PHYSICAL REVIEW D VOLUME 20, NUMBER 12 15 DECEMBER 1979

Interaction between classical and quantum systems: A new approach to quantum measurement. III. Illustration

S. R. Gautam, T. N. Sherry, and E. C. G. Sudarshan

Center for Particle Theory, Department of Physics, The University of Texas at Austin, Austin, Texas 78712

(Received 31 July 1978)

Following the approach to quantum measurement, proposed in an earlier paper, in which the apparatus is treated as a purely classical instrument to be described by the laws of classical mechanics we examine some simple experiments for the purpose of illustration. The Stern-Gerlach experiment is examined in detail, within our approach, and we see explicitly how the relevant information is transferred to the apparatus variables. We also examine a magnetic interferometer arrangement and a crossed-field arrangement. Consideration of these experiments leads us to the inclusion of an extra procedural step in our approach. This extra step is necessary so that an observer may measure the classical apparatus variables at will after the interaction is completed.

I. INTRODUCTION

In our previous papers[1-3] we introduced a new approach to the treatment of measurement in quantum theory. The unusual feature of this approach is to treat the apparatus as a classical system, truly described by classical mechanics. The system to be observed is, however, a quantum system. Thus our approach requires that we couple together classical and quantum systems.

The proposal, which we forwarded in Ref. 2, is an attempt to construct an alternative theory of the quantum measurement process. It does not yet have the status of a theory, as its logical consistency is yet to be demonstrated. Once that is accomplished, then a decision as to which theory is correct can only be made on the basis of agreement with experiment.

We shall refer to Refs. 2 and 3 as I and II, respectively. In I we concentrated on setting up the formalism and introducing the tools necessary in the detailed description of the approach. The formalism introduced was one which allows the direct interaction of a classical and quantum system. In II we examined the effect of placing certain restrictions on the coupling; in particular, the requirement that the apparatus observables retain their classical integrity, despite interacting with the quantum system.

In this paper we focus our attention on the measurement aspect of the model: Exactly how does our model implement the measurement process? In I we suggested a necessary requirement for a measurement: Unambiguous information on the values of quantum system observables should be transferred to the observables of the apparatus. Of course, this transfer is to occur as a direct result of the interaction between the apparatus and the quantum system. We must, furthermore, address the problem of how the observer is to "read" the apparatus, after the interaction has occurred. Does the principle of integrity guarantee that the apparatus will be classical from the point of view of an outside observer? Another interesting question which must be posed is whether the outside observer will interact with the quantum-enlarged apparatus system, or with the conventional classical system, and in fact whether it makes any difference which happens.

The approach we take in this paper is one of illustration. We pursue the theoretical description of an idealized experiment, which is related to the Stern-Gerlach experiment.[4] We have previously shown in II that such an experiment can be described by our model so that the principle of integrity is satisfied. In this paper we wish to see how a measurement results.

We also discuss two variants of the "Stern-Gerlach (SG) experiment." The first is a magnetic interferometer,[5,6] where magnetic fields are introduced to recombine the split beams of the original experiment. Although there is no measurement resulting from this experiment, as the recombination undoes the effect of the first SG interaction, this is an interesting case to examine. It demonstrates that in our formalism a unique classical state can evolve in time into an effective mixture state, and back into a unique classical state. "Effective mixture state" means that from the viewpoint of the observable sector of the theory the state is a mixture state.

The second variant is a "crossed-field" experiment where two magnetic fields, with field gradients orthogonal, are used in place of the original field.

Before we discuss these example we review briefly the formalism of our approach in Sec. II. We introduce the model, with which we illustrate our ideas, in Sec. III. Sections IV and V are given over to a discussion of the experiments listed above. In Sec. VI we shall address the problem of the observer's interaction with the apparatus, and we will suggest a way in which it can be understood.

II. REVIEW OF THE APPROACH

In this section we review our approach to quantum measurement as developed in I and II. We first review the results of I. The main thrust of that paper was in setting up a formalism which would allow the direct interaction of a classical system with a quantum system. The main tool used was, in effect, a new way to view classical systems. It was described how one could envisage a classical system embedded in a much larger quantum structure (insofar as the dynamical variables are noncommuting); and yet the observable part of this larger quantum-mechanical system would mimic exactly in its behavior the original classical system.

Let us denote the dynamical variables of the simple classical system by $(q_1, \ldots, q_n)$ and $(p_1, \ldots, p_n)$. The Hamiltonian is a function of these phase-space points, $H(q,p)$. Their development in time is given by Hamilton's equations

$$\dot{q}_i = \frac{\partial H(q,p)}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H(q,p)}{\partial q_i}. \tag{2.1}$$

Specifying initial conditions $q_i(t=0)=q_i^0$ and $p_i(t=0)=p_i^0$ then determines fully the future time development of the system.

We can describe the quantum system in which we find the above classical system embedded as follows. This description involves the following:

(1) We use operators acting on a state space as the dynamical variables; for this quantum system we write them as

$$\omega = \{\omega^1, \ldots, \omega^{2n}\} \equiv \{q_1, \ldots, q_n; p_1, \ldots, q_n\}. \tag{2.2}$$

(2) We introduce operators conjugate to the ω^μ with respect to commutation, i.e., operators π^ν such that

$$[\omega^\mu, \pi^\nu] = \omega^\mu \pi^\nu - \pi^\nu \omega^\mu = i\delta^{\mu\nu}. \tag{2.3}$$

A representation of the π operators is

$$\pi^\nu = -i\frac{\partial}{\partial \omega^\nu}. \tag{2.4}$$

(3) The Hamiltonian operator is

$$\mathcal{H} = -\sum_j \left[\frac{\partial H(q,p)}{\partial q_j}\pi_j^p - \frac{\partial H(q,p)}{\partial p_j}\pi_j^q\right] \equiv \frac{\partial H(\omega)}{\partial \omega^\nu}\epsilon^{\mu\nu}\pi^\mu. \tag{2.5}$$

The Heisenberg picture is used to discuss the time development of the system. Using the Hamiltonian operator defined in (2.5), the equations of motion of the ω^μ operators

$$\dot{\omega}^\mu(t) = -i[\omega^\mu(t), \mathcal{H}] \tag{2.6}$$

mimic exactly the classical equations (2.1).

(4) The analog of the initial values of the usual classical description is the specification of the state of the quantum system in the Heisenberg picture. This is made more precise by the following choice of state:

$$|\psi\rangle = |\omega_0\rangle = |q^0, p^0\rangle, \tag{2.7}$$

where the initial values seen earlier are the eigenvalues of the ω^μ on this state.

(5) We distinguish between the observable and unobservable parts of the system by invoking a superselection principle: The set of operators $\{\omega^1, \ldots, \omega^{2n}\}$ are superselecting operators. The immediate consequences of this principle are that the conjugate π^ν operators are unobservable, and that the algebra of observables generated by the ω^μ is commutative. We note here that our use of superselection does not follow the conventional usage.[7,8] For example, the Hamiltonian (2.5) is not an observable. As a result no superselection rule applies. This usage of superselection has been discussed in I in more detail.

The observable sector of the resulting quantum theory exactly mimics the simple classical system first discussed. Our proposal was to use this model to couple together a classical apparatus and a quantum system. First we construct the "quantum-enlarged" apparatus system, in the manner described above. The enlarged system is then coupled to the quantum system under investigation.

