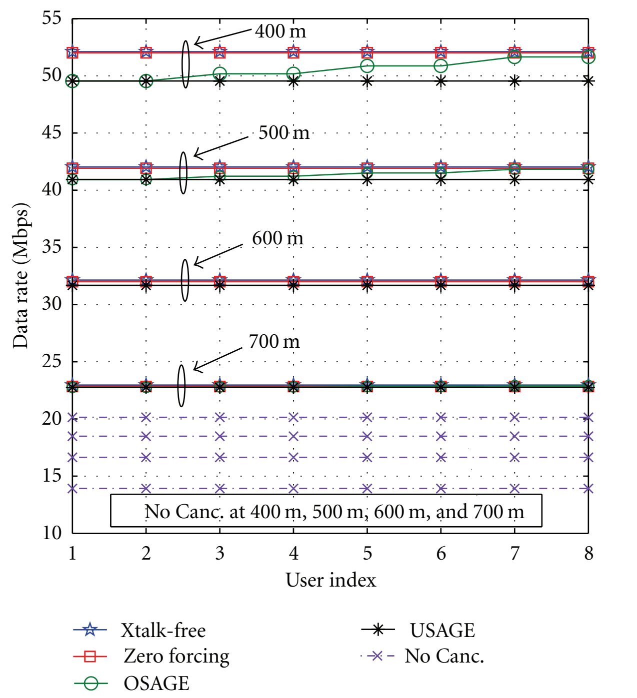

Figure 7: Upstream data rate of each user with OSAGE and USAGE algorithms for self-crosstalk cancellation under equidistant scenario (all users are assumed to be at the same line length).

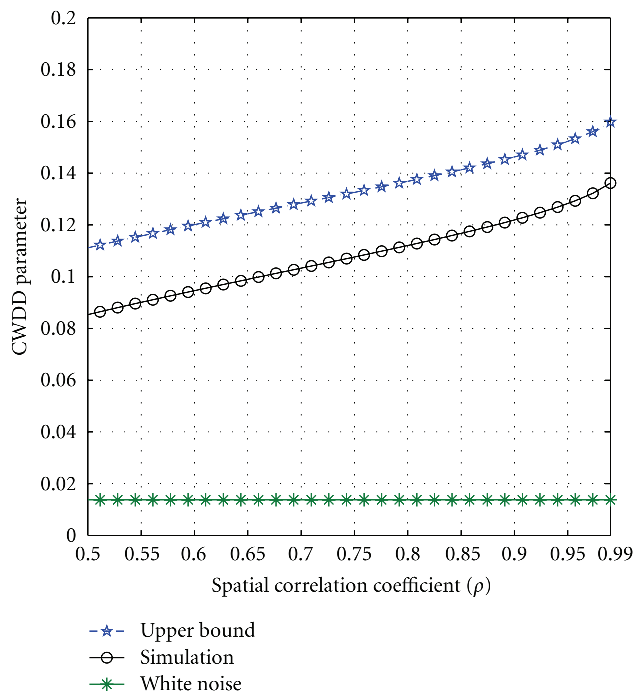

Figure 9: CWDD parameter of the modified channel (after noise whitening) at a 5 MHz tone and 300 m line length with alien PSD of −100 dBm/Hz versus correlation coefficient.

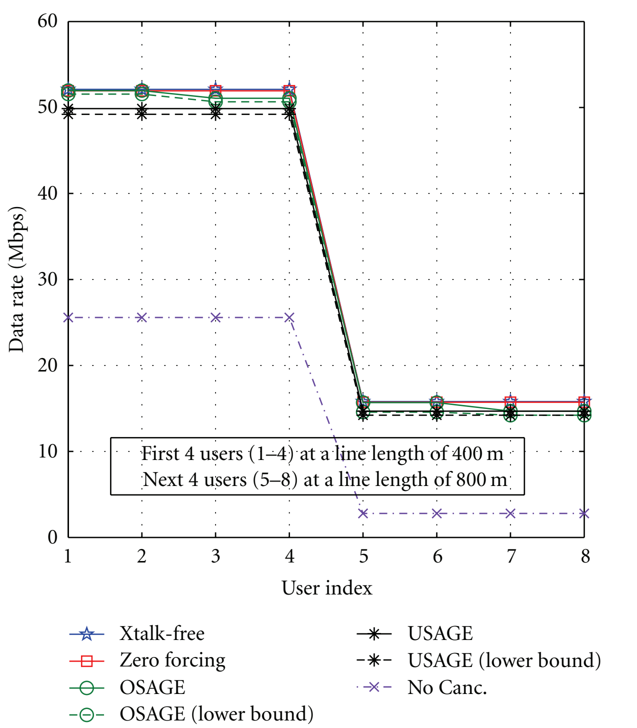

Figure 8: Upstream data rate of each user with OSAGE and USAGE algorithms for self-crosstalk cancellation under near-far scenario (first four users are at 400 m while the next four are at 800 m).

