Semi-blind equalization at the symbol rate with application to OFDM

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Abstract

Blind equalization algorithms for non-minimum-phase channels are generally based on oversampling the output of the system. In this paper, we consider a symbol-rate sampled scheme that requires the periodic transmission of a few zeros within the data sequence. Sampling at the symbol rate relaxes some of the identi4ability conditions, and ensures that the additive noise is uncorrelated. We develop a least-squares approach for the blind equalization of such a system, that works well with very short observation windows. The price paid for eliminating the oversampling is the reduced throughput. We derive the necessary and su7cient conditions for the identi4ability of the system. Our simulations show that channels with near-common zeros can be equalized better using the proposed scheme, as compared to existing schemes. Also, the proposed method is seen to perform much better than the recently proposed methods (IEEE Trans. Signal Process. 47 (1999) 2007) in terms of bit error rate, for short observation windows, and requires much less stringent identi4ability conditions. However, the computational complexity of the method is high. The method can be applied directly to the blind equalization of orthogonal frequency division multiplexing systems that use a zero pre4x instead of a cyclic pre4x.

Keywords: Blind estimation; OFDM (orthogonal frequency division multiplexing); SISO (single input single output); Deterministic

1. Introduction

The problem of blind estimation is to estimate, from the observed output alone, the input and the impulse response of the system. It is well known that non-minimum-phase systems cannot be estimated from the second-order statistics (SOS) of the input and output [8,9]. However, if the input is cyclostationary, it has been shown that the SOS contains the phase information necessary to identify the channel [2,11]. Normally, blind estimation techniques make use of the cyclostationarity of oversampled digital communication signals. Alternatively, they use multiple antennas at the receiver. Either way, the problem is transformed to one of identifying an SIMO system [7,12].

There are certain limitations associated with the equalization of oversampled systems. If the polyphase components of the oversampled channel share common zeros, the system is unidenti4able [7,11,12]. Near-common zeros lead to drastic deterioration in performance. Again, a complication that would arise in practice, and is usually ignored in research work, is that oversampling at a rate faster than the baud rate would generally lead to the corresponding samples

Nomenclature

$y(n)$	channel output	$\mathbf{s}_i$	input vector for ith block
$h(n)$	channel coe7cients	$\mathbf{n}$	vector of AWGN
$s(n)$	input to channel	$\mathbf{Y}_i$	Toeplitz matrix of output for ith block
$y_i(n)$	channel output for the ith block		
$s_i(n)$	input to channel for the ith block	$\mathbf{Y}_G$	Toeplitz block matrix of output
$H(z)$	z transform for channel coe7cients	$\mathbf{H}_G$	matrix of channel coe7cients
$Y_i(z)$	z transform for output, ithe block	$\mathbf{S}_G$	stacked matrix of inputs
$S_i(z)$	z transform for input, ithe block	$\mathbf{s}_G$	vector of stacked inputs for diFerent blocks
L	length of impulse response of the channel	$\mathbf{G}_r$	stacked matrix of channel coe7cients
N	block size	*m fi*	number of possible combinations of n elements out of m
K	number of blocks used		
$\mathbf{H}$	channel matrix		
$\mathbf{y}_i$	output vector for ith block	**E**	summation

of the additive noise being correlated. Under these conditions, the noise samples cannot be assumed to be iid. Many methods require this assumption (or, otherwise, knowledge of the actual statistics of the noise), and others may suFer from bias or increased variance when it becomes invalid.

In this paper, we examine a semi-blind strategy that works with the symbol-rate sampled output. $L - 1$ zeros (where L is the length of the impulse response) are transmitted at the beginning of every block of N symbols. It then becomes possible to develop deterministic approaches[1] for channel and symbol estimation.

A possible application of the method is in block transmission techniques such as orthogonal frequency division multiplexing **(OFDM) [1,6].** Several recent papers make use of similar techniques for OFDM systems. In [5], a blind method of estimating an OFDM system making use of the cyclic pre4x is described. The method assumes knowledge of the second-order cyclostationary statistics of the input. However, the redundancy introduced by the cyclic pre4x is not su7cient for blind identi4cation in a deterministic framework. Reddy et al. **[10]** and Giannakis et al. **[3]** consider a scheme for OFDM systems that uses a zero pre4x instead of the usual cyclic pre4x. In [4], deterministic methods for the blind estimation of such a scheme are presented.[2] The methods require at least as many blocks as the number of symbols per block (the number of subcarriers, in the case of OFDM). In OFDM systems, which typically use a large number of subcarriers (sometimes hundreds or thousands), this would mean an estimation delay of the order of hundreds or thousands of symbols, which might not be acceptable for rapidly fading channels.

Here, we present a deterministic method that is attractive because it performs well with comparatively very short data lengths. The method requires a minimum of only two blocks for estimation. However, the computational complexity is signi4cantly higher than that of [4].

