THREE DIMENSIONAL BOUNDARY LAYER ON A YAWED SEMI-INFINITE FLAT PLATE WITH OR WITHOUT SUCTION AND INJECTION

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1. Introduction

The boundary layer on yawed infinite cylinder and on yawed infinite flat plate was first studied by Sears.[1] It was shown that for an incompressible flow over such a yawed body, the chordwise flow, both inside and outside the boundary layer, is independent of the spanwise flow. This 'principle of independence' introduced a considerable mathematical simplification. In order to solve the problem by momentum integral method earlier workers took both the boundary layer thicknesses δ_x, δ_y for chordwise and spanwise profiles to be the same, but greater accuracy is achieved by taking δ_x, δ_y, as different as has been shown by Wild,[2] Cooke,[3] and Wilkinson.[4] They have used the Pohlhausen technique assuming fourth degree profiles for the spanwise and chordwise flows. Besides, following the usual pattern they take only one boundary condition at the surface. An excellent review of the literature on three-dimensional boundary layer is given in References (13) and (14).

In the present paper we have considered the boundary layer on a yawed semi-infinite plate for the main flows: Case (i) $U = 1 - x$, $V = \text{const.}$, Case (ii) $U = 1 + x$, $V = \text{const.}$ Following Bhatnagar and Jain,[5] we take two boundary conditions at the plate and compare the results of the chordwise flow by taking fourth, fifth and sixth degree profiles for (u/U). We find that a fifth degree profile predicts the point of separation closer to that predicted by Görtler's new series for the chordwise flow $U = 1 - x$, and a sixth degree profile gives better range of applicability for the chordwise flow $U = 1 + x$.

For the calculation of spanwise flow we have taken the boundary layer thicknesses for chordwise (δ_x) and spanwise (δ_y) flows as unequal. We have calculated the spanwise flow by assuming fourth, fifth and sixth degree profiles for (v/V). We have plotted the various flow characteristics against the chordwise distance in Figs. 1–8. In particular, we mention that for decelerating flow the x-component of the skin friction is less than its y-component at all points on the plate and decreases more rapidly as we move downstream, as expected due to the separation in the chordwise flow. The conclusion for the accelerating flow is in the reverse direction.

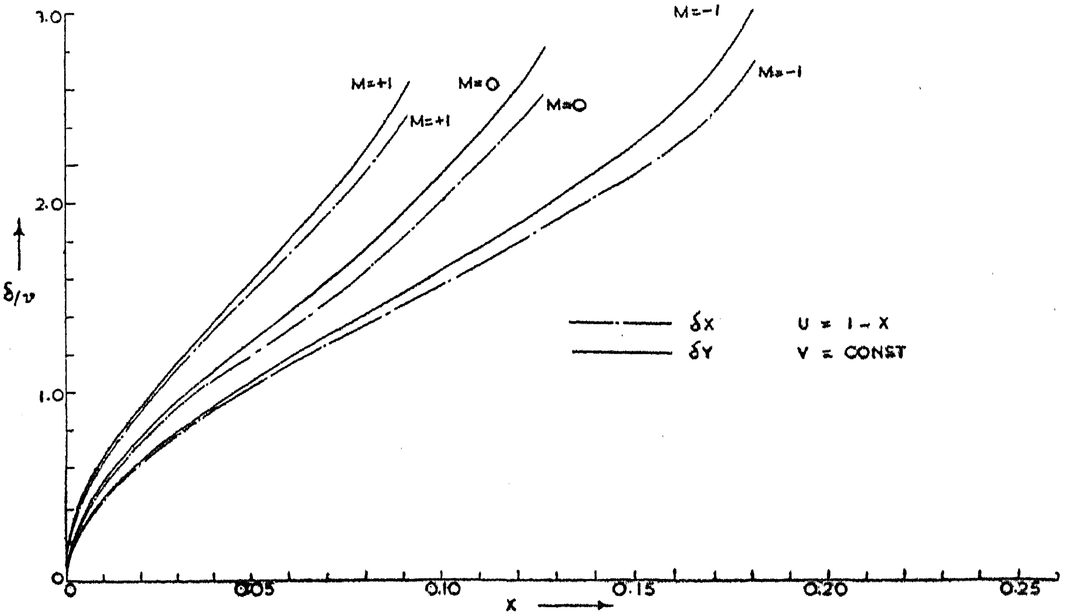

FIG. 1. Boundary layer thickness against chord-wise distance.

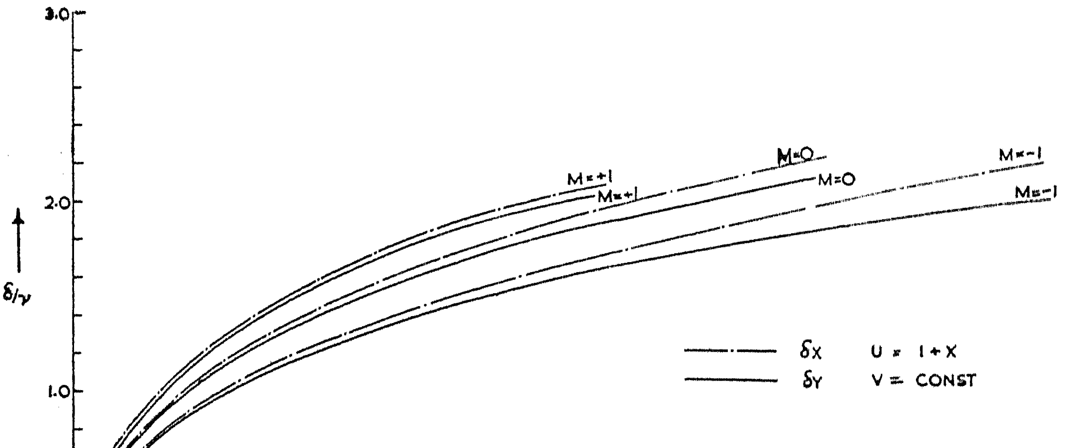

FIG. 2. Boundary layer thickness against chord-wise distance.

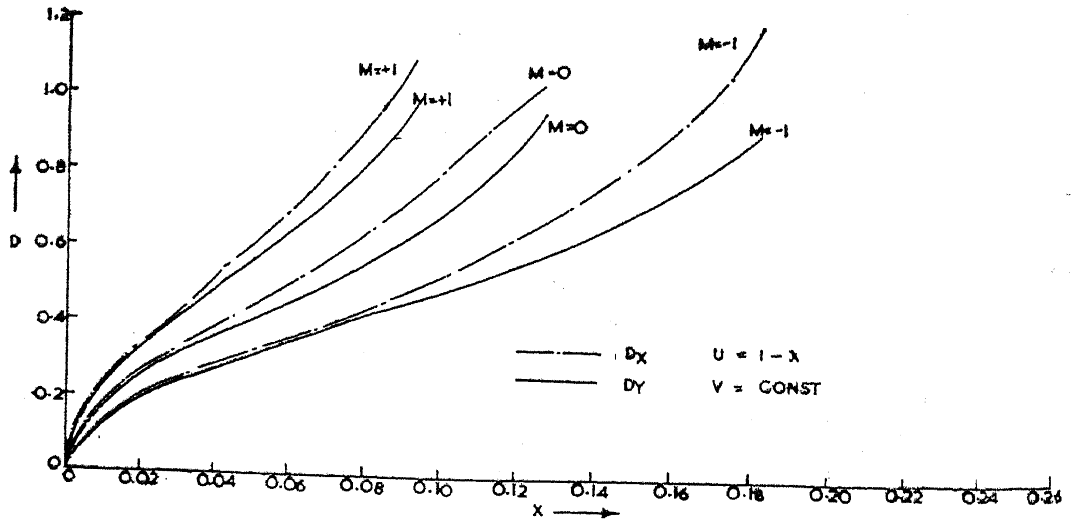

FIG. 3. Displacement thickness against chord-wise distance.

