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Quantum Field Theory of Interacting Tachyons

J. DHAR
Department of Physics and Astrophysics, University of Delhi, Delhi, India

AND

E. C. G. SUDARSHAN
Department of Physics, Syracuse University, Syracuse, New York, U. S. A. and Centre for Advanced Study in Physics, University of Delhi, Delhi, India.

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A quantum field theory of spin-0 particles traveling with speeds greater than that of light has been constructed. The theory constructed here is explicitly Lorentz-invariant; and the quanta of the field obey Bose statistics. Formalism developed for the free field has been extended to the case of interaction of these particles with nucleons. A new feature of theory is the occurrence of negative-energy particles; this is a necessary consequence of the relativistic invariance of the theory, since the distinction between positive and negative energies is not a relativistically invariant concept for such particles. The occurrence of these negative-energy particles does not, however, prevent the theory from being meaningful; the physical interpretation of the situation is provided by the postulate that *any process* involving negative-energy particles is to be identified with a physical process with *only* positive-energy particles traveling in the opposite direction, with the roles of emission and absorption interchanged. The scattering amplitudes are the same as in the usual theory with m^2 replaced by $-m^2$.

INTRODUCTION

IT has generally been believed that no particle can exceed the speed of light.[1] This has meant in turn that in formulating the quantum theory of fields it has been tacitly assumed that all the particles described by such fields belong to one of two classes: those which have a finite rest mass and travel with speeds less than the speed of light; and those which have zero rest mass and hence always travel with the speed of light. We may also consider a third class of particles: those which travel with speeds greater than the speed of light. If we try to ascribe a rest mass to such particles it will be pure imaginary, but this leads to no conceptual difficulties since these particles cannot be brought to rest. The real difficulty with such particles has been that the usual Lorentz transformation properties lead to negative energies in suitable frames. Several years ago it was shown how this difficulty may be overcome[2]; crucial to the resolution of the difficulty is the reinterpretation of "negative-energy particles traveling backward in time" to be positive-energy particles traveling forward in time. All the puzzles and paradoxes that have been put up by various people could be resolved using this basic idea, at least as far as classical theory is concerned. It also motivated two brilliant experiments[3] searching for these faster-than-light particles, which we shall call tachyons.[4] Both these experiments had negative results, but we believe that this should be interpreted to mean that, like particles of vanishing mass, tachyons carry no electric charge.[5]

It is now of interest to consider a quantum theory of

[1] H. Poincaré, Bull. Sci. Math. 28, 302 (1904); A. Einstein, Ann. Physik (Paris) 17, 891 (1905).
[2] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. 30, 718 (1962).
[3] T. Alväger, P. Erman, Nobel Inst. Report 1966 (unpublished); T. Alväger and N. M. Kreisler, Phys. Rev. 171, 1357 (1968).
[4] The name "tachyon" is the contribution of G. Feinberg, Phys. Rev. 159, 1089 (1967).
[5] E. C. G. Sudarshan, Proceedings of the Nobel Symposium, Lerum Sweden, 1968 (to be published). For an exactly solvable Hamiltonian model for the charged scalar theory using both positive- and negative-energy mesons, see E. C. G. Sudarshan, in *Theoretical Physics 1961* (W. A. Benjamin, Inc., New York, 1962).

tachyons. The quantum theory would have to incorporate, in a suitably amended form, our resolution of the negative energy difficulty of the geometric transformation. We know that this must imply fundamental differences in the principle of quantization since there is no invariant distinction between positive and negative frequencies. Arons and Sudarshan have shown how this is to be carried out; they introduce creation and destruction operators for both positive and negative energy tachyons.[6] The quantum field theory so obtained uses the physical reinterpretation postulate: *In any transition amplitude viewed from any frame, a negative-energy tachyon in the initial (final) state is to be replaced by an antitachyon in the final (initial) state with the opposite values of all additive dynamical variables.*

Since tachyons have spacelike momenta, their polarization states should furnish a unitary representation of the Lorentz group in three dimensions. This group is composed of rotations around the direction of the spatial momentum and the pure Lorentz transformations in the two directions perpendicular to it. Since this group is noncompact except for its one-dimensional representation, all other representations are infinite-dimensional. In other words, except for spin-0 tachyons, all other tachyons must have an infinite number of polarization states.[5]

It is advisable to study scalar (or pseudoscalar) tachyons to start with; in this case we can use the Klein-Goron Lagrangian to exhibit a quantum theory of the tachyon field. We have studied the free field and the interacting field. The latter case can be worked out with the same degree of consistency as the local field theory of ordinary quantized (scalar) fields in interaction. This theory is developed in detail in the subsequent sections of this paper. Consistent with the fundamental theorem on the connection between spin and statistics,[7] we find that spin-0 tachyons satisfy Bose statistics.

