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Spatial-temporal development of hadron-nucleus collisions

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In contrast to the usual S-matrix approach where only asymptotic reaction products are considered, we study the spatial-temporal development of hadronic reactions following Feinberg. Certain features of the development are generally applicable to a wide variety of reactions. The crucial aspect is the modification of the interactions which take place in rapid succession. The time scales involved make the effect practically unobservable except in nuclear hadronic cascades. After demonstrating the nature of the modifications in solvable models we apply it to hadronic cascades and obtain a good fit to the data on pion production in hadron-nucleus collision at incident projectile energies of 50, 100, and 200 GeV. This good fit is obtained without the use of any free parameters.

I. INTRODUCTION

The reaction rates in particle production and scattering processes are sufficiently great for us not to be able to follow their spatial-temporal development. Most of the time we are interested only in the asymptotic reaction products and an S-matrix formalism is more than adequate. On the other hand, theoretical models do give, in principle, detailed pictures of the time evolution. We have been concerned with time evolution of quantum-mechanical processes and, in particular, the short time deviations from the exponential time dependence in the decay of unstable systems. In general, these deviations all point to the system being in an "immature" nonequilibrium state for a characteristic time from the time of creation. In most cases we have looked at, this "Zeno effect" is not easily demonstrated by laboratory experiments. The reason is that characteristic times are such that the particles traverse only internuclear distances rather than interatomic distance during their "lapse time."

The situation is reminiscent of the study of the wavelengths of x rays. The wavelengths were so small that traditional ruled gratings could be used only with great difficulty and that too at almost grazing incidence. A suitable grating was, however, provided by the natural crystals. Following the same lead we are led to look to the nucleus as a theatre for Zeno to perform: We expect the effects of the nonequilibrium behavior at short times to affect the behavior of hadron-nucleus collisions. In our earlier phenomenological studies of hadron nucleus reactions we had in fact sought the spatial-temporal development of the particle production reaction. We generalize and extend these studies in this paper relating it to the Zeno effect.

The electromagnetic properties of a particle undergoing a sharp momentum change have been studied by Feinberg.[1] He has shown the following: (1) Immediately after such an interaction, the particle does not emit a photon of wave number k^0 for a time

$$T_{k^0} \simeq \frac{\gamma}{k^0} = \frac{1}{k^0}\,\frac{\epsilon_{p_2}}{m}\,,$$

where ϵ_{p_2} is the energy of the particle *after* collision. (2) Immediately after collision the state of the particle is "bare," i.e., $\phi_0 = a^\dagger(p_2)|0\rangle$ and it does not become "dressed" for a time T_k. (3) During this time if a second interaction took place then it does not emit bremsstrahlung photons directed along a cone about the intermediate momentum p_2. He has proved these result within the special framework of QED.

In this paper we would first investigate whether the same result holds true for other cases, for example the Lee model.[2] From it we can extract general features which are model independent. We also outline some models where the same result holds. We then make a connection with general properties of decay of an unstable particle.

II. LEE-MODEL CALCULATIONS

We restrict ourselves first to the case where the decay $V \to N\theta$ is energetically allowed. We use the standard notation of Ref. 3. The Hamiltonian is $H = H_0 + \delta H + H_{\rm int}$ with

$$H_0+\delta H=(m_V-\delta m_V)\mathfrak{N}^2\sum_p V^*(p)V(p)+m_N\sum_p N^*(p)N(p)+\sum_k \omega_k\theta^*(k)\theta(k)\,, \tag{1}$$

$$H_{\text{int}}=g\sum_{\kappa,p}\frac{f(k^2)}{\sqrt{2\omega_k}}[V^*(p)N(p-k)\theta(k)+\text{H. c.}]\,. \tag{2}$$

Introduce an external scattering center and also write H_I (in the configuration space) including an interaction with an external source θ^e

$$H_I+H_{\text{int}}+V(t)=g\int\int d^3x d^3y[V^*(\bar{y},t)N(\bar{y},t)\theta(\bar{y}-\bar{x},0)f(\bar{x},t)+\text{H. c.}]$$

$$+g\int\int d^3xd^3y[V^*(\bar{y},t)V(\bar{y},t)\theta^e(\bar{y}-\bar{x},0)f(\bar{x},t)+\text{H. c.}]\,. \tag{3}$$

At $t=-\infty$, start with a wave function of the V particle

$$|\phi_0\rangle\equiv|\phi(-\infty)\rangle=\int\rho(p_1)V^*(p_1)dp_1|0\rangle\,, \tag{4a}$$

where $\rho(p_1)$ is a Gaussian centered around q_1

$$\rho(p_1)=\frac{L}{\sqrt{\pi}}\exp[-L^2(p_1-q_1)^2]\,. \tag{4b}$$

Evolving this to time t gives us

$$|\phi(t)\rangle=T\exp\left[-i\int_{-\infty}^{t}H_I(t')dt'\right]|\phi(-\infty)\rangle\,. \tag{5}$$

