A MODEL FOR HYPERFRAGMENTS*

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AT the present time our knowledge on the structure of the nucleon-Λ hyperon interaction is quite poor since there are no direct measurements on $N - \Lambda$ scattering. That they interact strongly is inferred from the large production cross-section of strange particles and from the existence of hyperfragments which contain a bound Λ hyperon in a complex nucleus. There are several careful measurements on the properties of hyperfragments, especially on the binding energies[1]; hence it would be desirable to develop a systematic method of analysis which would relate the binding energies to the matrix elements of the $\Lambda - N$ interaction.

Meson theory suggests an extremely complex structure for this interaction. Since the Λ hyperon has spin $\frac{1}{2}$, the $\Lambda - N$ forces can be analysed in terms of triplet and singlet central forces, the spin orbit force, the tensor force and the quadratic spin-orbit force as in the $n - p$ system. But since the Λ hyperon is an isotopic singlet, Yukawa processes involving single pion emission by a Λ hyperon are not allowed ; consequently the asymptotic one-pion " tail " is absent here and the range of the $\Lambda - N$ forces is expected to be considerably shorter. Now the two-pion contribution to the potential is naturally more complicated and cannot be clearly separated from the one-kaon contribution. Also, at these short ranges it is no longer legitimate to treat the " actual " potential in terms of a $\Lambda - N$ system alone, since the reaction $\Lambda + N \rightarrow \Sigma + N$ is possible in addition to $\Lambda + N \rightarrow \Lambda + N$ and hence the interaction is better described in terms of a " potential matrix " for the hyperon-nucleon system ; and the off-diagonal terms of this potential matrix have a long range associated with a one-pion tail.

In contrast to this complexity the properties of hyperfragments would depend only on some average properties of the interaction ; a parallel instance is the dependence of low-energy scattering only on zero-energy scattering

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length and effective range. Of course, one may attempt[2] to solve the Schrödinger equation for the many-particle system using an associated interaction structure with unknown parameters, but this method requires for its practical implementation, some approximation technique like the variational method ; and even then, except for the simplest systems, to make the calculation practicable one has to invoke special models. We have a parallel instance in the theory of complex nuclei, but here the shell model has been a good first approximation to nuclear structure, at least as far as nuclear low-lying levels are concerned. Since one has to use a model anyway, it becomes interesting to see if it can be patterned after the theory of complex nuclei.

We have investigated this problem[3, 4] and developed in detail a model for hyperfragments. The physical postulates are that the system is well represented by a suitably antisymmetrized product of one-particle wave functions with two-particle interactions. We also make the assumption of $j-j$ coupling so that the one-particle wave functions may be classified as $S_{1/2}$, $P_{3/2}$, $P_{1/2}$, $D_{5/2}$, etc. The hyperon is always in the $S_{1/2}$ state at least for the low-lying states ; and all hyperfragments that have so far been identified correspond to nuclei belonging to the $S_{1/2}$ and $P_{3/2}$ shells. The two-particle interactions are assumed to lead to negligible configuration mixing so that the low-lying levels are almost pure configurations ; this assumption is certainly more accurate for the energy eigen values than for the wave functions themselves. In the absence of inter-particle forces the ground state of the model is many fold degenerate, but the interactions split this ; and the low-lying levels of the model with interactions are all obtained from this level ; they all hence correspond to the same set of one-particle configurations coupled in different fashions.

Having specified the model, one could proceed to calculate the energy levels in terms of the two-body interaction parameters. Since the nucleon binding energies are known, we shall take advantage of this to subtract out the ground state energy of the nucleon group ; since the $\Lambda - N$ interaction is somewhat weaker than the $N - N$ interaction for the first excited states of the hyperfragments, the nucleon group is in its ground state. In any case this is true of the ground state ; and for S-shell nuclei there are no bound excited states anyway. This restriction to hyperfragment states with unexcited nucleon groups permit us to label these states simply by the two spins which in general have two values corresponding to the parallel and the antiparallel spin orientations of the Λ and the spin of the nucleon group.

In the actual calculations, the so-called "Projection Theorem"[5] furnishes a powerful tool, in that it relates in our model, the matrix elements of two-particle operators like the interaction energy for the many-particle wave functions in terms of the simpler matrix elements for two-particle configurations and coefficient of fractional parentage. The method is now useful in those cases where the expression in terms of the fractional parentage coefficients have a summable form so that closed expressions result. Fortunately all the hyperfragments that we discuss have this structure and so one can work with these closed expressions. The details and derivations may be found elsewhere.

