ZEEMAN EFFECT OF HYPERFINE STRUCTURE IN INTERMEDIATE FIELDS

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THE theory of the Zeeman effect of ordinary fine structure in intermediate and high fields, developed by Heisenberg and Jordan,[1] Darwin[2] and Mensing,[3] has been modified by Goudsmit and Bacher[4] to suit the case of hyperfine structure and its Zeeman effect. This theory has been verified in the cases of Thallium II and III and Bismuth II and III by Green and Wulff[5] at field strengths of 14700, 32500 and 43350 gauss, using a large grating as the resolving instrument. The complete Paschen-Back effect for Hg^{199} has also been observed by Green.[6] Jackson and Kuhn[7] have observed an incomplete Paschen-Back effect in the case of potassium at 730 gauss, the results being in good agreement with theory. In the present work, the Zeeman effect of the hyperfine structure of the mercury green line Hg I λ 5461 Å has been studied at field strengths up to 4000 gauss. On account of the complicated structure of the line, attention has been mostly focussed on one particular satellite $+0{\cdot}753$ cm.$^{-1}$, which arises from the transition $7^3S_1\, f = \frac{3}{2} \rightarrow 6^3P_2\, f = \frac{5}{2}$ in the odd isotope Hg^{199} with a nuclear spin of $\frac{1}{2}$.[8] The results show a good agreement with theory both as regards separations as well as intensities. Thus while the complete Paschen-Back effect is the best spectroscopic evidence for the correctness of an assignment of nuclear spin, the Zeeman effect of hyperfine structure at lower fields can also provide substantial support to the value of the spin otherwise deduced.

Theoretical

The calculation of the expected Zeeman effect is based on the following theory. The positions of the levels in the magnetic field are found by writing down the equations :*

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1 Heisenberg and Jordan, *Zeits. f. Physik*, 1926, **37**, 263.
2 Darwin, *Proc. Roy. Soc.*, 1927, **115**, 1.
3 Mensing, *Zeits, f. Physik*, 1926, **39**, 24.
4 Goudsmit and Bacher, *Zeits. f. Physik*, 1930, **66**, 16.
5 Green and Wulff, *Phys. Rev.*, 1931, **38**, 2176 and 2186.
6 ———, *Ibid.*, 1936, **50**, 126.
7 Jackson and Kuhn, *Proc. Roy. Soc.*, 1938, **165**, 303.
8 Schüler and Jones, *Zeits. f. Physik*, 1932, **74**, 640.
* There is a misprint in connection with this formula in the original paper of Goudsmit and Bacher.

$$- X_{M_I+1, M_J-1} \frac{A}{2} (I + M_I + 1)(J - M_J + 1)$$

$$+ X_{M_I M_J} [E - A M_I M_J - M_J g(J) OH]$$

$$- X_{M_I-1, M_J+1} \frac{A}{2} (I - M_I + 1)(J + M_J + 1) = 0, \qquad \text{(I)}$$

[there being as many equations as there are values of M_I and M_J corresponding to a given $M_F = M_I + M_J$] and equating their determinant to zero and solving the resulting equation for E. Here E is the distance of the level from the centre of gravity of the hyperfine multiplet level. In the case of the satellite under consideration which belongs to Hg^{199}, $I = \frac{1}{2}$, so that $M_I = \pm \frac{1}{2}$, and we have only two equations of the type (I) and their determinant equated to zero yields a quadratic. The values so obtained for E are given below. $g(J)$ has been calculated on the assumption that Russell-Saunders coupling exists. In these equations O is the Larmor frequency. Expressing E and O in cm.$^{-1}$, $O = 4 \cdot 67 \times 10^{-5}$ cm.$^{-1}$.