We derive the necessary and su7cient conditions required for identi4ability of the system, assess the

[1] We use the term "deterministic" to connote a method that does not use knowledge of the actual source statistics for system estimation. A characteristic of such a method is that, provided certain identi4ability conditions are met, and there is no noise present, the channel and the input can be perfectly reconstructed from a 4nite length of observations. A "statistical" method, on the other hand, though using additional information about source statistics, would give perfect performance only asymptotically. Hence, when we do have system noise, and only short data lengths, deterministic methods may be expected to perform better.

[2] As part of this work, we had independently developed the subspace method proposed earlier in [4]. Because of its earlier publication, however, we limit our discussion in this paper to only a comparative performance evaluation of the subspace method, and the least-squares approach of this paper.

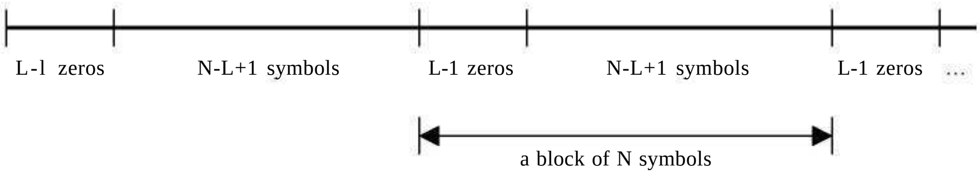

Fig. 1. Transmission format.

computational complexity of the proposed method, and discuss the advantages of a symbol-rate sampled scheme for blind identi4cation, as compared to an oversampled scheme. We compare performance with that of the subspace method for channel estimation in [4] using Monte Carlo simulations, where we 4nd that the proposed method performs better in terms of bit error rate (BER), when we use small numbers of blocks. We also compare performance with a standard blind estimation technique relying on oversampling of the output [12], and con4rm that the proposed scheme works better when the channels have near-common zeros.

The paper is organized as follows: In Section 2 we present the proposed scheme of transmission, and formulate the problem of identi4cation mathematically. In Section 3 we propose schemes for the blind identi4cation of the system, derive the identi4ability conditions, and also compare the symbol-rate sampled and oversampled schemes for blind equalization. Simulation results are presented in Section 4. Finally, we draw some conclusions about the proposed method in Section 5.

2. System model and problem formulation

The transmission scheme: We propose that $L-1$ zeros be transmitted at the beginning of every block of N symbols, where L is the length of the channel impulse response. The scheme is shown in Fig. 1.

The system model: The output of a linear, time-invariant channel, sampled at the symbol rate, may be represented by the convolution

$$y(n) = h(n) * s(n), \tag{1}$$

where y represents the output sequence, h the channel impulse response, and s the sequence of transmitted symbols, and the symbol $*$ denotes convolution.

Since $L-1$ zeros are transmitted at the beginning of every block of N transmitted symbols, a block of N observed outputs maybe represented as in the following equation:

$$\mathbf{y} = \mathbf{H}\mathbf{s}, \tag{2a}$$

where

$$\mathbf{H} = Sylv([h(L-1)\,h(L-2)\cdots h(0)], N-L+1), \tag{2b}$$

$$\mathbf{y} = [y(N-1)\,y(N-2)\cdots y(0)]^T, \tag{2c}$$

$$\mathbf{s} = [s(N-L)\cdots s(0)]^T, \tag{2d}$$

$y(i)$ is the ith observation, $s(i)$ the ith transmitted symbol, $h(i)$ the impulse response coe7cients, and $Sylv([h(L-1)\cdots h(0)], k)$ is a Sylvester matrix with k columns, de4ned as follows:

$$Sylv([h(L-1)\,h(L-2)\cdots h(0)], k) = \begin{bmatrix} h(L-1) & & & \\ h(L-2) & h(L-1) & & \\ \vdots & h(L-2) & \ddots & \\ \vdots & \vdots & \ddots & h(L-1) \\ \vdots & \vdots & & h(L-2) \\ h(0) & \vdots & & \vdots \\ & h(0) & & \vdots \\ (0) & & & \vdots \\ & & & h(0) \end{bmatrix}. \tag{3}$$

$\mathbf{H}$ is a matrix of dimensions $N \times (N-L+1)$, and full column rank.

The matrix representation in (2a) is equivalent to the polynomial representation

$$Y(z) = H(z)S(z), \tag{4a}$$

where

$$Y(z) = y_{N-1}z^{-(N-1)} + \cdots + y_1 z^{-1} + y_0, \tag{4b}$$

$$H(z) = h_{L-1}z^{-(L-1)} + \cdots + h_1 z^{-1} + h_0, \tag{4c}$$

$$S(z) = s_{(N-L)}z^{-(N-L)} + \cdots + s_1 z^{-1} + s_0. \tag{4d}$$

In the presence of additive noise, (2a) is modi4ed to

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{5}$$

where $\mathbf{n} = [n(N-1) \cdots n(0)]^T$ is a vector containing samples of noise, which is assumed to be white and gaussian.

Problem formulation: The problem of interest here is to separate H and **s** from y, or $H(z)$ and $S(z)$ from $Y(z)$, with only minimal knowledge of the characteristics of the channel and the transmitted symbols.