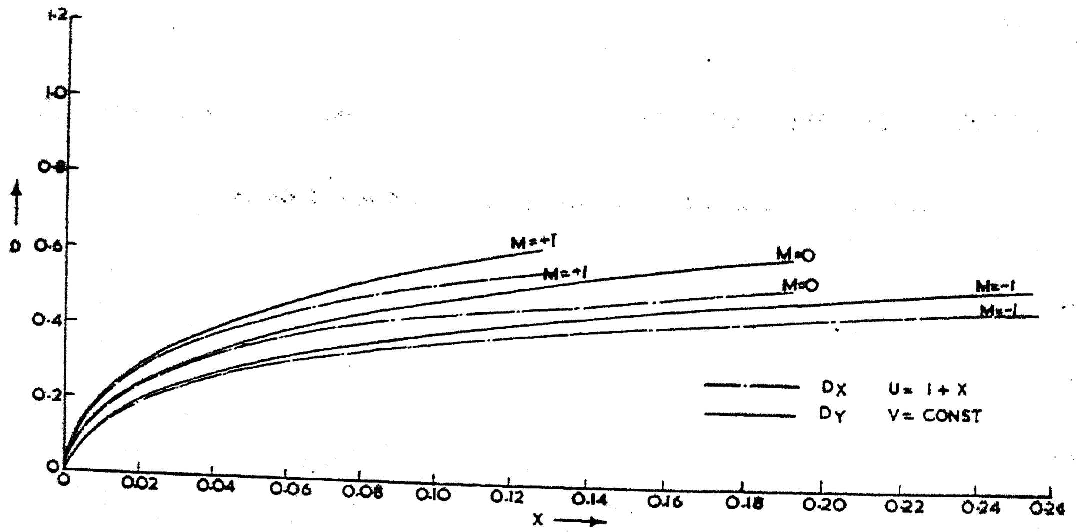

FIG. 4. Displacement thickness against chord-wise distance.

In Part B we discuss the effect of imposing suction and injection on the boundary layer flows discussed in Part A. To simplify the analysis we have represented the effect of suction or injection by a parameter $M = w_0\delta_x/\nu$ and considered only those normal suction or injection velocities w_0 which conform to $w_0 = \pm \nu/\delta_x$ ($M = \pm 1$), *i.e.*, the magnitude of the suction or injection velocity is inversely proportional to the boundary layer thickness in the chordwise direction.

The analysis predicts that the suction increases the skin friction for both the accelerating and decelerating flows considered here. Further,

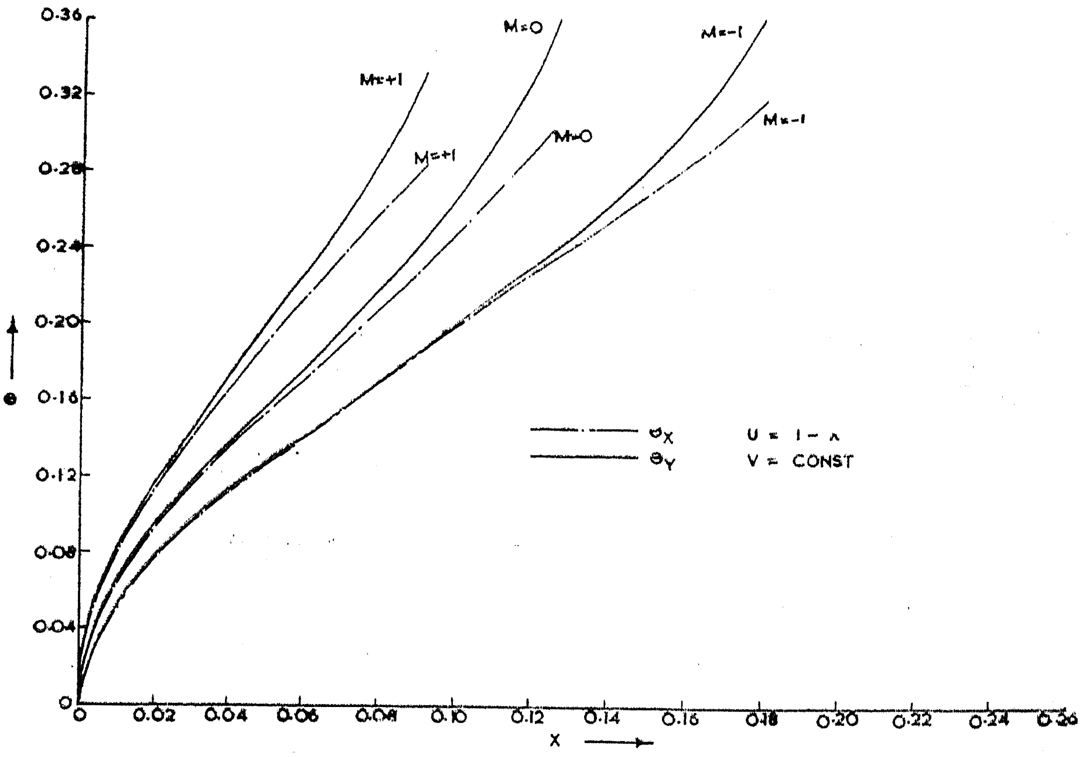

FIG. 5. Momentum thickness against chord-wise distance.