I. FREE TACHYON FIELD

The free scalar tachyon field satisfies the equation of motion:

$$(\partial^2/\partial x^{02}-\nabla^2-m^2)\phi(x)=0. \tag{1.1}$$

The general solution of (1.1) may be written in the form

$$\begin{aligned}\phi(x)&=\frac{1}{(2\pi)^{3/2}}\int d^4k\; a(k)\delta(k^2+m^2)e^{-ik_0x_0+i\mathbf{k}\cdot\mathbf{x}}\\&=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{2\omega}\{a(\omega,\mathbf{k})e^{-i\omega x_0+i\mathbf{k}\cdot\mathbf{x}}\\&\qquad+a(-\omega,-\mathbf{k})e^{i\omega x_0-i\mathbf{k}\cdot\mathbf{x}}\},\end{aligned} \tag{1.2}$$

where

$$\omega=+(\mathbf{k}^2-m^2)^{1/2}.$$

Since $\mathbf{k}$, k_0 are real, it follows that

$$\mathbf{k}^2>m^2. \tag{1.3}$$

Canonical quantization requires

$$\begin{aligned}&\delta(x^0-y^0)[\phi^\dagger(x),\phi(y)]=0,\\&\delta(x^0-y^0)[\phi^\dagger(x),\phi(y)]=i\bar{\delta}(\mathbf{x}-\mathbf{y})\delta(x^0-y^0).\end{aligned} \tag{1.4}$$

In the second equation the function $\bar{\delta}$ that appears on the right-hand side is the "filtered δ function," which does not contain any spatial momenta violating Eq. (1.3):

$$\begin{aligned}\bar{\delta}(\mathbf{x})&=\frac{1}{(2\pi)^3}\int d^3k\; e^{i\mathbf{k}\cdot\mathbf{x}}\theta(\mathbf{k}^2-m^2)\\&=\delta(\mathbf{x})-\frac{1}{2\pi^2}\int_0^m dk\; k^2\frac{\sin kr}{kr}.\end{aligned} \tag{1.5}$$

From (2) and (4) we obtain the commutation relations

$$\begin{aligned}&[a(\omega,\mathbf{k}),a(\omega',\mathbf{k}')]=0,\\&[a(\omega,\mathbf{k}),a^\dagger(\omega',\mathbf{k}')]=2\omega\delta(\mathbf{k}-\mathbf{k}'),\\&[a(\omega,\mathbf{k}),a^\dagger(-\omega',\mathbf{k}')]=0,\\&[a(-\omega,\mathbf{k}),a^\dagger(-\omega',\mathbf{k}')]=-2\omega\delta(\mathbf{k}-\mathbf{k}').\end{aligned} \tag{1.6}$$

Since *all* wave fields appropriate to (1) for $\phi(x)$ are expanded in terms of the annihilation operators, we have to introduce the conjugate field $\phi^\dagger(x)$ which contains the creation operators. The complete tachyon field is

$$\begin{aligned}\chi(x)&=(1/\sqrt{2})[\phi(x)+\phi^\dagger(x)]\\&=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{2\omega}\Big\{\frac{1}{\sqrt{2}}[a(\omega,\mathbf{k})+a^\dagger(-\omega,-\mathbf{k})]e^{-i\omega x_0+i\mathbf{k}\cdot\mathbf{x}}\\&\qquad+\frac{1}{\sqrt{2}}[a^\dagger(\omega,\mathbf{k})+a(-\omega,-\mathbf{k})]e^{i\omega x_0-i\mathbf{k}\cdot\mathbf{x}}\Big\}.\end{aligned} \tag{1.7}$$

Using (1.6) we can write down the general commutation relation for the tachyon field (1.7):

$$\begin{aligned}[\chi(x),\chi(y)]&=-\frac{i}{(2\pi)^3}\int\frac{d^3k}{\omega}\sin\omega(x_0-y_0)e^{i\mathbf{k}\cdot(\mathbf{x}\cdot\mathbf{y})}\\&=i\Delta(x-y).\end{aligned} \tag{1.8}$$

The invariant function $\Delta(x-y)$ so defined contains all

[6] M. E. Arons and E. C. G. Sudarshan, Phys. Rev. **173**, 1622 (1968).
[7] E. C. G. Sudarshan, Proc. Ind. Acad. Sci. **A67**, 284 (1968).

frequencies, but only spatial momenta satisfying the inequality (3). This function has the properties

$$\Delta(x-y)\delta(x^0-y^0)=0\,, \tag{1.9a}$$

$$[(\partial/\partial y^0)\Delta(x-y)]\delta(x^0-y^0)=\bar{\delta}(\mathbf{x}-\mathbf{y})\delta(x^0-y^0)\,, \tag{1.9b}$$

$$(\partial^2/\partial x^{02}-\nabla^2-m^2)\Delta(x-y)=0\,, \tag{1.9c}$$

$$\Delta(x-y)=-\Delta(y-x)\,. \tag{1.9d}$$

As a consequence we obtain the equal-time commutation relations

$$[\chi(x),\chi(y)]\delta(x^0-y^0)=0\,,\quad [\chi(x),\dot{\chi}(y)]\delta(x^0-y^0)=i\bar{\delta}(\mathbf{x}-\mathbf{y})\delta(x^0-y^0)\,. \tag{1.10}$$