$$6^3P_2: \quad M_F = \tfrac{3}{2}; \; 2E = -\frac{A}{2} + \tfrac{9}{2} OH - \sqrt{6\tfrac{1}{4} A^2 - \tfrac{9}{2} AOH + \tfrac{9}{4} O^2H^2}$$

$$M_F = \tfrac{1}{2}; \; 2E = -\frac{A}{2} + \tfrac{3}{2} OH - \sqrt{6\tfrac{1}{4} A^2 - \tfrac{3}{2} AOH + \tfrac{9}{4} O^2H^2}$$

$$M_F = -\tfrac{1}{2}; \; 2E = -\frac{A}{2} - \tfrac{3}{2} OH - \sqrt{6\tfrac{1}{4} A^2 + \tfrac{3}{2} AOH + \tfrac{9}{4} O^2H^2}$$

$$M_F = -\tfrac{3}{2}; \; 2E = -\frac{A}{2} - \tfrac{9}{2} OH - \sqrt{6\tfrac{1}{4} A^2 + \tfrac{9}{2} AOH + \tfrac{9}{4} O^2H^2}$$

$$7^3S_1: \quad M_F = \tfrac{3}{2}; \; E = \frac{A}{2} + 2\,OH$$

$$M_F = \tfrac{1}{2}; \; 2E = -\frac{A}{2} + 2\,OH + \sqrt{2\tfrac{1}{4} A^2 - 2AOH + 4\,O^2H^2}$$

$$M_F = -\tfrac{1}{2}; \; 2E = -\frac{A}{2} - 2\,OH + \sqrt{2\tfrac{1}{4} A^2 + 2AOH + 4\,O^2H^2}$$

$$M_F = -\tfrac{3}{2}; \; E = \frac{A}{2} - 2\,OH.$$

The values of E having been found, the expected Zeeman-pattern is computed, remembering that $\Delta M_F = 0$ for π-components and $\Delta M_F = \pm 1$ for σ-components. The component lines are here referred to the centre of gravity of the field-free hyperfine multiplet.

The values of E thus obtained being substituted in (I), we can solve them for the X's when the normalizing condition

$$\sum_{M_F = \text{const.}} X^2_{M_I M_J} (I + M_I)!\,(I - M_I)!\,(J + M_J)!\,(J - M_J)! = 1 \qquad \text{(II)}$$

is added. When the X's have been thus found, the intensities of the Zeeman components are calculated according to the following formulæ:

Transition $J \rightarrow J'$ $(J' = J - 1)$

π-Components:

$$M'_F = M_F \qquad \text{Int.} = 4\ [\sum_{M_F \text{ const.}} X_\mu X'_\mu\ (I+M_I)!\ (I-M_I)!\ (J+\mu)!\ (J-\mu)!]^2$$

σ-Components:

$$M'_F = M_{F-1} \qquad \text{Int.} = (\sum X_\mu X'_{\mu-1}\ (I+M_I)!\ (I-M_I)!\ (J+\mu)!\ (J-\mu)!]^2$$

$$M'_F = M_{F+1} \qquad \text{Int.} = (\sum X_\mu X'_{\mu+1}\ (I+M_I)!\ (I-M_I)!\ (J+\mu)!\ (J-\mu)!]^2,$$

where $\mu = M_J$ such that M_F is constant.

Transition $J \rightarrow J'$ $(J' = J)$

π-Components

$$M'_F = M_F \qquad \text{Int.} = 4\ [\sum X_\mu X'_\mu (I+M_I)!\ (I-M_I)!\ (J+\mu)!\ (J-\mu)!]^2$$

σ-Components:

$$M'_F = M_F - 1 \quad \text{Int.} = [\sum X_\mu X'_{\mu-1}\ (1+M_I)!\ (I-M_I)!\ (J+\mu)!(J-\mu+1)!]^2$$

$$M'_F = M_F + 1 \quad \text{Int.} = [\sum X_\mu X'_{\mu+1}\ (I+M_I)!\ (I-M_I)!\ (J+\mu+1)!\ (J-\mu)!]^2$$

'A' for the 6^3P_2 level is $303 \cdot 2 \times 10^{-3}$ cm.$^{-1}$ and that for the 7^3S_1 level is $713 \cdot 3 \times 10^{-3}$ cm.$^{-1}$. With these values the separations of the various Zeeman components of the satellite $+0 \cdot 753$ cm.$^{-1}$ from its field-free position, as also their intensities, have been calculated and the results are given in the tables below; the intensities are given within brackets.

