Structure of proton decay in SO(n) family unification

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We show that for all SO(n), $n>10$, family unification schemes there exist no gauge currents relevant for baryon decay other than those already appearing in the SO(10) model. The effect of Yukawa interactions for baryon-number-violating processes in SO(n) models is also investigated.

I. INTRODUCTION

It is well known that when the SU(3) color and SU(2)$\times$U(1) electroweak interactions are unified in an SU(5) grand unified model,[1] there exist new gauge bosons (X and Y) which can mediate baryon decay.[1-3] The model predicts decay modes such as

$$p \to e^+\pi^0\ , \tag{1.1}$$

$$p \to \mu^+K^0\ , \tag{1.2}$$

$$p \to \mu^+\pi^0\ ,\ e^+K^0\ . \tag{1.3}$$

Reactions (1.1) and (1.2) occur directly via the gauge currents, whereas this is not the case for reactions (1.3). The latter reactions, however, can occur from Yukawa interactions, or as a result of mixing from the fermionic mass matrix. It is usually believed that the mixing from the fermionic mass matrix is small and that the gauge interactions dominate the Yukawa interactions. Therefore, decay modes like (1.3) are expected to be suppressed. (Such a suppression has been named the "kinship hypothesis" by Wilczek and Zee[4] in the context of the general analysis of baryon decay.[4,5]) However, this result is not valid in general, since neither the theory nor the available data is sufficient to determine the fermion mass matrix or the strength of the Yukawa interactions.

Upon expanding the grand unified model from SU(5) to SO(10),[6] the above situation remains effectively unaltered, despite the introduction of new baryon-number-changing currents. This is essentially due to the fact that both the SU(5) and SO(10) models are single-family unification schemes.

There have been many proposals[7,8] recently for SO(n), $n>10$, models which contain more than a single family in one irreducible spinor representation of the group. [For the purposes of this paper, a "family" shall be identified with a 16-dimensional SO(10) spinor (of fixed chirality[9]) contained in the SO(n) spinor representation.] For these models, in addition to the SO(10) (vertical) interactions, it is well known that there appear interactions linking particles of different families in a horizontal fashion, i.e., horizontal interactions.[10] In these models one may further expect to see interactions connecting particles of different families in a nonhorizontal fashion, i.e., "diagonal interactions" (cf. Fig. 1). If some of the corresponding "diagonal" gauge bosons (X_D and Y_D) carry baryon number, then the relative rates of processes (1.1)–(1.3) would be affected. In particular, processes like (1.2) would be enhanced, and if mixing between X,Y and X_D,Y_D is present, processes like (1.3) can no longer be expected to be suppressed.

In this paper, we show that no gauge bosons X_D and Y_D exist in SO(n), $n>10$, models. In fact, for all SO(n), $n>10$, models, there exist no gauge currents which are relevant for baryon decay other than those already appearing in the SO(10) model. This is essentially due to the presence of conjugate families[11] (families which have $V+A$ weak interactions) which must be removed from the low-lying spectrum.[7,8] We find that the only diagonal gauge bosons which appear in SO(n), $n>10$, models are those which link families to conjugate families.

The situation is different in the Yukawa sector of these models. Here we find that there do exist Higgs boson carrying baryon number which mediate diagonal interactions between families. These bosons therefore provide a mechanism for enhancing processes like (1.2). If mixing occurs with the analogous bosons mediating vertical interactions, processes like (1.3) could be enhanced as well. Al-

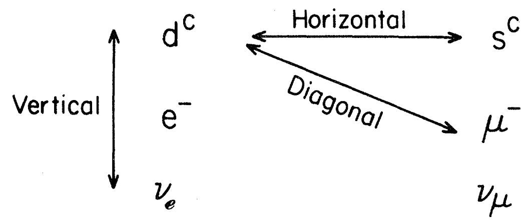

FIG. 1. Vertical, horizontal, and diagonal interactions are illustrated for particles belonging to two 5* representations of SU(5).

though the relative Yukawa coupling constants leading to processes (1.1)—(1.3) are determined in an SO(n) model, the mass matrix for the associated Higgs fields is unknown. Hence the relative rates of (1.1)—(1.3) are left arbitrary. In the Yukawa sector, as well as in the gauge sector, we find interactions which link families to conjugate families.

In Sec. II, we review some properties of SO(n) spinor representations[12,13] and their applications to family unification in grand unified models.[13] The proof that there exist no gauge currents which are relevant for proton decay other than those appearing in SO(10) is given in Sec. III. The Yukawa sector for SO(n), $n>10$, is examined in Sec. IV. In the above discussion the effect of mixing from the fermion mass matrix has been ignored. In the Appendix, we give several examples of mass matrices where this is justified.

II. SPINOR REPRESENTATIONS OF SO(n)

We begin with the case $n=2N$ ($N=$ integer).

A. $n=2N$

The SO($2N$) group contains two 2^{N-1}-dimensional irreducible spinor representations which are constructed explicitly below.