This tachyon field is self-conjugate (particles and antiparticles are identical); if we want to describe non-self-conjugate tachyons, we can write

$$\chi(x)=(1/\sqrt{2})[\chi_1(x)+i\chi_2(x)]$$
$$=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{2\sqrt{2}\omega}\{[a(\omega,\mathbf{k})+b^+(-\omega,-\mathbf{k})]e^{-i\omega x_0+i\mathbf{k}\cdot\mathbf{x}}+[b^+(\omega,\mathbf{k})+a(-\omega,-\mathbf{k})]e^{i\omega x_0-i\mathbf{k}\cdot\mathbf{x}}\}\,, \tag{1.11}$$
$$a(\omega,\mathbf{k})=(1/\sqrt{2})[a_1(\omega,\mathbf{k})+ia_2(\omega,\mathbf{k})],\quad b(\omega,\mathbf{k})=(1/\sqrt{2})[a_1(\omega,\mathbf{k})-ia_2(\omega,\mathbf{k})]\,,$$

where $\chi_1(x)$ and $\chi_2(x)$ are two self-conjugate tachyon fields. The general commutation relations are

$$[\chi(x),\chi^\dagger(y)]=i\Delta(x-y)\,, \tag{1.12}$$

with the creation and annihilation operators satisfying

$$\begin{aligned}
&[a(\omega,\mathbf{k}),a^\dagger(\omega',\mathbf{k}')]=2\omega\delta(\mathbf{k}-\mathbf{k}')\,,\\
&[a(\omega,\mathbf{k}),a^\dagger(-\omega',\mathbf{k}')]=0\,,\\
&[b(\omega,\mathbf{k}),b^\dagger(\omega',\mathbf{k}')]=2\omega\delta(\mathbf{k}-\mathbf{k}')\,,\\
&[a(-\omega,\mathbf{k}),a^\dagger(-\omega',\mathbf{k}')]=-2\omega\delta(\mathbf{k}-\mathbf{k}')\,,\\
&[a(\omega,\mathbf{k}),b^\dagger(\omega',\mathbf{k}')]=0\,,
\end{aligned} \tag{1.13}$$

and so on.

To complete the quantization of the free field, we must also introduce the vacuum state $|0\rangle$ by requiring that it be annihilated by all the annihilation operators:

$$a(\omega,\mathbf{k})|0\rangle=a(-\omega,-\mathbf{k})|0\rangle=b(\omega,\mathbf{k})|0\rangle=b(-\omega,-\mathbf{k})|0\rangle=0\,. \tag{1.14}$$

We have immediately a Fock representation of the field χ in terms of the particles and the antiparticles. For the self-conjugate field we have only one kind of particle.

The total number of particles minus the number of antiparticles is given by

$$Q=\int\frac{d^3k}{2\omega}[a^\dagger(\omega,\mathbf{k})a(\omega,\mathbf{k})-a^\dagger(-\omega,\mathbf{k})a(-\omega,\mathbf{k})-b^\dagger(\omega,\mathbf{k})b(\omega,\mathbf{k})+b^\dagger(-\omega,\mathbf{k})b(-\omega,\mathbf{k})]\,. \tag{1.15}$$

This coincides with the field-theoretic expression

$$Q=i\int[\Phi_a{}^\dagger(x)\dot{\Phi}_a(x)-\dot{\Phi}_a{}^\dagger(x)\Phi_a(x)-\Phi_b{}^\dagger(x)\dot{\Phi}_b(x)+\dot{\Phi}_b{}^\dagger(x)\Phi_b(x)]=\int d^3x\,j^0(x)\,, \tag{1.16}$$

where

$$j^\mu(x)=i\{\Phi_a{}^\dagger(x)\partial^\mu\Phi_a(x)-[\partial^\mu\Phi_a(x)]\Phi_a(x)-\Phi_b(x)\partial^\mu\Phi_b(x)+[\partial^\mu\Phi_b(x)]\Phi_b(x)\}\,, \tag{1.17}$$

and the fields Φ_a and Φ_b are given by

$$\begin{aligned}
\Phi_a(x)&=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{2\omega}\{a(\omega,\mathbf{k})e^{-i\omega x_0+i\mathbf{k}\cdot\mathbf{x}}+a(-\omega,-\mathbf{k})e^{i\omega x_0-i\mathbf{k}\cdot\mathbf{x}}\}\,,\\
\Phi_b(x)&=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{2\omega}\{b(\omega,\mathbf{k})e^{-i\omega x_0+i\mathbf{k}\cdot\mathbf{x}}+b(-\omega_1-\mathbf{k})e^{i\omega x_0-i\mathbf{k}\cdot\mathbf{x}}\}\,.
\end{aligned} \tag{1.17$'$}$$

$j^\mu(x)$ is the charge-current four-vector satisfying the usual continuity equation

$$\partial_\mu j^\mu(x)=0\,. \tag{1.18}$$

It is understood that the expressions (16), (17), and (19) are normal-ordered.