TABLE I

π-components ($+0 \cdot 753$ cm.$^{-1}$)

H in gauss	$10^3 \times \Delta\nu$ in cm.$^{-1}$ (Intensity) A	B*	C*	D	I_A/I_D
2000	− 60·8 (3·23)	− 7·9 (0·30)	+ 34·5 (0·22)	+ 69·1 (1·80)	1·9
2500	− 74·3 (3·49)	− 5·2 (0·31)	+ 46·8 (0·21)	+ 87·4 (1·68)	2·1
3000	− 87·1 (3·76)	− 0·7 (0·31)	+ 60·5 (0·19)	+106·0 (1·56)	2·4
3500	− 99·9 (4·05)	+ 5·7 (0·31)	+ 75·4 (0·18)	+128·9 (1·46)	2·8
4000	−110·2 (4·34)	+13·9 (0·31)	+ 91·4 (0·17)	+144·1 (1·37)	3·2

* Not observed because of their relatively small intensity.

TABLE II

σ-components (+ 0·753 cm.$^{-1}$)

H in gauss	$\Delta\nu \times 10^3$ cm.$^{-1}$ (Intensity)						$\frac{\Delta\nu_{de}}{\Delta\nu_{cf}}$
	a*	b*	c*	d	e	f	
2000	− 177·8 (0·88)	− 133·2 (0·97)	− 96·8 (0·63)	+ 109·1 (0·98)	+ 159·8 (1·06)	+ 200·4 (0·67)	1·25
2500	− 218·0 (0·89)	− 162·4 (0·95)	− 119·3 (0·59)	+ 138·5 (1·03)	+ 204·0 (1·05)	+ 253·5 (0·64)	1·33
3000	− 256·4 (0·89)	− 190·3 (0·91)	− 141·0 (0·56)	+ 168·6 (1·07)	+ 250·1 (1·02)	+ 307·5 (0·61)	1·42
3500	− 293·8 (0·88)	− 216·5 (0·87)	− 162·3 (0·53)	+ 199·6 (1·12)	+ 297·6 (1·00)	+ 362·6 (0·58)	1·50
4000	− 327·6 (0·88)	− 241·5 (0·83)	− 183·0 (0·49)	+ 231·3 (1·17)	+ 346·8 (0·97)	+ 418·5 (0·55)	1·61

* Not observed due to overlapping.

Tables III and IV give the computed Zeeman effect of the hyperfine structure component $-0\cdot315$ cm.$^{-1}$ of λ 5461 Å arising from the transition $7\,^3S_1\, f=\frac{1}{2} \rightarrow 6\,^3P_2\, f=\frac{3}{2}$ in the odd isotope Hg^{199} for the above intermediate field strengths.

TABLE III

π-components ($-0\cdot315$ cm.$^{-1}$)

H	$\Delta\nu \times 10^3$ cm.$^{-1}$ (Intensity)	
2000	− 41·4 (13·05)	+ 39·2 (13·67)
2500	− 52·0 (12·99)	+ 48·6 (13·75)
3000	− 62·5 (12·94)	+ 57·6 (13·86)
3500	− 73·2 (12·88)	+ 66·5 (13·94)
4000	− 83·9 (12·83)	+ 75·0 (14·03)

TABLE IV

σ-components ($-0\cdot315$ cm.$^{-1}$)

H	$\Delta\nu \times 10^3$ cm.$^{-1}$ (Intensity)			
2000	− 209·1 (3·05)	− 130·7 (9·50)	+ 124·5 (10·42)	+ 206·9 (3·67)
2500	− 261·2 (2·99)	− 164·2 (9·36)	+ 154·7 (10·52)	+ 257·8 (3·76)
3000	− 313·3 (2·93)	− 198·1 (9·23)	+ 184·5 (10·59)	+ 308·4 (3·86)
3500	− 365·1 (2·88)	− 233·0 (9·08)	+ 214·0 (10·68)	+ 358·4 (3·94)
4000	− 416·8 (2·83)	− 266·5 (8·93)	+ 243·2 (10·76)	+ 407·9 (4·03)

