folder 12

THE DIFFRACTION OF LIGHT BY HIGH FREQUENCY SOUND WAVES: PART I.

BY C. V. RAMAN
AND
N. S. NAGENDRA NATH.
(From the Department of Physics, Indian Institute of Science, Bangalore.)

Received September 28, 1935.

1. *Introduction.*

As is well known, Langevin showed that high frequency sound-waves of great intensity can be generated in fluids by the use of piezo-electric oscillators of quartz. Recently, Debye and Sears[1] in America and Lucas and Biquard[2] in France have described very beautiful experiments illustrating the diffraction of light by such high-frequency sound-waves in a liquid. Amongst the experimenters in this new field, may be specially mentioned R. Bär[3] of Zürich who has carried out a thorough investigation and has published some beautiful photographs of the effect. The arrangement may be described briefly as follows. A plane beam of monochromatic light emerging from a distant slit and a collimating lens is incident normally on a cell of rectangular cross-section and after passing through the medium emerges from the oppsite side. Under these conditions, the incident beam will be undeviated if the medium be homogeneous and isotropic. If, however, the medium be traversed by high-frequency sound-waves generated by introducing a quartz oscillator at the top of the cell, the medium becomes stratified into parallel layers of varying refractive index. Considering the case in which the incident beam is parallel to the plane of the sound-waves, the emerging light from the medium will now consist of various beams travelling in different directions. If the inclination of a beam with the incident light be denoted by θ, it has been found experimentally that the formula

$$\sin \theta = \pm \frac{n\lambda}{\lambda^*}, \quad n \text{ (an integer)} \geqslant 0 \qquad \text{.. .. (1)}$$

is in satisfactory agreement with the observed results, where λ and λ^* are the wave-lengths of the incident light and the sound wave in the medium

[1] P. Debye and F. W. Sears, *Proc. Nat. Acad. Sci.* (Washington), 1932, **18**, 409.

[2] R. Lucas and P. Biquard, *Jour. de Phys. et Rad.*, 1932, **3**, 464.

[3] R. Bär, *Helv. Phys. Acta*, 1933, **6**, 570.

PROCEEDINGS OF THE INDIAN ACADEMY OF SCIENCES
V.2 p.406-412 1935 REF (2)

respectively. With sound waves of sufficient intensity, numerous orders of these diffraction spectra have been obtained ; a wandering of the intensity amongst these orders has also been noticed by Bär[3] when the experimental conditions are varied.

Various theories of the phenomena have been put forward by Debye and Sears,[1] by Brillouin,[4] and by Lucas and Biquard.[2] The former have not presented quantitative results and it is hard to understand from their theory as to why there should be so many orders and why the intensity should wander between the various orders under varying experimental conditions. In Brillouin's theory, the phenomenon is attributed to the reflection of light from striations of the medium caused by the sound waves. We know, however, from the work of Rayleigh that the reflection of light by a medium of varying refractive index is negligible if the variation is gradual compared with the wave-length of light. Under extreme conditions, we might perhaps obtain the Brillouin phenomenon, but the components of reflection should be very weak in intensity compared to the transmitted ones. As one can see later on in this paper, the whole phenomenon including the positions of the diffracted beams and their intensities can be explained by a simple consideration of the transmission of the light beam in the medium. Lucas and Biquard attribute the phenomenon to an effect of mirage of light waves in the medium. In what way the relation (1) enters in their theory is not clear. The wandering of the intensities of the various components observed by Bär has not found explanation in any of the above theories.

We propose in this paper a theory of the phenomenon on the simple consideration of the regular transmission of light in the medium and the phase changes accompanying it. The treatment is limited to the case of normal incidence. The formula (1) has been established in our theory. Also, a formula for the intensities of the various components has been derived. It is found that the above results are in conformity with the experimental results of Bär.[3]

2. *Diffraction of light from a corrugated wave-front.*

The following theory bears a very close analogy to the theory of the diffraction of a plane wave (optical or acoustical) incident normally on a periodically corrugated surface, developed by the late Lord Rayleigh.[5] He showed therein that a diffraction phenomenon would ensue in which the positions of the various components are given by a formula similar to (1)

---

[4] L. Brillouin, "La Diffraction de la Lumiere par des Ultra-sons", *Act. Sci. et Ind.*, 1933, 59.