The expressions for the energy and momentum of the field are

$$\begin{aligned}
H&=\int d^3x[\dot{\Phi}_a{}^\dagger(x)\dot{\Phi}_a(x)+\nabla\Phi_a{}^\dagger(x)\cdot\nabla\Phi_a(x)+\dot{\Phi}_b{}^\dagger(x)\dot{\Phi}_b(x)+\nabla\Phi_b{}^\dagger(x)\cdot\nabla\Phi_b(x)\\
&\qquad-m^2\Phi_a{}^\dagger(n)\Phi_a(x)-m^2\Phi_b{}^\dagger(x)\Phi_b(x)]\,,\\
\mathbf{P}&=-\int d^3x[\dot{\Phi}_a{}^\dagger(x)\nabla\Phi_a(x)+\nabla\Phi_a{}^\dagger(x)\dot{\Phi}_a(x)+\dot{\Phi}_b{}^\dagger(x)\nabla\Phi_b(x)+\nabla\Phi_b{}^\dagger(x)\dot{\Phi}_b(x)]\,.
\end{aligned} \tag{1.19}$$

These yield, in terms of the particle operators,

$$\begin{aligned}
H&=\int\frac{d^3k}{2\omega}\,\omega[a^\dagger(\omega,\mathbf{k})a(\omega,\mathbf{k})+a^\dagger(-\omega,\mathbf{k})a(-\omega,\mathbf{k})+b^\dagger(\omega,\mathbf{k})b(\omega,\mathbf{k})+b^\dagger(-\omega,\mathbf{k})b(-\omega,\mathbf{k})]\,,\\
\mathbf{P}&=\int\frac{d^3k}{2\omega}\,\mathbf{k}[a^\dagger(\omega,\mathbf{k})a(\omega,\mathbf{k})-a^\dagger(-\omega,\mathbf{k})a(-\omega,\mathbf{k})+b^\dagger(\omega,\mathbf{k})b(\omega,\mathbf{k})-b^\dagger(-\omega,\mathbf{k})b(-\omega,\mathbf{k})]\,.
\end{aligned} \tag{1.20}$$

By virtue of the commutation relations (10) and (13), we can verify the standard relations

$$[\chi(x),Q]=\chi(x),$$
$$[\chi(x),H]=i\dot{\chi}(x), \quad (1.21)$$
$$[\chi(x),\mathbf{P}]=i^{-1}\nabla\chi(x).$$

The theory so constructed is manifestly invariant under the Poincaré group. The inequality (3) is a relativistically invariant requirement as long as the field equation (1) is implied. The "filtered" commutator function $\Delta(x)$ defined by (8) is also an explicitly covariant function by virtue of the fact that $\Delta(x)$ satisfies Eq. (9c), so that it gets contributions only from the mass shell.

The present theory has the unfamiliar features of having negative-energy particles. We would like to consider as physical only those particles which have positive energy. These negative-energy particles, however, are a necessary consequence of relativistic invariance for faster-than-light particles, since the distinction between positive energy and negative energy is not relativistically invariant for such particles. Any attempt at avoiding Fock states for negative-energy particles violates the relativistic invariance of the theory.

The same problem occurs in the classical theory of tachyons also, though in this case we do not deal with state vectors. A close study of the physical framework showed that the negative-energy particles travel backwards in time. Hence a process involving negative-energy particles is physically indistinguishable from another process involving positive-energy particles traveling in the opposite direction, with the roles of emission and absorption interchanged. This provides the basis for the physical interpretation of our theory. Any *process* involving negative-energy particles is to be identified with a physical process with only positive-energy particles traveling in the opposite direction, with the roles of emission and absorption interchanged.[6]

It is most important to note that this interchange of the roles of emission and absorption can be done *only for processes* (i.e., transition amplitudes) *but not for states*. Any attempt to do it for the states would lead to violation of relativistic invariance of the theory.[8] When the roles of emission and absorption are interchanged, the negative-energy particle in the initial state corresponds physically to a positive-energy antiparticle in the final (*not* the initial) state. Such a transformation requires the consideration of a process which has both initial and final states; it would not be possible to give such a physical interpretation if we had to deal with states alone without considering processes.

As long as we restrict attention to positive energy particles alone, the states of the system can be given the usual invariant probability interpretation.

[8] S. Tanaka, Progr. Theoret. Phys. (Kyoto) **24**, 171 (1960); G. Feinberg, Phys. Rev. **159**, 1089 (1967).