In the expansion of this to second order in the interaction, we restrict our attention to the term containing H_{int} and v. We define

$$M(t)\equiv\langle 0|N(p_2-k)\theta(k)|\phi(t)\rangle$$

$$=-\left\langle 0\left|N(p_2-k)\theta(k)\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''T[H_{\text{int}}(t')V(t'')]\right|\phi_0\right\rangle$$

$$=-g^2\left\langle 0\left|N(p_2-k)\theta(k)\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''T\left[\int d^3x\int d^3y\,[V^*(y)N(y)\theta(y-x)f(x)\right.\right.\right.$$

$$+N^*(y)V(y)\theta^*(y-x)f^*(x)]_{t'}$$

$$\left.\left.\times\int d^3z\int d^3\omega[V^*(z)V(z)\theta^e(z-\omega)f(\omega)+\text{H. c.}]_{t''}\right]\right|\phi_0\right\rangle$$

$$=-g^2\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''\int dxdydzd\omega\,\langle N\theta,p_2-k,k|T[N^*(x)_{t'}V(x)_{t'}\theta^*(x-y)_{t'}V^*(z)_{t''}V(z)_{t''}]|V\rangle$$

$$\times\theta^e(z-\omega)f^*(y)f(\omega)|\Phi_0\rangle\,. \tag{6}$$

In a diagrammatic interpretation this looks like Fig. 1. A simple calculation yields

$$M=g^2\theta^e(p_2-p_1)f^*(k)f(p_2-p_1)\int_{-\infty}^{t}dt_1\int_{-\infty}^{t}dt_2\theta(t_1-t_2)R_2^-\exp[-i(\epsilon_{p_2}-\epsilon_{p_2-k}-k)t_1-i(\epsilon_{p_1}-\epsilon_{p_2})t_2]\,. \tag{7}$$

Here $\epsilon_p=(\bar{p}^2+m^2)^{1/2}$ is the energy. We notice that this agrees exactly with Eq. (1.10) in Ref. 1, with just a new expression for R_2^-

$$R_2^-=N^*(p_2-k)\frac{1}{p_{20}-m_V+i\epsilon}V(p_1)\,. \tag{8}$$

Since this is the only difference, we can directly calculate and get

$$M(t)=-g^2\theta^e(p_2-p_1)f^*(k)f(p_2-p_1)\,\frac{R_2^-\{1-\exp[-i(\epsilon_{p_2}-\epsilon_{p_2-k}-k-i\epsilon)t]\}}{\epsilon_{p_2}-\epsilon_{p_2-k}-k}\,\Delta_L(\epsilon_{q_1}-\epsilon_{p_2-k}-k)\,, \tag{9a}$$

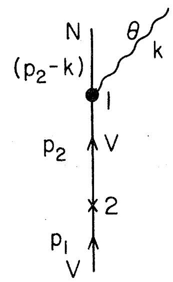

FIG. 1. Scattering of a V particle on an external source with the emission of a θ particle (bremsstrahlung).

with

$$\Delta_L(\xi)=\frac{Le^{-L^2\xi^2}}{\sqrt{\pi}}\,. \tag{9b}$$

For later convenience we define the cutoff time

$$T_k=(\epsilon_{p_2}-\epsilon_{p_2-k}-k)^{-1}\,. \tag{9c}$$

Hence we get a suppression factor similar to Feinberg's. Let $v_0=q_1/\epsilon_{q_1}$, $v=p/\epsilon_p$, $\bar{p}=\bar{p}_2-\bar{k}$; and θ_0 and θ the angles of the vector $\bar{k}$ with $\bar{q}_1$ and $\bar{p}$. Then one can easily derive the following formulas:

$$\epsilon_{q_1}-\epsilon_{q_1-k}-k=-\frac{2\epsilon_{q_1}k}{\epsilon_p+\epsilon_{q_1-k}}\,(1-v_0\cos\theta 0)\,, \tag{10a}$$

$$\epsilon_{p_2}-\epsilon_{p_2-k}-k=-\frac{2\epsilon_p k}{\epsilon_{q_1}+\epsilon_{p+k}}\,(1-v\cos\theta)\,, \tag{10b}$$

$$T_k=\frac{\epsilon_{q_1}+\epsilon_{p_2}}{2\epsilon_{p_2-k}k(1-v\cos\theta)}\,. \tag{10c}$$

