

DIVERGENCELESS CURRENTS AND
K-MESON DECAY*

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The striking successes of the \underline{V} - \underline{A} theory for weak interactions raise a difficult point of prin-

ciple. The chirality invariance¹ which led to the universal \underline{V} - \underline{A} four-fermion interaction was postulated for the bare fermions and it is not clear why the strong interactions modify the results so little for the physical fermions. The evidence from β decay indicates that the pion renormalization effects are approximately the same (within 15%) for the \underline{V} and \underline{A} parts of the interaction. The fact that the coupling constants for μ meson and O^{14} decay are so close (within a few percent) implies that the renormalization effects are essentially nonexistent for the \underline{V} interaction (and *a fortiori* for the \underline{A} interaction). It appears therefore as if the four-fermion interaction actually couples the positive chiral states of the physical fermions.

In order to explain the remarkable agreement between the coupling constants for μ -meson and O^{14} decay, Feynman and Gell-Mann² have proposed that the ordinary (i.e., strangeness-conserving) \underline{V} part of the interaction be described in terms of a divergenceless current. This would eliminate pion renormalization effects and has experimental consequences which can be tested.³ It seems difficult to extend this attractive idea to other (i.e., strangeness-nonconserving) lepton interactions, since we do not know how to write down any other divergenceless currents than the ordinary vector one. This is hardly a conclusive argument, and one might suspect that if we understood the strong interactions and the origin of particle masses better, that we could construct such currents. It seems worthwhile, therefore, to look for experiments that can test directly whether the "current"

responsible for a particular decay process is divergenceless or not. In this connection, it is to be noted that the currents for ordinary (e, ν) and (μ, ν) \underline{A} interactions are not divergenceless, since $\pi \rightarrow \mu + \nu$ does occur, and since there is no enormous pseudoscalar component in beta-decay.⁴ We wish to point out that one may test the role of divergenceless currents in strangeness nonconserving (μ, ν) interactions by looking for certain striking effects in the angular correlations in $K_{\mu 3}$ decay.

If we assume that only positive chiral states are coupled, then all (μ, ν) interactions with $\Delta S = \pm 1$ arise from a weak-interaction Hamiltonian

$$H_w = \int_{\lambda} [\bar{\psi}_{\mu} \alpha^{\lambda} (1 + \alpha_5) \psi_{\nu}] - \int_{\lambda} \dagger [\bar{\psi}_{\nu} \alpha^{\lambda} (1 + \alpha_5) \psi_{\mu}]. \quad (1)$$

Here \int_{λ} is an unknown function of baryon and meson fields with the transformation properties under proper Lorentz transformations of a vec-

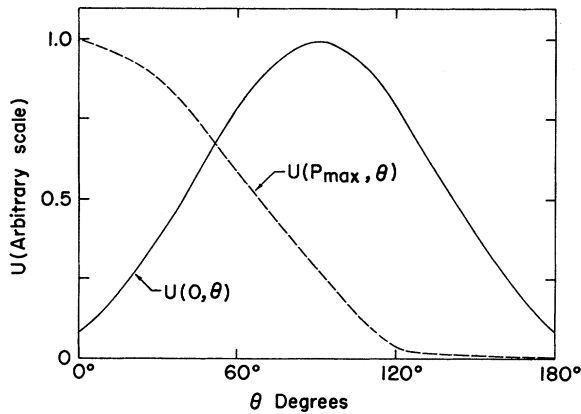


FIG. 1. Pion-neutrino correlation function from K_{μ_3} decay for two values of the pion momentum on the assumption of a "divergenceless" current.

tor. Adopting the convention of calling the K meson pseudoscalar, we may split \mathcal{J}_λ into polar and axial vectors,

$$\mathcal{J}_\lambda = \mathcal{J}_\lambda^{(V)} + \mathcal{J}_\lambda^{(A)}. \quad (2)$$

Since parity is conserved in strong interactions, we see immediately that only $\mathcal{J}_\lambda^{(A)}$ contributes to $K \rightarrow \mu + \nu$ while only $\mathcal{J}_\lambda^{(V)}$ contributes to $K \rightarrow \pi + \mu + \nu$. Now we may be certain that $\mathcal{J}_\lambda^{(A)}$ is not divergenceless, since the $K \rightarrow \mu + \nu$ decays do occur. The hypothesis we wish to test is therefore

$$\alpha^\lambda \mathcal{J}_\lambda = 0. \quad (3)$$

If (3) is correct, we shall have an interesting parallel with the ordinary lepton interactions.