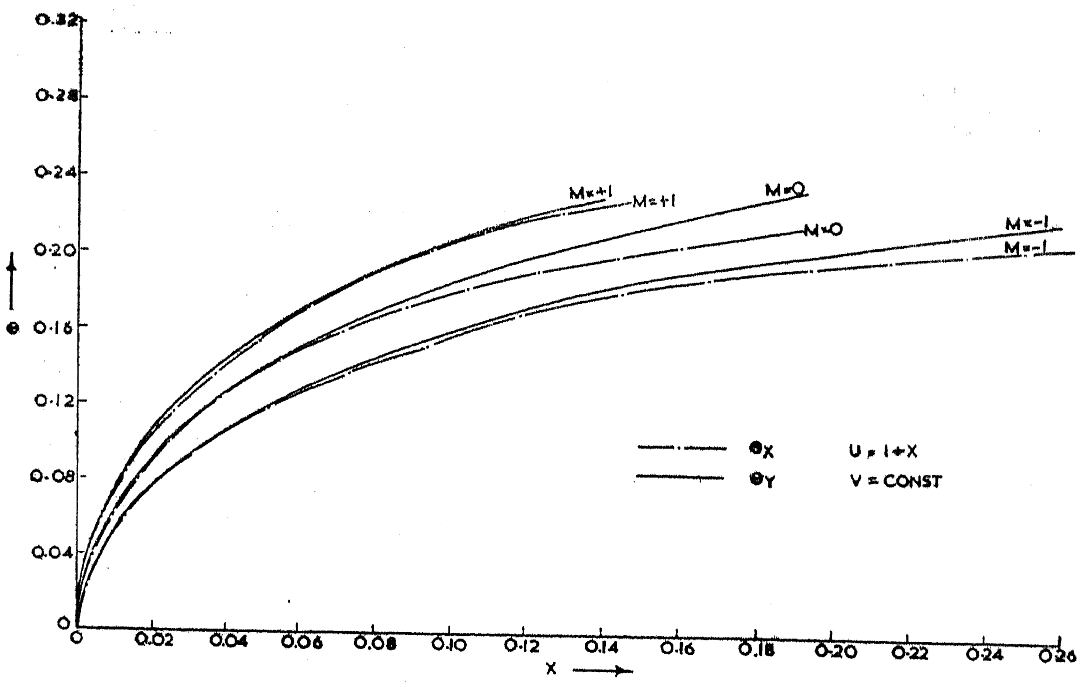

FIG. 6. Momentum thickness against chord-wise distance.

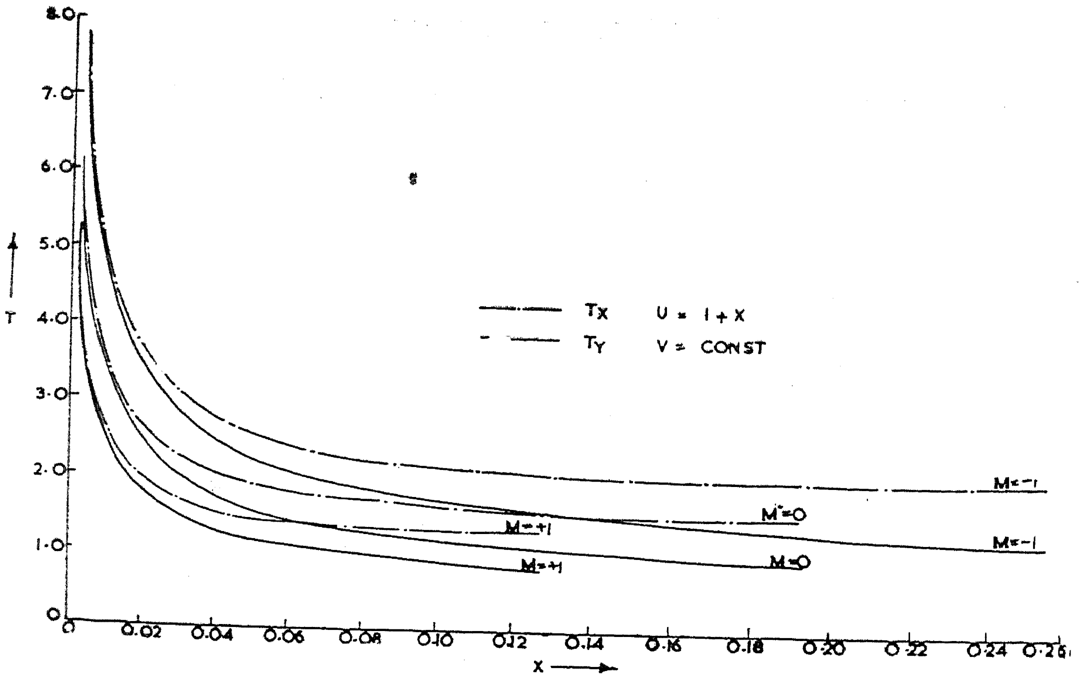

FIG. 7. Skin friction against chord-wise distance.

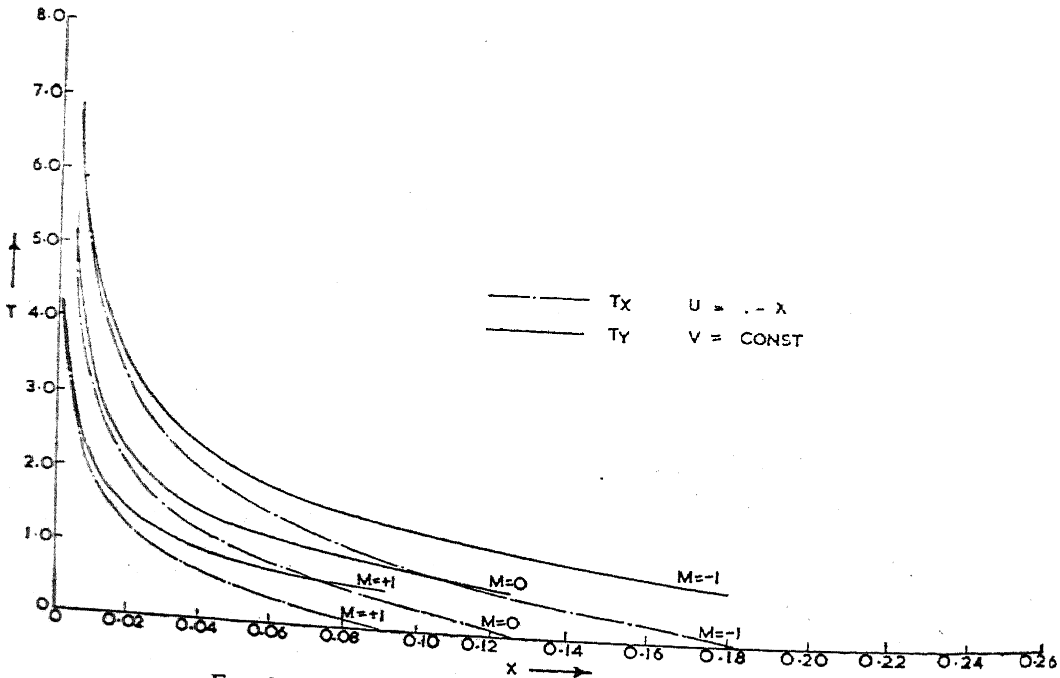

FIG. 8. Skin friction against chord-wise distance.

suction delays the separation in the decelerating flow and increases the range of applicability of the profiles for the accelerating flow.

2. Equations of the Problem

The boundary layer equations for incompressible flow in three dimensions are[6]:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \tag{2.1}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} \tag{2.2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2.3}$$

where

x, y are curvilinear orthogonal co-ordinates on the surface of the body,

z is the co-ordinate normal to the body,

p is pressure, μ is coefficient of viscosity, ρ is density,

ν is kinematic viscosity (μ/ρ).

