"UNIVERSAL" RESISTANCE JUMP OF VORTICES AT THE MELTING TRANSITION

A.G. Rojo, [a] A.K. Sood [b] and C.A. Balseiro [c]

[a] Department of Physics, The University of Michigan, Ann Arbor, MI 48109-1120, U.S.A

[b] Department of Physics, Indian Institute of Science, Bangalore 560 1012, India

[c] Centro Atómico Bariloche and Instituto Balseiro, 8400-Bariloche, Argentina

We consider the jump in resistance at the melting transition, which is experimentally observed to be constant, independent of magnetic field (vortex density). We present an explanation of this effect based on vortex cuttings, and universalities of the structure factor at the freezing transition (the Hansen–Verlet criterion).

The subject of vortex dynamics in high Tc superconductors **is** of topical interest [1,2]. In recent years, new aspects of the phase diagram of the vortex lattice have been elucidated through experiment. Perhaps one of the salient recent experimental findings is the first order melting transition of the vortex lattice, first observed in transport measurements **[3]** and later confirmed in a number of equilibrium properties [4]. It is now well accepted that in clean BiSrCuO and in untwined YBaCuO samples the melting transition of the vortex lattice for an applied magnetic field parallel to the c-axis is first order and that the superconducting coherence is simultaneously lost in all directions at the melting temperature. A striking regularity is observed in the transport experiments: for different values of the applied magnetic field, which is proportional to the areal density of vortices, the in-plane resistivity $\rho_{ab}(T)$ decreases with decreasing temperature until it jumps (essentially) discontinuously to zero at the melting temperature T_m, with $\rho_{ab}(T_m)$ independent of field. Moreover, recent studies of the melting transition in YBaCuO show that the jump in the out-of-plane resistivity ρ_c is also field independent.

The resistance due to vortex motion is given by

$$\rho = \left(\frac{\phi_0}{c}\right)^2 \frac{n}{\eta}, \tag{1}$$

with n the number of vortices per unit area and η the viscosity. If in the above equation one replaces the viscosity by its value due to the frictional force the electrons flowing through the normal core, one obtains a resistance that increases linearly with the magnetic field. Using general arguments from the theory of liquids at the freezing point, we will show below that, if the vortex cutting dominates the viscosity, the value of ρ at the melting transition **is** constant, independent of $\boldsymbol{n}$. This universality constitutes, we argue, an evidence of general universalities in the structure factor $S(q)$ of the liquid phase at the melting point as first pointed out by Hansen and Verlet for normal liquids [5].

Verlet's criterion for freezing is the counterpart of Lindeman's criterion for melting. Verlet observed that the rescaled curves of $S(q)$ are nearly identical for a variety of liquids that freeze to the same crystalline structure. The only scale is q_m, the wave vector at the first peak in $S(q)$, with $S(q_m) \simeq 2.85$.

While Lindeman's criterion tells us a way of anticipating melting by looking at mean square displacements of the atoms in the solid phase [7], Verlet's criterion provides a way of predicting freezing by looking at the structure factor in the liquid phase. Ramakrishnan and Yussouf *[6]* have offered a theoretical explanation of the criterion based on an ab-initio density functional theory. While Lindeman's criterion has been used and corroborated for the case of the solid–liquid transition of vortices [1], Verlet's criterion has not been widely addressed in this context. The density functional theory applied to vortices has **shown** that at high fields, the correlations are effectively two dimensional and $S(q_{max}) \sim 5$ at freezing [8,19].

More recently Löwen et *al.* [9] made the empirical observation of a *dynamical* criterion for freezing of colloidal liquids. For these systems the ratio of the long-time and short-time diffusion coefficients of the colloidal particles D_L/D_S has a universal value close to 0.1 at the freezing temperature of a variety of systems. It has been shown that, for the colloidal liquids, universalities in $S(q)$ at freezing imply universalities in the long time diffusion constant [10, 11].

In the present paper, we will assume the validity of Verlet's criterion for vortices. However, we wish to emphasize that there are still important differences in the the diffusion coefficients of vortices and colloidal particles which stem from the different topological properties of these systems. Vortices are extended objects that can entangle.