Let us denote the quantum variables by $\{\xi\}$, and the undisturbed quantum Hamiltonian by $X(\eta)$. Then the Hamiltonian operator for the coupled "enlarged apparatus" and quantum system is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}, \tag{2.8}$$

where

$$\mathcal{H}_0 = \frac{\partial H(\omega)}{\partial \omega^\nu}\epsilon^{\mu\nu}\pi^\mu + X(\eta) \tag{2.9a}$$

and

$$\mathcal{H}_{\text{int}} = \phi(\omega, \pi; \xi; t). \tag{2.9b}$$

We should point out that we are restricting our attention, at this stage of the program, to closed systems about which we have maximum knowledge allowed by theory. Our quantum-mechanical systems will then be described by elementary quantum mechanics so that, for example, the time development will be effected by a unitary transformation, and state vectors rather than density matrices are employed.

Finally in I we addressed the question as to what restrictions should be placed on the interaction Hamiltonian (2.9b). There were two general requirements:

(1) A measurement is achieved if unambiguous information concerning the values of certain variables of the quantum system being examined can be "stored" in the variables of the classical apparatus;

(2) after the interaction has occurred the apparatus must be "classical" in some sense.

The second requirement is rather vague as stated. As we saw in I, the classical nature of the state of the apparatus is not retained when arbitrary interactions with quantum systems are envisaged. However, the remaining classical property is a statement about the classical observables, which form a commuting set: The $\omega^\mu(t)$ are observable for all times t. We proposed that the requirement (2) above be applied to this property of the apparatus system. This proposal was formulated in both weak and strong forms as follows:

Weak form: After the interaction has ceased, the apparatus observables should retain their classical integrity. While the interaction is taking place no such restriction is enforced.

Strong form: The apparatus observables should retain their classical integrity at all times.

We called either requirement the "principle of integrity." Requiring that the interactions satisfy this principle weakly is the weakest requirement which we can impose if we wish the apparatus to be "classical," in any sense, after interacting with the quantum system.

We note that for the uncoupled classical system this property is automatically satisfied because the Hamiltonian (2.5) is at most linear in the unobservable π^μ. For a general time-independent interaction

$$\mathcal{H}_{\text{int}} = \Phi(\omega, \pi; \xi), \tag{2.10}$$

this result no longer follows. Clearly, if the coupling function Φ is quadratic (or higher) in the unobservables π^μ the apparatus variables $\omega^\mu(t)$ will not be observables for times $t>0$. If Φ is linear in π^μ it may occur that the principle of integrity is satisfied even in the presence of some interactions of the form

$$\mathcal{H}_{\text{int}} = \phi^\mu(\omega; \eta')\pi^\mu + h(\omega; \hat{\epsilon}), \tag{2.11}$$

where $\{\eta'\}$ and $\{\hat{\epsilon}\}$ are subsets of the quantum variables. This form, however, is not sufficient to guarantee that the apparatus observables retain their classical integrity. Both the primary coupling functions ϕ^μ and the secondary coupling function h depend on unspecified quantum variables.

In II we set out to examine what constraints the strong form of the principle of integrity places on the primary and secondary coupling functions. The resulting constraints did not give further restrictions on the functional form of ϕ^μ and h, but were in the form of criteria which could be used to check different models. We called these the integrity criteria.

As it was not obvious that any interactions exist which satisfy the principle of integrity we applied the integrity criteria to a simple example. The model we examined, and to which we return in this paper, consists of a quantum spin system and a freely moving classical particle as the apparatus. It is loosely related to the Stern-Gerlach experiment. For this model we found a nontrivial interaction which obeyed the integrity criteria. We defer a discussion of the form of the interaction to the following section where we discuss the model in detail.

III. A SIMPLE MODEL

The purpose of this paper is to illustrate how our model provides a theoretical description of an idealized measurement experiment. The experiment we examine is related to the well-known Stern-Gerlach experiment. However, the setup we use is subtly different from the conventional Stern-Gerlach experiment. In this section we introduce the apparatus and the quantum system and we discuss the formulation of the experiments we later examine. We include in the following two sections the solutions for the various cases examined.

The quantum system we are going to examine is a very simple system. Its dynamical variables have discrete spectra only, and furthermore the system is inert. It is a quantum spin system characterized by the three spin operators S_1, S_2, and S_3, and the total spin squared $S^2 = S_1^2 + S_2^2 + S_3^2$. These operators satisfy the commutation relations

$$[S_i, S_j] = i\epsilon_{ijk}S_k, \quad [S^2, S_i] = 0. \tag{3.1}$$

Since the system is inert, the Hamiltonian vanishes—the system does not change in time if left undisturbed. The state of the system is specified by choosing an eigenstate of, for example, S^2 and S_1, which constitute a complete commuting set of operators of the system.

For the classical apparatus we also choose a simple system; namely a freely moving classical particle of mass m. The quantum-enlarged apparatus system, following the outline in Sec. II, is characterized by dynamical variables

$$\omega^\mu = \{\vec{q}_i, \vec{p}_j\}$$

and their (unobservable) conjugate operators

$$\pi^\mu = \{\pi_i^q, \pi_i^p\} .$$

The ω^μ are superselecting operators, and they generate the algebra of observables for the apparatus. As a consequence, the conjugate π^ν operators are not observable. The state of the apparatus can be specified to be an eigenstate of the observables, which form a commuting set. (We assume that the state space has been appropriately extended to include such non-normalizable states.) The time development of this system is given by the Hamiltonian

$$\mathcal{H} = \frac{1}{m} \vec{p} \cdot \vec{\pi}^q \tag{3.2}$$

which is derived from the usual classical free-particle Hamiltonian $(1/2m)p^2$ using the prescription (2.5). We note that the Hamiltonian operator (3.2) is not an observable.

We must choose a suitable interaction between the apparatus and the quantum spin system. To describe such an interaction we envisage the quantum system being carried along as internal degrees of freedom by the (electrically neutral) particle whose translatory degrees of freedom are classical. The internal degrees of freedom may be considered to give rise to a magnetic moment, $\vec{\mu} = \gamma\vec{S}$, for the particle. The interaction is induced by causing the classical particle to pass through an inhomogeneous magnetic field, as in the Stern-Gerlach experiment.[4]

The coupling between the apparatus and the quantum system is specified by requiring that the apparatus observables satisfy the correct classical equations of motion, namely

$$\dot{p}_i(t) = F_i \quad \text{and} \quad \dot{q}_i(t) = \frac{1}{m} p_i(t) .$$

At the purely classical level the potential energy of the particle in a magnetic field $\vec{B}(q)$ is $\vec{\mu} \cdot \vec{B}(q)$. The force exerted on the particle is then

$$\vec{F} = -\vec{\nabla}(\vec{\mu} \cdot \vec{B}) = -\gamma \vec{S} \cdot \vec{\nabla} B_i . \tag{3.3}$$

The choice of primary coupling functions

$$\phi_j^q = 0 , \quad \phi_j^p = F_j \tag{3.4}$$

then ensures the correct equations of motion for the apparatus observables. The full Hamiltonian is then

$$\mathcal{H} = \frac{1}{m} \vec{p} \cdot \vec{\pi}^q - \gamma S_i [\vec{\nabla} B_i(q)] \cdot \vec{\pi}^p . \tag{3.5}$$

This form can be derived from the classical Hamiltonian

$$\frac{1}{2m} p^2 + \vec{\mu} \cdot \vec{B}(q) \tag{3.6}$$

by using the prescription (2.5). We note that we have chosen a vanishing secondary coupling function. As we saw in II for this example we could transform such a coupling term to zero.