For a yawed wing of infinite span, all the variables are independent of y and the equations reduce to

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \tag{2.4}$$

$$u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} \tag{2.5}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{2.6}$$

where x and y, in this case, are measured along chordwise and spanwise directions respectively.

Boundary conditions are:

(i) $u = v = w = 0$ at $z = 0$ (no suction or injection) (2.7)

$u = v = 0$, $w = w_0$ at $z = 0$ (in the presence of suction or injection) (2.7′)

(ii) $u \to U$, $\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial z^2} = \ldots = 0$ as $z \to \infty$ (2.8)

$v \to V$, $\frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} = \ldots = 0$ as $z \to \infty$. (2.8′)

In the present paper we consider the following main flows:

Case (i) $U = 1 - x$, $V = \text{const.}$ (2.9)

Case (ii) $U = 1 + x$, $V = \text{const.}$ (2.10)

On integrating equation (2.4) and equation (2.5) w.r.t. z from 0 to δ_x and 0 to δ_y respectively and using equations (2.6), (2.7), (2.8) we get the momentum integral equations which include the terms for suction or injection as:

$$\frac{d}{dx}\int_0^{\delta_x} u^2 dz - U\frac{d}{dx}\int_0^{\delta_x} u dz = UU'\delta_x - w_0(x)U - \nu\left(\frac{\partial u}{\partial z}\right)_{z=0} \tag{2.11}$$

and

$$\frac{d}{dx}\int_0^{\delta_y} u(v - V)\,dz = -w_0(x)V - \nu\left(\frac{\partial v}{\partial z}\right)_{z=0} \tag{2.12}$$

where

δ_x is the chordwise boundary layer thickness,

δ_y is the spanwise boundary layer thickness,

and

$w_0(x)$ is the velocity normal to the surface which is negative for suction and positive for injection.

3. METHOD OF SOLUTION

We observe that equation (2.11) is the momentum integral equation for a two-dimensional case which can be integrated by assuming polynomial expression for the velocity distribution (u/U) in terms of the normal distance from the wall. Knowing the form of u, and assuming a polynomial expression for the velocity distribution (v/V) we integrate equation (2.12). Following Bhatnagar and Jain, we evaluate the coefficients by taking two boundary conditions on the surface, *viz.*,

$$\left.\begin{aligned}
&\text{(i) } UU' + \nu\left(\frac{\partial^2 u}{\partial z^2}\right)_{z=0} \\
&\quad = w_0(x)\left(\frac{\partial u}{\partial z}\right)_{z=0} && \text{(in the presence of suction or injection)} \\
&\quad = 0 && \text{(no suction or injection)} \\
&\text{(ii) } \nu\left(\frac{\partial^3 u}{\partial z^3}\right)_{z=0} \\
&\quad = w_0(x)\left(\frac{\partial^2 u}{\partial z^2}\right)_{z=0} && \text{(in the presence of suction or injection)} \\
&\quad = 0 && \text{(no suction or injection)}
\end{aligned}\right\} \qquad (3.1)$$

for chordwise flow and

$$\left.\begin{aligned}
&\text{(i) } \nu\left(\frac{\partial^2 v}{\partial z^2}\right)_{z=0} \\
&\quad = w_0(x)\left(\frac{\partial v}{\partial z}\right)_{z=0} && \text{(in the presence of suction or injection)} \\
&\quad = 0 && \text{(no suction or injection)} \\
&\text{(ii) } \nu\left(\frac{\partial^3 v}{\partial z^3}\right)_{z=0} \\
&\quad = w_0(x)\left(\frac{\partial^2 v}{\partial z^2}\right)_{z=0} && \text{(in the presence of suction or injection)} \\
&\quad = 0 && \text{(no suction or injection)}
\end{aligned}\right\} \qquad (3.2)$$

for the spanwise flow.

PART A

4. Chordwise Flow

First, we concentrate on the chordwise flow in the absence of suction or injection and consider polynomial expressions for (u/U) of degrees 4, 5 and 6 to examine which profile gives better agreement with the exact results. We shall use the profile of this degree in Part B when we consider suction or injection.

(i) *Fourth degree profile*

Let

$$\frac{u}{U} = \sum_{s=1}^{4} a_s \eta_x^{\ s}$$

where

$$\eta_x = \frac{z}{\delta_x}. \tag{4.1}$$

From the boundary conditions (3.1) and (2.8) with $w_0 = 0$ we have

$$a_1 = \frac{N+4}{3}; \quad a_2 = -\frac{N}{2}; \quad a_3 = 0;$$

$$a_4 = \frac{N-2}{6}; \quad N = \frac{U'\delta_x^{\ 2}}{\nu}. \tag{4.2}$$

The equation (2.11) with (4.1) and $w_0 = 0$ reduces to

$$\frac{dN}{dx} = -2\frac{U'}{U}\frac{17N^3 + 211N^2 - 2002N + 7560}{42N^2 + 33N - 812} \tag{4.3}$$

which on integration gives

$$12.890365 - 4\, ln \frac{U}{U_0} = 3\cdot193875\, ln\,(N + 19\cdot5844)$$
$$+0\cdot903062 ln\ (N^2 - 7\cdot172635N + 22\cdot707053)$$
$$-\,0.670427\, \tan^{-1}\frac{N - 3\cdot586317}{3\cdot137731} \tag{4.4}$$

where we have used the condition that $N = 0$ when $x = 0$.

For $U = 1 - x$, the point of separation is given by $x_S = 0\cdot112$ and for $U = 1 + x$, the range of applicability* is given by $x_R = 0\cdot132$.

(ii) *Fifth degree profile*

Let

$$\frac{u}{U} = \sum_{s=1}^{5} a_s \eta_x^{\ s},$$

where

$$\eta_x = \frac{z}{\delta_x},$$

* For definition refer to p. 7 in reference 5.

and

$$\left.\begin{aligned} a_1 &= \frac{3N+20}{12}; \quad a_2 = -\frac{N}{2}; \quad a_3 = 0 \\ a_4 &= \frac{3N-10}{6}; \quad a_5 = \frac{4-N}{2}; \quad N = \frac{U'\delta_x^2}{\nu} \end{aligned}\right\}. \tag{4.6}$$

The equation (2.11) with (4.5) and $w_0 = 0$ reduces to

$$\frac{dN}{dx} = -\frac{U'}{U}\frac{1692N^3 + 39264N^2 - 662320\,N + 3326400}{2115N^2 + 4500N - 124000} \tag{4.7}$$

which on integration gives

$$\begin{aligned} 3\cdot889228 - ln\frac{U}{U_0} \\ &= 0\cdot802432\, ln\,(N + 35\cdot709317) \\ &\quad + 0\cdot223784\, ln\,(N^2 - 12\cdot503641N + 55\cdot054385) \\ &\quad - 0\cdot122953 \tan^{-1}\frac{N - 6\cdot251820}{3\cdot996149} \end{aligned} \tag{4.8}$$

assuming $N = 0$ when $x = 0$

For $U = 1 - x$, the point of separation is given by $x_s = 0\cdot127$

For $U = 1 + x$, the range of applicability is given by $x_R = 0\cdot087$.