What we have been able to achieve by the periodic transmission of the zeros are: (i) forcing the multiplicative property on the polynomials as in (4a), and (ii) forcing full column rank on the channel coe7cients matrix, as de4ned in (2a). These properties are crucial for the methods of blind estimation presented here.

3. Blind channel estimation

In this section, we propose two approaches to obtain blind estimates of H and **s** appearing in (2a). The estimation is carried out over K blocks of data. We assume that the channel impulse response remains unchanged over the transmission of K blocks, and that the application of interest would permit an algorithmic delay of KN symbols, or K blocks.

In what follows, the subscripts of $y_i(n)$, $Y_i(z)$, $\mathbf{y}_i$, $\mathbf{Y}_i$, etc. refer to the block indices.

3.1. Separation of the zeros of $H(z)$ and $S(z)$

We 4rst discuss a simple method of estimating the system that gives an intuitive understanding of the problem. The zeros of $H(z)$ and $S(z)$ can be separated as follows: Find the zeros of $Y(z)$ for each received block. In the absence of noise, the zeros that repeat in every block are the zeros of $H(z)$. In the presence of noise, these zeros may be determined by plotting the zeros of $Y(z)$ for every block, as in Fig. 2, so that the densest regions (i.e. zeros with small variances) correspond to the zeros of $H(z)$, or by maximizing a suitably formed cost function of the polynomial over the complex plain. As one example, the cost function

$$c(z) = \frac{1}{(1/K)\sum_{i=1}^{K}|Y_i(z)|^2} \tag{6}$$

is plotted in Fig. 3.

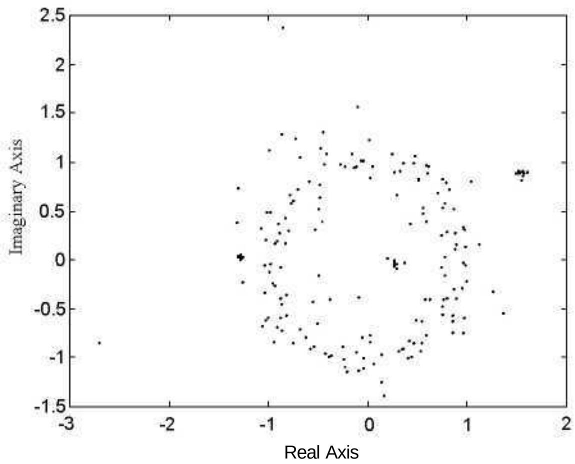

Fig. 2. Plot of the roots of $Y(z)$; 10 blocks, 20 symbols per block, 38 dB. Channel roots at: $1.54 + j0.9, -1.27 + j0.02, 0.27 - j0.04$.

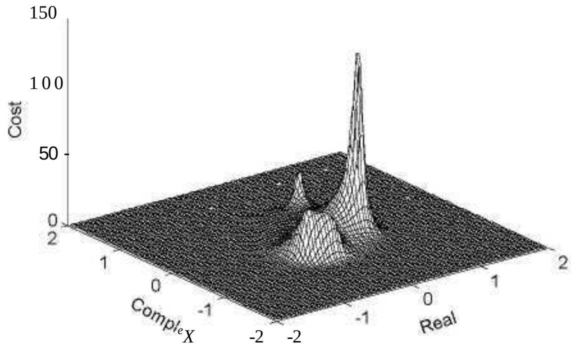

Fig. 3. Two-dimensional plot of the cost function (6), with SNR = 27 dB, 20 symbols per block, over 5 blocks. Channel roots inside the unit circle: $-0.5 - j0.6, 0.5 - j0.25, 0.4 + j0.75$.

It is observed from the plot in Fig. 2 that we have the densest clusters of points around the channel zero locations, and it is easy to pick out these zeros.

We also note a phenomenon that occurs when N is increased. The roots of typical polynomials representing the symbol sequence, viz., $S(z)$ (which we call the signal zeros), tend to concentrate in a band around the circumference of the unit circle. As N increases, the band becomes narrower, and tends to the unit circle. This means that as N increases, the density of the signal roots in the band increases. This makes it dif4cult, or impossible, even at moderate SNRs, to pick out the channel zeros from the composite set of channel zeros and signal zeros, based on the density of the clusters. One would expect this to happen at SNRs at which the spread in the channel roots created by the noise is equal to or greater than the natural spread of the roots of $S(z)$. Hence, performance can be expected to deteriorate with the block size, for a given number of blocks and SNR.

We also expect that as we increase the number of blocks used for the estimation, for a given SNR and block size, the performance would saturate, the saturation level being lower for higher SNRs and for higher block sizes.

These observations become obvious from Fig. 2, and are reLected in the simulation results, reported in Section 4.

If we know a priori that the channel does not have unit circle zeros, performance could be improved by placing a constraint on the search for the channel zeros. The densest areas away from the unit circle would yield the channel zeros. This can be directly done in the methods of separation of zeros. But it may not be possible to introduce such a constraint easily into the cross relations with zero pre4x (ZPCR) or subspace with zero pre4x (ZPSS) [4] approach discussed subsequently.