For convenience we adopt a formalism commonly used in many-body physics.[12–14] Let us introduce a set of N fermionic creation and annihilation operators $a_i^\dagger$ and a_i, $i=1,2,\ldots,N$. They satisfy the usual anticommutation relations

$$\{a_i,a_j^\dagger\}=\delta_{ij}\;,$$
$$\{a_i,a_j\}=\{a_i^\dagger,a_j^\dagger\}=0\;. \tag{2.1}$$

The Hermitian combinations

$$\Gamma_{2i}\equiv a_i+a_i^\dagger \tag{2.2}$$

and

$$\Gamma_{2i-1}\equiv -i(a_i-a_i^\dagger)\;,\quad i=1,2,\ldots,N \tag{2.3}$$

define a rank-$2N$ Clifford algebra, since from (2.1),

$$\{\Gamma_A,\Gamma_B\}=2\delta_{AB}\;,\quad A=1,2,\ldots,2N\;. \tag{2.4}$$

By forming commutators of the Γ's, we obtain the generators for the SO($2N$) group

$$\Sigma_{AB}=\frac{1}{2i}[\Gamma_A,\Gamma_B]\;. \tag{2.5}$$

They consist of linear combinations of all possible bilinear products $a_i^\dagger a_j$, a_ia_j, and $a_i^\dagger a_j^\dagger$. In particular, the operators

$$\begin{aligned}\tau^i_j&=a_i^\dagger a_j-\tfrac{1}{2}\delta_{ij}\\&=\frac{1}{4i}(\Sigma_{2j,2i}+\Sigma_{2j-1,2i-1}\\&\qquad+i\Sigma_{2i-1,2j}+i\Sigma_{2j-1,2i})\end{aligned} \tag{2.6}$$

generate the U(N) subgroup of SO($2N$), since from (2.1) (Ref. 15)

$$[\tau^i_j,\tau^k_l]=\delta_{kj}\tau^i_l-\delta_{il}\tau^k_j\;. \tag{2.7}$$

The N-dimensional Cartan subalgebra of the group is spanned by

$$H_i\equiv\tau^i_i=\tfrac{1}{2}\Sigma_{2i-1,2i}\;. \tag{2.8}$$

A central element Γ_0 [the "SO($2N$)-chirality operator"] can be formed by taking the product of all the elements of the Cartan subalgebra

$$\Gamma_0\equiv(-2)^N H_1H_2\cdots H_N\;. \tag{2.9}$$

It serves to label the two irreducible spinor representations.

In constructing the spinor representations, let us first define a U(N)-singlet state $|0\rangle$. The SO($2N$) spinor with SO($2N$) chirality equal to $+1$ is given by

$$\begin{aligned}|\psi_{+(2N)}\rangle&=\psi|0\rangle+\tfrac{1}{2}\psi^{ij}a_i^\dagger a_j^\dagger|0\rangle\\&\quad+\tfrac{1}{24}\psi^{ijkl}a_i^\dagger a_j^\dagger a_k^\dagger a_l^\dagger|0\rangle+\cdots\;,\end{aligned} \tag{2.10}$$

where the indices $i,j,k\cdots$ are completely antisymmetric. The SO($2N$) spinor with SO($2N$) chirality equal to -1 is given by

$$|\psi_{-(2N)}\rangle=\psi^i a_i^\dagger|0\rangle+\tfrac{1}{6}\psi^{ijk}a_i^\dagger a_j^\dagger a_k^\dagger|0\rangle+\cdots\;. \tag{2.11}$$

Each term in (2.10) and (2.11) corresponds to an

$N!/[l!(N-l)!]$-dimensional representation of U(N) ($l=$ the number of indices in $\psi^{i_1 i_2 \cdots i_l}$). Both series (2.10) and (2.11), of course, terminate when $l>N$, yielding two 2^{N-1}-dimensional representations of SO($2N$).

In applying (2.10) and (2.11) to grand unified models, the coefficients $\psi^{ij\cdots k}$ are identified with two-component Weyl spinors. In particular, the 16-dimensional representation $|\psi_{+(10)}\rangle$ of SO(10) is given the particle assignment

$$\psi=\nu_{eL}^c\ ,$$
$$\psi^{\alpha\beta}=\epsilon^{\alpha\beta\gamma}u_{L\gamma}^c\ ,\quad \psi^{4\alpha}=u_{L\alpha}\ ,\quad \psi^{5\alpha}=d_{L\alpha}\ ,\quad \psi^{45}=e_L^+\ ,$$
$$\psi^{45\alpha\beta}=\epsilon^{\alpha\beta\gamma}d_{L\gamma}^c\ ,\quad \psi^{5123}=e_L^-\ ,\quad \psi^{1234}=\nu_{eL}\ , \tag{2.12}$$

where α, $\beta=1,2,3$ is a color index and we adopt the convention that all particles are left handed. With this assignment the SU(3) color and U(1)$_{\rm em}$ subgroups are generated, respectively, by

$$S_\beta^\alpha=\tau_\beta^\alpha-\tfrac{1}{3}\delta_{\alpha\beta}\sum_\gamma H_\gamma \tag{2.13}$$

and

$$Q=-\tfrac{1}{3}\sum_\alpha H_\alpha+H_4\ . \tag{2.14}$$

In addition, the weak hypercharge and baryon minus lepton ($B-L$) number operators can be identified with

$$Y=-\tfrac{1}{3}\sum_\alpha H_\alpha+\tfrac{1}{2}H_4+\tfrac{1}{2}H_5 \tag{2.15}$$

and

$$B-L=-\tfrac{2}{3}\sum_\alpha H_\alpha\ , \tag{2.16}$$

respectively.