There are four S-shell hyperfragments which have been identified ${}_{\Lambda}He^3$, ${}_{\Lambda}H^4$, ${}_{\Lambda}He^4$, ${}_{\Lambda}He^5$. The first and last are isotopic singlets and the other two form a doublet ; they correspond to the deuteron, the triton, He^3 and the alpha particle. The experimental[1] values for the binding energies are respectively 0·12 Mev., 2·20 Mev., 2·36 Mev. and 3·08 Mev. There are three parameters[3] occurring in the expression for the Λ-binding energy, namely, the one-particle energy of the hyperon (T_{Λ}) and the singlet and triplet matrix elements (S_0, S_1) of the (charge-independent) $N - \Lambda$ interaction for the configuration $S_{1/2}\,S_{1/2}$. The possible spin assignments and theoretical expressions for the binding energies in terms of these parameters deduced from the binding energies for the antiparallel spins are $T_{\Lambda} = 8\cdot 28$ Mev., $S_0 = -4\cdot 88$ Mev., $S_1 = -2\cdot 16$ Mev. While charge independence demands that ${}_{\Lambda}H^4$ and ${}_{\Lambda}He^4$ should give the same binding energy, experiments give different values agreeing within experimental error ; in computing the parameters for the model, we have averaged over these two values. For the parallel spin configuration the model parameters cannot be found since one notices from Table I that the expressions for the binding energies form a degenerate set which is inconsistent with the experimental values. This data thus indicates antiparallel configuration of the hyperon spin and the spin of the nucleon group and in particular demands that ${}_{\Lambda}He^4$ has spin 0. This last result is particularly important since the large rate of the reaction $K^- + He^4 \rightarrow {}_{\Lambda}H^4 + \pi^-$ then argues for a pseudoscalar kaon.[2] This large rate makes it irrelevant to know if there is a loosely bound excited state[6] of spin 1 for ${}_{\Lambda}He^4$; but from the parameters deduced above and the expressions for the binding energies, according to Table I, one finds that there are no bound excited states for any of the S-shell hyperfragments. One also notices that the $\Lambda - N$ interaction is definitely spin-dependent and attractive and the ratio $S_0/S_1 \sim 2$ is consistent with the previous estimates by Dalitz and Downs.[7] It turns out that the systems $nn\Lambda$ and ${}_{\Lambda}He^3$ are unbound.

A similar analysis can be carried through for P-shell hyperfragments. There are thirteen hyper nuclides belonging to the $P_{3/2}$-shell and among them there are a sufficiently large number of events[1] on four (${}_{\Lambda}Li^7$, ${}_{\Lambda}Li^8$, ${}_{\Lambda}Be^9$,

TABLE I

S-Shell Λ⁻ binding energy in terms of two-particle interaction parameters and the hyperfragment spin assignments

(A = 3, 4, 5)

Hyperfragment	J	Coefficients of T_Λ	Coefficients of S_0	Coefficients of S_1
$_\Lambda H^3$	½ (antiparallel)	1	3/2	½
	3/2 (parallel)	1	0	2
$_\Lambda H^4$, $_\Lambda He^4$	0 (antiparallel)	1	3/2	3/2
	1 (parallel)	1	½	5/2
$_\Lambda He^5$	½	1	1	3

$_\Lambda Li^9$) and the parameters P_2', P_1', (defining the $\Lambda - N$ interaction for the configuration $S_{1/2}$, $P_{3/2}$ for states with $J = 2, 1$ respectively) are deduced from a least-squares fit to those. In this case it is advantageous to subtract the binding energy of the $_\Lambda He^5$ hyperfragment so that the one-particle contribution disappears. The analysis is too lengthy to be reproduced here but the comparison between theory and experiment for the binding energies is given in Table II. Using these parameters one can predict the first excited states of these hyperfragments and the binding energies of the other $P_{3/2}$-shell hyperfragments. A comparison of the latter prediction with experiment is given in Table III. One notices that while the fit is satisfactory, it is difficult to differentiate between the parallel and the anti-parallel configurations on the basis of binding energy data alone.

The same model may be applied to calculate other properties; for example one may develop a projection formula for the magnetic moments of hyperfragments,[4] but these do not appear to be amenable to immediate measurement. More useful would be a correlation of the decay dynamics. For mesic decay modes, especially in the lighter hyperfragments, pion rescattering corrections may be ignored in a first approximation; and it is then possible to write the weak decay interaction as a two-particle operator with the pion wave function treated as an "external" field; this calculation is quite analogous to the treatment of radiation processes in atomic systems to lowest order.

TABLE II

Comparison between theory and experiment for the binding energies of $P_{3/2}$ shell hyperfragments (all energies given in Mev.)

Hyperfragment	"Reduced" Binding Energy	Theory	
		Anti-parallel spins	Parallel spins
$_{\Lambda}Li^{i}$	−2·38	−2·18	−2·10
$_{\Lambda}Li^{8}$	−3·03	−3·16	−3·14
$_{\Lambda}Be^{9}$	−3·52	−3·49	−3·53
$_{\Lambda}Li^{9}$	−4·12	−4·14	−4·19

TABLE III

Predicted and measured binding energies for rare P-shell hyper fragments

Hyperfragment	Experiment	Theory	
		Antiparallel	Parallel
$_{\Lambda}He^{7}$	3·0 ·7	4·73	4·84
$_{\Lambda}Be^{8}$	6·6 ·6	6·24	6·22
$_{\Lambda}Be^{11}$	9·9 ·6	9·18	9·37
$_{\Lambda}Be^{12}$	9·8 ·6	9·73	9·75
$_{\Lambda}c^{13}$	10·8 ·6	10·10	10·10

For non-mesic decays also one can (to a poorer approximation) use a two-particle decay operator. These calculations are now being completed, but our object in mentioning them is to point out that these are tests of the model independent of binding energy determinations, though there is no guarantee that the model should be adequate for decay processes ; again, the situation as far as beta and gamma transition matrix elements in complex nuclei furnishes a parallel.