The coupling functions ϕ_j^p are still arbitrary in as much as the form of the magnetic field $\vec{B}(q)$ is unspecified. In II we investigated what form of the coupling functions, or, equivalently, what form of $\vec{B}(q)$, is allowed if the strong form of the principle of integrity is to be satisfied. When applied to the above model the integrity criteria led to the result that the gradients $(\partial/\partial q_i)\vec{B}(q)$ must all point in the same direction. In other words, only magnetic fields of the form

$$\vec{B}(q) = \vec{B}(0) + F(q)\hat{n} , \tag{3.7}$$

where $F(0) = 0$, can be used to induce the interaction between the apparatus and the quantum system.

We found that the form (3.7) for the magnetic field does not allow a treatment of the conventional Stern-Gerlach experiment. The actual experiment makes use of an inhomogeneous magnetic field produced by suitably shaped fixed magnets. The field pattern in the space between these pole pieces is necessarily rather complicated. Also, in that region, in absence of currents, Maxwell's homogeneous equations are satisfied, giving us

$$\vec{\nabla} \cdot \vec{B}(q) = 0 \quad \text{and} \quad \vec{\nabla} \times \vec{B}(q) = 0 . \tag{3.8}$$

Magnetic fields of the form (3.7) can only satisfy Eqs. (3.8) if they are homogeneous. Such fields, clearly, do not induce an interaction term in the Hamiltonian (3.5).

However, our aim in this section is to describe a simple, idealized, even if impractical, interaction between the apparatus and the quantum system. In fact, we could choose a magnetic field $\vec{B}(q)$ of the simple form (3.7) which does satisfy Maxwell's homogeneous equations. We choose a further simplified form of (3.7), namely

$$\vec{B}(q) = B(q_1, q_2)\hat{n} , \tag{3.9}$$

where $\hat{n}$ is a constant unit vector pointing in the q_3 direction. If we have a nonvanishing current distribution

$$\vec{J} = (J_1(q_1, q_2), J_2(q_1, q_2), 0) , \tag{3.10}$$

Maxwell's equations can be solved[3] to yield

$$B(q_1, q_2) = \int_0^{q_2} J_1(q_1, q_2')dq_2' + \int_0^{q_1} J_2(q_1', 0)dq_1' . \tag{3.11}$$

For the purposes of the calculations in the following sections we use the following explicit form:

$$B(q_1, q_2) = aq_1 + b ,$$

where a and b are constants independent of q_1, q_2, and q_3.

IV. ILLUSTRATION: THE STERN-GERLACH EXPERIMENT

In this section we illustrate our approach to the measurement of quantum systems by taking a detailed look at the model described in Sec. III. This analysis demonstrates how information on quantum variables can be stored in the variables of the classical apparatus.

A. The Stern-Gerlach experiment

The first application of the model is to describe a Stern-Gerlach experiment. In this experiment the heavy particle passes through a single region of inhomogeneous magnetic field. Inside this region the classical and quantum degrees of freedom interact. Outside of this region the classical and quantum degrees of freedom do not couple. In our treatment we ignore the contributions from the edge region. We later justify this approximation by treating the edges and taking the appropriate limit.

Thus, the magnetic field which we use in our calculations to induce the interaction between the apparatus and the spin system is, in line with our discussion of Sec. III,

$$\vec{B}(q) = B(q_1, q_2)\hat{n} , \tag{4.1}$$

where

$$B(q_1, q_2) = \begin{cases} 0, & q_2 < y_0 \\ aq_1 + b, & y_0 \leq q_2 \leq y_1 \\ 0, & y_1 < q_2 \end{cases} \tag{4.2}$$

and $\hat{n}$ is a unit vector pointing in the positive q_3 direction. The classical particle moves to the right along the q_2 axis and is initially situated at $q_2 < y_0$. The points y_0 and y_1 define the boundaries of the region of the inhomogeneous magnetic field.

The magnetic field (4.1) is, in fact, not differentiable at the boundary points y_0 and y_1. This, however, is an artifact of our approximation and a harmless one at that as will be seen when we treat explicitly the edge effects. Thus in taking derivatives of $B(q_1, q_2)$ to define the field gradients (and it is these which induce the interaction), we define

$$\frac{\partial}{\partial q_1} B(q_1, q_2) = \begin{cases} 0, & q_2 < y_0 \\ a < 0, & y_0 \leq q_2 \leq y_1 \\ 0, & y_1 < q_2 \end{cases} \tag{4.3}$$

and

$$\frac{\partial}{\partial q_2} B(q_1, q_2) = \frac{\partial}{\partial q_3} B(q_1, q_2) = 0 .$$

The time development of the combined apparatus and quantum system is given by the Hamiltonian operator

$$\mathcal{H} = \frac{1}{m} \vec{p} \cdot \vec{\pi}^q - \gamma \frac{\partial}{\partial q_i} (\vec{B} \cdot \vec{S}) \pi_i^p , \tag{4.4}$$

Here, the gradients of $\vec{B}$ are given by (4.3) and from our choice of B the quantum variable coupled to the apparatus is $\vec{S} \cdot \hat{n} = S_3$. As stated in Sec. III, the state of the apparatus, initially, is an eigenstate of the six apparatus variables $\vec{q}$ and $\vec{p}$,

$$|\psi\rangle = |\hat{q}_i, \hat{p}_i\rangle . \tag{4.5}$$

The initial conditions of the experimental setup are incorporated by the choice

$$\hat{q}_2 < y_0, \quad \hat{p}_2 > 0 , \qquad \hat{q}_1 = \hat{q}_3 = \hat{p}_1 = \hat{p}_3 = 0 . \tag{4.6}$$

For the initial state of the quantum system we choose an eigenstate of S^2 and S_1, namely

$$|\phi\rangle = |s, S_1 = s'\rangle .$$

Then the initial state of the combined system is $|\Psi\rangle = |\psi\rangle \otimes |\phi\rangle$.

We treat the time development in the Heisenberg picture. The operator equations of motion then are very simple, namely

$$\begin{aligned} &\dot{p}_1(t) = -\gamma \frac{\partial}{\partial q_1} BS_3, \quad \dot{q}_i(t) = \frac{1}{m} p_i(t), \quad i = 1, 2, 3 , \\ &\dot{p}_i(t) = 0, \quad i = 2, 3 , \\ &\dot{\pi}_i^p(t) = -\frac{1}{m} \dot{\pi}_i^q(t), \quad \pi_i^q(t) = 0 , \\ &\dot{S}_3(t) = 0 , \\ &\dot{S}_1(t) = \gamma \frac{\partial B}{\partial q_1} \pi_1^p(t) S_2(t) , \\ &\dot{S}_2(t) = -\gamma \frac{\partial B}{\partial q_1} \pi_1^p(t) S_1(t) . \end{aligned} \tag{4.7}$$