We now consider the possibility of associating $|\psi_{-(10)}\rangle$ with another family of particles. From (2.14) and (2.15), the particles in $|\psi_{-(10)}\rangle$ have opposite electroweak charge assignments from those of $|\psi_{+(10)}\rangle$. In order that the particles in $|\psi_{-(10)}\rangle$ give the usual $V-A$ weak interactions, they must therefore be made right-handed. A left-handed multiplet $|\psi_{-(10)}\rangle$ would produce $V+A$ weak interactions.[7] We shall refer to the latter as conjugate families.[8,11]

If we wish to incorporate more than one generation of particles into a single irreducible spinor representation of SO($2N$), we must take $N\geq 7$. The SO(12) group is ruled out for the following reason (cf. Ref. 16): Let $|\psi^+_{(12)}\rangle$ denote the 32-dimensional spinor representation of SO(12). It can be decomposed into SO(10) spinors according to

$$|\Psi_{+(12)}=|\psi_{+(10)}\rangle+a_6^\dagger|\psi_{-(10)}\rangle\ . \tag{2.17}$$

We require that all the particles in $|\Psi_{+(12)}\rangle$ are left-handed. This implies that $|\lambda_{-(10)}\rangle$ is associated with a conjugate family, which must be removed from the low-lying spectrum. (This can be done via a suitable mass matrix.) Consequently, we are left with just one (low-lying) family of particles in $|\Psi_{+(12)}\rangle$. On the other hand, an irreducible spinor representation of SO(14) contains two (low-lying) families. Here

$$|\Psi_{+(14)}\rangle=|\psi_{+(10)}\rangle+a_6^\dagger|\lambda_{-(10)}\rangle+a_7^\dagger|\eta_{-(10)}\rangle +a_6^\dagger a_7^\dagger|\chi_{+(10)}\rangle\ . \tag{2.18}$$

Once again, $|\lambda_{-(10)}\rangle$ and $|\eta_{-(10)}\rangle$ are associated with conjugate families which must be removed from the low-lying spectrum, leaving the two families $|\psi_{+(10)}\rangle$ and $|\chi_{+(10)}\rangle$. Here $|\psi_{+(10)}\rangle$ is given by (2.12) and $|\chi_{+(10)}\rangle$ is given by

$$\chi=\nu_{\mu L}^c\ ,$$
$$\chi^{\alpha\beta}=\epsilon^{\alpha\beta\gamma}c_{L\gamma}^c\ ,\quad \chi^{4\alpha}=c_{L\alpha}\ ,\quad \chi^{5\alpha}=s_{L\alpha}\ ,\quad \chi^{45}=\mu_L^+\ ,$$
$$\chi^{45\alpha\beta}=\epsilon^{\alpha\beta\gamma}s_{L\gamma}^c\ ,\quad \chi^{5123}=\mu_L^-\ ,\quad \chi^{1234}=\nu_{\mu L}\ . \tag{2.19}$$

In general, an irreducible spinor representation of SO($10+2M$) contains a total of 2^{M-1} (low-lying) families.

B. $n=2N+1$

To generalize the algebra associated with the SO($2N$) group to that associated with SO($2N+1$), we note from (2.9) that

$$\Gamma_0{}^2=1\ ,\quad \{\Gamma_0,\Gamma_A\}=0\ . \tag{2.20}$$

Thus with the inclusion of Γ_0, the rank-$2N$ Clifford algebra can be extended to a rank-($2N+1$) Clifford algebra. To generate the SO($2N+1$) group we simply let the indices of Σ_{AB} [cf. Eq. (2.5)] run from 0 to $2N$. The new generators which appear in the SO($2N+1$) group are

$$\Sigma_{2i,0}=2H_i\Gamma_0\Gamma_{2i-1} \tag{2.21}$$

and

$$\Sigma_{2i-1,0}=-2H_i\Gamma_0\Gamma_{2i}\ . \tag{2.22}$$

Since these generators mediate transitions between the two spinor representations $|\psi_{+(2N)}\rangle$ and $|\psi_{-(2N)}\rangle$ of SO(2N), $|\psi_{+(2N)}\rangle$ and $|\psi_{-(2N)}\rangle$, separately, do not form representations of SO($2N+1$). Instead their sum

$$|\psi_{(2N+1)}\rangle=|\psi_{+(2N)}\rangle+|\psi_{-(2N)}\rangle \tag{2.23}$$

gives an irreducible representation of the group. Thus, the two 2^{N-1}-dimensional representations of SO(2N) combine to form one 2^N-dimensional representation of SO($2N+1$).