(iii) *Sixth degree profile*

Let

$$\frac{u}{U} = \sum_{s=1}^{6} a_s \eta_x^{\,s}$$

where

$$\eta_x = \frac{z}{\delta_x}, \tag{4.9}$$

and

$$\left.\begin{aligned} &a_1 = \frac{N+10}{5}; \quad a_2 = -\frac{N}{2}; \quad a_3 = 0; \\ &a_4 = N - 5; \quad a_5 = 6 - N; \quad a_6 = \frac{3N-20}{10}; \\ &N = \frac{U'\delta_x^2}{\nu} \end{aligned}\right\}. \tag{4.10}$$

The equation (2.11) with (4.9) and $w_0 = 0$ reduces to

$$\frac{dN}{dx} = -\frac{U'}{U}\frac{168N^3 + 6288N^2 - 164532N + 1081080}{210N^2 + 855N - 29550} \qquad (4.11)$$

which on integration gives

$$4 \cdot 365268 - ln\frac{U}{U_0}$$
$$= 0 \cdot 803906 \, ln\,(N + 56 \cdot 702042)$$
$$+ 0 \cdot 223047 \, ln\,(N^2 - 19 \cdot 273471N + 113 \cdot 487995)$$
$$- 0 \cdot 064675 \tan^{-1}\frac{N - 9.636735}{6.373487} \qquad (4.12)$$

under the condition that $N = 0$ when $x = 0$.

For $U = 1 - x$, the point of separation is given by $x_s = 0 \cdot 136$

For $U = 1 + x$, the range of applicability is given by $x_R = 0 \cdot 193$.

To decide about the choice of the velocity profile we compare the predicted point of separation with the other known results in the following table:—

Name of the method	Position of the point of separation
Th. V. Karman and Millikan[7] ..	0·102
Howarth[8]	0·120
Meksyn[9]	0·124
Schlichting[10]	0·138
Pohlhausen[11]	0·160
Görtler (New Series)[12]	0·126
Present method:	
Fourth degree profile	0·112
Fifth degree profile	0·127
Sixth degree profile	0·136

(4.13)

From (4.13) it is evident that the present approach gives excellent results for retarding flows with fifth degree profile because in this case the position of the point of separation is nearly the same as that predicted by Görtler's new series.

For accelerating flows we observe that the sixth degree profile has greater range of applicability than the fourth or fifth degree profiles as shown by the following table:—

Name of the method	Range of applicability
Karman-Pohlhausen: Fourth degree profile	0.164
Present method:	
Fourth degree profile	0.132
Fifth degree profile	0.087
Sixth degree profile	0.193

(4.14)

Hence, we conclude that a fifth degree profile with two boundary conditions on the plate gives better results for retarding flows and a sixth degree profile with two boundary conditions on the plate gives better results for accelerating flows.

5. Spanwise Flow

Case (a).—Fourth degree profile, for the cases (2.9) *and* (2.10)

Let

$$\frac{v}{V} = \sum_{s=1}^{4} b_s \eta_y^{\,s}$$

where

$$\eta_y = \frac{z}{\delta_y}. \tag{5.1}$$

From the boundary conditions (3.2) and (2.8′) with $w_0 = 0$ we have

$$b_1 = \frac{4}{3}; \quad b_2 = 0; \quad b_3 = 0; \quad b_4 = -\frac{1}{3}. \tag{5.2}$$

For both the cases,

$$k\left(=\frac{\delta_y}{\delta_x}\right) > 1 \quad \text{and} \quad < 1,$$

the equation (1.12), on using (3.1) reduces to the form

$$\frac{d}{dx}(\mathrm{UF}\delta_x) = -\frac{4}{3}\frac{\nu}{k\delta_x}, \tag{5.3}$$

where

$$\mathrm{F} = -\frac{2}{5}k - \frac{\mathrm{N}-12}{30} + \frac{\mathrm{N}-8}{54k} - \frac{5\mathrm{N}-28}{5670k^4}, \quad \text{for } k > 1$$

and

$$\mathrm{F} = -\frac{\mathrm{N}+4}{27}k^2 + \frac{\mathrm{N}}{42}k^3 - \frac{\mathrm{N}-2}{405}k^5, \quad \text{for } k < 1. \tag{5.4}$$

When $k > 1$, the equation (5.3) with (4.3) reduces to

$$\frac{dk}{d\mathrm{N}} = [2268\,(25\mathrm{N}^3 - 178\mathrm{N}^2 + 1190\mathrm{N} - 7560)\,k^6 - 189\,(9\mathrm{N}^4 + 900\mathrm{N}^3 - 7330\mathrm{N}^2 + 36960\mathrm{N} - 90720)\,k^5 + 105\,(9\mathrm{N}^4 + 800\mathrm{N}^3 - 9642\mathrm{N}^2 + 29824\mathrm{N} - 2016)\,k^4 - (45\mathrm{N}^4 + 3700\mathrm{N}^3 - 30954\mathrm{N}^2 + 146720\mathrm{N} - 211680)\,k] \times [2\mathrm{N}\,(17\mathrm{N}^3 + 211\mathrm{N}^2 - 2002\mathrm{N} + 7560) \times \{2268k^5 + 105\,(\mathrm{N}-8)\,k^3 - 4\,(5\mathrm{N}-28)\}]^{-1} \tag{5.5}$$

with $k = 1$ when $\mathrm{N} = 0$.

When $k < 1$, the equation (5.3) with (4.3) reduces to

$$\frac{dk}{d\mathrm{N}} = [7560\,(42\mathrm{N}^2 + 33\mathrm{N} - 812) + 210\,(9\mathrm{N}^4 + 500\mathrm{N}^3 - 4482\mathrm{N}^2 + 17920\mathrm{N} + 30240)\,k^3 - 135\mathrm{N}\,(9\mathrm{N}^3 + 600\mathrm{N}^2 - 5194\mathrm{N} + 22680)\,k^4 + 14\,(9\mathrm{N}^4 + 650\mathrm{N}^3 - 5550\mathrm{N}^2 + 25060\mathrm{N} - 15120)\,k^6] \times [2\mathrm{N}\,(17\mathrm{N}^3 + 211\mathrm{N}^2 - 2002\mathrm{N} + 7560)\,\{-420\,(\mathrm{N}+4)\,k^2 + 405\mathrm{N}k^3 - 70\,(\mathrm{N}-2)\,k^5\}]^{-1} \tag{5.6}$$

with $k = 1$ when $\mathrm{N} = 0$.

Near $\mathrm{N} = 0$, (5.5) and (5.6) admit the following solutions:

$$k = 1 - 0{\cdot}031698\mathrm{N} - 0{\cdot}002712\mathrm{N}^2 + 0{\cdot}000891\mathrm{N}^3 + 0{\cdot}000963\mathrm{N}^4 - \ldots, \text{ for } k > 1 \tag{5.7}$$

and

$$k = 1 - 0\cdot031698\text{N} - 0\cdot002712\text{N}^2 - 0\cdot000659\text{N}^3 - 0\cdot000048\text{N}^4 - \ldots, \text{ for } k < 1 \quad (5.8)$$

We have calculated the starting values from (5.7) and (5.8) to perform the numerical integration of (5.5) and (5.6) by Milne's method.