[5] Lord Rayleigh, *Theory of Sound* (Vol. 2), page 89.

and their relative intensities are given by a formula similar to the one we have found.

Consider a beam of light with a plane wave-front emerging from a rectangular slit and falling normally on a plane face of a medium with a rectangular cross-section and emerging from the opposite face parallel to the former. If the medium has the same refractive index at all its points, the incident beam will emerge from the opposite face with its direction unchanged. Suppose we now create layers of varying refractive index in the medium, say by suitably placing a quartz oscillator in the fluid. If the distance between the two faces be small, the incident light could be regarded as arriving at the opposite face with variations in the phase at its different parts corresponding to the refractive index at different parts of the medium. The change in the phase of the emerging light at any of its parts could be simply calculated from the optical lengths found by multiplying the distance between the faces and the refractive index of the medium in that region. This step is justified for $\int \mu(x, y, z) ds$ taken over the actual path is minimum, *i.e.*, it differs from the one taken over a slightly varied hypothetical path by a differential of the second order. So, the incident wave-front becomes a periodic corrugated wave-front when it traverses a medium which has a periodic variation in its refractive index. The origin of the axes of reference is chosen at the centre of the incident beam projected on the emerging face, the boundaries of the incident beam being assumed to be parallel to the boundaries of the face. The X-axis is perpendicular to the sound-waves and the Z-axis is along the direction of the incident beam of light. If the incident wave is given by

$$Ae^{2\pi i\nu t}$$

it will be

$$Ae^{2\pi i\nu \{t - L\mu(x)/c\}}$$

when it arrives at the other face where L is the distance between the two faces and $\mu(x)$ the refractive index of the medium at a height x from the origin. It is assumed that the radii of curvature of the corrugated wave-front are large compared with the distance between the two faces of the cell. If μ_0 be the refractive index of the whole medium in its undisturbed state, we can write $\mu(x)$ as given by the equation

$$\mu(x) = \mu_0 - \mu \sin \frac{2\pi x}{\lambda^*}$$

ignoring its time variation, μ being the *maximum variation* of the refractive index from μ_0.

The amplitude due to the corrugated wave at a point on a distant screen parallel to the face of the medium from which light is emerging whose join with the origin has its x-direction-cosine l, depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2} e^{2\pi i\{lx - \mu L \sin(2\pi x/\lambda^*)\}/\lambda}\, dx$$

where p is the length of the beam along the X-axis. The real and the imaginary parts of the integral are

$$\int_{-p/2}^{p/2} \{\cos ulx \cos(v \sin bx) - \sin ulx \sin(v \sin bx)\} dx$$

and

$$\int_{-p/2}^{p/2} \{\sin ulx \cos(v \sin bx) + \cos ulx \sin(v \sin bx)\} dx$$

where $u = 2\pi/\lambda$, $b = 2\pi/\lambda^*$ and $v = u\mu L = 2\pi\mu L/\lambda$.

We need the well-known expansions

$$\cos(v \sin bx) = 2\sum_0^\infty{}' J_{2r} \cos 2rbx$$

$$\sin(v \sin bx) = 2\sum_0^\infty J_{2r+1} \sin \overline{2r+1}\, bx$$

to evaluate the integrals, where $J_n [= J_n(v)]$ is the Bessel function of the nth order and a dash over the summation sign indicates that the co-efficient of J_0 is half that of the others. The real part of the integral is then

$$2\sum_0^\infty{}' J_{2r} \int_{-p/2}^{p/2} \cos ulx \cos 2rbx\, dx - 2\sum_0^\infty J_{2r+1} \int_{-p/2}^{p/2} \sin ulx \sin \overline{2r+1}\, bx\, dx$$

or

$$\sum_0^\infty{}' J_{2r} \int_{-p/2}^{p/2} \{\cos(ul + 2rb)x + \cos(ul - 2rb)x\} dx$$

$$+ \sum_0^\infty J_{2r+1} \int_{-p/2}^{p/2} \{\cos(ul + \overline{2r+1}\, b)x - \cos(ul - \overline{2r+1}\, b)x\} dx$$