We note that in the absence of noise, the channel zeros can be separated out for any block size, provided the input sequence is su7ciently diverse. This is because the channel zeros of the various blocks would be superimposed upon each other, while the input zeros, however closely spaced, would not be the same for every block. Hence, in the absence of noise, the clusters of input zeros cannot be as dense as the clusters of channel zeros.

Identi0ability: In the absence of noise, the roots of $H(z)$ are separable from the roots of $\{Y_i(z);\ i = 1;\dots;K\}$; $K \geq 2$, iF $\{S_i(z);\ i=1;\dots;K\}$ do not share any common roots.

Digital communication signals are su7ciently diverse to make such sharing of zeros extremely unlikely.

3.2. A cross relations approach (ZPCR)

In this section we propose a deterministic method, similar to that of the cross relations (CR) approach [12], which we call the ZPCR method. Note that the outputs of any two blocks may be written as

$$y_i(n) = h(n) * s_i(n); \tag{7}$$

$$y_j(n) = h(n) * s_j(n), \tag{8}$$

where the subscripts refer to the block indices.

Now, $h(n)$ may be eliminated from the two equations as follows. From (8), using the commutativity of the convolution operator, we have

$$\begin{aligned} s_i(n) * y_j(n) &= s_i(n) * h(n) * s_j(n) \\ &= h(n) * s_i(n) * s_j(n). \end{aligned} \tag{9}$$

Using (7), this reduces to

$$s_i(n) * y_j(n) = s_j(n) * y_i(n). \tag{10}$$

This is the required cross relation. It may be written in a matrix form as

$$\mathbf{Y}_i s_j = \mathbf{Y}_j s_i \quad \forall i, j, \tag{11}$$

where $\mathbf{Y}_i$ is a Toeplitz matrix formed from the observations as follows:

$$\begin{aligned} \mathbf{Y}_i = Sylv([&y_i(N-1)\ y_i(N-2)\ \cdots\ y_i(0)], \\ &N-L+1), \end{aligned} \tag{12}$$

with dimensions $(2N-L) \times (N-L+1)$.

Stacking the equations for all pairs of i and j, we have

$$\mathbf{Y}_G s_G = 0; \tag{13}$$

where $\mathbf{Y}_G$ is a Toeplitz block matrix of dimensions ${}^K C_2 \times (2N-L) \times K(N-L+1)$ (where C represents the combinations operator), with each row containing a pair of matrices from the matrix set $\{Y_i; i = 1, \ldots, K\}$ located at suitable positions:

$$Y_G = \begin{bmatrix} Y_2 & -Y_1 & & & \\ Y_3 & & -Y_1 & & \\ \vdots & & & \ddots & \\ Y_K & & & & -Y_1 \\ & Y_3 & -Y_2 & & \\ & Y_4 & & \ddots & \\ & \vdots & & & \\ & Y_K & & & -Y_2 \\ & & \ddots & & \\ & & & Y_K & -Y_{K-1} \end{bmatrix}, \tag{14}$$

and sG is obtained by stacking $\{s_i; i = 1, \ldots, K\}$ one below the other. Eq. (13) may be solved to get the transmitted symbols directly.

Effect of noise: In the absence of noise, the method gives the exact solution with a minimum of two blocks of data. In the presence of noise, more blocks would have to be used to obtain good performance. Eq. (13) may then be solved in the least-squares sense to get an estimate of the transmitted symbols.

Identifiability:

Theorem 1. *The necessary and sufficient condition for the solution of* (13) *to be unique up to a scale factor is that the polynomials* $\{S_i(z); i = 1, \ldots, K\}$, $K \geq 2$, *do not share common zeros.*

Proof of the sufficient condition. It is easy to see that YG may be written in terms of the channel coefficients and the input as

$$Y_G = H_G \times S_G, \tag{15a}$$

where HG is a block diagonal matrix given by

$$H_G = \mathrm{diag}(G_r, G_r \cdots G_r), \tag{15b}$$

with G_r repeated ${}^K C_2$ times along the diagonal. The $(N + N' - 1) \times (2N' - 1)$ matrix G_r is given by

$$G_r = \mathrm{Sylv}([h(L-1)h(L-2)\cdots h(0)], 2N, -1), \tag{15c}$$

where the matrix Sylv(a, k) is defined in (3), $N' = N - L + 1$, and SG is a block matrix defined similar to Y_G in (14), with the Y_i replaced by the $(2N' - 1) \times (N - L + 1)$ matrix S_i, given by

$$S_i = \mathrm{Sylv}([s_i(2N-2)\cdots s_i(0)], N-L+1). \tag{15d}$$

If $\hat{S}_G$ is a solution of (13), it follows that

$$H_G S_G \times \hat{s}_G = 0. \tag{16}$$

Since HG is full column rank, (13) implies that

$$S_G \hat{s}_G = 0 \tag{17}$$

or

$$s_j * \hat{s}_i = s_i * \hat{s}_j, \tag{18}$$

implying further that

$$S_i(z)\hat{S}_j(z) = S_j(z)\hat{S}_i(z). \tag{19}$$

Using $Z\{P(z)\}$ to denote the zeros of the polynomial $P(z)$, (19) implies the following relationship between the zeros of $S_1(z)$ and those of the other polynomials:

$$\begin{aligned} Z\{S_1(z)\} &\in \bigcap_{j=2}^{K} Z\{S_j(z)\hat{S}_1(z)\}, \\ &= \bigcap_{j=2}^{K} Z\{S_j(z)\} \cup Z\{\hat{S}_1(z)\} \\ &= Z\{\hat{S}_1(z)\}, \end{aligned} \tag{20}$$

if $S_i(z), i = 1, \ldots, K$, do not have any common roots. This means that

$$Z\{S_1(z)\} \in Z\{\hat{S}_1(z)\}. \tag{21}$$

Since $S_1(z)$ has at most $N - 1$ zeros, it follows that

$$Z\{S_1(z)\} = Z\{\hat{S}_1(z)\} \tag{22}$$

and

$$S1(z) = a \times \hat{S}i(z) \tag{23}$$

for some scalar *a*. This completes the proof.

Necessary condition: If $\{Si(z); i = 1,\ldots,K\}$ share a common root $Z0$, then $Si(z) = S\text{-}[(z)(z - Z0)$ and $Z0$ becomes an unidenti4able parameter of the system. To see this, let $\hat{S}_t(z) = S[(z)(z - z'_o)$. Then $S'_t(z)$ and any $Z'Q$ satisfy (17) and (10). Hence, a necessary condition for the input to be uniquely identi4able is that $\{Si(z); i = 1\ldots,K\}$ do not share any common roots. •

Computational complexity: The minimization involved in getting the least-squares solution to (13), could be implemented by obtaining an extreme eigenvector of the matrix $Y_G^T Y_G$, where Y_G has dimensions ${}^KC_2 \times (2N - L) \times K(N - L + l)$. This would require $O(K^2N^2)$ Lops. Since YG is extremely sparse, the formation of $Y_G^T Y_G$ would not have a signi4cant computational count. Hence, the computational complexity of the method would be $O(K^2N^2)$.

3.3. Remarks

We conclude this section by making some general observations pertaining to the comparison of the methods proposed here with the blind algorithms based on oversampling:

(i) Oversampled systems are not identi4able if the channels resulting from the oversampling have common zeros [7,11,12]. This problem does not arise here. However, performance deteriorates at even moderate SNRs if the channel has zeros on the unit circle.

(ii) Most schemes for blind equalization assume that the noise is uncorrelated. This assumption is not valid when the output is sampled at rates higher than the baud rate.

(iii) Overestimation of the impulse response length does not cause these methods to break down, unlike the standard methods based on oversampling of the output [7,11,12]. Overestimation would mean transmitting $L' > L$ zeros per block, and choosing $L' - 1$ zeros of $Y(z)$ to be the zeros of $H(z)$.

Table 1
Computational complexity of the methods

	Method	Complexity
1	ZPSS	$O(L2 + KN^2)$
2	ZPCR	$O(K^4N^3)$
3	Subspace (standard)	$O(M^2L^2+N^2)$
4	CR (standard)	$O(M^2L^2+M^5LN^2)$

(iv) In the ZPCR method presented, we have to work over a number of blocks to get the performance the CR method in the oversampling framework [12] gives with a single block of the same size. This introduces a large delay in the equalization.

(v) For blocks of the same length, the computational complexity of the methods presented here are much greater than that of the standard methods. Table 1 gives a comparison of the complexity of the various methods.

Summarizing, there are de4nite trade-oFs between the methods proposed here and those based on the oversampled approach. A careful choice needs to be made for selection of the appropriate method in a given application.

We also suggest that when the block size is large, the ZPCR method could still be used to advantage as follows. The choice of the estimation technique at any time can be made by determining whether the oversampled channel has common zeros. (This can be done by testing the rank of an appropriate matrix.) If the channel does have common zeros, the ZPCR method could be used, and otherwise, a method exploiting oversampling could be used. Thus the presence of the zeros in a ZP-OFDM system can be exploited for better estimation.

4. Simulations and performance

Extensive computer simulations have been carried out to assess the performance of the methods, and to compare them with existing methods. Non-minimum-phase channels with impulse responses spanning four symbol periods, and a 16 QAM inputs were used.

The channels used represent a mix of conditions regarding the location of zeros with respect to the unit circle, and are listed in Table 2.

Table 2
Locations of channel zeros

Channel No.	Location of zeros	Position with respect to unit circle
1	1;5414 + j0.9000; -1.2790 + j0.01 63; 0.2706 - j0.0395	In, out
2	-0.7059 - j0.1 525; j; 0.2860 - j0.5046	In, on, out
3	0.5 - j0.5; -0.6 + j0.2; -0.1176 + j0.4706	In
4	40.7072;0.7072;0.1249	In
5	50.882 — jl.471,—0.8 + j1.6; —1.31 -jO.175	Out
6	60.384+jl.92,2-j,1.31 +j0.175	Out
7	-1, + 1; 0.7071 +j0.7071	On
8	80.5477+ j0.8367; 0.9487 +j0.3 162; 0.7746-jO.6325	On

For comparison with the proposed ZPCR method, we have used the method for channel estimation presented in [4], referred to hereafter as the ZPSS method. Equalization in the latter case was carried out using the pseudoinverse of the appropriate matrix formed from the estimated channel coe7cients.