The smallest SO($2N+1$) group which is applicable to grand unfied models is SO(11). The SO(11) spinor representation contains the two spinors $|\psi_{+(10)}\rangle$ and $|\psi_{-(10)}\rangle$ of SO(10). As was the case in SO(12), only one of the SO(10) spinors, $|\psi_{+(10)}\rangle$ (using the conventions of Sec. II A) can be identified with a (low-lying) family. The number of such families in SO(13) is two. They correspond to $|\psi_{+(10)}\rangle$ and $|\chi_{+(10)}\rangle$ in the expansion

$$|\Psi_{(13)}\rangle=|\psi_{+(10)}\rangle+|\psi_{-(10)}\rangle +a_6^\dagger|\chi_{-(10)}\rangle+a_6^\dagger|\chi_{+(10)}\rangle . \tag{2.24}$$

In general, we can identify a total of 2^M (low-lying) families within a spinor representation of SO($2M+11$).

III. GAUGE INTERACTIONS IN SO(n) MODELS

As in the previous section, we identify (low-lying) families with SO(10) spinors of a fixed chirality. In what follows, we shall assume, as a first approximation, that there is no induced mixing into the gauge sector via the fermion mass matrix.

The general expression for the fermion–gauge-field interaction in SO(n) grand unified theories is given by

$$g\langle\psi_{(n)}|\gamma^0\gamma^\mu A_\mu|\psi_{(n)}\rangle\ ,\quad A_\mu\equiv A_\mu^{AB}\Sigma_{AB}\ , \tag{3.1}$$

where γ^μ ($\mu=0-3$) spans the Clifford algebra associated with the Lorentz group, g is the gauge coupling constant, and A_μ^{AB} are the gauge-boson fields. For the case $n=10$, the terms in A_μ known to mediate proton decay are

$$X_\mu^\alpha\tau_4^\alpha\ ,\quad Y_\mu^\alpha\tau_5^\alpha\ ,\quad Y'^\alpha_\mu a_\alpha a_5\ ,\quad Z_\mu^\alpha a_\alpha a_4\ , \tag{3.2}$$

and their Hermitian conjugates. In particular, their interactions with the components of $|\psi_{+(10)}\rangle$ are given by

$$\begin{aligned}&gX_\mu^\alpha(-\epsilon_{\alpha\beta\gamma}\bar{u}_{L\beta}^c\gamma^\mu u_{L\gamma}+\bar{d}_{L\alpha}\gamma^\mu e_L^+-\overline{e_L^-}\gamma^\mu d_{L\alpha}^c)+gY_\mu^\alpha(-\epsilon_{\alpha\beta\gamma}\bar{u}_{L\beta}^c\gamma^\mu d_{L\gamma}+\bar{u}_{L\alpha}\gamma^\mu e_L^+-\bar{\nu}_{eL}\gamma^\mu d_{L\alpha}^c)\\&\quad+gY'^\alpha_\mu(\epsilon_{\alpha\beta\gamma}\bar{u}_{L\beta}\gamma^\mu d_{L\gamma}^c+\bar{u}_{L\alpha}^c\gamma^\mu e_L^-+\bar{\nu}_{eL}^c\gamma^\mu d_{L\alpha})+gZ_\mu^\alpha(-\epsilon_{\alpha\beta\gamma}\bar{d}_{L\beta}\gamma^\mu d_{L\gamma}^c-\bar{u}_{L\alpha}^c\gamma^\mu\nu_{eL}+\bar{\nu}_{eL}^c\gamma^\mu u_{L\alpha})+\text{H.c.}\ ,\end{aligned} \tag{3.3}$$

where we have used (2.12).

We now show that after enlarging to an SO(n), $n>10$, grand unified model, no additional gauge bosons appear which could mediate proton decay. We begin with the case $n=2N$.

A. $n=2N$

From the discussion in Sec. II A, the smallest relevant SO(2N) group is SO(14). The new generators obtained by enlarging the group from SO(10) to SO(14) are linear combinations of

$$\begin{gathered}H_6,\ H_7,\ a_6a_7,\ \tau_6^7\ ,\\ \tau_6^i,\ \tau_7^i,\ a_6a_i,\ a_7a_i\ ,\end{gathered} \tag{3.4}$$

and Hermitian conjugates, where i runs from 1 to 5. From (2.18), the 64-dimensional spinor of SO(14) contains two families $|\psi_{+(10)}\rangle$ and $|\chi_{+(10)}\rangle$ and two conjugate families $|\lambda_{-(10)}\rangle$ and $|\eta_{-(10)}\rangle$. The first four generators in (3.4) are SO(10) neutral, i.e., they commute with all the SO(10) generators. The first two mediate interactions within a given family or conjugate family, whereas a_6a_7 (τ_6^7), along with its Hermitian conjugate, is responsible for horizontal interactions between different families (conjugate families). The remaining generators mediate diagonal interactions between families and conjugate families. Clearly, none of the new generators are relevant for proton decay. Thus, with regards to proton decay, the gauge sector of the SO(14) model is identical to that of the SO(10) model.

Similar results hold for all SO(2N) theories. All generators not belonging to the SO(10) subgroup are associated with either "SO(10) neutral processes" or interactions which connect families to conjugate families.