We can simplify this further in two cases. If $p_2\ll m_v$, in the rest frame of p_2 one gets (K^0 being the momentum of the θ particle in the rest frame of v particle)

$$T_k\simeq\frac{1}{k}=\frac{1}{k^0}\,. \tag{11a}$$

If $p_2\gg m$, $|p_2-k|\gg m$, and $k_\perp\ll k_\parallel\lesssim p_2$ where $\bar{k}_\parallel$ and $\bar{k}_\perp$ are components of $\bar{k}$ parallel and perpendicular to $\bar{p}_2$, one gets

$$\begin{aligned}T_k&=\frac{2p_2k_\parallel(p_2-k_\parallel)}{M_V{}^2k_\parallel{}^2+k_\perp{}^2p_2{}^2+m_\theta{}^2p_2{}^2-m_\theta{}^2k_\parallel p_2}\\&=2\,\frac{\epsilon_{p_2}}{m_v}\,\frac{k_\parallel^0(1-k_\parallel^0/m_v)}{k_\parallel^{02}+k_\parallel^{02}+m_\theta{}^2-(m_\theta{}^2/m_v{}^2)k_\parallel^0}\\&\sim\frac{\gamma}{k^0+m_\theta{}^2/2k^0}\sim\frac{\gamma}{(k^{02}+m_\theta{}^2)^{1/2}}\,.\end{aligned} \tag{11b}$$

Now we look at the case where $V\to N\theta$ is energetically not allowed. In this case there is no $V\to N\theta$ decay except in between two scatterings (where V is virtual). Therefore, we will calculate $\phi(t)$ for the V particle coming from $t=-\infty$ and compare it to a "bare" V particle starting at $t=0$ and evolving ($t=0$ is the instant of the first scattering). We are interested in calculating (see Fig. 2)

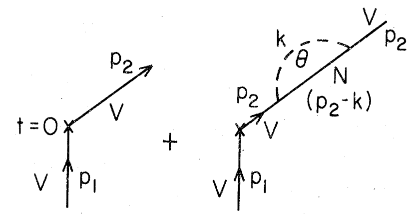

FIG. 2. Generation of a "meson cloud" around a "bare" V particle after scattering.

$$\Delta|\phi(t)\rangle=-\int_{-\infty}^{t}V(t')dt'|\phi_0\rangle+\int\int d^3kd^3p_2\,M(t)N^*(p_2-k)\theta^*(k)|0\rangle\,, \tag{12a}$$

with

$$M(t)=-\left\langle 0\left|N(p_2-k)\theta(k)\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''T[H_{\text{int}}(t')v(t'')]\right|\phi_0\right\rangle \tag{12b}$$

$$=g^2\theta^e(p_2-p_1)f^*(k)f(p_2-p_1)\,\frac{R_2^-\{\exp[-i(\epsilon_{p_2}-\epsilon_{p_2-k}-k)t]-1\}}{\epsilon_{p_2}-\epsilon_{p_2-k}-k}\,\Delta_L(\epsilon_{q_1}-\epsilon_{p_2-k}-k)\,, \tag{12c}$$

and comparing it with

$$|\tilde{\phi}(t)\rangle=\left(1-i\int_0^t H_{\text{int}}(t')dt'\right)V^*(p_2)|0\rangle\,, \tag{13}$$

which is the V particle created at $t=0$ and evolved to the time t. For convenience we define

$$E\equiv\{\exp[-i(\epsilon_{p_2}-\epsilon_{p_2-k}-k)t]-1\}/(\epsilon_{p_2}-\epsilon_{p_2-k}-k)\,. \tag{14}$$

$\tilde{\phi}(t)$ is just the part of Fig. 2 after the scattering and can be calculated to be

$$|\tilde{\phi}(t)\rangle = V^*(p_2)|0\rangle - ig\int d^3k N(p_2-k)\theta(k)\frac{1}{p_{20}-m_v+i\epsilon}\theta^*(k)f^*(k)N^*(p_2-k)V^*(p_2)|0\rangle E\,. \tag{15}$$

At this point we can notice the similarity between $\Delta|\phi(t)\rangle$ and $|\tilde{\phi}(t)\rangle$ which we can make precise by writing out the two terms in $\Delta|\phi(t)\rangle$ using the expression for R_2^- [Eq. (8)], and noticing that the two Δ_L functions that arise are almost equal. Hence the first term in $\Delta|\phi(t)\rangle$ becomes

$$-i\int_{-\infty}^{t}\nu(t')dt'|\phi_0\rangle = \int d^3p_2 M_0\Delta_L(\epsilon_{p_2}-\epsilon_{q_1})V^*(p_2)|0\rangle\,, \tag{16a}$$

$$M_0 = -ig\theta^e(p_2-p_1)f(p_2-p_1)V(p_2)V^*(p_1)\,. \tag{16b}$$

The second term becomes

$$\int d^3p_2[-ig\theta^e(p_2-p_1)f(p_2-p_1)V(p_2)V^*(p_1)]ig\int d^3kE\Delta_L(\epsilon_{q_1}-\epsilon_{p_2-k}-k)N(p_2-k)\theta(k)$$
$$\times\frac{1}{p_{20}-m_v+i\epsilon}N^*(p_2-k)\theta^*(k)V^*(p_2)|0\rangle\,. \tag{16c}$$