The physical interpretation of processes involving negative-energy particles together with the possibility of the change of sign of the energy of tachyons under Lorentz transformations means that a physical process of a certain type in one Lorentz frame may appear as a process of a different type in another Lorentz frame. For example, a process of elastic scattering of two particles may appear as the decay of a particle into three particles in another Lorentz frame.

II. GREEN'S FUNCTIONS AND CONTRACTION FUNCTIONS

As a preliminary to considering interactions of tachyon fields, we wish to consider some mathematical questions. If we consider the inhomogeneous equation

$$(\partial^2/\partial x^{02}-\nabla^2-m^2)\chi(x)\equiv K_x\chi(x)=\xi(x), \quad (2.1)$$

then a solution for $\chi(x)$ is given by

$$-\int G(x-y)\xi(y)d^4y,$$

where $G(x-y)$ is any Green's function satisfying

$$K_xG(x-y)=-\delta(x-y). \quad (2.2)$$

The general solution is

$$\chi(x)=\chi_0(x)-\int d^4y\, G(x-y)\xi(y), \quad (2.3)$$

where $\chi_0(x)$ is any solution of the homogeneous equation

$$K_x\chi_0(x)=0. \quad (2.4)$$

We may rewrite (2.3) in the form

$$\chi_0(x)=\chi(x)+\int d^4y\, G(x-y)\xi(y), \quad (2.5)$$

which shows that for the same solution $\chi(x)$, as we change the Green's function $G(x-y)$ we also change $\chi_0(x)$.

If $G(x-y)$ and $G_1(x-y)$ are any two Green's functions, (G_1-G) is a solution of the homogeneous equation, so that

$$\int [G_1(x-y)-G(x-y)]\xi(y)d^4y$$

is a solution of the homogeneous equation. The change from G to G_1 without any change of the physical field $\chi(x)$ thus implies a change from the asymptotic field $\chi_0(x)$ to

$$\chi_0(x)-\int d^4y[G(x-y)-G_1(x-y)]\xi(y). \quad (2.6)$$

We shall have occasion to make use of this result in the next section.

Two simple choices for the Green's function are

$$G(x-y)=\bar{\Delta}(x-y)=\frac{P}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot(x-y)}}{k^2+m^2} \quad (2.7)$$

and

$$G_1(x-y)=\Delta_F(x-y)=\frac{1}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot(x-y)}}{k^2+m^2+i\epsilon}. \quad (2.8)$$

The change of Green's function from (2.7) to (2.8) without change of the Heisenberg field $\chi(x)$ implies the change from the asymptotic field $\chi_0(x)$ to

$$\chi_0(x)-\frac{\frac{1}{2}i}{(2\pi)^3}\int d^4y\int d^4k\, e^{-ik\cdot(x-y)}\xi(y)\delta(k^2+m^2)$$

$$=\chi_0(x)-\tfrac{1}{2}i\int d^4y\,\Delta^{(1)}(x-y)\xi(y), \quad (2.9)$$

where $\Delta^{(1)}$ is the symmetric invariant solution of the free-field equation:

$$\Delta^{(1)}(x)=\frac{1}{(2\pi)^3}\int d^4k\, e^{-ik\cdot x}\delta(k^2+m^2). \quad (2.10)$$

This is an essential result for the calculation of the scattering amplitude in perturbation theory.

Let us now calculate the contraction functions for the tachyon fields. Care must be exercised in its evaluation because of the inequality (1.3) that is satisfied by the spatial momenta. Since the contraction functions involve discontinuous functions of time, they get contributions from momenta off the mass shell and, consequently, (1.3) is no longer an invariant restriction. The contraction function is defined by

$$\tau(x-y)=\langle 0|T(\chi^\dagger(x),\chi(y))|0\rangle \quad (2.11)$$
$$=T(\chi^\dagger(x),\chi(y))-N(\chi^\dagger(x),\chi(y)).$$

Making use of (1.11) and (1.13) we obtain

$$\tau(x-y)=-\tfrac{1}{2}i\frac{1}{(2\pi)^3}\epsilon(x_0-y_0)\int\frac{d^3k}{\omega}e^{+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$
$$\times\sin\omega(x_0-y_0)\theta(\mathbf{k}-m^2)$$
$$=\frac{i}{(2\pi)^4}P\int d^4k\frac{e^{-ik\cdot(x-y)}}{k^2+m^2}\theta(\mathbf{k}^2-m^2). \quad (2.12)$$

This contraction function is *not* relativistically invariant because of the appearance of the factor $\theta(\mathbf{k}^2-m^2)$, which depends on the Lorentz frame. We can write

$$\tau(x-y)=i\bar{\Delta}(x-y)-iD(x-y;\eta), \quad (2.13)$$

where we have introduced the timelike vector η with components (1,0,0,0), and $D(x-y;\eta)$ is the function

$$D(x-y;\eta)=\frac{1}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot(x-y)}}{(k^2+m^2)}$$
$$\times\theta(k^2+m^2-(\eta\cdot k)^2). \quad (2.14)$$

The singularity of the integrand is integrable and we need not bother to indicate a positive imaginary infinitesimal in the denominator.