The times at which the classical particle enters and leaves the magnetic field are determined by the eigenvalues of

$$p_2(t) = p_2(0) = \hat{p}_2$$

and

$$q_2(t) = q_2(0) + \frac{t}{m} \hat{p}_2 = \hat{q}_2 + \frac{t}{m} \hat{p}_2 .$$

in the state $|\psi\rangle$, and they are, respectively,

$$t_0 = \frac{m}{\hat{p}_2}(y_0 - \hat{q}_2),$$

and (4.8)

$$t_1 = t_0 + \frac{m}{\hat{p}_2}(y_1 - y_0).$$

For times $t > t_1$ the solutions to the remaining equations are

$$\begin{aligned} p_1(t) &= p_1(0) - a\gamma S_3(0)(t_1 - t_0), \\ q_1(t) &= q_1(0) + \frac{t}{m} p_1(0) \\ &\quad - \frac{a\gamma}{m} S_3(0)(t_1 - t_0)\left(t - \frac{t_0 + t_1}{2}\right), \\ \pi_i^q(t) &= \pi_i^q(0), \\ \pi_i^p(t) &= \pi_i^p(0) - \frac{t}{m}\pi_i^q(0), \\ S_3(t) &= S_3(0), \\ S_1(t) &= S_1(0)\cos\Phi + S_2(0)\sin\Phi, \\ S_2(t) &= -S_1(0)\sin\Phi + S_2(0)\cos\Phi, \end{aligned} \tag{4.9}$$

where the operator Φ is given by

$$\Phi = a\gamma\left[\pi_1^p(0)t - \pi_1^q(0)\frac{t^2}{2m}\right].$$

To find out about the measurement aspect of the interaction we need only concentrate upon the effect of the quantum system on the apparatus observables, which in this case means $p_1(t)$ and $q_1(t)$. Nevertheless, we have included the other solutions for later use. The apparatus observables $p_1(t)$ and $q_1(t)$ depend functionally on the quantum variables $S_3(0)$. In fact, information has been stored in the classical observables. Let us consider the action of $q_i(t)$ on the state $|\psi\rangle \otimes |\phi\rangle$. Clearly

$$q_2(t)|\psi\rangle \otimes |\phi\rangle = \left(\hat{q}_2 + \frac{t}{m}\hat{p}_2\right)|\psi\rangle \otimes |\phi\rangle,$$

and (4.10)

$$q_3(t)|\psi\rangle \otimes |\phi\rangle = 0.$$

However, we notice that $|\psi\rangle \otimes |\phi\rangle$ is not an eigenstate of $q_1(t)$ [though it is an eigenstate of $q_1(0)$] as is clear from the Eqs. (4.9). We can expand the state in terms of the eigenstates of $q_1(t)$, and we find

$$\begin{aligned} q_1(t)\{|\psi\rangle \otimes |\phi\rangle\} &= q_1(t)|\psi\rangle \otimes \sum_{s''} \langle s, S_3 = s'' | s, S_1 = s'\rangle |s, S_3 = s''\rangle \\ &= q_1(t)\sum_{s''} a_{s'',s'}|\psi\rangle \otimes |s, S_3 = s''\rangle \\ &= -\frac{a\gamma}{m}(t_1 - t_0)\left(t - \frac{t_1 + t_0}{2}\right)\sum_{s''} a_{s'',s'}\, s''|\psi\rangle \otimes |s, S_3 = s''\rangle. \end{aligned} \tag{4.11}$$

From the action of $q_1(t)$ on the state $|\psi\rangle \otimes |\phi\rangle$, we deduce that the position of the classical particle at time $t > t_1$ depends on the action of $S_3(0)$ on the quantum state $|\phi\rangle$. Thus if we know the position of the particle, we know the value of $S_3(0)$ on the internal quantum state.

If we consider an initial beam of classical particles each with an internal quantum system in the same state $|\phi\rangle$, then Eq. (4.11) tells us that the trajectory of the initial beam has split into $(2s+1)$-phase correlated beams, which are spatially separated, as a result of the interaction. The resultant beams are characterized by the states

$$|\psi\rangle \otimes |s, S_3 = s''\rangle \quad \text{for } s'' = s, s-1, \ldots, -s, \tag{4.12}$$

and on each of these the eigenvalues of $q_1(t)$ is

$$-\frac{a\gamma}{m}(t_1 - t_0)\left(t - \frac{t_0 + t_1}{2}\right)s'' \quad \text{for } t > t_1.$$

This treatment in the Heisenberg picture allows us to deduce that in the corresponding Schrödinger picture analysis the initial state $|\psi\rangle \otimes |\phi\rangle$ evolves into, at time t, a superposition over distinct q_1 eigenstates. Since the apparatus observables are also superselecting operators, each of the q_1 eigenstates in the superposition belongs to a different superselected sector of the state space. Such a state is not a unique classical state. We call them effective mixture states. This state is effectively a mixture state as the relative phases in the superposition are not measurable. The example above demonstrates that within our formalism a unique classical state can evolve in time into an effective mixture state.

We have satisfied the requirement imposed in the introduction so that the interaction might lead to a measurement. Unambiguous information on the values of the quantum variable $S_3(0)$ has been stored in the observables q_1 and p_1 of the apparatus. In the first case above, with just one particle, what is measured by q_1 is the value of S_3 on the internal spin system. In the case of a beam of particles what is actually measured is the eigenvalue of S^2 which is

$$s(s+1)=\tfrac{1}{4}[(2s+1)^2-1],$$

by counting the number of $2S+1$ resultant beams. However, we must emphasize at this point that for a true measurement to occur an observer must read the information from the apparatus. We return to a discussion of this question in Sec. VI.

B. The edge regions

In Sec. IV A to simplify the analysis we made use of an approximation—we simply ignored the possible effects of the magnetic field at the edges of the region in which it was nonzero. We now wish to justify the use of that approximation.

We shall examine the model Stern-Gerlach experiment of the previous subsection with the following magnetic field:

$$B(q_1q_2)\hat{n}=f(q_2)(aq_1+b)\hat{n}, \tag{4.13}$$

where

$$f(q_2)=\begin{cases}0, & q_2\leq y_0\\ \dfrac{q_2-y_0}{y_1-y_0}, & y_0<q_2<y_1\\ 1, & y_0\leq q_2\leq y_2\\ \dfrac{y_3-q_2}{y_3-y_2}, & y_2<q_2<y_3\\ 0, & y_3\leq q_2.\end{cases} \tag{4.14}$$

With this choice the magnetic field tapers off to zero linearly in the edge regions. This function is once differentiable everywhere, although $f'(q_2)$ is not continuous. The Hamiltonian for the interaction is,

$$\mathcal{H}_{\rm op}=\frac{1}{m}\vec{p}\cdot\vec{\pi}^q-\gamma\left(\frac{\partial B}{\partial q_2}\right)S_3\pi_2^p-\gamma\left(\frac{\partial B}{\partial q_1}\right)S_3\pi_1^p. \tag{4.15}$$

In the regions $q_2\leq y_0$, $y_1\leq q_2\leq y_2$, and $y_3\leq q_2$ the analysis is as in Sec. IV A. It is only in the regions $y_0<q_2<y_1$ and $y_2<q_2<y_3$ that the edge effects enter the problem.