Integrating the above, we obtain

$$p \sum_0^\infty{}' J_{2r} \left\{ \frac{\sin(ul + 2rb)p/2}{(ul + 2rb)p/2} + \frac{\sin(ul - 2rb)p/2}{(ul - 2rb)p/2} \right\}$$

$$+ p \sum_0^\infty J_{2r+1} \left\{ \frac{\sin(ul + \overline{2r+1}\, b)p/2}{(ul + \overline{2r+1}\, b)p/2} - \frac{\sin(ul - \overline{2r+1}\, b)p/2}{(ul - \overline{2r+1}\, b)p/2} \right\} \quad \ldots \quad (2)$$

The integral corresponding to the imaginary part of the diffraction integral is zero. One can see that the magnitude of each individual term of (2) attains its highest maximum (the other maxima being negligibly small compared to the highest) when its denominator vanishes. Also, it can be seen that when any one of the terms is maximum, all the others have negligible values as the numerator of each cannot exceed unity and the denominator is some integral non-vanishing multiple of b which is sufficiently large. So the maxima of the magnitude of (2) correspond to the maxima of the magnitudes of the individual terms. Hence the maxima occur when

$$ul \pm nb = 0 \quad n(\text{an integer}) \geqslant 0 \quad \ldots \quad (3)$$

where n is any even or odd positive integer. The equation (3) gives the directions in which the magnitude of the amplitude is maximum which correspond also to the maximum of the intensity. If θ denotes the angle between such a direction in the XZ-plane along which the intensity is maximum and the direction of the incident light, (3) can be written as

$$\sin\theta = \pm \frac{n\lambda}{\lambda^*} \quad \ldots \quad (4)$$

remembering that $u = 2\pi/\lambda$ and $b = 2\pi/\lambda^*$. This formula is identical with the formula (1) given in the first section. The magnitudes of the various components in the directions given by (4) can be calculated if we know,

$$J_n \text{ or } J_n(v) \text{ or } J_n(2\pi\mu L/\lambda).$$

Thus the relative intensity of the mth component to the nth component is given by

$$\frac{J_m^2(v)}{J_n^2(v)} \quad \text{where } v = 2\pi\mu L/\lambda.$$

In the undisturbed state of the medium there is no variation of the refractive index, *i.e.*, $\mu = 0$. In this case all the components vanish except the zero component for

$$J_m(0) = 0 \text{ for all } m \neq 0 \text{ and } J_0(0) = 1.$$

In the disturbed state, the relative intensities depend on the quantity v or $2\pi\mu L/\lambda$ where λ is the wave-length of the incident light, μ is the maximum variation of the refractive index and L is the path traversed by light in the medium. We have calculated the relative intensities of the various components which are observable for values of v lying between 0 and 8 at different steps (Fig. 1).

Fig. 1 shows that the number of observable components increases as the value of v increases. When $v = 0$, we have only the central component. As v increases from 0, the first orders begin to appear. As v increases still more, the intensity of the central component decreases steadily and the first orders increase steadily in their intensity till they attain maximum intensity when the zero order will nearly vanish and the second orders will have just appeared. As v increases still more, the zero order is reborn and increases