And so we get

$$\Delta|\phi(t)\rangle = \int d^3p_2 M_0|\tilde{\phi}(t)\rangle\Delta_L(\epsilon_{p_2}-\epsilon_q)\,. \tag{16d}$$

This looks as if the particle was "created bare" at $t=0$ and evolved into $|\tilde{\phi}(t)\rangle$. In fact, here we can state the result in the following manner:

Irrespective of whether the V particle is stable or unstable, the natural $V\to N\theta$ transition is depressed by a factor of

$$[1-\exp(it/T_k)]$$

at time t small compared to the characteristic time T_k and with the characteristic inverse transition rate g^2T_k. The time t is measured from the instant of the *last* reaction in which the V particle took part. It is as if the particle is "fatigued" by its last interaction and needs a time of the order of T_k to "recover."

So far we have found the suppression in production of particles between two inelastic collisions. Now we can take the particle defined by $|\tilde{\phi}(t)\rangle$ in Eq. (13) and let is scatter from a new scattering center located at a distance l_0 from the point of creation of the particle. The result can be obtained by calculating

$$|\phi'(t)\rangle = T\exp\left[-i\int_{t_1}^{t}V(t')dt'\right]|\tilde{\phi}(t_1)\rangle\,,$$

where the size of the wave packet L is much less than the distance between the two scattering centers l_0 and

$$0<L\ll t_1<t\,.$$

Then we can calculate the matrix element

$$M'(t) = \langle 0|V(p_3-k')\theta(k')|\phi'(t)\rangle\,,$$

which is the amplitude for an inelastic collision with the second scattering center. The calculations are similar to the previous ones. There are two processes, one where the θ is emitted after the second scattering. This is suppressed at time t by

$$1-\exp[-i(\epsilon_{p_3}-\epsilon_{p_3-k'}-k')t-\epsilon t]\,. \tag{17}$$

The other is where θ is emitted between $t=0$ and $t=t_0=l_0/v_{p_2}$ = time at which the second collision takes place. This is suppressed by

$$1-\exp[-i(\epsilon_{p_2}-\epsilon_{p_2-k'}-k')t_0-\epsilon t_0]\,. \tag{18}$$

This can be interpreted by saying that, in two rapid successive scatterings, the probability of the second scattering is reduced because the particle does not get enough time to become "dressed" and so does not interact fully with the second scattering center.

We can again define, similar to Eq. (9c)

$$T_k = (\epsilon_{p_2}-\epsilon_{p_2-k'}-k')^{-1}\,, \tag{19a}$$

and simplify it in the same way to

$$T_k \simeq \frac{1}{k^0}\text{ for }p_2\ll m_v \tag{19b}$$

and

$$T_k \simeq \frac{\epsilon_{p_2}}{mv}\frac{1}{(k'^{02}+m_\theta{}^2)^{1/2}}\,. \tag{19c}$$

At this point, we can see that our result is quite general. Consider a particle created at time $t=-\infty$, and evolved to time t by Eqs. (4a), (4b), and (5).

If we use this state as the initial state in a scattering process happening at time t, we get

$$S_{fi}=\langle f|S|i\rangle=\langle f|S|\phi(t)\rangle\rangle, \tag{20}$$

where S is the scattering matrix and $|f\rangle$ is any final state. But instead of this if we create the particle at $t=0$ and evolve it to $t>0$, i.e., if we use $|\tilde{\phi}(t)\rangle\rangle$ in Eq. (13) as our initial state, then we get

$$S_{fi'}=\langle f|S|\tilde{\phi}(t)\rangle\rangle . \tag{21}$$

From the results we have got within the approximation that t is much larger than the length of the wave packet L, it is clear that the only difference between (20) and (21) is a factor of

$$a(t)=\left[1-\exp\left(-i\frac{t}{T_k}\right)\right], \tag{22}$$

where T_k is given by Eq. (19a) for a fixed value of k. Hence the probability for the second reaction is suppressed by a factor $|a(t)|^2$. If a range of values of k is allowed, one has to average over it to give the actual suppression factor. (This we do when we apply it to hadron-nucleus collisions.) The suppressing factor can therefore be seen to be independent of the specific nature of the collision, and is just a kinematical factor.

Essentially the same kind of suppression factor arises in the amplitude for a process initiated by a particle created at time t_1 interacting at time t_2. We have employed these kinematic factors in the explicit calculation (see below).

III. OTHER INTERESTING MODELS

While we have used the Lee model to fix our ideals and to be in a position to solve the model exactly, to the extent that the number of channels is finite, any model of interactions will lead to the same kind of Zeno effect. (We can trace this result back to Dirac's "variation of constants" method of doing time-dependent perturbation theory.) It must also be clear from the method of derivation that there is no reason for the two successive interactions to be the same or different; in either case we find the reduced tendency to interact immediately after an interaction.