For the region $y_0<q_2<y_1$ the equations of motion for the apparatus observables are

$$\begin{aligned}\dot{p}_1(t)&=-\frac{a\gamma}{y_1-y_0}[q_2(t)-y_0]S_3(t),\\ \dot{p}_2(t)&=-\frac{a\gamma}{y_1-y_0}\left(q_1(t)+\frac{b}{a}\right)S_3(t),\\ \dot{p}_3(t)&=0,\\ \dot{q}_i(t)&=p_i(t)/m,\quad i=1,2,3\\ \dot{S}_3(t)&=0.\end{aligned} \tag{4.16}$$

These equations can be written as follows:

$$\begin{aligned}&\left(\frac{d^4}{dt^4}-\beta^4\right)p_i(t)=0,\quad i=1,2\\ &\left(\frac{d^4}{dt^4}-\beta^4\right)q_1(t)=\frac{b}{a}\beta^4,\\ &\left(\frac{d^4}{dt^4}-\beta^4\right)q_2(t)=-y_0\beta^4,\end{aligned} \tag{4.17}$$

where we have used

$$\beta^2=\frac{-a\gamma}{m(y_1-y_0)}S_3(0).$$

At times t such that $t_0<t<t_1$, the solutions to these equations are

$$\begin{aligned}p_2(t)&=\frac{p_2(t_0)-p_1(t_0)}{2}\cosh\beta(t-t_0)+\beta m\frac{\alpha_1}{2}\sinh\beta(t-t_0)+\frac{p_2(t_0)+p_1(t_0)}{2}\cos\beta(t-t_0)-\beta m\frac{\alpha_2}{2}\sin\beta(t-t_0),\\ q_2(t)&=\frac{p_2(t_0)-p_1(t_0)}{2m\beta}\sinh\beta(t-t_0)+\frac{\alpha_1}{2}\cosh\beta(t-t_0)+\frac{p_2(t_0)+p_1(t_0)}{2m\beta}\sin\beta(t-t_0)+\frac{\alpha_2}{2}\cos\beta(t-t_0)+y_0,\\ p_1(t)&=-\frac{p_2(t_0)-p_1(t_0)}{2}\cosh\beta(t-t_0)-\frac{\beta m\alpha_1}{2}\sinh\beta(t-t_0)+\frac{p_2(t_0)+p_1(t_0)}{2}\cos\beta(t-t_0)-\frac{\beta m\alpha_2}{2}\sin\beta(t-t_0),\\ q_1(t)&=-\frac{p_2(t_0)-p_1(t_0)}{2m\beta}\sinh\beta(t-t_0)-\frac{\alpha_1}{2}\cosh\beta(t-t_0)+\frac{p_2(t_0)+p_1(t_0)}{2m\beta}\sin\beta(t-t_0)+\frac{\alpha_2}{2}\cos\beta(t-t_0)-\frac{b}{a},\end{aligned} \tag{4.18}$$

where

$$\begin{aligned}\beta&=+\left(\frac{-a\gamma S_3(0)}{y_1-y_0}\right)^{1/2},\quad S_3(0)\text{ diagonal}\\ \alpha_1&=[q_2(t_0)-y_0]-\left(q_1(t_0)+\frac{b}{a}\right),\\ \alpha_2&=[q_2(t_0)-y_0]+\left(q_1(t_0)+\frac{b}{a}\right).\end{aligned} \tag{4.19}$$

These solutions are interesting for the following reasons. If we let $y_1\to y_0$, that is, we let the edge region get smaller and smaller, the solutions in (4.18) behave as

$$\begin{aligned}p_i(t)&\to p_i(t_0),\quad i=1,2\\ q_i(t)&\to q_i(t_0),\quad i=1,2.\end{aligned} \tag{4.20}$$

In other words, if we let the linear edge get smaller, its effect on the apparatus observables tends to zero. Furthermore, in this limit the gradients of $B(q_1, q_2)$ approach the values we used earlier in (4.3). It is in this sense that our earlier approximation is justified.

V. ILLUSTRATION

In this section we illustrate our approach further. We consider experimental arrangements which are slightly more complicated than that of the previous section. To the right of the inhomogeneous magnetic field we consider other regions where the magnetic field is nonzero, and these magnetic fields induce further interactions between the apparatus and the quantum system.

A. The magnetic interferometer

In this subsection we investigate an interaction which will undo the effects of the "Stern-Gerlach" experiment described in Sec. IV. That is, after the classical beam has passed through the inhomogeneous magnetic field we introduce an interaction which returns the variables of the classical apparatus to the form they would have had in the absence of any interaction—we recombine the beams. The interesting question is whether the internal quantum system returns to its original state or not.[9]

The experimental arrangement we are discussing is called the magnetic interferometer.[5,6] As depicted in Fig. 1, there are three disjoint regions of space where the magnetic field gradient is nonzero, separated by two regions of zero magnetic field. The first region of the nonzero magnetic field, labeled region II and the following region of no magnetic field, labeled region III, are as in the Stern-Gerlach model set up of Sec. IV. Region IV, immediately after these, is chosen with the magnetic field antiparallel to that of region II, but with the same magnitude and twice the spatial extent of region II. This has the effect of bending the divergent beams so that in the middle of the region IV the beams are moving parallel to each other. Thus, if we had just I, II, III and $\frac{1}{2}$IV the emerging trajectories of the classical particles would be parallel. The final region VI is identical to the region II and is separated from region IV by a region of zero magnetic field, the same as region III in extent. The effect of the final region is to cause the converging beams to coalesce.

The details of the analysis of this experiment follow much as in Sec. IV. The Heisenberg equations of motion for the dynamical variables are

$$\begin{aligned}
&\dot{S}_3(t)=0\,,\\
&\dot{q}_i(t)=\frac{1}{m}\,p_i(t)\,,\quad i=2,3\\
&\dot{p}_i(t)=0\,,\quad i=2,3\\
&\dot{p}_1(t)=-\phi\,,\\
&\dot{q}_1(t)=p_1(t)/m\,,\\
&\dot{S}_1(t)=i[S_1(t),\,\phi]\pi_1^p(t)\,,\\
&\dot{S}_2(t)=i[S_2(t),\,\phi]\pi_1^p(t)\,,\\
&\dot{\pi}_i^q(t)=0\,,\quad \text{for } i=1,2,3\\
&\dot{\pi}_i^p(t)=-\pi_i^q(t)/m\,,
\end{aligned}\tag{5.1}$$

where

$$\phi=\begin{cases} a\gamma S_3 & \text{in regions II and VI}\\ -a\gamma S_3 & \text{in region IV}\\ 0 & \text{elsewhere}\,.\end{cases}\tag{5.2}$$

Here Φ is the gradient $\partial B/\partial q_1$ of the magnetic field for the interferometer.

It is a rather straightforward matter to solve

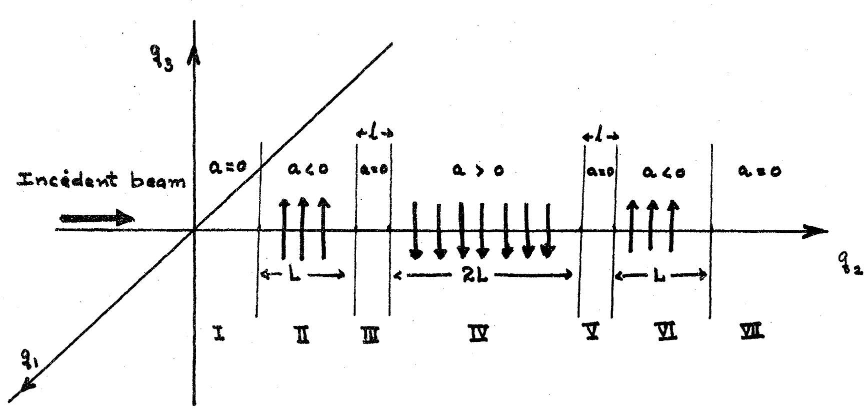

FIG. 1. The magnetic field arrangement for the magnetic interferometer. The solid arrows denote the direction of the magnetic field gradient $(\partial/\partial q_1)\vec{B}(q)$.

these equations in the various regions. The choice of the magnetic fields in the different regions, and the sizes of these regions, guarantees that the solutions at a time after the interaction is completed are

$$
\begin{aligned}
&p_i(t)=p_i(0)\,,\\
&q_i(t)=q_i(0)+\frac{t}{m}\,p_i(0)\,,\\
&\pi_i^q(t)=\pi_i^q(0)\,, \qquad\qquad (5.3)\\
&\pi_i^p(t)=\pi_i^p(0)-\frac{t}{m}\,\pi_i^q(0)\,,\\
&S_i(t)=S_i(0)\,.
\end{aligned}
$$

These solutions tell us that not only have the classical beams recombined, but also the quantum states also interfere coherently—the quantum state finally is the same as it was initially. Thus all the effects of the first interactions have been undone.