B. $n = 2N+1$

From the discussion in Sec. II B, the smallest relevant SO$(2N+1)$ group is SO(13). The generators in SO(13) which are not in the SO(10) subgroup are linear combinations of

$$H_6,\ a_6,\ \tau_6^i,\ a_6 a_i\ , \tag{3.5}$$

and Hermitian conjugates. The first two generators in (3.5) are SO(10) neutral. The generator H_6 mediates interactions within families (and conjugate families), whereas a_6 and $a_6^\dagger$ produce horizontal interactions between different families (and conjugate families) [cf. Eq. (2.24)]. Once again, τ_6^i and $a_6 a_i$, along with their Hermitian conjugates, mediate diagonal transitions between families and conjugate families. Thus, again, we find that none of the new generators are relevant for proton decay. It is straightforward to reproduce these results for any SO$(2N+1)$ model.

In conclusion, the gauge sector for SO(n) theories containing more than one family in a single irreducible representation is identical to that of SO(10) theories as far as baryon decay is concerned.

Note that in carrying out a similar analysis for SU(n) family unification schemes, one encounters the following difficulty: Unlike in SO(10) models, an SU(5) family is associated with a reducible representation, namely 5^*+10, of the group. In an SU(n) model containing many 5^* and 10 representations of SU(5), there exist no criteria for determining which 5^* and which 10 combine to form an individual family. The question of whether baryonic gauge currents are vertical or diagonal depends crucially on the particular choice of combinations made.

IV. YUKAWA INTERACTIONS IN SO(n) MODELS

In most treatments of baryon decay in grand unified models the Yukawa sector is not considered to play a dominant role. This is due to the assumptions that (a) the gauge couplings are larger than the Yukawa couplings and (b) the masses of the Higgs boson and gauge bosons are of the same order of magnitude. (a) follows in an SU(5) **[SO(10)] theory when a single 5- (10-) dimensional** Higgs multiplet is used to give mass to both the fermions and W and Z bosons. Since neither assumption (a) nor (b) is required in the most general grand unified model, we feel that a study of Yukawa interactions leading to baryon decay may be relevant.

In what follows we shall repeat the analysis of the previous section for the Yukawa sector of SO(n), $n>10$, models. Unlike in the gauge sector, here we find that there exist diagonal interactions between families which lead to baryon decay. However, since it is possible to write down Yukawa interactions linking different irreducible spinor representations of the group, such interactions may already be incorporated in an SO(10) model. Essentially the only difference we find between the Yukawa sector of SO(10) and SO(n), $n>10$, models is that the arbitrariness of the relative coupling strengths of the different families in the former gets fixed in the latter. We again assume no additional induced mixing from the fermionic mass matrices.

The general expression for the Yukawa interaction Lagrangian in SO(n) models is[16,17]

$$\kappa^{(i)}\langle \psi^*_{(n)} \,|\, C^{-1}B_{(n)}\Phi^i \,|\, \psi_{(n)}\rangle + \text{H.c.}\ , \tag{4.1}$$

$$\Phi^i = \phi_{A_1A_2\cdots A_i}\Gamma_{A_1}\Gamma_{A_2}\cdots\Gamma_{A_i}\ ,$$

where $\kappa^{(i)}$ are the Yukawa couplings $\phi_{A_1A_2\cdots A_i}$ are scalar Higgs fields and C is the charge-conjugation operator associated with the Lorentz group. $B_{(n)}$ is the analogous charge-conjugation operator associated with the SO(n) group. It satisfies the relation[17]

$$\hat{\Sigma}^T_{AB}\hat{B}_{(n)} = -\hat{B}_{(n)}\hat{\Sigma}_{AB}\ ,\quad n=2N \text{ or } 2N+1\ , \tag{4.2}$$

where a caret indicates a $2^N\times 2^N$ matrix representation and T stands for transposition. Equation (4.2) is satisfied with the choice[16]

$$B_{(2N)} = B_{(2N+1)} = \Gamma_1\Gamma_3\cdots\Gamma_{2N-1}\ . \tag{4.3}$$

In theories where more than one irreducible spinor is present, we may include Yukawa interactions which link the different spinors. For instance, in an SO(10) model with two families, **corresponding to $|\psi_{+(10)}\rangle$ and $|\chi_{+(10)}\rangle$ [cf. Eqs. (2.12) and (2.19)], the following interactions are possible[18]:**

$$\kappa_{11}^{(i)}\langle \psi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Phi^i \,|\, \psi_{+(10)}\rangle + \kappa_{12}^{(i)}\langle \psi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Phi^i \,|\, \chi_{+(10)}\rangle$$
$$+\kappa_{21}^{(i)}\langle \chi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Phi^i \,|\, \psi_{+(10)}\rangle + \kappa_{22}^{(i)}\langle \chi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Phi^i \,|\, \chi_{+(10)}\rangle + \text{H.c.} \tag{4.4}$$

Higgs fields which give nonzero contributions to (4.4) are Φ^i, $i=1,3,5$, corresponding to 10-, 120-, and 126-dimensional representations, respectively. It is known that the corresponding matrices $K^{(i)}$,

$$K^{(i)}=\begin{bmatrix}\kappa_{11}^{(i)} & \kappa_{12}^{(i)}\\ \kappa_{21}^{(i)} & \kappa_{22}^{(i)}\end{bmatrix}, \tag{4.5}$$

are symmetric for $i=1,5$ and antisymmetric for $i=3$.