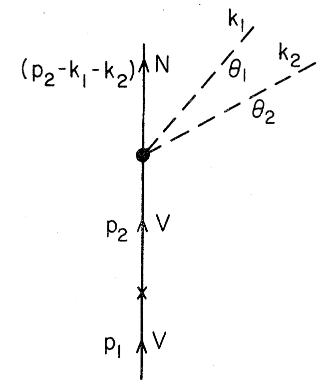

FIG. 3. Bremsstrahlung of two species of θ particles.

In order to apply the same considerations to the decays of the kind $n\to pe\nu$, we can look at a prototype Lee model with two species of particles θ_1 and θ_2 coupled to the V particle with $V\to N\theta_1\theta_2$ allowed energetically, and with $m_{\theta_1}\neq m_{\theta_2}$.

In this case the Hamiltonian will be H_0+H_I with

$$H_I=\lambda\int\frac{dkf(k^2)}{\sqrt{2\omega_k}}\int dp[V^*(p)N(p-k)\theta_1(k_1)\theta_2(k_2)+N^*(p-k)\theta_2^*(k_2)\theta_1^*(k_1)V(p)] \tag{23a}$$

(where $k=k_1+k_2$) or in configuration space

$$H_I=\lambda\int dxdydz[f(x-y)f(x-z)V^*(x)N(x)\times a_1(y)a_2(z)+\text{H. c.}]. \tag{23b}$$

For seeing the essential result (and also with a view to possible application to weak interactions), one might choose a point interaction. It can be easily generalized. So we choose

$$H_I=\lambda f(x)V^*(x)N(x)a_1(x)a_2(x). \tag{23c}$$

Now we introduce an external scatterer

$$H_I^e=\lambda\int f(k)V^*(p-k)V(p)a^3(k). \tag{23d}$$

Going through the same procedure as before, one finds (see Fig. 3):

$$M=g^2\theta^e(p_2-p_1)f^*(k)f(p_2-p_1)\times\int_{-\infty}^{t}dt_1\int_{-\infty}^{t}dt_2\theta(t_1-t_2)R_2^-\exp[-i(\epsilon_{p_2}-\epsilon_{p_2-k_1}-k_1-k_2)t_1-(\epsilon_{p_1}-\epsilon_{p_2})t_2]. \tag{24}$$

Note that again we get an expression similar to (9), but with $k=k_1+k_2$.

The final answer after doing the integrations is

$$M=ig^2\theta^e(p_2-p_1)R_2^-\frac{1-\exp(-i\Delta Et)}{\Delta E}\Delta_L(E), \tag{25a}$$

with

$$\Delta E = \epsilon_{p_2} - \epsilon_{p_2-k_1-k_2} - k_1 - k_2 \qquad (25b)$$

and

$$E = \epsilon_{q_1} - \epsilon_{(p_2-k_1-k_2)} - k_1 - k_2 \,. \qquad (25c)$$

This gives

$$T_k = \frac{1}{\epsilon_{p_2} - \epsilon_{p_2-k_1-k_2} - k_1 - k_2} \,. \qquad (26a)$$

Intuitively we realize that this will give suppression at both ends of the β-decay energy spectrum. Again in the same limit as Sec. II we get

$$T_k = \frac{\gamma}{\epsilon_{k_1} + \epsilon_{k_2}} = \frac{1}{\epsilon_{k_1} + \epsilon_{k_2}} \frac{\epsilon_{p_2}}{M_v} \,. \qquad (26b)$$

IV. RELATION TO ZENO'S PARADOX IN QUANTUM MECHANICS

Start with a state $|M\rangle$ at $t=0$ and let is evolve. The "nondecay amplitude" at t is given by

$$a(t) = \langle M | e^{-iHt} | M\rangle \,. \qquad (27)$$

Let $Q(t) = |a(t)|^2$ and $R(z) = (H - zI)^{-1}$ = resolvent of H, the Hamiltonian. Thus we can write

$$a(t) = \frac{1}{2\pi i} \int_C e^{-izt} \beta(z) dz$$

$$= \frac{1}{2\pi i} \int_C \frac{e^{-izt}}{\gamma(z)}\, dz \,. \qquad (28)$$

But for the Lee model we know from Ref. 2 [formula (7.6)] that

$$\gamma(z) = z - M_V + \frac{1}{\pi} \int_0^\infty \frac{|f(\lambda)|^2}{\lambda - z}\, d\lambda \,, \qquad (29)$$

where M_V = mass of "bare" V.