One might at this point raise the question of the validity of a shell model with two-particle interactions only,[6] even as a first approximation in such a region of low mass numbers ; one may also point out that in the case of the $\Lambda - N$ interaction, the three body forces $N - \Lambda - N$ brought about by the exchange of a pion with each of the nucleons may be important.[2] With regard to these points one has to remember that in setting up the model one has tacitly assumed the existence of sufficiently bound configuration of the particles so that the possibility of finding a particle far from the rest of the nucleus is fairly small ; and a substantial part of the many-body forces (which are equally well present in ordinary nuclei) is already used up in setting up these one-particle eigen functions. In this sense, we are working not so much with two-particle interactions but rather with a two-particle correlation approximation. It is perhaps unfortunate that we are thus deprived of knowing the exact two-particle interaction, but then the interaction parameters that we find are the combinations relevant for the structure of the many-body system. The general effect of many-particle correlations is to prevent the two-particle interaction parameters from having any significant dependence on the two-body potential and has thus the same qualitative nature as the Pauli principle and the "hard" core ; in view of these we believe that the present model is a fair representation of the physical situation and that the interaction parameters are suitable averages of the true two-particle interaction in the neighbourhood of these average separation. We hope to develop these considerations in a more quantitative fashion so as to make more specific inferences about the $\Lambda - N$ potential.

However, a test of the degree of validity of the model for hyperfragments is suggested[8]: one could see how good is a similar model for the corresponding S and P shell nuclei. (Usually this region of atomic numbers is considered "off limits" for the shell-model !) We have three binding energies for the S-shell: the deuteron, triton He^3 and alpha particle. With a notation similar to that used for hyperfragments, we have

$$2T_N + S_1' = -2\cdot 23 \text{ Mev.}$$

$$3T_N + \frac{3}{2}S_1' + \frac{3}{2}S_0' = -8\cdot 49 \text{ Mev.}$$

$$4T_N + 3S_1' + 3S_0' = -28\cdot 30 \text{ Mev.}$$

From which we get $T_N = +5\cdot 67$ Mev., $S_1' = -13\cdot 56$ Mev. and $S_0' = -3\cdot 43$ Mev. Notice that the one-particle energy is smaller here (corresponding to a much stronger binding) as compared with the value obtained for T_Λ. If we now try to predict the virtual singlet level of the $n-p$ system we find that its "excitation energy" is far too high ; it then appears that the model fails here. But the reason for the failure is easily found; in contrast to the triton and the alpha particle the deuteron is a rather "open" structure. The wave functions are spread out much farther than the one-particle wave functions appropriate to the S-shell. The residual two-particle interaction has of course the one-pion tail in this case and the spread out wave functions exploit this attractive tail and get further bound. If this explanation is physical, then the relation between the interaction parameters and the one-particle energy for the deuteron should contain a smaller energy ; if we put for this binding energy the value zero, we get an altered set of parameters and these bring the predicted singlet level nearer zero energy. It is still too high, but this must also happen since the unbound singlet level should obtain more "extra binding". Similar studies have been made for the $P_{3/2}$ and shell nuclei also.[9, 10] And the fit to binding energies is to better than two per cent. The lesson of these studies is that for "compact" (sufficiently bound) nuclei the model is adequate.

This analysis now raises the question to what extent our predictions for the hyperfragments is modified by these "extraordinary" corrections ; this is especially important since ${}_{\Lambda}H^3$ is only loosely bound and one might see if these corrections could remove the inconsistency for one parallel spin configurations. One verifies immediately that for an attractive tail the corrections go in the wrong direction (since for consistency we need the binding energy of about 0·5 Mev.); on the other hand, a repulsive tail behaves in a very different manner from an attractive tail and is relatively unimportant for a one-particle wave function which was sufficiently compact to start with. For the P-shell hyperfragments all of the binding energies are considerably larger. It is to be emphasized that the corrections are qualitatively expected to be much less important since no long range tail for the Λ —N interaction is believed to exist ; and the one-particle wave function being an extended wave function (corresponding to the large positive value of T_{Λ}), the residual change in them is unimportant.

This, then, is our model. The main conclusions derived from this analysis are the spin dependence of the Λ — N interaction the zero spin of ${}_{\Lambda}He^4$ and the consequent odd parity of the Kaon ; and finally the comparative weakness of the Λ — N interaction and the consequent slowly varying extended wave function of the Λ^- hyperon. This last point is particularly relevant in connection with the omission of pion rescattering corrections in correlating the decay dynamics.

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