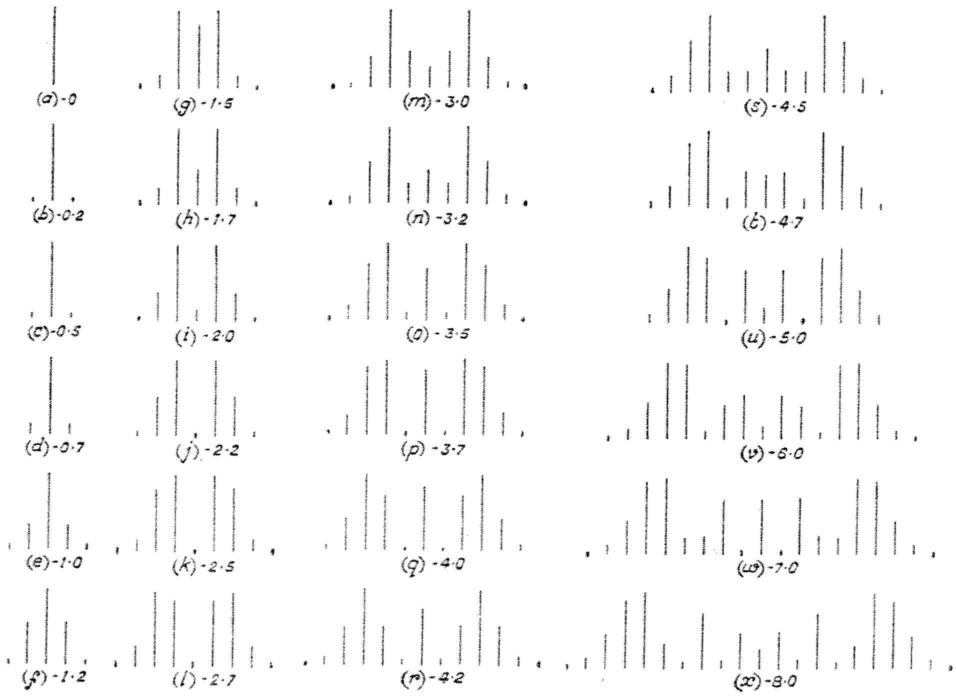

FIG. 1.

Relative intensities of the various components in the diffraction spectra.

(For tables, see Watson's *Bessel Functions and Report of the British Association*, 1915.)

in its intensity, the first orders fall in their intensity giving up their former exalted places to the second orders, while the third orders will have just appeared and so on.

Our theory shows that the intensity relations of the various components depend on the quantity v or $2\pi\mu L/\lambda$. Thus an *increase* of μ (*i.e.*, an increase of the supersonic intensity which creates a greater variation in the refractive index of the medium) or an *increase* of L or a *decrease* of λ should give similar effects *except* in the last case where the directions of the various beams will be altered in accordance with (4).

3. *Interpretation of Bär's Experimental Results.*

(*a*) *Dependence of the effect on the supersonic intensity.*—Bär has observed that only the zero order (strong) and the first orders (faint) are present when the supersonic intensity is not too great. He found that more orders appear as the supersonic intensity is increased but that the intensity of the zero order decreases while the first orders gain in their intensity. Increasing the supersonic intensity more, he found that the first order would become very faint while the second and third orders will have about the same intensity. The figures 1*a* of his paper may very well be compared with our figures 1(*c*), 1(*h*) and 1(*k*). Thus, we are able to explain the appearance of more and more components and the wandering of the intensity amongst them as the supersonic intensity is increased, in a satisfactory manner.

(*b*) *Dependence of the effect on the wave-length of the incident light.*—We have already pointed out that the effects due to an increase of μ caused by an increase of supersonic intensity are similar due to those with a decrease of λ except for the fact that the positions of the components of the emerging light alter in accordance with (4). Bär has obtained two patterns of the phenomenon by using light with wave-lengths 4750Å and 3650Å. He obtained, using the former seven components and using the latter eleven components in all. He also observed great variations in the intensities of the components. Not only is the increase in the number of components an immediate consequence of our theory, but we can also find the pattern with 3650Å if we assume the pattern with 4750Å. The pattern with the latter in Bär's paper shows a strong resemblance to our figure 1(*p*) for which $2\pi\mu L/\lambda$ is 3·7. Thus we can calculate $2\pi\mu L/\lambda$ when λ is 3650Å. It comes to about 4·8. Actually our figure for which $2\pi\mu L/\lambda$ is 4·8 closely corresponds to Bär's pattern with 3650Å.

(*c*) *Dependence of the effect on the length of the medium which the light traverses.*—It is clear from our theory that an increase of L corresponds to an increase of v and that the effects due to this variation would be similar to those with an increase of the supersonic intensity. But the basis of our theory does not actually cover any large change in L. However, we should find more components and the wandering of the intensity amongst the various components.

4. *Summary.*

(*a*) A theory of the phenomenon of the diffraction of light by sound-waves of high frequency in a medium, discovered by Debye and Sears and Lucas and Biquard, is developed.

(*b*) The formula

$$\sin\,\theta = \pm\,\frac{n\lambda}{\lambda^*} \quad n \text{ (an integer)} \geqslant 0$$

which gives the directions of the diffracted beams from the direction of the incident beam and where λ and λ^* are the wave-lengths of the incident light and the sound wave in the medium, is established. It has been found that the relative intensity of the mth component to the nth component is given by

$$J_m{}^2(2\pi\mu L/\lambda)\ /\ J_n{}^2(2\pi\mu L/\lambda)$$

where the functions are the Bessel functions of the mth order and the nth order, μ is the maximum variation of the refractive index and L is the path traversed by light. These theoretical results interpret the experimental results of Bär in a very gratifying manner.