We immediately notice that this is identical to Eq. (25) of Ref. 4. But for this case it has been proven in Ref. 4 that

$$\dot{Q}(t) \sim -|t|^{1/2} \text{ as } t \to 0 \,. \qquad (30)$$

In the case of the Lee model, the quantity denoted by $-\dot{Q}(t)$ is the decay rate to *all* the various energies possible. As such it is an integral of the quantity $g^2 f^*(k) f(p_2 - p_1) M$ which had the characteristic $[1 - \exp(-it/T_k)]$ suppression at short times.

The Zeno effect is thus the composite of two effects. First, we have the kinematic suppression effect due to the integral of $\exp(it/T_k)$ expressing the interaction-picture phase of the daughter state with respect to time. This is the quantity we are primarily interested in and which we apply to hadron-nucleus collisions in the next section. Folded onto this effect is a sum over all configurations of the daughter states—this in turn leads to the $-\alpha|t|^{\beta-1}$ behavior of the decay amplitude. Hence, to the extent we demonstrate the "immaturity" and "fatigue" of interacting particles we are indirectly verifying the Zeno effect also.

V. APPLICATIONS TO HIGH-ENERGY HADRON-NUCLEUS COLLISIONS[5,6]

We have developed elsewhere[7] a new phenomenological space-time model for hadron-nucleus collisions. We shall refer to it as the BCT model. We note that the phenomenon of "maturity" proposed there follows naturally from our calculations here. We calculate $dn/d\eta$ and R and show that they agree closely with the data at three energies (50, 100, and 200 GeV). This gives a simple theoretical basis for the BCT model.

We start with Eq. (9), but recognize that we can now consider the production of an arbitrary number of pions. The value of T_k will, of course, depend upon the specific reaction channel.

We have, from (9), the suppression factor for a given total k (compare Sec. III):

$$a(t) = 1 - \exp(-it/T_k) \,, \qquad (31a)$$

with

$$\omega^{-1} = T_k = \frac{\gamma}{(k^2 + m_\pi^2)^{1/2}} = \frac{\gamma}{E} \,. \qquad (31b)$$

We have to average this suppression over the range of energies of the pions produced. The effect of this averaging is similar to that of a "fuzzy" edge in an interference pattern from a slit. It damps out the oscillations in the factor $a(t)$ beyond the first one. Only the central maximum and the first dip survive.

Various phenomenological analyses[8,9] indicate that the pions produced in the collision are bunched in clusters which eventually break up into three pions on the average. Let the rapidity of the incident particle be Y, and the average rapidity of the pion in a given cluster be y. We will assume[9] that the cluster has a typical range of ~1 unit of rapidity or the energies of pions in the c.m. of the cluster are spread over a range of $\Delta' \sim 0.2$ GeV. We have varied the value of Δ' over a wide range (0.1 to 0.6 GeV) and found that the final result is very insensitive to it. In our final calculations we have used the value $\Delta' = 0.2$ GeV. We average $a(t)$ over this range to get

$$a(t) = 1 - \exp(-i\omega_0 t)\exp(-\Delta' t/\gamma) \,, \qquad (32a)$$

where $\omega_0 = E_\pi/\gamma$ = energy of the pion in the rest frame of the cluster divided by the dilation factor γ in going from this frame to the lab frame. Hence

$$\omega_0 = \frac{m_{\Pi T}}{\cosh(y)} \,, \qquad (32b)$$

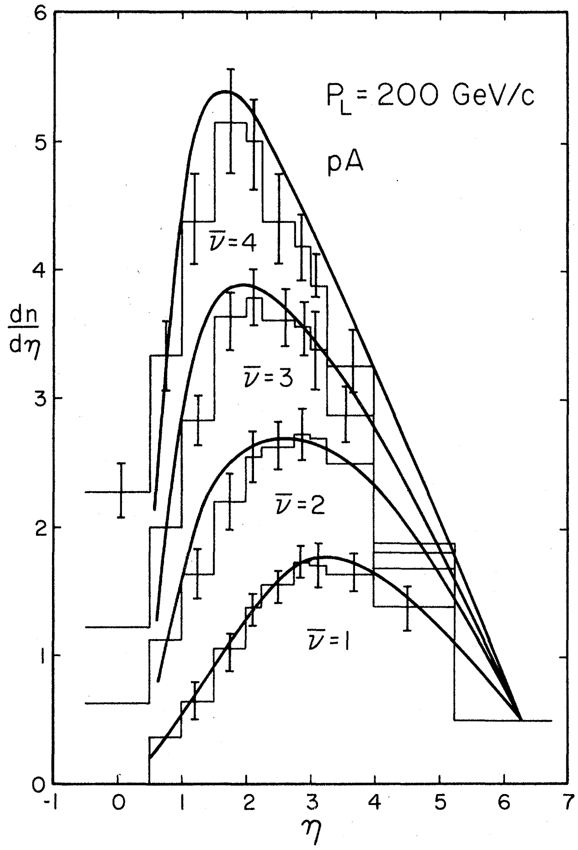

FIG. 4. Variation of $dn/d\eta$ as function of η and $\bar{\nu}_p$ for collision at 200 GeV. The $\bar{\nu}_p = 1$ curve is the input. Histograms are the data of Ref. 12.