It is important to note that the contraction function (2.12) is not a Green's function, since the components with spatial momenta not satisfying the inequality (1.3) are not contained in the contraction function.

III. INTERACTING TACHYON FIELDS

As an example of tachyon interactions, we consider the coupling of a self-conjugate tachyon to a pair of fermion fields:

$$\mathcal{L}_{\text{int}}=g\bar{\psi}(x)\psi(x)\chi(x). \quad (3.1)$$

The usual symmetrization of the fermion fields is understood and will not be explicitly indicated. This interaction would lead to single tachyon emissions or absorptions. The energy-momentum conservation laws would permit these emissions or absorptions to become real processes:

$$\text{fermion}\rightarrow\text{fermion}+\text{tachyon}. \quad (3.2)$$

However, such a reaction is forbidden as a physical process unless the initial fermion is in motion with a suitable kinetic energy. This is a characteristic property of tachyons: An ordinary particle stable in its own rest system can decay into *itself* and a tachyon in flight. Such a possibility affords a natural method of detection of tachyons in high-energy reactions.

We can now consider the effects to second order in the interaction (3.1). The simplest of these is the "Compton" scattering of a tachyon by the fermion. The calculation proceeds in the usual fashion and leads to a relativistically invariant scattering amplitude:

$$F=\bar{u}(A+\gamma\cdot QB)u,$$
$$A=\tfrac{1}{2}g^2M[(1/s-M^2)+(1/u-M^2)], \quad (3.3)$$
$$B=\tfrac{1}{2}g^2[(1/s-M^2)-(1/u-M^2)].$$

It is understood that we are talking about elastic scattering of positive-energy tachyons. In (3.3) the quantities s and u are the squares of the center-of-mass energies in the direct channel and in the channel with the tachyons crossed. This same amplitude describes also the decay of a fermion into itself and a pair of tachyons. It is also possible to obtain the reactions with the tachyons "crossed" by going to a Lorentz frame which would tend to give both the mesons negative energy. Unlike the case of reactions involving ordinary particles only, where crossed reactions can only be related to each other by analytic continuation through a set of unphysical states, the present theory relates reactions with tachyon lines crossed by suitable Lorentz transformations.

The other second-order process that is entailed by the interaction (3.1) is the "Möller" scattering of two fermions with the exchange of a tachyon. To compute this we calculate the tachyon field generated by the first fermion at the position of the second fermion. In the previous section we have calculated the meson field produced by the scalar source:

$$\chi(x)=\chi_0(x)-g\int d^4y\, G(x-y)\bar{\psi}(y)\psi(y)\,. \tag{3.4}$$

The second term is the contribution of the fermion at the point y. The interaction of this term with the fermion field at the point x is given by

$$-g^2\bar{\psi}(x)\psi(x)\int d^4y\, G(x-y)\bar{\psi}(y)\psi(y)\,.$$

The complete interaction is

$$-\tfrac{1}{2}g^2\int d^4x\int d^4y\, \bar{\psi}(x)\psi(x)G(x-y)\bar{\psi}(y)\psi(y)\,. \tag{3.5}$$

The factor $\frac{1}{2}$ is added to compensate for the double counting. Choosing Δ_F as the Green's function according to (2.8) we get for the effective interaction

$$-\tfrac{1}{2}g^2\int d^4x\int d^4y\int d^4k\, \bar{\psi}(x)\psi(x)e^{ik\cdot x}\frac{1}{k^2+m^2+i\epsilon}\times e^{-ik\cdot y}\bar{\psi}(y)\psi(y)\,. \tag{3.6}$$

This leads to the standard form for the fermion-fermion scattering by exchange of a scalar meson. The only difference is that this amplitude develops a pole in the physical region at a value of the momentum transfer squared equal to $-m^2$, reflecting the possibility of reaction (3.2) occurring as a physical process.

It is interesting to note that (3.6) involves the invariant Green's function rather than the noninvariant contraction function. This means that the tachyon-fermion Yukawa interaction is not simply expressible as a trilinear interaction in the *interaction picture*, though the original interaction in the Heisenberg picture is trilinear.

Comparison with (2.13) shows that there should be a direct fermion interaction (whose structure and coupling are determined by the Yukawa interaction) which should be added to the trilinear interaction so as to produce a relativistically invariant result. The total interaction in the interaction picture has the form

$$g\int d^4x\, \bar{\psi}(x)\psi(x)\chi(x)+\tfrac{1}{2}g^2\int d^4x\int d^4y \times\bar{\psi}(x)\psi(x)D(x-y;\eta)\bar{\psi}(y)\psi(y)\,. \tag{3.7}$$

The second term is nonlocal in form; but the sum of these two interactions restores Lorentz invariance to the scattering amplitudes. For second-order predictions this is verified by inspection; and the first-order predictions are unaltered by it.