From (5.7) and (5.8) we conclude that $k > 1$ for retarding flows and $k < 1$ for accelerating flows.

Tables I and II give the values of k for some selected values of N for the two cases of the main flows which we have considered.

Case (i).—

$$\text{U} = 1 - x, \quad \text{V} = \text{const.}; \quad k > 1;$$

Case (ii).—

$$\text{U} = 1 + x, \quad \text{V} = \text{const.}, \quad k < 1.$$

TABLE I

$-$N=	0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0
k=	1·0031	1·0062	1·0093	1·0123	1·0182	1·0240	1·0297	1·0571	1·0844	1·1131

TABLE II

+N=	0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0
k=	0·9968	0·9936	0·9902	0·9868	0·9798	0·9725	0·9648	0·9188

In case (i) we have calculated the values upto $\text{N} = -4$ only, as at this value the chordwise flow undergoes separation.

Similarly in case (ii) we have considered the values of N up to $\text{N} = 2$ only in view of the range of applicability of the assumed profile for the chordwise flow.

Case (*b*): *Fifth degree profile*

For the case (i) $\text{U} = 1 - x$, $\text{V} = \text{const.}$

We calculate the spanwise flow by assuming a fifth degree profile for (v/V):

$$\frac{v}{\text{V}} = \sum_{s=1}^{5} b_s \eta_y{}^s,$$

where

$$\eta_y = \frac{z}{\delta_y}, \tag{5.9}$$

and

$$b_1 = \frac{5}{3}; \quad b_2 = 0; \quad b_3 = 0; \quad b_4 = -\frac{5}{3}; \quad b_5 = 1. \tag{5.10}$$

Here the equation determining k is

$$\begin{aligned}\frac{dk}{dN} = [&- 166320\,(1269N^3 - 15132N^2 + 207160N - 1663200)\,k^7 \\ &+ 4158\,(846N^4 + 159552N^3 - 2344240N^2 + 18265600N \\ &- 66528000)\,k^6 - 330\,(5940N^4 + 106974N^3 - 21398400N^2 \\ &+ 104936000N - 20160000)\,k^5 + 220\,(1269N^4 + 198720N^3 \\ &- 3032136N^2 + 20769280N - 46569600)\,k^2 \\ &- 27\,(3807N^4 + 591084N^3 - 9035880N^2 + 61479200N \\ &- 133056000)\,k] \times [4\,(423N^4 + 9816N^3 - 165580N^2 \\ &+ 831600N)\,\{- 166320k^6 - 330\,(15N - 200)\,k^4 \\ &+ 880\,(3N - 28)\,k - 135\,(9N - 80)\}]^{-1}.\end{aligned} \tag{5.11}$$

As in case (*a*), here also $k > 1$ and $k = 1$ when $N = 0$.

Table III gives the values of k for some chosen values of N from $N = 0$ to $N = -6{\cdot}6$ as in the present case separation takes place at $N = -20/3$.

TABLE III

$-N=$ 0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0	5·0	6·0	6·6
$k=$1·0014	1·0029	1·0043	1·0057	1·0086	1·0114	1·0142	1·0280	1·0420	1·0564	1·0715	1·0875	1·09[illegible]

Case (*c*): *Sixth degree profile*

For the case (ii), $U = 1 + x$, $V =$ const., we assume

$$\frac{v}{V} = \sum_{s=1}^{6} b_s \eta_y^{\,s}$$

with

$$\eta_y = \frac{z}{\delta_y}, \tag{5.12}$$

and

$$b_1 = 2;\ b_2 = 0;\ b_3 = 0;\ b_4 = -5;\ b_5 = 6;\ b_6 = -2. \qquad (5.13)$$

The differential equation for k takes the form:

$$\begin{aligned}\frac{dk}{d\mathrm{N}} = {} & [45\,(126\mathrm{N}^4 + 28251\mathrm{N}^3 - 697524\mathrm{N}^2 + 5919180\mathrm{N} \\ & - 10810800)\,k^8 - 780\,(42\mathrm{N}^4 + 9333\mathrm{N}^3 - 230982\mathrm{N}^2 \\ & + 1937916\mathrm{N} - 3243240)\,k^7 + 1456\,(42\mathrm{N}^4 + 9207\mathrm{N}^3 \\ & - 228693\mathrm{N}^2 + 1885200\mathrm{N} - 2702700)\,k^6 \\ & - 3575\,(42\mathrm{N}^4 + 8577\mathrm{N}^3 - 217248\mathrm{N}^2 + 1621620\mathrm{N})\,k^4 \\ & + 4290\,(42\mathrm{N}^4 + 7317\mathrm{N}^3 - 194358\mathrm{N}^2 + 1094460\mathrm{N} \\ & + 5405400)\,k^3 + 3603600\,(42\mathrm{N}^2 + 171\mathrm{N} - 5910)] \\ & \times [4\,(42\mathrm{N}^4 + 1572\mathrm{N}^3 - 41133\mathrm{N}^2 + 270270\mathrm{N}) \\ & \times \{-315\,(3\mathrm{N} - 20)\,k^7 + 4680\,(\mathrm{N} - 6)\,k^6 \\ & - 7280\,(\mathrm{N} - 5)\,k^5 + 10725\mathrm{N}k^3 - 8580\,(\mathrm{N} + 10)\,k^2\}]^{-1}.\end{aligned} \qquad (5.14)$$

In the present case $k < 1$ and the starting value is $k = 1$ when $\mathrm{N} = 0$. The variations of k with N at chosen values of N are recorded in Table IV upto $\mathrm{N} = 5$ taking into considerations the range of applicability of the chordwise flow.

TABLE IV

N= 0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0	5·0
k=0·9993	0·9985	0·9978	0·9970	0·9955	0·9940	0·9925	0·9848	0·9766	0·9673	0·9561

6. CHARACTERISTICS OF FLOW

We give below the characteristics of flow for the cases (*b*) and (*c*) considered in § 5.

Case (*b*) *Ratarding Flow:*

Boundary layer thickness:

$$\frac{\delta_x}{\sqrt{\nu}} = \sqrt{\frac{\mathrm{N}}{\mathrm{U}'}}\,;\quad \delta_y = k\delta_x. \qquad (6.1)$$

Skin friction:

$$\mathrm{T}_x \equiv \frac{\tau_x}{\mu}\sqrt{\nu} = \mathrm{U}\,\frac{3\mathrm{N}+20}{12}\sqrt{\frac{\mathrm{U}'}{\mathrm{N}}}\,;\quad \mathrm{T}_y \equiv \frac{\tau_y}{\mu}\sqrt{\nu} = \frac{5}{3k}\sqrt{\frac{\mathrm{U}'}{\mathrm{N}}}. \tag{6.2}$$

Displacement thickness:

$$\mathrm{D}_x \equiv \frac{\delta_x^*}{\sqrt{\nu}} = \frac{20-\mathrm{N}}{60}\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}}\,;\quad \mathrm{D}_y \equiv \frac{\delta_y^*}{\sqrt{\nu}} = \frac{k}{3}\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}} \tag{6.3}$$

Momentum thickness:

$$\left.\begin{aligned} \theta_x &\equiv \frac{\theta_x^*}{\sqrt{\nu}} = \frac{124000-1500\mathrm{N}-423\mathrm{N}^2}{997920}\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}} \\ \theta_y &\equiv \frac{\theta_y^*}{\sqrt{\nu}} = \frac{775}{6237}\,k\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}} \end{aligned}\right\}. \tag{6.4}$$

These characteristics have been plotted against the chordwise distance in Figs. 1, 3, 5, 7.