For the case $i=1$ the Higgs components which mediate proton decay are

$$\phi_1,\phi_2,\ldots,\phi_6\,. \tag{4.6}$$

Substituting into the first term in (4.4), we find the following interactions amongst the first family of particles:

$$2(\phi_{2\alpha-1}-i\phi_{2\alpha})(\bar{u}_{\alpha R}e_L^+-\bar{d}_{\alpha R}\nu_{eL}^c)-2(\phi_{2\alpha-1}+i\phi_{2\alpha})(\bar{u}_{\alpha R}^c e_L^- +\bar{d}_{\alpha R}^c\nu_{eL})$$
$$+2\epsilon_{\alpha\beta\gamma}(\phi_{2\alpha-1}+i\phi_{2\alpha})(\bar{u}_{\beta R}d_{\gamma L}^c+\bar{u}_{\beta R}^c d_{\gamma L})+\text{H.c.}\,, \tag{4.7}$$

where α, β, γ sum from 1 to 3. The second and third terms in (4.4) can contribute to processes (1.2) and (1.3). If Φ^1 alone mediates all Yukawa interactions in an SO(10) model, then, some restrictions on the constants $\kappa_{ij}^{(1)}$ can come from the fermion mass matrix. In particular, if we insist that the Cabibbo angle is calculable from the mass matrix, then $\kappa_{22}^{(1)}>\kappa_{12}^{(1)}$, $\kappa_{11}^{(1)}=0$ (cf. Appendix) which leads to a suppression of (1.3). In general, however, many Higgs multiplets may contribute to Yukawa interactions and no specific form of the mass matrix is required. Consequently, the $\kappa_{jk}^{(i)}$'s are completely arbitrary in SO(10) theories. We now show that this is not the case in SO(n), $n>10$, models.

A. $n=2N$

Let us concentrate on the SO(14) model where two families and two conjugate families are contained in the spinor $|\Psi_{+(14)}\rangle$ [cf. Eq. (2.18)]. The Yukawa interaction is given by

$$\kappa^{(i)}\langle\Psi_{+(14)}^*|C^{-1}B_{(14)}\Phi^i|\Psi_{+(14)}\rangle+\text{H.c.} \tag{4.8}$$

The Higgs fields which give nonzero contributions to (4.8) are Φ^3 and Φ^7 corresponding to 364- and 1716-dimensinal representations, respectively. [The Yukawa couplings involving Φ^1 and Φ^5 are antisymmetric under the exchange of different spinors $|\Psi_{+(14)}\rangle$ and $|\chi_{+(14)}\rangle$, and hence do not contribute to (4.8).] In Φ^3, six 10-dimensional representations and the 120-dimensional representation of SO(10) can be identified. They are associated with the components

$$\phi_{13,11,A},\ \phi_{14,11,A},\ \phi_{13,12,A},\ \phi_{14,12,A}\,,$$
$$\phi_{11,12,A},\ \phi_{13,14,A},\ \text{and}\ \phi_{ABC}\,, \tag{4.9}$$

where A, B, C run from 1 through 10. The first four sets of Higgs fields in (4.9) contribute only to processes amongst individual families (i.e., they do not mix families). For example, the first set leads to the interactions

$$\kappa^{(3)}\phi_{13,11,A}[\langle\psi_{+(10)}^*|C^{-1}B_{(10)}\Gamma_A|\psi_{+(10)}\rangle$$
$$+\langle\chi_{+(10)}^*|C^{-1}B_{(10)}\Gamma_A|\chi_{+(10)}\rangle]+\text{H.c.} \tag{4.10}$$

On the other hand, the last three sets of Higgs fields in (4.9) contribute solely to processes which mix families. For example, the last set leads to the interactions

$$\kappa^{(3)}\phi_{ABC}[\langle\psi_{+(10)}^*|C^{-1}B_{(10)}\Gamma_A\Gamma_B\Gamma_C|\chi_{+(10)}\rangle$$
$$-\langle\chi_{+(10)}^*|C^{-1}B_{(10)}\Gamma_A\Gamma_B\Gamma_C|\psi_{+(10)}\rangle]+\text{H.c.} \tag{4.11}$$

In addition to (4.10) and (4.11), we can have Yukawa interactions which link families to conjugate families. In Φ^3 such interactions are mediated by

$$\phi_{11,A,B},\ \phi_{12,A,B},\ \phi_{13,A,B},\ \text{and}\ \phi_{14,A,B} \tag{4.12}$$

corresponding to four 45-dimensional representations of SO(10). The first set in (4.12) leads to the interactions

$$i\kappa^{(3)}\phi_{11,A,B}[\langle\psi_{+(10)}^*|C^{-1}B_{(10)}\Gamma_A\Gamma_B|\eta_{-(10)}\rangle$$
$$+\langle\eta_{-(10)}^*|C^{-1}B_{(10)}\Gamma_A\Gamma_B|\psi_{+(10)}\rangle]+\text{H.c.} \tag{4.13}$$

Processes analogous to (4.11)–(4.13) can be found with regards to the Higgs multiplet Φ^7.