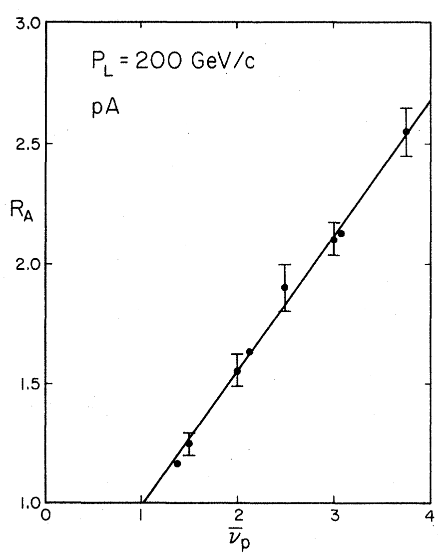

FIG. 5. Variation of the integrated multiplicity R_A with $\bar{\nu}_p$ for p-A collision at 200 GeV. Data points are from Ref. 12.

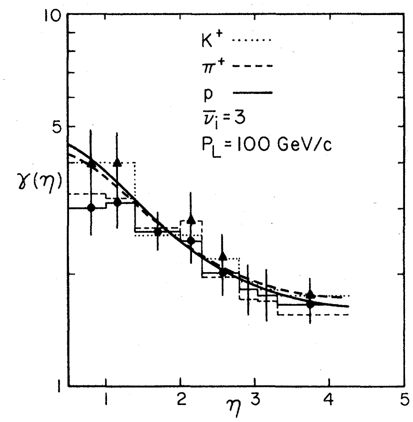

FIG. 6. The multiplicity ratio $\gamma(\eta) = (dn/d\eta)_{\bar{\nu}}/(dn/d\eta)_{\bar{\nu}=1}$ as a function of η at 100 GeV. The data points are from Ref. 13.

$$m_{\Pi T} = (m_\Pi{}^2 + p_T{}^2)^{1/2}. \tag{32c}$$

We define

$$\Delta'/\gamma = \Delta \simeq \frac{0.2\ \mathrm{GeV}/c^2}{\cosh(y)}. \tag{32d}$$

These give the suppression factor

$$Q(t) = 2\cos(\omega_0 t)\exp(-\Delta t) - \exp(-2\Delta t). \tag{33}$$

The time t between the successive collisions depends on the mean free path of the nucleon. We make the simple assumption that probability that the incident particle does not have a collision up to a distance x and then has one between x and $x + \delta x$ is given by

$$\delta(x)\,\frac{1}{d_0}\,\exp\left(-\frac{x}{d_0}\right),$$

where d_0 is the mean free path of the incident particle given approximately by $1.3/m_\Pi$. Hence, we average the suppression factor over the above distribution to get

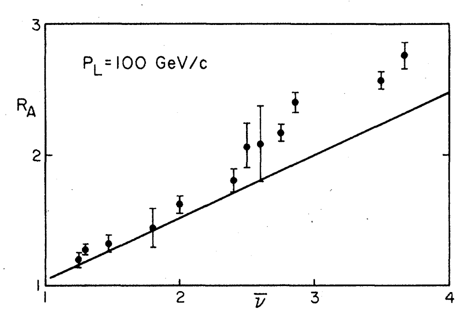

FIG. 7. R_A at $\bar{\nu}$ at 100 GeV. Data points are from Ref. 13 (see Ref. 14).

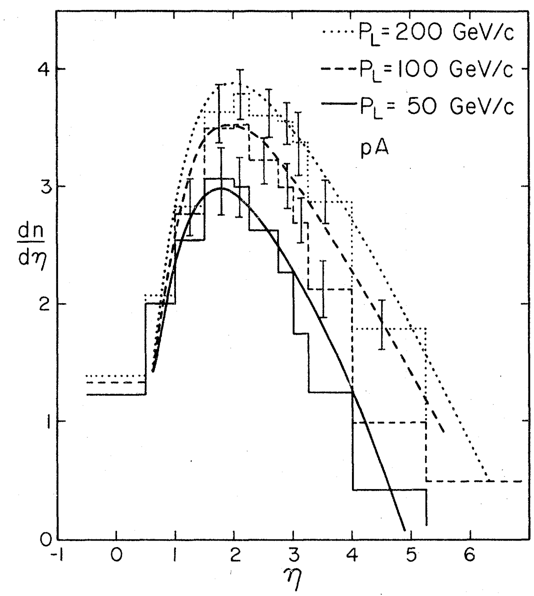

FIG. 8. Variation of $dn/d\eta$ as a function of η at three energies 50, 100, and 200 GeV. The data points are from Ref. 12.