Experimental

The low density mercury vacuum arc lamp employed in this investigation was of a special type. A primary arc carrying a current of 4 amps. was maintained between the water-cooled cathode K (Fig. 1) and the tungsten anode A 1. A low current (< ½ amp.) secondary arc was next started between K and the anode A 2; the vertical long tube (nearly 20 cms. long) with the low current arc was placed between the parallel pole-pieces of an electromagnet. The diameter of the flat pole-pieces was about 4 cms.; about 2 cms. of the arc-length in the centre of the magnetic field was considered to lie in a uniform field and was therefore isolated for study by properly screening the rest of the arc. The local heating of the Pyrex glass tube produced by the deflection of the arc in the magnetic field was small, because of the low current in the arc. The light of the arc in the field was passed through a nicol and then condensed on to an adjusted Fabry-Perot etalon of the Lesche type in front of which was placed a green-ray filter.

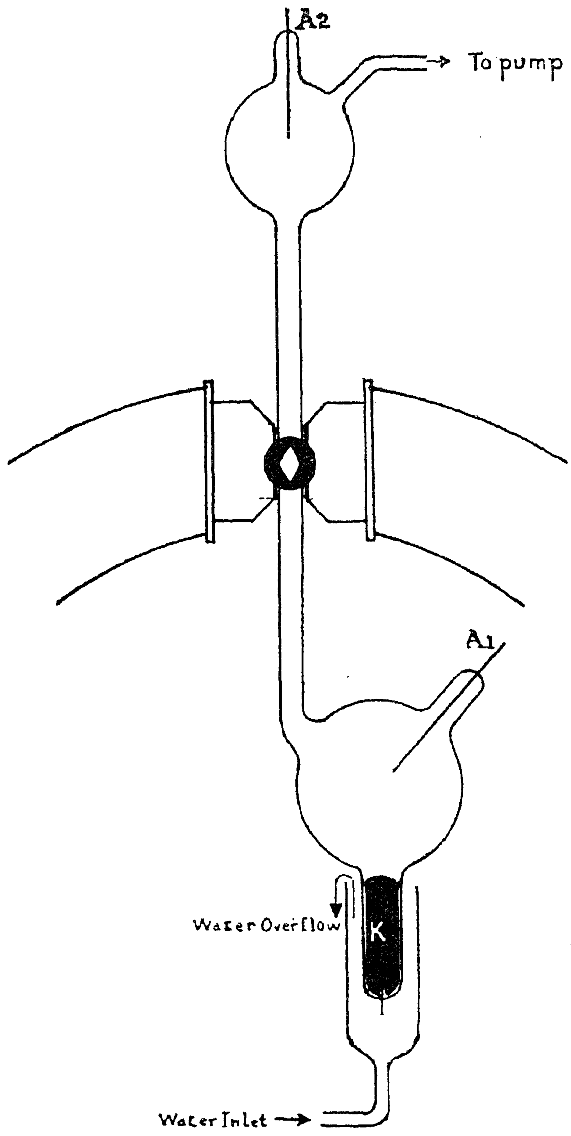

FIG. 1 Source

The invar distance pieces (2·5 mms. and 3 mms. thick) were selected so as to bring the satellite $+0 \cdot 753$ cm.$^{-1}$ of the mercury line Hg I λ 5461 Å into the middle of the pattern; in the immediate vicinity of this satellite there was no other component which could have vitiated our results. The Zeeman effect of the satellite was in several cases directly measured by means of a micrometer eyepiece in the observing telescope placed immediately after the Fabry-Perot etalon. But the comparison of the relative intensities of the components necessitated the photographing of the pattern, and this was done by a camera substituted in place of the telescope. Hypersensitive Panchromatic plates were used and a special fine grain developer was employed. Fig. 2 is a reproduction of a typical pattern showing the Zeeman π-components of the satellite $+0\cdot753$ cm.$^{-1}$ in the magnetic field. The intensity curves in Fig. 3 were obtained on a Moll Recording Microphotometer by the kind permission of Sir C. V. Raman at the Indian Institute of Science. The magnetising current was carefully controlled and maintained constant during measurement and photography. The field strength was measured directly by a flux-meter and a check on this value was obtained by estimating the field strength from the Zeeman splitting of the main line; these two independent methods gave values agreeing within 2 per cent. The results of visual measurements were consistent with theoretical expectations; the values obtained from a few of the photographed patterns whose intensity curves have been reproduced in Fig. 3, are given in Table V and compared with the results of theoretical calculation. In the case of the σ-components, only one half of them *d, e, f* could be observed as the other half *a, b, c* got merged with the main components. The ratio of the intensities of the π-components A and D, and the separation between them were plotted against the field strength H, to get their values at the particular field strengths employed; these calculated values were then compared with the observed values. Similarly in the σ-components, the ratio $\Delta\nu_{de}/\Delta\nu_{ef}$ has been obtained by the same method for any particular field.