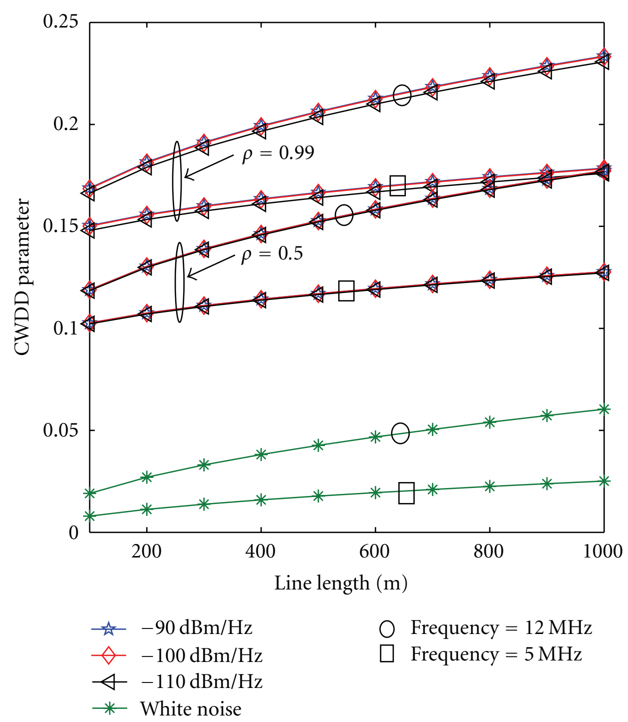

Figure 10: Upper bound on CWDD parameter of the modified channel (after noise whitening) versus line length for various parameters of alien crosstalk.

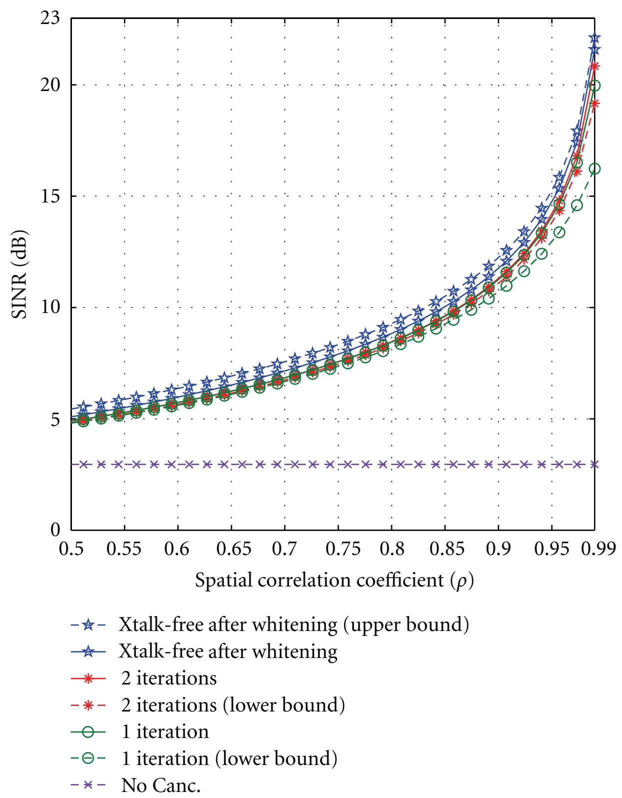

Figure 11: SINR performance of a user with OSAGE algorithm in the presence of alien crosstalk at a 10 MHz frequency tone and line length of 500 m with an alien PSD of −100 dBm/Hz versus correlation coefficient.