In the entangled phase diffusion implies cutting of vortex lines, an effect which is not present in suspended particles. For clean systems we show that vortex cutting, together with Verlet's criterion, implies a universality in the jump of both ρ_{ab} and ρ_c. We illustrate this by solving a simple model for vortex cutting, and show that the resistivity at the melting temperature is given by the normal resistivity and the structure factor only, and thus the observed universality in the resistivity jump implies the validity of Verlet's criterion.

If a current is applied, different vortices experience different net forces depending on their total length and on their orientation (in the entangled phase some spontaneously generated vortices percolate in directions perpendicular to the applied field). This implies velocity gradients, and a limiting center of mass velocity that depends on the vortex–vortex interactions even *in the* ubsence ofpinning impurities. This distinguishes the entangled vortex phase from a colloidal suspension, for which the limiting velocity in the absence of hydrodynamic interactions is given by the bare viscosity and is independent of the particle–particle interactions. Therefore, for a colloidal system in an infinite container the self difussion coefficient goes to zero at the freezing transition but the flow resistance is independent of interactions and is the same above and below the transition since the solid moves as a whole. One could overcome this by introducing a non-extensive pinning surface that is enough to pin the solid but does not affect the viscosity of the liquid.

We will first analyze the out of plane resistivity ρ_c. The mechanisms of dissipation when the current is parallel to the applied field originate in the entanglement of the field-induced vortices and the thermally generated vortex loops [14]. In this entangled phase, the structure of vortex lines percolates through the sample in the direction perpendicular to the external field [15]. The result is a "phantom mesh" of vortex lines that threads the sample in all directions and where each vortex diffuses by cutting the others. The topology of the phantom mesh allows different segments of the field-induced vortices to diffuse independently. This implies that the vortex motion is uncorrelated in the c-direction, as observed in the pseudo **DC** transformer experiments [16].

When a current flows along the c-axis, only those lines oriented in the direction perpendicular to the c-axis will contribute to the bulk resistivity. The resistivity can be written as

$$\rho_c = \left(\frac{\phi_0}{c}\right)^2 n_P \frac{D_P}{k_B T}, \qquad (2)$$

with n_P the density of percolating vortex lines, and D_P their diffusion coefficient. In the computation of D_P one needs to include the cutting of the percolating vortex lines with the field-induced vortices.

We stress that the viscosity in the present case can be obtained from the self-diffusion coefficient of the "horizontal" vortices, because, after acted upon by a force, they will move with respect to the field induced vortices. In other words, what we are really computing is the renormalized viscosity.

We compute the diffusion by considering the random walk of the center of mass of a vortex line. We introduce a discretization of the random walk, with step length ℓ, and we call τ_i the time elapsed at the i-th step. Neglecting vortex cutting all time steps are equal, with $\tau_i = \tau_0$, and the bare diffusion coefficient is given by $D_0^{(c)} = \ell^2/\tau_0$. This bare diffusion constant is related to the Bardeen–Stephen [12, 13] viscosity coefficient $\eta_0^{(c)} = k_B T/D_0^{(c)}$. The superscript "$c$" refers to the viscosity corresponding to the motion in the direction perpendicular to the c-axis. Written in terms of the normal state resistance ρ_{cN} in the c-direction we have $\eta_0^{(c)} = (\hbar/e)^2(\pi/2)/[\rho_{cN}\xi_c^2]$, with ξ_c an effective core radius for an horizontal vortex which will be of the order of magnitude of the distance between layers.

Assume now that there is a fraction n_c of steps, at which vortex cutting takes place, with a characteristic step time $\tau_\times$; the diffusion coefficient is then

$$D_P = \ell^2/\left[n_c\tau_\times + (1 - n_c)\tau_0\right]. \qquad (3)$$

The cutting frequency is

$$\frac{1}{\tau_\times} = \frac{1}{\tau_0} e^{-U_\times/k_B T} \qquad (4)$$

with $U_\times$ an effective barrier for cutting [17]. In the above equation we have assumed thermal activation for the vortex crossings. The Boltzman factor gives the relative probability of finding a vortex in a crossing configuration.