Case (c) Accelerating Flow:

Boundary layer thickness:

$$\frac{\delta_x}{\sqrt{\nu}} = \sqrt{\frac{\mathrm{N}}{\mathrm{U}'}}\,;\quad \delta_y = k\delta_x. \tag{6.5}$$

Skin friction:

$$\mathrm{T}_x \equiv \frac{\tau_x}{\mu}\sqrt{\nu} = \mathrm{U}\,\frac{\mathrm{N}+10}{5}\sqrt{\frac{\mathrm{U}'}{\mathrm{N}}}\,;\quad \mathrm{T}_y \equiv \frac{\tau_y}{\mu}\sqrt{\nu} = \frac{2}{k}\sqrt{\frac{\mathrm{U}'}{\mathrm{N}}}. \tag{6.6}$$

Displacement thickness:

$$\mathrm{D}_x \equiv \frac{\delta_x^*}{\sqrt{\nu}} = \frac{30-\mathrm{N}}{105}\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}}\,;\quad \mathrm{D}_y \equiv \frac{\delta_y^*}{\sqrt{\nu}} = \frac{2k}{7}\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}} \tag{6.7}$$

Momentum thickness:

$$\left.\begin{aligned} \theta_x &\equiv \frac{\theta_x^*}{\sqrt{\nu}} = \frac{98500-950\,\mathrm{N}-302\mathrm{N}^2}{900900}\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}} \\ \theta_y &\equiv \frac{\theta_y^*}{\sqrt{\nu}} = \frac{985}{9009}\,k\sqrt{\frac{\mathrm{N}}{\mathrm{U}'}} \end{aligned}\right\}. \tag{6.8}$$

These characteristics have been plotted against the chordwise distance in Figs. 2, 4, 6, 8.

We note the following important conclusions:—

Decelerating Flow:

(i) The ratio δ_x/δ_y changes by about 10 per cent. as we go from the leading edge to a distance $x = 0 \cdot 1266$ downstream along the plate.

(ii) $T_y > T_x$ at all points. Besides, T_y decreases more slowly than T_x as we go downstream.

(iii) $D_x > D_y$ at all points and further D_x increases more rapidly than D_y as we go downstream.

(iv) $\theta_y > \theta_x$ at all points and θ_y increases more rapidly than θ_x as we go downstream.

Accelerating Flow:

(i) The ratio δ_y/δ_x changes by about 6 per cent. as we go from the leading edge to a distance $x = 0 \cdot 1928$.

(ii) $T_y < T_x$ at all points and T_x decreases more slowly than T_y as we go downstream.

(iii) $D_x < D_y$ at all points and D_x increases more slowly than D_y as we go downstream.

(iv) $\theta_x < \theta_y$ at all points and θ_x increases more slowly than θ_y as we go downstream.

Finally it is interesting to compare the characteristics of decelerating and accelerating flows:—

(i) The variation in k is more pronounced in decelerating flow than in accelerating flow.

(ii) The chordwise skin friction is less than the spanwise skin friction for decelerating flow whereas the conclusion is reversed for accelerating flow.

(iii) The chordwise displacement thickness is greater than the spanwise displacement thickness for decelerating flow and is less for accelerating flow.

(iv) The spanwise momentum thickness is greater than the chordwise momentum thickness in both the cases.

PART B

In this part we study the effect of suction or injection on the yawed infinite plate with main flows, $U = 1 - x$, $V = \text{const.}$ and $U = 1 + x$, $V = \text{const.}$ Here again we use a fifth degree profile for the decelerating flow and a sixth degree profile for the accelerating flow and integrate the equations for two values of the suction or injection parameter

$$M\left(=\frac{w_0\delta_x}{\nu}\right) = \pm 1.$$

7. RETARDING FLOW

$U = 1 - x$, $V = \text{const.}$

(a) Chordwise Flow:

Let

$$\frac{u}{U} = \sum_{s=1}^{5} c_s \eta_x^{\,s},$$

where

$$\eta_x = \frac{z}{\delta_x}. \tag{7.1}$$

From the boundary conditions (3.1) and (2.8) we have

$$\left.\begin{aligned} c_1 &= \frac{(9+M)N+60}{D}; \quad c_2 = \frac{30M - 18N}{D};\\ c_3 &= \frac{M(10M - 6N)}{D};\\ c_4 &= \frac{(8N - 15M - 45)M + (18N - 60)}{D};\\ c_5 &= \frac{(24 - 3N + 6M)M + (36 - 9N)}{D} \end{aligned}\right\} \tag{7.2}$$

where

$$D = M^2 - 9M + 36; \quad M = \frac{w_0\delta_x}{\nu}; \quad N = \frac{U'}{\nu}\delta_x^{\,2}.$$

We consider the following two cases, one for suction corresponding to $M = -1$ and one for injection $M = +1$.

(i) *Suction with* $M = -1$:

The equation (2.11) with the help of (7.1) reduces to

$$\frac{dN}{dx} = -\frac{U'}{U}\frac{532N^3 + 12978N^2 - 140742N + 620928}{655N^2 + 2457\,N - 32760} \tag{7.3}$$

which on integration gives

$$\begin{aligned} 3.791124 - ln\frac{U}{U_0} &= 0{\cdot}812303\ ln\,(N + 33{\cdot}370561) \\ &\quad + 0{\cdot}218848\ ln\,(N^2 - 8{\cdot}975824N + 34{\cdot}975659) \\ &\quad - 0{\cdot}190143\ \tan^{-1}\frac{N - 4{\cdot}487912}{3{\cdot}851533} \end{aligned} \tag{7.4}$$

under the condition that $N = 0$ when $x = 0$.

Point of separation:

$$N = -7{\cdot}5,\ x_s = 0{\cdot}181 \tag{7.5}$$

(ii) *Injection with* $M = +1$:

The equation (2.11) with (7.1) reduces to

$$\frac{dN}{dx} = -\frac{U'}{U}\frac{10324N^3 + 233046N^2 - 5761470N + 33790680}{12905N^2 + 7425N - 922350} \tag{7.6}$$

which on integration gives

$$\begin{aligned} 3.989323 - ln\frac{U}{U_0} &= 0.794314\ ln\,(N + 39{\cdot}023353) \\ &\quad + 0{\cdot}227843\ ln\,(N^2 - 16{\cdot}450126N + 83{\cdot}873398) \\ &\quad - 0{\cdot}061722\ \tan^{-1}\frac{N - 8{\cdot}225063}{4{\cdot}027622} \end{aligned} \tag{7.7}$$

taking $N = 0$ when $x = 0$.