Experiment 1: First, we consider the ZPCR method using 4 blocks of 20 symbols each, with the SNR varying from 20 to 33 dB. The simulations were carried out for channels with impulse responses normalized to have unit norm, and with zeros distributed (i) only inside the unit circle, (ii) only outside the unit circle (iii) only on the unit circle, and (iv) inside, outside and on the unit circle. While the results, shown in Fig. 4, cannot be taken to be conclusive due to the small number of channels experimented with, they seem to indicate that the BER performance is dependent on the position of the roots. Non-minimum-phase channels are generally seen to have poorer performance than minimum-phase channels. Channels with zeros on the unit circle seem to yield the poorest performance.

Experiment 2: Here, we take up a comparison of the ZPCR and the ZPSS methods. The BER performance of the methods was evaluated for varying numbers of blocks. The experiment was carried out with an SNR of 30 dB, and a block length of 20 symbols. The results are shown in Fig. 5. We observe that for short observation windows, the ZPCR method outperforms the ZPSS method. In general, it needs far fewer blocks to give the same performance.

Experiment 3: Next, we compare the performance of the proposed method with respect to the number of blocks, at 30 and 35 dB. The results, depicted in

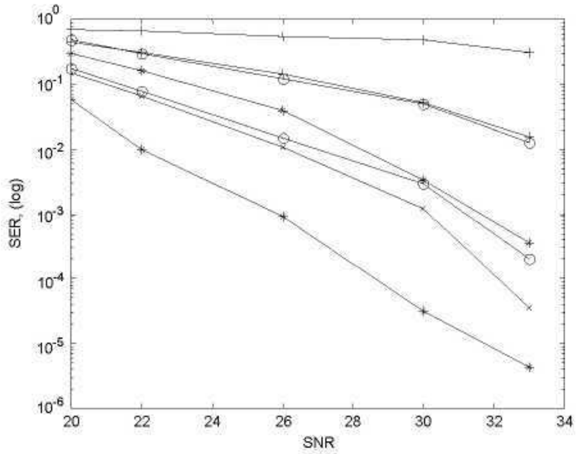

Fig. 4. BER performance of the CR method for various channels, at 26 dB, with 4 blocks of 20 symbols each. Channel roots: (+) only inside the unit circle, (x) only outside the unit circle, (o) only on the unit circle, (*) inside, outside and on the unit circle.

Fig. 6, indicate that for a particular block size, the BER performance saturates, and the saturation level is lower for a higher SNR.

Experiment 4: The dependence of performance on the block size is considered next. We used an output SNR of 26 dB. The ZPSS simulations were carried out with 20 and 30 blocks of data, and the block size varying from 19 to 40 symbols. The ZPCR simulations were carried out on Channel 2, with 4 blocks, and the block size varying from 10 to 50 symbols. The results are presented in Figs. 7 and 8.

We observe that performance actually deteriorates with increasing block size, as conjectured earlier. This could be ascribed to the increasing density of roots, as

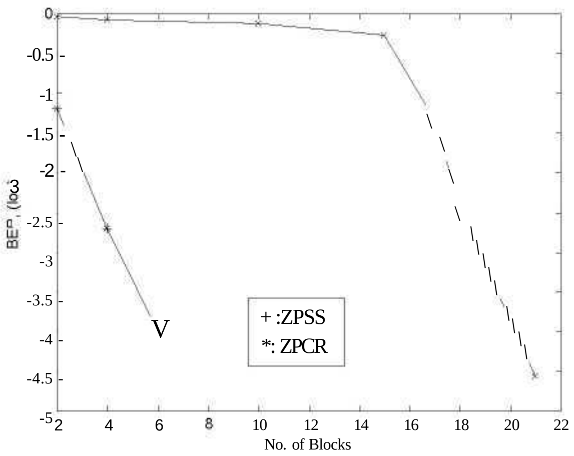

Fig. 5. Symbol error rate performance at 30 dB, with 20 symbols per block, on a Rayleigh frequency selective channel with uniform power pro4le $L = 4$.

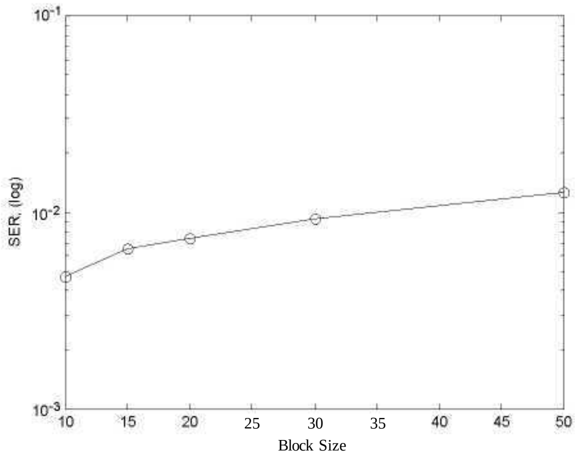

Fig. 7. ZPCR method: symbol error rate performance for 26 dB, 4 blocks, Channel 2.