If $k_B T < U_\times$, and assuming that n_c is of the order of the density of field-induced vortices n, the resistivity is given by

$$\rho_c = \left(\frac{\phi_0}{c}\right)^2 \frac{1}{\eta_0^{(c)}} \left(\frac{a_0}{\xi_z}\right)^2 e^{-U_\times/k_B T} \tag{5}$$

where $a_0 = n^{-1/2}$ is the mean distance between field induced vortices along the c-direction and $\xi_z = n_P^{-1/2}$ is the mean distance between percolating vortices in the direction perpendicular to the field. In what follows, we argue that Verlet's criterion implies that ρ_c is universal (in a weak sense) at the melting temperature. The argument is two-fold: *i* — The universality of the structure factor $S(q)$ indicates that the scaled correlation lengths are independent of density at T_m, and therefore the factor $(a_0/\xi_z)^2$ is density independent at freezing. *ii* — The exponential factor in equation (5) represents an effective probability of cutting which will be also universal at melting due to Verlet's criterion. (See the solution of the simplified one-dimensional model below.)

We conclude that the out-of-plane resistivity ρ_c is independent of the vortex density at the melting temperature T_m. This weak universality is a consequence of Verlet's criterion.

The arguments presented above are valid for good quality single crystals for which vortex pinning is weak. Evidence from both experiments and numerical simulations have shown that intrinsic disorder destroys the first order transition. For example, upon irradiation, the magnitude of the jump decreases. In the cleanest YBCO samples, the jump in ρ_{ab} is about a fifth of the normal resistivity at the critical temperature. **A** smaller jump in the resistivity at the melting transition is an indication of disorder. In our formulation, a large disorder can be incorporated in a modified particle–particle correlation function which will retain its universal properties as long as the transition remains first order. Consequently, to the extent that Verlet's criterion remains valid, the universality in the resistivity jump is preserved.

Now let us turn to a simplified one-dimensional model of interacting diffusing particles. The purpose of this discussion is to justify using a probability of cutting which is universal at melting. We consider the following Fokker–Planck equation, describing diffusing interacting particles

$$\frac{\partial}{\partial t} P(\{x_i\}, t) = \left\{ \sum_{i=0}^{N} D_i \frac{\partial^2}{\partial x_i^2} - \sum_{i=0}^{N} \frac{D_i}{k_B T} \frac{\partial}{\partial x_i} F_i \right\} \times P(\{x_i\}, t), \tag{6}$$

with $F_i = -\delta_{i,0}\left[f + \sum_{j=1}^{N} V'(x_j - x_0)\right] + (1 - \delta_{i,0}) V'(x_i - x_0)$, V being an interaction potential between particle zero and the rest of the N particles. Note that there is no interaction between particle i land particle j for $i \neq 0 \neq j$. Also, the diffusion constant is D_0 for particle 0, and $D_i \ll D_0$ for $i \neq 0$. We added a drag force f acting on particle zero, and **computed** the resulting mean drift velocity of particle zero $\langle v \rangle$ by solving the Focker–Planck equation imposing periodic boundary conditions [18] on a length L. The result is $\langle v \rangle = f/\eta$, with

$$\frac{1}{\eta} - \frac{1}{\eta_0} = \frac{L}{\int_0^L dx e^{-\sum_{i=1}^{N} V(x-x_i)/k_B T}} \times \frac{L}{\int_0^L dx e^{+\sum_{i=1}^{N} V(x-x_i)/k_B T}}, \tag{7}$$

with $\eta_0 = k_B T / D_0$ Note that, since particle zero diffuses faster than the other particles, we take the configuration of the rest of the particles as described by the equilibrium distribution in ***the*** absence of the force f. **Also,** if the potential is monotonically decreasing with distance with $V(0) \gg k_B T$ and the range of the interaction is much smaller than the inter-particle separation,

$$\frac{1}{L} \int_0^L dx e^{+\sum_{i=1}^{N} V(x-x_i)/k_B T} \sim \frac{N}{L} \lambda e^{V(0)/k_B T}, \tag{8}$$

with λ the range of the interaction. We can now identify the integral in equation (7) which involves the negative exponential as a partition function Z, and defining $P_\times = [\exp -V(0)/k_B T]/Z$, we obtain

$$\frac{\eta_0}{\eta} = \frac{1}{n\lambda} P_\times. \tag{9}$$

In this simplified model $P_\times$ represent the probability of finding particle zero on top of particle i This probability can be extracted from the dimensionless pair correlation function $g(r)$ or its Fourier transform $S(q)$: $P_\times = \int d(q/q_m)[S(q/q_m) - 1]$. This results from the fact that in this simplified model $g(r) = [\exp -V(r)/k_B T]/A$, with A a normalization constant. The jump in the in-plane resistivity is also independent of the external field.