In II we considered, as well as the couplings discussed in this paper, a π-independent secondary coupling function for the Stern-Gerlach model. In that case the above result does not hold, i.e., the quantum variables $S_1(t)$ and $S_2(t)$ are not the same as $S_1(0)$ and $S_2(0)$, although they are related by a unitary transformation. However, as we saw in II, the secondary coupling function could be transformed away by a unitary transformation.

In the magnetic interferometer described in this section there has been no measurement. At no stage could information on the state of the quantum system be transferred to an observer. We will return again to discuss this question in Sec. VI.

B. The crossed-field experiment

In this subsection we examine a crossed-field experiment. The simple experimental arrangement of Sec. IV is extended by including another region of inhomogeneous magnetic field to the right of the original setup. The field gradient in this additional region is chosen to be orthogonal to that of the first region. Thus, in the no-edge approximation we have a magnetic field

$$\vec{B}(q_1,q_2,q_3)=B_1(q_1,q_2)\,\hat{n}+B_2(q_2,q_3)\,\hat{n}'\,, \qquad (5.4)$$

where

$$\hat{n}\cdot\hat{n}'=0\,,\quad B_1(q_1,q_2)\text{ is as in (4.2)} \qquad (5.5)$$

and

$$B_2(q_2,q_3)=\begin{cases}0\,, & q_2\leq y_2\\ cq_3+d\,, & y_2<q_2<y_3\\ 0\,, & y_3\leq q_2\end{cases} \qquad (5.6)$$

with $c < 0$, such that $y_0<y_1<y_2<y_3$; thus, the regions of the support of B_1 and B_2 are disjoint, as shown in Fig. 2.

We have discussed an interaction derived from a magnetic field of this type in II. There we were interested in finding the most general interaction allowed within our model "Stern-Gerlach" arrangement which satisfied the integrity criteria—that is, which allowed the observables of the classical apparatus (in this case the particle) to remain classical after the interaction. We derived the result that the magnetic field gradients $(\partial/\partial q_i)\vec{B}(\vec{q})$ must all be unidirectional leading to the following form for the magnetic field:

$$\vec{B}(\vec{q})=\vec{B}(0)+B(\vec{q})\,\hat{n}\,,$$

where $\hat{n}$ is a constant unit vector, provided the magnetic field were to be analytic. On the other hand, if the magnetic field was not restricted to be analytic, the necessary integrity criteria required the field gradients $f_i(\vec{q})$ to be unidirectional in every disjoint region of support. With the choice we have used, in Sec. IV, for the form of the magnetic field, namely $\vec{B}(0)$ parallel to $\hat{n}$, we have in

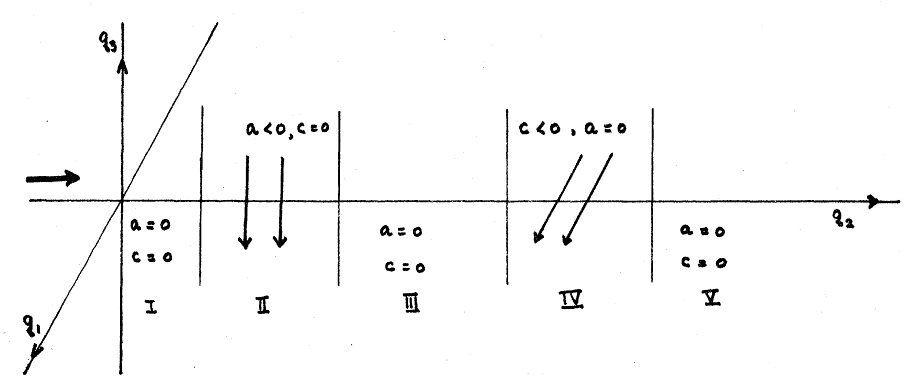

FIG. 2. The magnetic field arrangement for the crossed-field experiment. The solid arrows denote the direction of the magnetic field gradients $(\partial/\partial q_1)\vec{B}(q)$ and $(\partial/\partial q_3)\vec{B}(q)$.

this case as the allowed form for the magnetic field:

$$\vec{B}(\vec{q}) = \sum_m B^m(\vec{q})\,\hat{n}_m\,, \tag{5.7}$$

where $\hat{n}_m$ are constant unit vectors and the support regions of $B^m(\vec{q})$ for different m are disjoint. However, it is not yet clear if this form for the magnetic field is sufficient—does it allow the apparatus observables to remain classical? In II, we gave an argument that this would not be the case. In the present section we will demonstrate the result explicitly.

Clearly the inhomogeneous magnetic field in this experiment is not analytic as is seen from the Eqs. (5.6). In fact at certain points it is not even differentiable. However, because the linear edge model of Sec. IV B reduces in the limit of zero edge to the treatment of Sec. IV A, we are justified in ignoring the nondifferentiable points. The magnetic field does take the form (5.7) allowed by the (necessary) integrity criteria.

The analysis of the experiment through the first three regions of space is identical to that given in Sec. IV A, for the Stern-Gerlach case. In the fourth region we can concentrate simply on the apparatus observables. Their equations of motion are

$$\begin{aligned}
\dot{p}_3(t) &= -c\gamma S_1(t)\,,\\
\dot{q}_3(t) &= \frac{1}{m}\,p_3(t)\,,\\
\dot{p}_2(t) &= 0\,,\\
\dot{q}_2(t) &= \frac{1}{m}\,p_2(t)\,,\\
\dot{p}_1(t) &= 0\,,\\
\dot{q}_1(t) &= \frac{1}{m}\,p_1(t)\,,\\
\dot{S}_1(t) &= 0\,.
\end{aligned} \tag{5.8}$$

The corresponding solutions for a time t, where $t_2 < t < t_3$, are

$$S_1(t) = S_1(t_2)\,, \tag{5.9}$$

$$\begin{aligned}
p_3(t) &= p_3(0) - c\gamma(t-t_2)S_1(t_2)\,,\\
q_3(t) &= q_3(0) + \frac{t}{m}\,p_3(0) - \frac{c\gamma}{2m}(t-t_2)^2 S_1(t_2)\,,
\end{aligned} \tag{5.10}$$

$$\begin{aligned}
p_2(t) &= p_2(0)\,,\\
q_2(t) &= q_2(0) + \frac{t}{m}\,p_2(0)\,,\\
p_1(t) &= p_1(t_2)\,,\\
q_1(t) &= q_1(t_2) + \frac{t-t_2}{m}\,p_1(t_2)\,.
\end{aligned} \tag{5.11}$$

Let us concentrate on the solutions (5.9) and (5.10). The spin operators $S_1(t_2)$, which occurs in these solutions, is given by Eq. (4.9). Furthermore, it is a function of the unobservable operators $\pi_1^p(0)$ and $\pi_1^q(0)$. As a consequence, $S_1(t_2)$ lies in the unobservable sector of the algebra of operators—it is unobservable even in principle. From (5.10) we see that the apparatus variables $p_3(t)$ and $q_3(t)$ also are unobservable for $t > t_2$. In the language of our approach the apparatus has lost its classical integrity.