The choice of the Green's function in (3.5) was up to us; and reflects the ambiguity in the choice of the tachyon field "produced by the source $g\bar{\psi}(x)\psi(x)$." If we change the Green's function this contribution would change. We have chosen Δ_F to ensure the proper one-particle structure to the scattering amplitude. Such a structure, together with the fact that the tachyon mass is pure imaginary, leads to several simple methods to search for tachyons in high-energy particle reactions.

IV. REDUCTION OF THE S MATRIX

To develop a covariant perturbation theory for the interaction (3.1), we work in the interaction picture and consider the expression

$$S=T\left[\exp\left(i\int W(x)d^4x\right)\right], \tag{4.1}$$

where $W(x)$ is the interaction in the interaction picture. Starting with the interaction

$$g\bar{\psi}(x)\psi(x)\chi(x)$$

in the Heisenberg picture, we get two terms in the interaction picture:

$$W(x)=g\bar{\psi}(x)\psi(x)\chi(x)+\tfrac{1}{2}g^2\bar{\psi}(x)\psi(x)\times\int d^4y\, D(x-y)\bar{\psi}(y)\psi(y)\,. \tag{4.2}$$

We have already seen that the sum of these two terms is Lorentz-invariant, but neither of them is Lorentz-invariant separately.

To obtain the scattering amplitudes from (4.1), we need to proceed to a normal-ordering operation. Before doing that we recall from Sec. II that there is still some freedom in the definition of the asymptotic field; compare the discussion following (2.6), and, in particular, (2.9). We make the choice

$$\chi_{\text{in}}(x)=\chi_0(x)+\tfrac{1}{4}ig\int d^4y\, \Delta^{(1)}(x-y)\bar{\psi}(y)\psi(y)\,. \tag{4.3}$$

This choice of the asymptotic field is equivalent to replacing (4.2) by

$$W_1(x)=g\bar{\psi}(x)\psi(x)\chi_{\rm in}(x)+\tfrac{1}{2}g^2\bar{\psi}(x)\psi(x)\times\int d^4y\, D_1(x-y;\eta)\bar{\psi}(y)\psi(y)\,, \quad (4.4)$$

where

$$D_1(x-y;\eta)=D(x-y;\eta)-\tfrac{1}{2}i\Delta^{(1)}(x-y)\,. \quad (4.5)$$

The reduction of the S matrix now proceeds as in the usual theory: We rewrite (4.1) in a normal-ordered expansion for the asymptotic field $\chi_{in}(x)$. The coefficients of appropriate normal-ordered operators yield the various transition amplitudes. The results so calculated of course contain the standard divergences and would have to be subjected to a suitable renormalization before physically meaningful results can be extracted. There is essentially no difference between the renormalization of this theory and one in which the tachyon field is replaced by an ordinary scalar meson field; we shall content ourselves, therefore, with a derivation of the unrenormalized covariant perturbation expansion.

A particularly simple amplitude is the Möller scattering amplitude; to second order there are no divergences in the perturbation calculation of this amplitude. We get two contributions to this amplitude: one in second order from the first term of (4.4) and one in first order from the second term of (4.4). These are, respectively,

$$S^{(2)}=\frac{(-i)^2}{2!}g^2\int d^4x\int d^4y\,{:}\bar{\psi}(x)\psi(x)\tau(x-y)\bar{\psi}(y)\psi(y){:}\,,$$

$$S^{(1)}=\tfrac{1}{2}(-i)g^2\int d^4x\int d^4y\,{:}\bar{\psi}(x)\psi(x)D_1(x-y;\eta)\bar{\psi}(y)\psi(y){:}\,. \quad (4.6)$$

The sum of these two terms leads to the familiar expression

$$\tfrac{1}{2}(-i)^2g^2\int d^4x\int d^4y\,{:}\bar{\psi}(x)\psi(x)[i\Delta_F(x-y)]\bar{\psi}(y)\psi(y){:}\,. \quad (4.7)$$

This expression is of the same form as would have been derived from an interaction of the form

$$W_0(x)=g\bar{\psi}(x)\psi(x)\chi(x)$$

but with a contraction function

$$\tau_0(x)=i\Delta_F(x-y)\,.$$

This simple property, demonstrated here only for second-order amplitudes, is true to any order in perturbation theory. To fourth order we have verified it by direct calculation in the Appendix; a general combinatorial argument can be given to any order in perturbation theory.

The situation here is very similar to quantum electrodynamics in the radiation (Coulomb) gauge.[9] In this case, for each value of the momentum there are only two types of photons which are both transverse. The contraction function of two such Maxwell field operators is not covariant. Hence we have to add the direct Coulomb interaction between the electric charge densities with the coupling strength $\frac{1}{2}e^2$. The net result of all this is that the perturbation series can be developed *as if* the contraction function were covariant and as if there were longitudinal and scalar photons.