The resistivity ρ_{ab} can be written as

$$\rho_{ab} = \left(\frac{\phi_0}{c}\right)^2 n \frac{D}{k_B T}, \tag{10}$$

with $n = B/\phi_0$ the density of field induced vortices along the direction of the applied field B, D is the corresponding diffusion coefficient. The diffusion coefficient D can be calculated following the out-of-plane case, with an analogous results. The diffusion constant, or more precisely, the viscosity, is reduced by a factor proportional to n due to vortex cutting, canceling the prefactor of n in the above equation.

For the vortex case the quantity $n\lambda$ defined in the above one-dimensional calculation is identified with $n\xi^2$, ξ being the core radius. Finally, if in our equation for the resistivity $\rho_{ab} = (\phi_0/c)^2 n/\eta = (\phi_0/c)^2 P_\times/(\eta_0^{(ab)}\xi^2)$ we replace the expression for the Bardeen–Stephen viscosity, $\eta_0^{(ab)} = (\hbar/e)^2(\pi/2)/(\rho_{abN}\xi^2)$, with ρ_{abN} the normal state resistivity, we obtain for the resistivity jump at T_m

$$\frac{\Delta\rho_{ab}(T_m)}{\rho_{abN}} = P_\times[S(q/q_m)], \qquad (11)$$

where $P_\times[S(q/q_m)]$ means that $P_\times$ is a functional of $S(q/q_m)$ and is universal at melting. This equation is analogous to equation (5) and constitutes the main result of this work.

The calculation of the probability $P_\times$ of finding vortices in the cutting configuration requires the knowledge of the of the exact correlation functions $g(r)$. An estimate can be extracted from the calculations of the "cage" model described in Ref. [1]. Within that simple an entanglement length l_z is defined as the distance along the field required for a vortex to diffuse a distance of the order of the vortex–vortex separation a_0: $l_z \sim g\phi_0/(k_BTB)$, with g the tilt energy, and $a_0^2 \sim \phi_0/B$. From the above discussion, we estimate

$$P_\times \sim a_0/l_z = \frac{k_BT_m}{g}\left(\frac{B}{\phi_0}\right)^{1/2}. \qquad (12)$$

On the other hand, within the cage model, a melting temperature can be extracted by applying Lindemann's criteron on the solid phase [1]: $k_BT_m = c_L^2\varepsilon_0(m_\perp/m_z)^{1/2}(\phi_0/B)^{1/2}$, with $m_\perp$ and m_z being respectively the in-plane effective mass and the out-of-plane effective mass, and $\epsilon_0 = (m_z/m_\perp)g$ a coupling constant giving the interaction energy per unit length for the vortex lines. Also, c_L is the Lindemann constant, which for vortices is of order 0.1–0.2 [2]. Collecting this with equation (12) we obtain

$$P_\times \sim c_L^2(m_\perp/m_z)^{1/2}. \qquad (13)$$

If we assume that the anisotropy on the effective mass is the same as that of the normal resistivities, we obtain $\Delta\rho(T_m)/\rho_N \sim 10^{-2}$ for YBaCuO and $\Delta\rho(T_m)/\rho_N \sim 10^{-4}$ for BISCO, which should be compared with the jumps observed in experiments of 0.1 [3] and 10^{-4} [21] respectively.

An alternative estimate of the relative jump at melting can be can be computed from the expression $P_\times = \exp - U_\times/k_BT_m$, and using results from experimental fits [22] and numerical simulations [23] that indicate $U_\times \sim k_BT_m$. For the observed $\Delta\rho(T_m)/\rho_N = 0.1$ we obtain $U_\times = 2.3k_BT_m$, whereas the numerical simulations give $U_\times = 7.5k_BT_m$. Even though there is a numerical discrepancy, the fact that the crossing energy is independent of field at the melting temperature constitutes, in our view, an additional element supporting the applicability of Verlet's criterion in the vortex state.

