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HERMITE AND SPECIAL HERMITE EXPANSIONS REVISITED

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1. Introduction. On a compact Riemannian manifold M consider a first-order pseudodifferential operator $p(X,D)$ that is positive and selfadjoint. Let λ_j, $j=0,1,2,\ldots$, be the sequence of its eigenvalues and $e_j(x)$ the corresponding eigenfunctions. The family $\{e_j\}$ then forms an orthonormal basis for $L^2(M)$. Let E_jf be the projection of f onto the jth eigenspace so that we have

$$f=\sum_{j=0}^{\infty} E_jf, \tag{1.1}$$

where the series converges in the L^2-norm. For functions f in $L^p(M)$, where p is other than 2, the above series may not converge to f in the L^p-norm, and one is led to consider the Bochner-Riesz means $S_\lambda^\delta f$.

The Bochner-Riesz means are defined by the equation

$$S_\lambda^\delta f(x)=\sum_{\lambda_j\leqslant\lambda}\left(1-\frac{\lambda_j}{\lambda}\right)^\delta E_jf(x), \tag{1.2}$$

and we want to know if $S_\lambda^\delta f$ converges to f in the L^p-norm as $\lambda\to\infty$. Let $\delta(p)=\max\{n|1/p-1/2|-1/2,0\}$ be the critical index, where n is the dimension of the manifold. Then a necessary condition for the convergence of $S_\lambda^\delta f$ to f in the L^p-norm is that $\delta>\delta(p)$. In [6] Sogge proved that this condition is also sufficient as long as $1\leqslant p\leqslant 2(n+1)/(n+3)$ or $p\geqslant 2(n+1)/(n-1)$. This result includes previously known results for the multiple Fourier series ($M=T^n$, the n-torus) and spherical harmonic expansions ($M=S^n$, the $(n+1)$-sphere).

Let us leave the premises of compact manifolds and proceed to noncompact situations. The simplest example is the case of the standard Laplacian $-\Delta$ on $\mathbb{R}^n$, and in this case the operator does not have point spectrum. The spectral decomposition is given by the Fourier transform, and one is led to consider the Bochner-Riesz means

$$S_R^\delta f(x)=(2\pi)^{-n/2}\int_{|\xi|\leqslant R}\left(1-\frac{|\xi|^2}{R^2}\right)^\delta \hat{f}(\xi)e^{ix\cdot\xi}d\xi, \tag{1.3}$$

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