In the case of tachyon fields the covariant effective contraction function looks as if there were tachyons with complex energies in the range of 0 to $\pm im$. But, in fact, there are no states with these imaginary energies; instead we have the direct interaction through $D(x-y)$ with this form factor.

V. DISCUSSION

The study outlined in this paper has substantiated the view that the *usual* objections to the possibility of faster-than-light particles in relativistic quantum theory can be overcome. There may be objections of a fundamental nature which are not yet known to us which may make them physically inconsistent, but there is no reason why we must not search for such particles.

In this paper we have shown how to construct a quantum field theory of spin-zero tachyons and their interactions. It was necessary to construct a space of particles of both positive and negative energies. The physical processes, however, describe only positive-energy particles. It is thus clear that the factorization of a transition amplitude into initial and final states cannot be relativistically invariant if tachyons are included. This can be done only in the enlarged space containing both positive- and negative-energy states.

We have shown in this paper that the theory of interacting tachyons leads to processes in which the tachyon-exchange diagrams have the usual one-particle structure though the steps leading to this structure are nontrivial. Once such a structure is obtained, we could use it to search for tachyons; if the scattering amplitude between two ordinary particles has a "resonance" peak for a fixed value of the momentum transfer, independent of the center-of-mass energy, we have evidence for a tachyon. This feature, though motivated by perturbation theory, is valid as long as there are tachyons which are strongly coupled.

The most interesting feature of tachyons is the possibility that a particle which is stable "at rest" may emit a tachyon when the particle is moving with sufficient speed. This would be an alternate method of detecting tachyons. A search for such events in bubble-

[9] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965), Chap. 14.

chamber and spark-chamber pictures of high-energy collision processes is clearly important.

If tachyons exist they are probably neutral. If they do exist we ought to find them. If we do not find them we ought to be able to find out why they could not exist. So far we have found no reason why they could not exist.

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APPENDIX

The fourth-order terms which correspond to the exchange of two tachyons between the interacting nucleons are contained in $S^{(4)}$, and are given by

$$(-i)^4(\tfrac{1}{2}g^2)^2\int d^4x_1d^4x_1'd^4x_2d^4x_2'\tau(x_1-x_1')\tau(x_2-x_2')F(x_1,x_1',x_2,x_2')\,, \tag{A1}$$

where

$$\begin{aligned}F(x_1,x_1',x_2,x_2')= {:}[&(\bar\psi(x_1)+S_F(x_1-x_2)\psi(x_2))(\bar\psi(x_1')iS_F(x_1'-x_2')\psi(x_2'))\\ &+(\bar\psi(x_1')iS_F(x_1'-x_2)\psi(x_2))(\bar\psi(x_1)iS_F(x_1-x_2')\psi(x_2'))\\ &+(\bar\psi(x_1)iS_F(x_1-x_2)\psi(x_2))(\bar\psi(x_2')iS_F(x_2'-x_1')\psi(x_1'))\\ &+(\bar\psi(x_2)iS_F(x_2-x_1')\psi(x_1'))(\bar\psi(x_1)iS_F(x_1-x_2')\psi(x_2'))]{:}.\end{aligned} \tag{A2}$$

This is not a Lorentz-invariant quantity. There are, however, related contributions of the same order present in $S^{(2)}$ and $S^{(3)}$, which are given by

$$(-i)^2(\tfrac{1}{2}^2)g^2\int d^4x_1d^4x_1'd^4x_2d^4x_3D_1(x_1-x_1';\eta)D_1(x_2-x_2';\eta)F(x_1,x_1',x_2,x_2') \tag{A3}$$

and

$$(-i)^3(\tfrac{1}{2}g^2)^2\int d^4x_1d^4x_1'd^4x_2d^4x_2'[D_1(x_1-x_1';\eta)\tau(x_2-x_2')+\tau(x_1-x_1')D_1(x_2-x_2';\eta)]F(x_1,x_1',x_2,x_2')\,, \tag{A4}$$

respectively. The sum total of the three expressions (A1), (A3), and (A4) may be written as

$$\begin{aligned}&(-i)^4(\tfrac{1}{2}g^2)^2\int d^4x_1d^4x_1'd^4x_2d^4x_2'[\tau(x_1-x_1')+iD_1(x_1-x_1';\eta)][\tau(x_2-x_2')+iD_1(x_2-x_2';\eta)]F(x_1,x_1',x_2,x_2')\\ &\qquad=(-i)^4(\tfrac{1}{2}g^2)^2\int d^4x_1d^4x_1'd^4x_2d^4x_2'[i\Delta_F(x_1-x_1')][i\Delta_F(x_2-x_2')]F(x_1,x_1',x_3,x_2')\,.\end{aligned} \tag{A5}$$

(A5) is a Lorentz-invariant quantity, and is just of the kind which would be given by an interaction of the form

$$W_0(x)=g\bar\psi(x)\psi(x)\chi(x)$$

with the contraction function

$$\tau_0(x)=i\Delta_F(x)\,.$$