$$Q=\frac{1}{d_0}\int_0^\infty dx \exp\left(-\frac{x}{d_0}\right)[2\cos(\omega_0 t)\exp(-\Delta t) - \exp(-2\Delta t)]\,. \quad (34a)$$

Since the time t to travel a distance x at rapidity Y is given by $x/\tanh(Y)$, we write $\omega t = Fx$, and $\Delta t = Gx$ with

$$F=\frac{m_{\Pi T}}{\cosh(y)\tanh(Y)}\,, \quad (34b)$$

$$G \simeq \frac{0.2}{\cosh(y)\tanh(Y)}\,, \quad (34c)$$

Equations (34a), (34b), and (34c) give

$$Q=\frac{1}{d_0}\left\{2\left(\frac{1}{d_0}+\frac{G}{2}\right)\Big/\left[\left(\frac{1}{d_0}+\frac{G}{2}\right)^2+F^2\right]-\frac{1}{1/d_0+G}\right\}\,. \quad (35)$$

At any fixed value of y, there is a whole range of values of the transverse momentum possible. Thus we have to further average Q over the allowed values of $m_{\Pi T}$. The transverse-momentum distributions are used from the experimental data of Ref. 10, which fit well to a distribution $b^2\exp(-bp_T)$ with $b \simeq 5.9$ giving the average value of $\langle p_T\rangle \simeq 0.35$ GeV/c. We calculate the average over this distribution numerically.

Finally, we have to average this over the spectrum of pions produced. Following our previous work[7] we assume that the distribution of pions

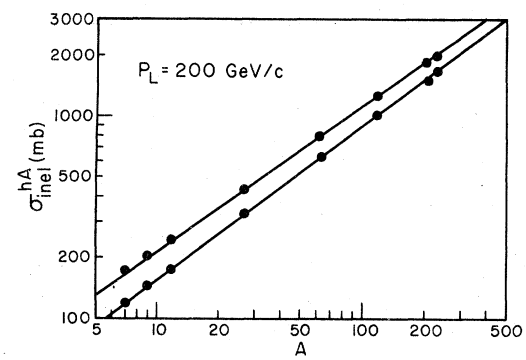

FIG. 9. Variation of $\sigma^{hA}_{\text{inel}}$ and $\sigma^{\Pi A}_{\text{inel}}$ as functions of A. Data points from Ref. 15. Solid lines are our curves.

produced is given by

$$V_M(y,Y)=c_0(1-e^{-y})^2 \quad \text{for } y \leq Y/2 \quad (36a)$$

$$= c_0(1-e^{-(Y-y)})^2 \quad \text{for } y > Y/2\,, \quad (36b)$$

$c_0 \simeq 1.87$.

This gives the average suppression factor

$$Q=\int_0^Y V_M(y,Y)Q(y,Y)dy \Big/ \int_0^Y V_M(y,Y)dz\,. \quad (37)$$

This is the "immaturity" factor we will use in the calculation. The calculation proceeds similar to the previous one (Ref. 7) with the new immaturity factor Q given above. The inelastic collision probabilities used are $P_{p(\bar{p})} \simeq 0.60$ and $P_\Pi \simeq 0.49$ based on newer data of Ref. 11. (These are slightly different from those used in the previous calculation.) We calculate the multiplicity distribution $dn/d\eta$ as a function of pseudorapidity η and the variable $\bar{\nu}$ defined as $\bar{\nu}=A\sigma^{hN}_{\text{inel}}/\sigma^{hA}_{\text{inel}}$. We also calculate the integrated multiplicity R_A as a function of $\bar{\nu}$. Our results are presented in Figs. 4 through 9. The predictions of our model agree well with the experimental data.

Our model gives basically a zero-parameter fit to the experimental data since we have not used the concept of "induced maturity" at all, and so do not have the single parameter λ used in Ref. 7.

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[14]The fits given in Ref. 12 for R_A are parametrized as $R_A = a + b\bar{\nu}_p$ with $a = 0.51$ and $b = 0.49$ at 50 GeV/c, $a = 0.45$ and $b = 0.53$ at 100 GeV/c, and $a = 0.39$ and $b = 0.63$ at 200 GeV/c. The fit at 100 GeV/c given in Ref. 13 is $R_A = 0.47 + 0.61\bar{\nu}$ which is not the same as given above in Ref. 12. Note that our predictions are $a = 0.54$ and $b = 0.46$ for 50 GeV/c, $a = 0.50$ and $b = 0.50$ for 100 GeV/c, and $a = 0.43$ and $b = 0.59$ for 200 GeV/c, which fit correctly the values given in Ref. 12. Since R_A is expected to increase slowly with incident energy, we believe our predictions fit the data correctly.
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