Thus we have wrongly applied the model in attempting to examine this experiment. Either we should pick an alternative apparatus, or we should modify the experiment, if we wish to describe it within our approach. As we shall discuss in the following section, we can describe this experiment if we require that an outside observer interact with the apparatus before it enters the crossed-field region.

VI. DISCUSSION OF THE EXPERIMENTS

When we proposed the present model for use in the description of measurement, the motivation for choosing the apparatus to be a classical system was that for such systems the question of measurement does not, in principle, present a problem. In principle we can know simultaneously all that can be known about the theory at any instant of time. Of course, a particular act of measurement may present technological problems, but these are not of concern to us at present. Then, if one could have a classical apparatus interacting with a quantum-mechanical system, in such a manner that information passes from the quantum system to the variables of the classical theory, we have taken the first step. The next important step is to discuss whether or not the hoped for simplification of the measurement process takes place—namely is it now possible to "measure at will" the classical variables of the apparatus. The information which has passed to the apparatus must, after all, eventually be available to an observer.

This step is not so easily achieved. We must remember that in causing the classical system to interact with the quantum system we had to embed it within a larger quantum structure. The observable sector of this quantum theory coincides with the classical theory, but it is accompanied by the unobservable sector. This unobservable sector is not an inert passenger—it has a definite role to play in the time development of the quantum system. We have also seen that following the interaction with a quantum system the state of the combined system is no longer an eigenstate of all of the apparatus observables. The question which arises at this point is whether the apparatus observables can be measured at will after the interaction has taken place.

The answer is that it cannot be done in general. Measurement of the apparatus observables will cause a disturbance to the unobservable operators. Since the state of the system is not an eigenstate of the observables, this measurement will affect the state. In complicated experiments this step may even have an effect on the quantum system.

It is only when the classical theory is formulated in the classical Hamiltonian language that its variables can be measured at will. Thus if we wish to make such a measurement (at will), we must first map the quantum-enlarged apparatus back to its purely classical form. This mapping will be the inverse of the embedding procedure used to write down, originally, the quantum-enlarged description of the apparatus. This step means, in effect, that we discard the unobservable sector of the quantum-enlarged apparatus. It will also entail the loss of phase correlations in the final state between the distinct eigenstates of the apparatus observables which occur in its expansion. The only information we retain is the "classical" state of the apparatus which is got by carrying out a partial trace over the quantum system states.

Once this is done, it is possible to measure at will the apparatus observables to extract the required information. After such a measurement has been carried out, we can reanalyze the apparatus and quantum system for further experiments, but the original experiment cannot be undone, due to the loss of phase correlations inherent in the step referred to above.

Let us illustrate this point with the treatment of the magnetic interferometer given in Sec. V A. Suppose we were to specify that after the first splitting, that is, in region III, a (true) measurement was to be made on the eigenvalue of S_3. This would necessitate first the projection back to the corresponding classical theory—in this case $2S+1$ beams of classical particles moving with momenta $\hat{p}_2$ and $\hat{p}_1 - a\gamma T s'$. The measurement can now be carried out to find the value of the classical variable p_1, and hence measure the values of s'. However, for the remainder of the experiment we begin with $(2S+1)$ classical beams, each carrying along an internal quantum system with state specified by the spin s and a particular value for S_3. The internal quantum spin states are now not phase correlated with each other, so that, if we do recombine the beams, the state of the quantum system will not be pure. The uncorrelated states will not interfere coherently—rather they will form a mixture state. In this case, the results of Sec. V A would not be valid.

In the treatment of the crossed-field experiment of Sec. V B, it is now clear that if an outside observer observes the beams of particles in the region of space between the crossed fields, then the difficulties encountered before do not arise. In this region we will now have $(2s+1)$ beams of classical particles each correlated to a particular quantum state specified by eigenvalues of S^2 and S_3. Each of the beams now interacts with the orthogonal magnetic field gradient in a manner exactly analogous to the treatment of the Stern-Gerlach experiment in Sec. IV.

We note that the principle of integrity is now satisfied. However, the experiment now describes two different measurements of noncommuting operators occurring sequentially. But the values are not held by the corresponding quantum operators simultaneously.

It remains to be shown how the mapping back to the classical Hamiltonian formulation is to be carried out. Examining a classical system on its own we found that the mapping proceeds via the eigenvalues of the observables in the state of the classical system.[2] However, for a classical system interacting with a quantum system, the state is, in general, not an eigenstate of the classical system observables. One can write the final state as a superposition over such eigenstates. Then it is clear that we can only proceed by mapping back to the usual classical formalism for each eigenstate which occurs. But in this step we unavoidably lose the phase correlations between the states in the final-state superposition. This occurs because in the Hamiltonian formulation of classical mechanics we cannot form superpositions of pure states. We may, in fact, view this loss of phase correlations as one aspect of the state collapse which occurs in any probabilistic theory.

Of course we must realize that an actual laboratory observation of a system, either classical or quantum, consists of two parts. The first part is the interaction between the observer and the system. This has the result of transforming the probability amplitude into a more complicated probability amplitude in which the various possible results are each correlated with a different state of the observer. The second part is the use of the result of an actual measurement as input to the theory, that is, resetting the state from the probability distribution to a particular realization. This second part is known as state collapse. As we have seen above it does occur not only in quantum systems, but can also occur for classical systems described by a probability distribution.

In the previous sections of this paper, and in parts I and II also, we concentrated upon the first—

the setting up of correlations and the development of the probability amplitudes. In this section we have touched on the second part, namely carrying out an observation for a single apparatus (single particle in the Stern-Gerlach case) and using that information in the subsequent development of the system.

There is another way in which we can interpret the mapping, as a result of which we discard the unobservable sector. Initially, we knew the value of $S_1(0)$ on the quantum state. We assumed that $S_2(0)$ and $S_3(0)$ were observable. To prepare the state, however, necessitated interactions of the type we have discussed here. Thus when an observation is made, and use is made of the information gained, we are justified in discarding the unobservable part of the theory and treating it as if it were pristine.

VI. DISCUSSION AND COMMENTS

In this paper we have achieved two main objectives. The first was to illustrate our new approach to the question of measurement in quantum physics by means of simple examples. The second was the gaining of a deeper understanding of measurement within our approach.