In conclusion, we can recover all the interactions (4.4) appearing in an SO(10) model [plus additional unwanted interactions (4.13)]. We note that (a) different SO(10) Higgs multiplets are associated with different Yukawa processes; namely, those which mix families, those which do not, and those which link families to conjugate families, and (b) the arbitrariness of the relative coupling strengths $\kappa_{ab}^{(i)}$ in the SO(10) model gets fixed for SO(14). We find that all interactions occur with equal strengths. Although the arbitrariness in relative coupling strengths is removed, a new arbitrariness in the relative masses of the different Higgs multiplets appear. Furthermore, mixing may occur between different Higgs components leading to processes like (1.3). Owing to the ambiguity in the Higgs mass matrix, no conclusions concerning the Yukawa contributions to processes (1.1)–(1.3) in an SO(14) theory can be drawn.

We believe the above qualitative conclusions can be drawn in all the SO(2N) models.

B. $n=2N+1$

We again specialize to the two-family unification group SO(13), but we believe our results are qualitatively the same for all SO($2N+1$) models. The Yukawa interaction is given by

$$\kappa^{(i)}\langle \Psi^*_{(13)} \,|\, C^{-1}B_{(13)}\Phi^i \,|\, \Psi_{(13)}\rangle + \text{H.c.} , \qquad (4.14)$$

where $B_{(13)}$ is given by (4.3) and $|\,\Psi_{(13)}\rangle$ contains the two families and two conjugate families of SO(13) [cf. Eq. (2.24)]. The Higgs fields which give nonzero contributions to (4.14) are Φ^2, Φ^3, and Φ^6. They correspond to 78-, 286-, and 1716-dimensional representations, respectively. [The Yukawa couplings involving all other Higgs multiplets are antisymmetric under the exchange of differerent spinors $|\,\Psi_{(13)}\rangle$ and $|\,\chi_{(13)}\rangle$, and hence do not contribute to (4.14).] Note that the same total number of Higgs fields contribute for both the SO(13) and SO(14) models. Furthermore, we shall show below that there are no qualitative differences between the Yukawa sectors of the two models.

In examining the Higgs multiplet Φ^2 we note that there exist three 10-dimensional representations of SO(10) corresponding to

$$\phi_{11,A},\ \phi_{12,A},\ \text{and}\ \phi_{0A},\quad A=1-10 . \qquad (4.15)$$

The first two sets of Higgs fields in (4.15) contribute solely to processes amongst individual families (i.e., they do not mix families). For example, the first set leads to the interactions

$$\kappa^{(2)}\phi_{11,A}[\langle \psi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Gamma_A \,|\, \psi_{+(10)}\rangle + \langle \chi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Gamma_A \,|\, \chi_{+(10)}\rangle] + \text{H.c.} \qquad (4.16)$$

The last set of Higgs fields in (4.15) contribute solely to processes which mix families, i.e.,

$$i\kappa^{(2)}\phi_{0A}[\langle \psi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Gamma_A \,|\, \chi_{+(10)}\rangle + \langle \chi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Gamma_A \,|\, \psi_{+(10)}\rangle] + \text{H.c.} \qquad (4.17)$$

The interactions (4.16) and (4.17) are similar to (4.10) and (4.11) of the SO(14) model. As in the SO(14) model, there exist additional Yukawa interactions linking families to conjugate families. These are mediated, for example, by the 45-dimensional multiplet ϕ_{AB}, $A,B=1-10$, belonging to Φ^2. Substituting into (4.14), we find these interactions to be given by

$$-i\kappa^{(2)}\phi_{AB}[\langle \psi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Gamma_A\Gamma_B \,|\, \chi_{-(10)}\rangle + \langle \chi^*_{-(10)} \,|\, C^{-1}B_{(10)}\Gamma_A\Gamma_B \,|\, \psi_{+(10)}\rangle + \langle \psi^*_{-(10)} \,|\, C^{-1}B_{(10)}\Gamma_A\Gamma_B \,|\, \chi_{+(10)}\rangle + \langle \chi^*_{+(10)} \,|\, C^{-1}B_{(10)}\Gamma_A\Gamma_B \,|\, \psi_{-(10)}\rangle] + \text{H.c.} , \qquad (4.18)$$

which is analogous to (4.13).

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APPENDIX

In this paper we have ignored the effect of induced mixing from the fermion mass matrix.[19] Here we give two examples where this effect is small.

In an SO(n), $n > 10$, model where all families are contained in a single irreducible representation of the group, the fermion mass matrix is symmetric. For simplicity we shall only be considering real mass matrices. Also we shall specialize to two-family unification schemes [i.e., SO(13) and SO(14)].