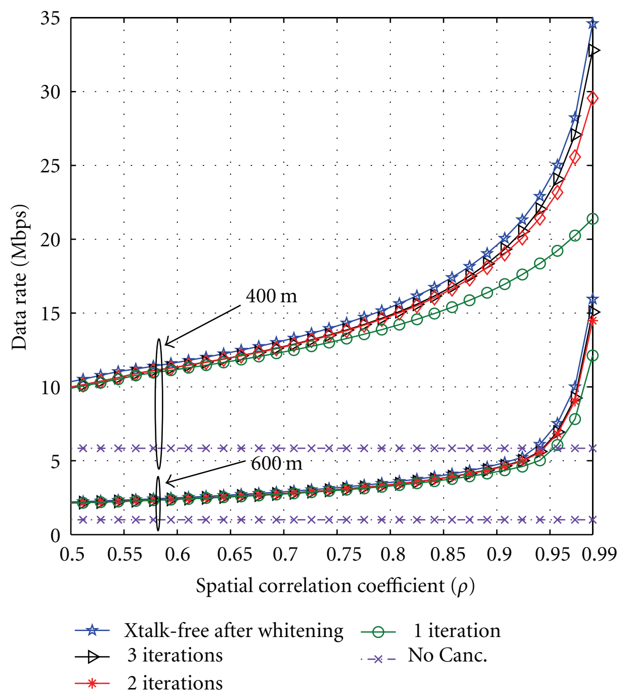

Figure 12: Upstream data rate of a user with OSAGE algorithm in the presence of alien crosstalk for equidistant scenario (all users are assumed to be located at a length of either 400 m or 600 m) with an alien PSD of −100 dBm/Hz versus correlation coefficient.

low correlation (which corresponds to a lower data rate improvement after noise whitening).

6. Conclusions

We have investigated the use of an iterative receiver based on the SAGE algorithm for crosstalk cancellation in upstream VDSL. The proposed receiver was shown to be practically feasible and computationally efficient. By employing user ordering, a lower bound (after each iteration) on the performance of the SAGE receiver was derived. Since the derived bound depends only on the binder size, the longest line, and crosstalk-free SNR, it is easier for designer to predict the data rate and thus achieve a specified quality of service. Analytical and simulation results confirm that the SAGE-based receiver operates close to self-crosstalk-free performance with a single iteration (and complexity comparable to that of the linear ZF receiver) while eliminating the need for channel inversion. An upper bound on the CWDD parameter of the modified channel (after noise whitening) was obtained. This CWDD property after noise whitening was shown to be retained, a fact that was exploited for the cancellation of alien crosstalk. Performance of the proposed receiver was shown to be dependent on noise correlation and alien crosstalk power. As such mitigation of alien crosstalk requires only one iteration for high alien power and/or low noise correlation while a few more iterations may sometimes be required for the case of low alien noise power and/or high noise correlation.

Appendix

In this appendix, we present the bounds on modulus of the determinant of the covariance matrix and its principal and nonprincipal submatrices. For simplicity, we assume equal alien powers and thermal noises at each TP. Applying Hadamard inequality ($|\det(\mathbf{R})| \leq (\prod_{i=1}^{N}\sum_{j=1}^{N}|r_{ij}|^2)^{1/2}$) yields an upper bound

$$|\det(\mathbf{R})| \leq \left[\left(\sigma_v^2+\sigma_a^2\right)^2+(N-1)\rho^2\sigma_a^4\right]^{N/2}. \quad \text{(A.1)}$$

Similarly, an upper bound on the modulus of determinant of the principal submatrix (size $(N-1)\times(N-1)$) can be obtained as

$$\begin{aligned}\gamma_r &= \left|\det\left(\overline{\mathbf{R}_{ii}^{1/2}}\right)\right| \leq \sqrt{\left|\det\left(\overline{\mathbf{R}_{ii}}\right)\right|} \\ &\leq \left[\left(\sigma_v^2+\sigma_a^2\right)^2+(N-2)\rho^2\sigma_a^4\right]^{(N-1)/4}.\end{aligned} \quad \text{(A.2)}$$