By general arguments of invariance, we may write the meson matrix element in K_{μ_3} decay as

$$\langle \pi | \mathcal{J}_\lambda^{(V)}(x) | K \rangle = [i f_V p_{K\lambda} + i g_V (p_K - p_\pi)_\lambda] \times \exp[i(p_K - p_\pi) \cdot x] \quad (4)$$

Here f_V and g_V are functions of $(p_K - p_\pi)^2$; in the c.m. system $p_K = (0, m_K)$, $p_\pi = (p, E)$, so that f_V and g_V may be taken as functions of p . The distribution $W(p, \theta)$ in \underline{p} and in the angle θ between neutrino and pion is then given by⁵

$$\begin{aligned} W(p, \theta) d\mathbf{p} d\cos\theta &= (1 - x^2 - y^2)^2 (m_K - E)^2 p^2 E^{-1} (1 + x \cos\theta)^{-4} \\ &\times x^2 [f_V(p)]^2 U(p, \theta) d\mathbf{p} d\cos\theta, \end{aligned} \quad (5)$$

where

$$U(p, \theta) = \sin^2\theta + \frac{y^2}{x^2} \left[1 + \frac{(m_K - E) g_V}{m_K f_V} (1 + x \cos\theta) \right]^2, \quad (6)$$

$$x = \frac{p}{(m_K - E)}, \quad y = \frac{m_\mu}{(m_K - E)}.$$

Thus, without using the "divergenceless" hypothesis, we obtain for \underline{U} just $\sin^2\theta$ plus an unknown positive correction function, asymmetric about $\theta = 90^\circ$. If we made a "reasonable" guess on the behavior of g_V/f_V , we would expect:

(a) For low \underline{p} , $x \rightarrow 0$, so that the second term in \underline{U} would dominate and \underline{U} would become constant in θ .

(b) For high \underline{p} , we have $y^2/x^2 \rightarrow 0.24$ so that we would expect the unknown correction term to be somewhat less important than the $\sin^2\theta$ term.

(c) Since there is no reason to assume g_V and f_V are the same for the $K_{\mu_3}^0$ and $K_{\mu_3}^\pm$ decays, the decay rate, spectrum shape, and angular correlations would be different in the two cases.

Now, let us suppose that the hypothesis (3) holds. Then from (4) we see that

$$g_V/f_V = \frac{-p_K \cdot (p_K - p_\pi)}{(p_K - p_\pi)^2} = \frac{m_K}{(m_K - E)} \left(\frac{1}{1 - x^2} \right). \quad (7)$$

This leads to a unique formula for the angular correlation; inserting (7) in (6) we obtain

$$U(p, \theta) = \sin^2\theta + \left(\frac{y}{1 - x^2} \right)^2 (x + \cos\theta)^2. \quad (8)$$

This just reverses all our former expectations; we now make the following predictions:

(a') For low \underline{p} , the $(1/x^2)$ singularity has disappeared and we get

$$U(0, \theta) = \sin^2\theta + 0.087 \cos^2\theta, \quad (9)$$

so that the $\sin^2\theta$ term is dominant (see Fig. 1).

(b') The ratio g_V/f_V becomes large at high \underline{p} , and at \underline{p}_{\max} ,

$$U(\underline{p}_{\max}, \theta) = \sin^2\theta + 5.15 (0.90 + \cos\theta)^2, \quad (10)$$

so that the correction term is dominant (see Fig. 1).

(c') Since (7) hold for both $K_{\mu_3}^0$ and $K_{\mu_3}^\pm$,

the angular correlations should be the same for both decay types at any given p . Of course, the spectra and total decay rates may still be entirely different.

It may be noted that (8) holds for K_{e3} decays if we replace m_μ in \underline{y} by m_e . This has the effect of eliminating the second term in \underline{U} at all but the highest (i.e., within a few tenths Mev of E_{\max}) pion energies. The K_{e3} mode may serve as a test of our assumption about the type of coupling, but not of the "divergenceless" assumption (3).

Experiments to check (8) in $K_{\mu 3}$ decay should be possible; perhaps the most favorable case would be for stopped K^+ in heavy liquid bubble chambers. If (8) turns out to be incorrect, this will mean that neither $\mathcal{J}_\mu^{(V)}$ nor $\mathcal{J}_\mu^{(A)}$ are divergenceless although, of course, they may be approximately so. If (8) is verified, this will be a striking confirmation of the Feynman-Gell-Mann idea. Furthermore, if we call the hyperon parity positive, in the "global" limit there is a vector, but not an axial vector, strangeness non-conserving current; verification of (8) would then perhaps be an indication that the global approximation is not too bad, and that the \underline{K} particle is pseudoscalar relative to hyperons. Of course, even if (3) held exactly there would still be a renormalization of the coupling constants for leptonic hyperon decay, proportional to the hyperon-nucleon mass differences.

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¹E. C. G. Sudarshan and R. E. Marshak, Proceedings of the Padua-Venice Conference, September, 1957 (unpublished), and Phys. Rev. **109**, 1860 (1958); cf. also R. P. Feynman and M. Gell-Mann, [Phys. Rev. **109**, 193 (1958)] who essentially make the same hypothesis that only the positive chiral states of the Dirac particle fields are coupled in weak interactions.

²R. P. Feynman and M. Gell-Mann, reference 1; see also S. S. Gerstein and J. B. Zeldovich, Zhur. Eksptl. i Teoret. Fiz. U.S.S.R. **29**, 698 (1955) [translation: Soviet Phys. JETP **2**, 576 (1956)].

³M. Gell-Mann (to be published).

⁴M. L. Goldberger and S. B. Treiman (private communication).

⁵Our formula (except for a change of sign inside the brackets for \underline{U}) is identical with that of A. Pais and S. B. Treiman [Phys. Rev. **105**, 1616 (1957)].