To illustrate our approach we discussed an experiment loosely based on the Stern-Gerlach experiment. We made this choice as the Stern-Gerlach experiment is conceptually quite simple, and so it allows a relatively simple interpretation within our model. The original experiment[4] was a striking verification of the validity of quantum physics for the microworld. Our treatment differs from the conventional treatment afforded this experiment in the literature.[4,10] We treat the particle motion as strictly classical. However, in the conventional treatments, the complete experiment is treated at the quantum level, i.e., even the translatory motion is quantum mechanical. To facilitate the calculations, the mass of the particle is then assumed to be so large that the particle motion can be closely approximated classically.[11]

We differ from the conventional approach for two reasons. First, if we wish to treat the particle as

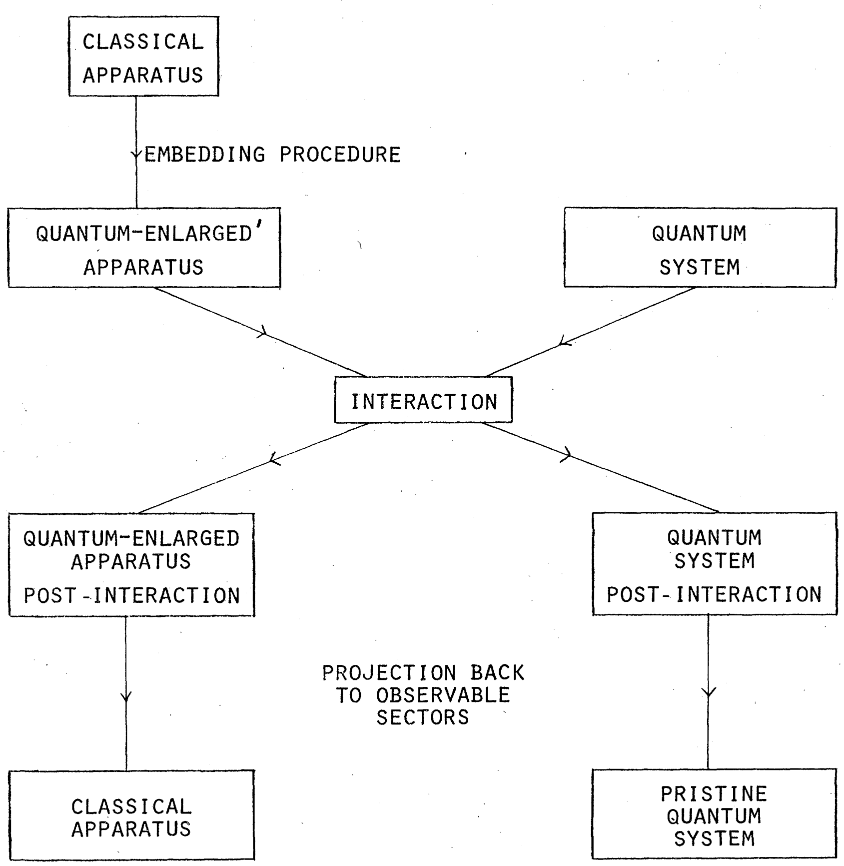

FIG. 3. Schematic diagram of a typical measurement process within our approach, illustrating the different stages in the procedures.

being the apparatus, and this is usually the case, then it should be strictly a classical system, and not only in the limit of its mass becoming very large. Secondly, if it is incorrect to treat the particle as classical, for instance if it is not a heavy atom in the experiment, then we cannot use the particle as the apparatus. In such a case, we should treat the particle and the internal spin system as the quantum system, and introduce an appropriate external apparatus admitting a classical description. Thus, our description can only represent the Stern-Gerlach experiment if the particle motion is truly classical.

Our analysis showed that our approach to the measurement problem can account for the qualitative features of the Stern-Gerlach experiment. Of course, the particular inhomogeneous magnetic field we used to induce the interaction does not correspond to the arrangement in the Stern-Gerlach experiment.

We also examined a magnetic interferometer arrangement, and verified, within the context of our model, the coherent interference of the quantum spin systems provided no outside observation was performed. On the other hand we examined a crossed-field arrangement, where the magnetic field gradients in two disjoint regions are orthogonal to each other, and in this case we found that our approach cannot be used to describe the experiment unless an outside observer makes an irreversible measurement on the apparatus, i.e., the particle, in the region between the magnetic fields.

As to the understanding of the meaning of measurement within the approach, we have been led to consider a mapping which undoes the effects of the embedding procedure *after* the interaction has occurred. In this way the apparatus will be described by a Hamiltonian system after the interaction. This mapping induced by the measurement does involve the loss of phase information. This phase information, however, lies in the unobservable part of the state space, and as such is not felt in the observable sector of this configuration. We do not know how to cause an information transfer without also having an unreversed interaction.

The propagation in crossed fields followed by reversed crossed fields deserves special mention. Standard quantum theory prediction would require that the phase information is not destroyed in the intermediate region. Our theory also makes the same statement *if no act of measurement has been performed* in the intermediate region. Under this situation the relative phase between the two alternate paths, and as such unmeasurable, has observable consequences when the beams are again brought together. If on the other hand, an act of measurement were performed in between, the phase information is not just nonobservable but is actually destroyed. Consequently, the recombination by the reversed fields does not lead to coherent superposition but a mixture.

The effect of the act of measurement on the beam is to destroy the phase; it has no observable consequence at that location since the phase was in the unobservable sector. But the destruction of the phase information reflects itself in the pattern formed in a subsequent interaction.

We sketch, in Fig. 3, a schematic description of a typical measurement process within our approach.

ACKNOWLEDGMENTS

This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-S-05-3992.

[1]E. C. G. Sudarshan, Pramāna 6, 117 (1976).

[2]T. N. Sherry and E. C. G. Sudarshan, Phys. Rev. D 18, 4580 (1978); E. C. G. Sudarshan, T. N. Sherry, and S. R. Gautam, in *Particles and Fields*, edited by D. H. Boal and A. N. Kamal (Plenum, New York, 1978), Proceedings of the 1977 Banff Summer Institute on Particles and Fields.

[3]T. N. Sherry and E. G. C. Sudarshan, Phys. Rev. D 20, 857 (1979).

[4]See, for example, N. Ramsey, *Molecular Beams* (Oxford Univ. Press, London, 1955); and D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951) for discussions of the Stern-Gerlach experiment.

[5]R. P. Feynman, *Lectures on Physics* (Addison Wesley, Reading, Mass., 1966), Vol. III, Chap. V and VI.

[6]M. Scully, R. Shea, and J. D. McCullen, University of Arizona report, 1978 (unpublished).

[7]We use a superselection principle rather than a superselection rule. The distinction to be drawn is that we do not require explicitly that a superselecting operator be a constant of the motion. If the time development operator is an observable there is no distinction. The conventional usage is explained in Ref. 8.

[8]G. C. Wick, A. S. Wightman, and E. P. Wigner, Phys. Rev. 88, 101 (1952).

[9]The recombing of beams split in a Stern-Gerlach experiment was discussed extensively by E. P. Wigner, Am. J. Phys. 31, 6 (1963).

[10]K. Gottfried, *Quantum Mechanics* (Benjamin, New York, 1966), Vol. 1.

[11]In Ref. 10, Gottfried has examined this experiment in some detail. His approach is quite similar to ours. First, he shows that the center-of-mass motion is essentially a classical motion. This arises from an

assumption that dividing the wave function by a macroscopic length yields a vanishingly small result. However, he also makes the statement that "for all known observables, $\langle \vec{r}', \vec{R}' | A | \vec{r}, \vec{R} \rangle$ vanishes when $|\vec{R} - \vec{R}|$ is macroscopic." This statement is tantamount to our superselection principle. In our formalism, the particular amplitude mentioned automatically vanishes for apparatus observables, as only unobservables can connect the states $|\vec{r}', \vec{R}'\rangle$ and $|\vec{r}, \vec{R}\rangle$ if $|\vec{R} - \vec{R}'|$ is macroscopic.