TABLE V

	Fig. 3	H in gauss	$\Delta\nu_{AD}$ in cm.$^{-1}$		I_A/I_D	
			Calc.	Obs.	Calc.	Obs.
π-components				$\pm$ 0·005		$\pm$ 0·3
	1	2127	0·134	0·137	1·95	2·0
	2	2596	0·168	0·164	2·14	2·2
	3	4032	0·256	0·256	3·23	3·0
σ-comp.			Calc		Obs.	
	4	3700	$\frac{\Delta\nu_{de}}{\Delta\nu_{ef}} = 1\cdot56$		$\frac{\Delta\nu_{de}}{\Delta\nu_{ef}} = 1\cdot5$	

Table V shows a satisfactory agreement between the calculated and observed values in the separations as well as the relative intensities of the Zeeman components of the satellite $+ 0 \cdot 753$ cm.$^{-1}$. The separations of the components for field-strengths below 4000 gauss nearly coincide with the values computed on the basis of the weak field theory which gives $\pm \frac{(7), (21), 33, 47, 61}{30}$ OH with relative intensities (1) : (9) : 3 : 4 : 3 respectively; and small deviations have been observed only above 4000 gauss. But the relative intensities of the π-components A and D, which should be equal according to the weak field theory, show agreement only with the results of the theory for intermediate fields. Because of the proximity of the other satellites the Zeeman effect study of $- 0 \cdot 315$ cm.$^{-1}$ was carried out at field strengths below 2500 gauss, and no deviation either in separation or in intensity from the values given by the weak field theory was observable. The computed results set forth in Tables III and IV for intermediate fields show little deviation at these field strengths from the weak field theory, which gives $\pm \frac{(13), 41, 67}{30}$ OH for the separations with relative intensities (4) : 3 : 1 respectively. With the field strengths employed, the intensities of the Zeeman components of $- 0 \cdot 315$ cm.$^{-1}$ computed for intermediate fields show no observable deviation from the results for weak fields.

In conclusion we thank Prof. B. Venkatesachar for initiating this work and giving us helpful suggestions. We are also indebted to Prof A. Venkat Rao Telang for his kind encouragement.

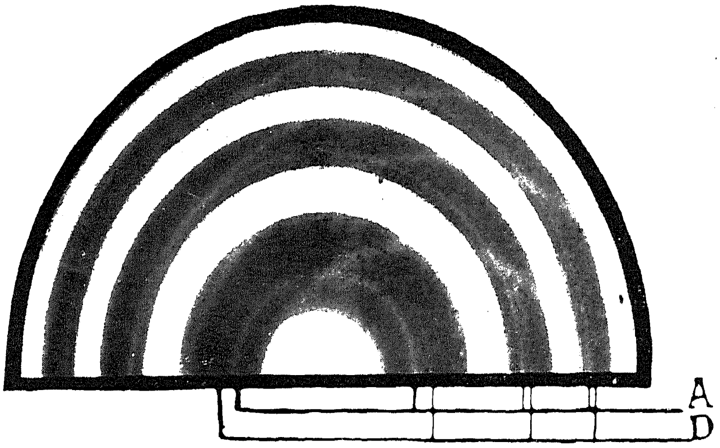

FIG. 2
π- components (A & D) of $+0.753$ cm.$^{-1}$ showing their relative intensities in a magnetic field H = 4032 gauss