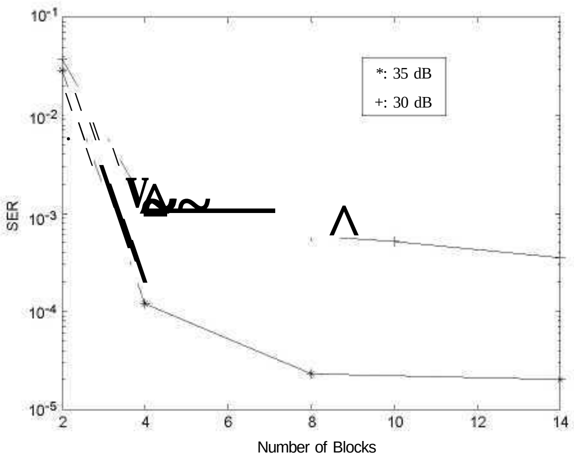

Fig. 6. Symbol error rate performance of the ZPCR method, with 20 symbols per block, on a Rayleigh frequency selective channel with uniform power pro4le $L = 4$.

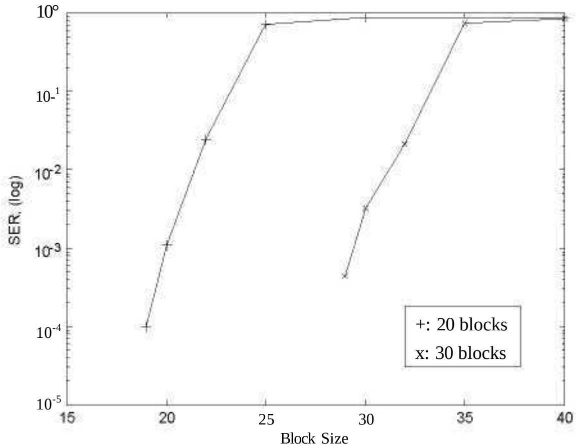

Fig. 8. Subspace method: symbol error rate performance for 26 dB, Channel 1.

mentioned in Section 3.1. It could also be explained by the fact that the ratio of known symbols (zeros) to unknown symbols (data) is reduced when the block size increases.

Experiment 5: Finally, it is interesting to compare the proposed semi-blind scheme with a standard time domain method for the blind equalization of over-sampled systems, for a given size of the observation window.

First, we used a well-conditioned channel, with an impulse response duration equal to four symbol periods. The proposed ZPCR method was simulated with 4 blocks of 20 symbols, and the standard CR method [2], with 80 symbols, and an oversampling factor of 4. The performance is depicted in Fig. 9. CR clearly performs better for a given size of the observation window.

We next used a channel with the duration of impulse response equal to 10 symbol periods. The impulse response was the truncated sum of two scaled

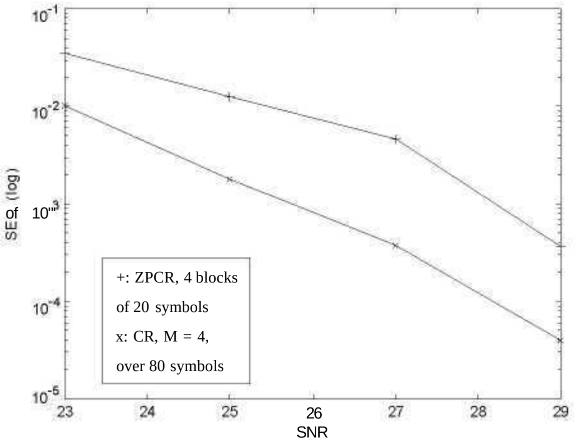

Fig. 9. Symbol error rate performance for a channel without near-common zeros, and an impulse response length of 4.

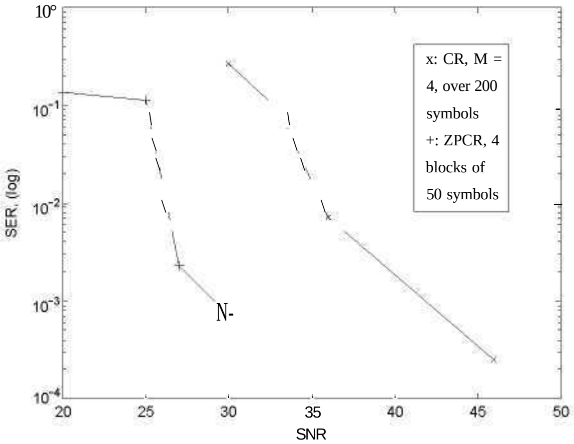

Fig. 10. Symbol error rate performance for a channel with near-common zeros and impulse response of length 10.

and shifted sinc functions. When oversampled by a factor of 4, the resulting four-phase channel impulse response components tend to have near-common zeros. We may, therefore, expect a signi4cant performance degradation in the oversampled approach. This is noticed clearly in Fig. 10, where we 4nd that the method presented here performs much better.