Example 1. Here we assume that the fermion mass matrices have the following form:

$$M_i = \begin{bmatrix} 0 & b_i \\ b_i & c_i \end{bmatrix}, \quad i = e,u,d , \tag{A1}$$

where M_e denotes the electron-muon mass matrix, M_u denotes the u-c quark mass matrix, etc. [In the SO(14) model, (A1) can be obtained from nonzero vacuum expectation values for Higgs components $\phi_{7,8,s}$, $\frac{1}{2}(\phi_{13,11,s} - i\phi_{13,12,s})$, $\phi_{11,12,s}$, $\phi_{13,14,s}$, $(\phi_{1,2,s} + \phi_{3,4,s} + \phi_{5,6,s})$, $s = 9,10$, belonging to the multiplet Φ^3.[20]] A realistic mass matrix for the neutrinos must contain Majorana contributions[21] and is consequently 4×4. We shall assume

$$M_\nu = \begin{bmatrix} 0 & M_u \\ M_u & kM_u \end{bmatrix}, \tag{A2}$$

where $k = m_u / m_{\nu_e}$ (m_u and m_{ν_e} are the u-quark and neutrino masses, respectively). (A1) and (A2) are diagonalized by real orthogonal matrices with mixing angles given by[22]

$$\begin{aligned} \vartheta_e &\cong b_e / c_e = \left[\frac{m_e}{m_\mu} \right]^{1/2}, \\ \vartheta_d &\cong b_d / c_d = \left[\frac{m_d}{m_s} \right]^{1/2}, \\ \vartheta_u &\cong b_u / c_u = \left[\frac{m_u}{m_c} \right]^{1/2}. \end{aligned} \tag{A3}$$

From the neutrino mass matrix (A2) it follows that $\vartheta_u = \vartheta_\nu$, where ϑ_ν denotes the mixing amongst the two light neutrinos. Consequently, all mixing angles are computed from the fermion masses and the induced mixing into gauge and Yukawa sector is small.

In an SO(10) model with only one set of Higgs scalars Φ^i contributing to the Yukawa interaction [cf. Eq. (4.4)], (A1) is obtained by setting $\kappa_{11}^i = 0$ in Eq. (4.4). Owing to the fact that the same vacuum expectation value gives mass to both e and μ, one obtains the relation

$$\frac{m_e}{m_\mu} \cong \left[\frac{\kappa_{12}^i}{\kappa_{22}^i} \right]^2 . \tag{A4}$$

It follows that $\kappa_{22}^i > \kappa_{12}^i$, and hence as mentioned in Sec. IV, processes (1.3) are suppressed in such a model. We also note that this model gives the mass relations

$$\frac{m_e}{m_\mu} = \frac{m_u}{m_c} = \frac{m_d}{m_s} = \frac{m_{\nu_e}}{m_{\nu_\mu}} . \tag{A5}$$

Example 2. We now assume a more general form for the mass matrix

$$M_i = \begin{bmatrix} a_i & b_i \\ b_i & c_i \end{bmatrix}, \quad i = e,u,d , \tag{A6}$$

with the following additional restrictions

$$M_d = M_e . \tag{A7}$$

[(A6) and (A7) are obtained in the SO(14) model from nonzero expectation values of the Higgs components $\phi_{13,11,s}$, $\phi_{13,12,s}$, $\phi_{11,12,s}$, $\phi_{13,14,s}$, $s = 9,10$.] Again M_ν is given by (A2). From (A7) and (A2) it follows that the mixing angles ϑ_i, $i = u,e,d,\nu$, fulfill the constraints

$$\vartheta_u = \vartheta_\nu , \quad \vartheta_d = \vartheta_e . \tag{A8}$$

Upon replacing the current eigenstates by the mass eigenstates in the gauge sector [cf. Eq. (3.3)], the only combination of mixing angles that appears is

$$\vartheta_d - \vartheta_u . \tag{A9}$$

(A9) is just the Cabibbo angle for weak interactions. Consequently, the effects of mixing in the gauge sector is small. However, in the Yukawa sector, in addition to (A9) the following combinations also appear:

$$\vartheta_d + \vartheta_u , \; 2\vartheta_d , \; 2\vartheta_u . \tag{A10}$$

This is due to the fact that for SO(n), $n > 10$, Yukawa interactions between different families are symmetric. Consequently, in this example there may be mixing effects in the Yukawa sector in addition to the diagonal interactions discussed in Sec. IV.

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[14]The treatment given here is a special case of the more general approach given by Ryan and Sudarshan in Ref. 12, where parafermi operators are utilized. In addition to the spinor representations, all representations are obtainable in the more general approach.

[15]Note, in general, we can set $\tau^i_j = a_i^\dagger a_j - \eta\delta_{ij}$, for any operator η which commutes with $a_i^\dagger a_j$ and still preserve the commutation relations (2.7). This is associated with the fact that $U(n) \approx SU(n) \times U(1)$, locally, and different choices for η correspond to different choices for the U(1) generator. The choice $\eta = \frac{1}{2}$ is particularly convenient since the average value of τ^i_i over any representation is zero. The corresponding expresssion for τ^i_j can be written $\tau^i_j = \frac{1}{2}[a_i^\dagger, a_j]$, and this form can readily be generalized to the parafermi case [cf. Ryan and Sudarshan (Ref. 12)].

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[22]Note that upon diagonalizing (A1) the resulting eigenvalues get opposite sign. This sign can be absorbed in a redefinition of the fermion fields; however, horizontal gauge interactions are not invariant under this redefinition.