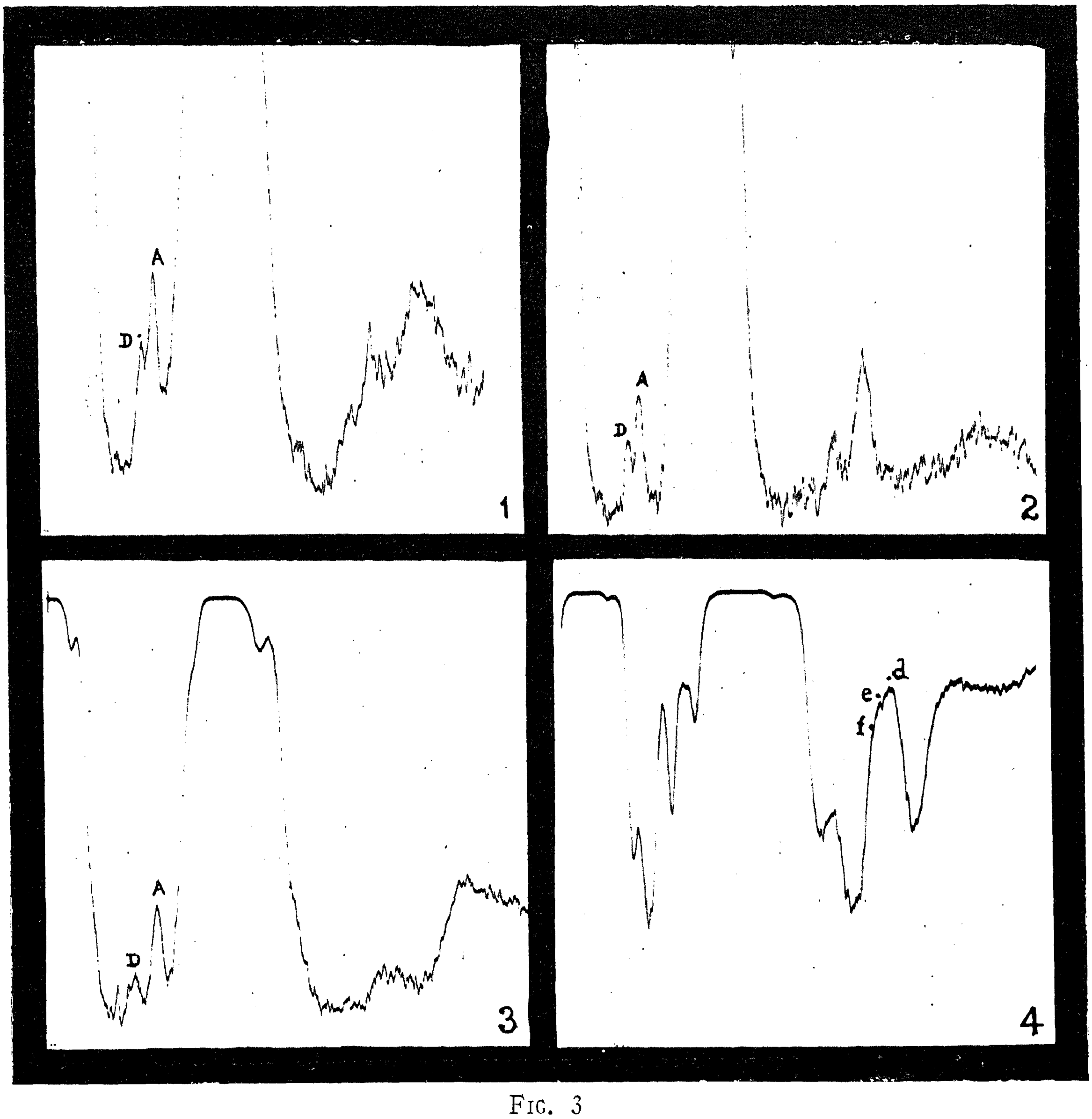

FIG. 3
Microphotographs of π- and σ-components