Experiment 6: Finally, we simulate the dependence of the CR and the ZPCR methods on a Rayleigh frequency selective channel with a uniform power pro4le. The results are depicted in Fig. 11. We observe that

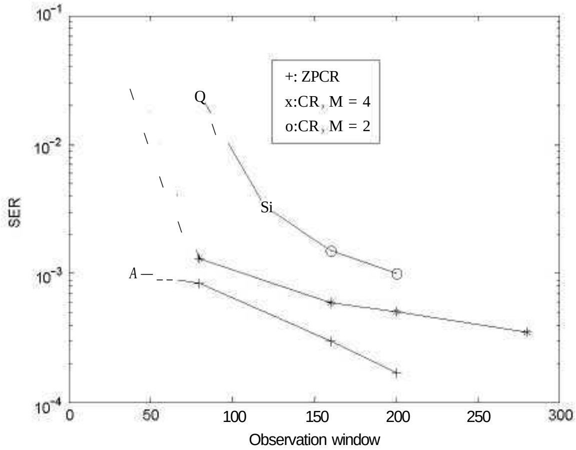

Fig. 11. Symbol error rate performance for a Rayleigh frequency selective channel with uniform power pro4le, and an impulse response length of 4, at 30 dB SNR.

CR performs better than ZPCR for an oversampling factor of 4, but worse for an oversampling factor of 2.

5. Conclusions

In this paper, we have presented a scheme for the semi-blind equalization of a system using the symbol-rate sampled output. A number of zero-valued samples are transmitted between blocks of symbols, and the equalization is carried out over several (two or more) of these blocks. Algorithms have been developed for the scheme on the lines of the cross relations [12] and subspace [7] methods for oversampled systems.

The necessary and su7cient conditions under which identi4cation is possible have been derived.

Simulation studies have been carried out to assess the performance of the methods. In the OFDM context, the proposed method is seen to perform better than recently proposed symbol-rate methods [5,4].

It is also shown to perform better than existing blind schemes based on oversampling, when the channels have near-common zeros. This provides the primary motivation and application for the work reported here.

The computational complexity of the method is considerable. It may be possible, however, to exploit the Hankel structure of the matrices to reduce the complexity.

We observe that performance can be improved by increasing the number of blocks used, but not by increasing the block size for a given number of blocks. For a given channel response length, throughput can be increased only by increasing the block size. This means that we have a tradeoF between throughput and performance.

In order to obtain acceptable performance even at high SNRs, the method requires observations over several blocks. The resulting increase in data sizes means that the computational complexity blows up. Hence, we also have a tradeoF between computational complexity and performance.

There is also a considerable delay in recovering the transmitted symbols.

Due to the large data lengths required for channel identi4cation, the methods of this paper could be useful where the channel can be assumed to remain stationary over large periods of time, such as slowly fading channels.

References

[1] L.J. Cimini, Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing, IEEE Trans. Commun. COM-33 (7) (July 1985) 665-675.

[2] W.A. Gardiner (Ed.), Cyclostationarity in Communications and Signal Processing, IEEE Press, New York, 1994.

[3] G.B. Giannakis, A. Scaglione, S. Barbarossa, Redundant 4lterbank precoders and equalizers. Part I: uni4cation and optimal designs, IEEE Trans. Signal Process. 47 (July 1999) 1988-2006.

[4] G.B. Giannakis, A. Scaglione, S. Barbarossa, Redundant 4lterbank precoders and equalizers. Part II: blind channel estimation, synchronization and direct equalization, IEEE Trans. Signal Process. 47 (July 1999) 2007-2022.

[5] R.W. Heath, G.B. Giannakis, Exploiting input cyclostationarity for blind channel identi4cation in OFDM systems, IEEE Trans. Signal Process. 47 (March 1999) 848-856.

[6] S. Kapoor, D.J. Marchok, Y.F. Huang, Adaptive interference suppression in multiuser wireless OFDM systems using antenna arrays, IEEE Trans. Signal Process. 47 (December 1999) 3381-3391.

[7] E. Moulines, P. Duhamel, J.F. Cardoso, S. Mayrargue, Subspace methods for the blind identi4cation of multichannel FIR 4lters, IEEE Trans. Signal Process. 43 (February 1995) 516-525.

[8] S.J. Orfanidis, Optimum Signal Processing, Macmillan, New York, 1988.

[9] A. Papoulis, Probability, Random Variables and Stochastic Processes, 3rd Edition, McGraw Hill, New York.

[10] V.U. Reddy, B.K. Dey, S. Dasgupta, Optimal bases with zero pre4x—with and without predetection equalizer, IEEE Trans. Signal Process., submitted for publication.

[11] L. Tong, G. Xu, T. Kailath, Blind identi4cation and equalization of multipath channels: a frequency domain approach, IEEE Trans. Inform. Theory 41 (January 1995) 329-334.

[12] G. Xu, H. Liu, L. Tong, T. Kailath, A least squares approach to blind channel identi4cation, IEEE Trans. Signal Process. 43 (December 1995) 2982-2993.