For a lower bound on $|\det(\mathbf{R})|$, we first represent the bounds on minimum and maximum eigenvalues ($|\lambda_{R,\min}|$, $|\lambda_{R,\max}|$) in terms of mean (μ_{λ_R}) and variance ($\sigma_{\lambda_R}^2$) of eigenvalues along with the trace of the covariance matrix $\mathbf{R}$ [22] as

$$\begin{aligned}\mu_{\lambda_R}-\sigma_{\lambda_R}(N-1)^{1/2} &\leq \left|\lambda_{R,\min}\right| \leq \left(\frac{\mathrm{tr}(\mathbf{R}^{\dagger}\mathbf{R})}{N}\right)^{1/2}, \\ \mu_{\lambda_R} &\leq \left|\lambda_{R,\max}\right| \leq \mu_{\lambda_R}+\sigma_{\lambda_R}(N-1)^{1/2},\end{aligned} \quad \text{(A.3)}$$

where mean and variance are given by

$$\begin{aligned}
\mu_{\lambda_R} &= \frac{\operatorname{tr}(\mathbf{R})}{N} = \sigma_v^2 + \sigma_a^2, \\
\sigma_{\lambda_R}^2 &= \frac{\operatorname{tr}(\mathbf{R}^\dagger \mathbf{R})}{N} - \left|\mu_{\lambda_R}\right|^2 \\
&= \left(\sigma_v^2 + \sigma_a^2\right)^2 + (N-1)\rho^2\sigma_a^4 - \left|\mu_{\lambda_R}\right|^2 \\
&= (N-1)\rho^2\sigma_a^4.
\end{aligned} \tag{A.4}$$

Using (A.4) in (A.3), lower (l) and upper (u) bounds on minimum and maximum of eigenvalues are obtained as follows:

$$\begin{aligned}
\left|\lambda_{R,\min}^l\right| &= \sigma_v^2 + \sigma_a^2 - (N-1)\rho\sigma_a^2 \\
&\leq \left|\lambda_{R,\min}\right| \leq \left(\sigma_v^2 + \sigma_a^2 + (N-1)\rho^2\sigma_{a,i}^4\right)^{1/2} \\
&= \left|\lambda_{R,\min}^u\right|, \\
\left|\lambda_{R,\max}^l\right| &= \sigma_v^2 + \sigma_a^2 \\
&\leq \left|\lambda_{R,\max}\right| \leq \sigma_v^2 + \sigma_a^2 + (N-1)\rho\sigma_a^2 \\
&= \left|\lambda_{R,\max}^u\right|.
\end{aligned} \tag{A.5}$$

Now, we use [23, Theorem 1] to find a lower bound on $|\det(\mathbf{R})|$ as

$$|\det(\mathbf{R})| \geq \left|\lambda_{R,\min}^l\right|^{\Omega_R} \left|\lambda_{R,\max}^u\right|^{N-\Omega_R}, \tag{A.6}$$

where a parameter Ω_R is given by

$$\Omega_R = N\left[\frac{\left|\lambda_{R,\max}^u\right| - \left|\mu_{\lambda_R}\right|}{\left|\lambda_{R,\max}^u\right| - \left|\lambda_{R,\min}^l\right|}\right]. \tag{A.7}$$

An upper bound on the modulus of determinant of the non-principal submatrix can be represented in terms of eigenvalues ($\lambda_{\sqrt{R}}$) of $\mathbf{R}^{1/2}$ [24] as

$$\gamma_t = \left|\det\left(\mathbf{R}_{ji,j\neq i}^{1/2}\right)\right| \leq \frac{1}{2}\left(\left|\lambda_{\sqrt{R}}^u\right| - \left|\lambda_{\sqrt{R}}^l\right|\right)\left(\left|\lambda_{\sqrt{R}}^u\right|\right)^{N-2}. \tag{A.8}$$

Here, eigenvalues ($\lambda_{\sqrt{R}}$) of $\mathbf{R}^{1/2}$ can be represented in terms of eigenvalues (λ_R) of $\mathbf{R}$ using the eigenvalue decomposition as

$$\mathbf{R}^{1/2} = \mathbf{Q}\Lambda^{1/2}\mathbf{Q}^\dagger, \tag{A.9}$$

where $\mathbf{Q}$ is the matrix of eigenvectors and Λ is the diagonal matrix of eigenvalues of $\mathbf{R}$. Using (A.8) and (A.9) and eigenvalue bounds of (A.5), an upper bound on γ_t can be obtained as

$$\begin{aligned}
\gamma_t &= \left|\det\left(\mathbf{R}_{ji,j\neq i}^{1/2}\right)\right| \\
&\leq \frac{1}{2}\left(\sqrt{\left|\lambda_{R,\max}^u\right|} - \sqrt{\left|\lambda_{R,\min}^l\right|}\right)\left(\sqrt{\left|\lambda_{R,\max}^u\right|}\right)^{N-2}.
\end{aligned} \tag{A.10}$$

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