Our theory assumes a continuum approximation for the vortex degrees of freedom, and will be valid as long as l_z is larger than the interplane separation s. When l_z becomes of the order of s one attains the so called decoupled or "superentangled" regime, in which our approximations are no longer valid. Therefore, we expect deviations from the universal jump in the resistance for magnetic fields larger than $B_{x2} = g\phi_0/sk_BT$.

An experimental proof of the universalities in $S(q)$ could be obtained from neutron scattering experiments [20] performed at the melting temperature, or probably by decorating a sample rapidly quenched from the melting point.

In summary, we have presented an explanation for the universal jump in the resistivity at the melting transition of vortices in high temperature superconductors. The theory is based on universalities at the melting transition as reflected in the structure factor, together with vortex cutting dominating the viscosity of vortices.

Acknowledgements—A.G.R. acknowledges partial support from the National Science Foundation. We acknowledge conversations with Charlie Doering, Mark Dykman, Franco Nori, Hugo Safar and Len Sander. We thank V. Vinokur for very useful remarks.

REFERENCES

1. Crabtree, G. and Nelson, D., ***Physics Today*, 50,** 1997, 38.
2. Blatter, G., *et al.*, ***Rev. Mod. Phys.*, 66,** 1994, 1125.
3. Safar, H., *et al.*, ***Phys. Rev. Lett.*, 69,** 1992, 824; Kwok, W.K., *et al.*, ***Phys. Rev. Lett.*, 69,** 1993, 3370.
4. Pastoriza, H., Goffman, M.F., Arribere, A., de la Cruz, F., ***Phys. Rev. Lett.*, 72,** 1994, 2951.
5. Hansen, J.-P. and Verlet, L., ***Phys. Rev.*, 184,** 1969, 159.
6. Ramakrishnan, T.V. and Yussouf, M., ***Phys. Rev.*, 18,** 1979, 2775.
7. Ziman, J.M., ***Principles of the Theory of Solids***, Cambridge University Press, 1972, Ch. 2.
8. Menon, G.I., Dasgupta, C., Krishnamurthy, H.R., Ramakrishnan, T.V. and Sengupta, S., ***Rev. B*, 54,** 1996, 16192; Menon, G.I. and Dasgupta, C., Private communication.
9. Löwen, H., Palberg, T. and Simon, R., ***Phys. Rev. Lett.*, 70,** 1993, 1557.
10. Indrani, A.V. and Ramaswamy, S., ***Phys. Rev. Lett.*, 73,** 1994, 360.

11. Medina-Noyola, M., *Phys. Rev. Lett.*, **60,** 1988, 2705.
12. Tinkham, M., *Introduction to Superconductivity,* McGraw-Hill, New York, 1996, p. 163.
13. Bardeen, J. and Stephen, M.J., *Phys. Rev.*, **140,** 1965, A1197.
14. Jagla, E. and Balseiro, C., *Phys. Rev. Lett.*, 77, 1996, 1588.
15. Jagla, E. and Balseiro, C., *Phys. Rev. B,* **53,** 1996, 538; **53,** 1996, 15305.
16. Lopez, D. *et al., Phys. Rev. B*, **53,** 1996, 8895; Lopez, D., Righi, E.F., Nieva, G. and de la Cruz, F., *Phys. Rev. Lett.*, **76,** 1996,4034.
17. Marchetti, M.C. and Nelson, D., *Physica* C, **174,** 1991, 40; Obukov, **S.P.** and Rubenstein, M., *Phys. Rev. Lett.*, **65,** 1990, 1279.
18. Risken, **H.,** *The Fokker–Planck Equation,* Ch. 11, Springer-Verlag, 1984.
19. Sengupta, S. *et al.*. *Phys. Rev. Lett.*. **67,** 1991,3444.
20. Cubitt. R. *et al., Nature,* **365,** 1993, 407.
21. Fuchs, D.T., *et al.*, *Phys. Rev. B,* **54,** 1996, R796.
22. Fendirch, J.A. *et al., Phys. Rev. Lett.*, 74, 1995, 1210.
23. Wilkin, N.K. and Moore, **M.A.,** *Physica C*, **325-240.** 1994,2641.