Point of separation:

$$N = -6,\ x_s = 0{\cdot}0914. \tag{7.8}$$

(b) *Spawnise Flow*:

Let

$$\frac{v}{V} = \sum_{s=1}^{5} d_s \eta_y{}^s,$$

where

$$\eta_y = \frac{z}{\delta_y} \tag{7.9}$$

and

$$\left.\begin{aligned} d_1 &= \frac{60}{D}; \quad d_2 = \frac{30kM}{D}; \quad d_3 = \frac{10k^2M^2}{D}; \\ d_4 &= \frac{-15k^2M^2 - 45kM - 60}{D}; \\ d_5 &= \frac{6k^2M^2 + 24kM + 36}{D} \end{aligned}\right\} \tag{7.10}$$

where

$$k = \frac{\delta_y}{\delta_x}$$

and

$$D = k^2M^2 + 9kM + 36$$

(i) *Suction with* $M = -1$:

The equation (2.12) with the help of (7.9) reduces to

$$\begin{aligned} &(532N^4 + 12978N^3 - 140742N^2 + 620928N) [(7k^2 - 54k + 180) \\ &\quad \times \{1164240k^8 + (6930N-2640330) k^7 - (62370N-5717250) k^6 \\ &\quad + (249480N - 4241160) k^5 - (113850N - 1311750) k^4 \\ &\quad - (825N - 9405) k^3 - (15525N - 113565) k^2 \\ &\quad + (23820N-173100) k - (7020N-48780)\} - (k^3 - 9k^2 + 36k) \\ &\quad \times \{9313920k^7 + (48510N - 18482310) k^6 - (374220N \\ &\quad - 34303500) k^5 + (1447400N - 21205800) k^4 - (455400N \\ &\quad - 5247800) k^3 - (2475N - 28215) k^2 - (31050N - 227130) k \\ &\quad + (23820N - 173100)\}] \frac{dk}{dN} \end{aligned}$$

$$
\begin{aligned}
= &- (399\mathrm{N}^3 - 4042\mathrm{N}^2 + 37611\mathrm{N} - 310464)(k^3 - 9k^2 + 36k) \\
&\times [1164240k^8 + (6930\mathrm{N} - 2640330)k^7 - (62370\mathrm{N} - 5717250)k^6 \\
&+ (249480\mathrm{N} - 4241160)k^5 - (113850\mathrm{N} - 1311750)k^4 \\
&- (825\mathrm{N} - 9405)k^3 - (15525\mathrm{N} - 113565)k^2 + (23820\mathrm{N} \\
&- 173100)k - (7020\mathrm{N} - 48780)] + (532\mathrm{N}^4 + 12978\mathrm{N}^3 \\
&- 140742\mathrm{N}^2 + 620928\mathrm{N})(k^3 - 9k^2 + 36k)[6930k^7 - 62370k^6 \\
&+ 249480k^5 - 113850k^4 - 825k^3 - 15525k^2 + 23820k - 7020] \\
&- 388080(655\mathrm{N}^2 + 2457\mathrm{N} - 32760)(k^3 - 9k^2 + 36k - 60) \\
&\times (k^7 - 9k^6 + 36k^5). \qquad (7.11)
\end{aligned}
$$

for $k > 1$; and the starting value is $k = 1$ when $\mathrm{N} = 0$.

Table V gives the values of k for some chosen values of N from $\mathrm{N} = 0$ to $\mathrm{N} = -7{\cdot}5$, where the chordwise flow undergoes separation:

TABLE V

-N= 0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0	5·0	6·0	7·0	75·
k=1·0019	1·0037	1·0055	1·0073	1·0107	1·0140	1·0173	1·0323	1·0460	1·0589	1·0713	1·0836	1·0958	1·1019

(ii) *Injection with* $M = +1$:

The equation (2.12) with (7.9) reduces to

$$
\begin{aligned}
&(10324\mathrm{N}^4 + 233046\mathrm{N}^3 - 5761470\mathrm{N}^2 + 33790680\mathrm{N})[(7k^2 + 54k + 180) \\
&\times \{318780k^8 + (9702\mathrm{N} + 2321550)k^7 + (87318\mathrm{N} + 5592510)k^6 \\
&+ (349251\mathrm{N} - 8232840)k^5 - (265650\mathrm{N} - 4136550)k^4 \\
&+ (7095\mathrm{N} - 82665)k^3 + (19539\mathrm{N} - 228525)k^2 + (21396\mathrm{N} \\
&- 252780)k - (10476\mathrm{N} - 116820)\} - (k^3 + 9k^2 + 36k) \\
&\times \{2550240k^7 + (67914\mathrm{N} + 16250850)k^6 + (523908\mathrm{N} \\
&+ 33555060)k^5 + (1746255\mathrm{N} - 41164200)k^4 - (1062600\mathrm{N} \\
&- 16546200)k^3 + (21285\mathrm{N} - 247995)k^2 + (39078\mathrm{N} \\
&- 457050)k + (21396\mathrm{N} - 252780)\}] \frac{dk}{d\mathrm{N}}
\end{aligned}
$$

$$
\begin{aligned}
= &- (7743\mathrm{N}^3 - 109098\mathrm{N}^2 + 1958385\mathrm{N} - 16895340)(k^3 + 9k^2 + 36k) \\
&\times [318780k^8 + (9702\mathrm{N} + 2321550)k^7 + (87318\mathrm{N} + 5592510)k^6
\end{aligned}
$$

$$+ (349251\text{N} - 8232840)\, k^5 - (265650\text{N} - 4136550)\, k^4 + (7095\text{N} - 82665)\, k^3 + (19539\text{N} - 228525)\, k^2 + (21396\text{N} - 252780)\, k - (10476\text{N} - 116820)] + (10324\text{N}^4 + 233046\text{N}^3 - 5761470\text{N}^2 + 33790680\text{N})\,(k^3 + 9k^2 + 36k)\,[9702k^7 + 87318k^6 + 349251k^5 - 265650k^4 + 7095k^3 + 19539k^2 + 21396k - 10476] + 637560\,(12905\text{N}^2 + 7425\text{N} - 922350)\,(k^3 + 9k^2 + 36k + 60) \times (k^7 + 9k^6 + 36k^5) \tag{7.12}$$

for $k > 1$, and with $k = 1$ when $\text{N} = 0$ as the starting value.

Table VI gives the variations of k with N for some chosen values of N from $\text{N} = 0$ to $\text{N} = -6$, where the chordwise flow undergoes separation:

TABLE VI

$-\text{N} =$	0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0	5·0	6·0
$k =$	1·0012	1·0024	1·0036	1·0048	1·0072	1·0096	1·0120	1·0239	1·0360	1·0484	1·0613	1·0747

8. *ACCELERATING FLOW*

$\text{U} = 1 + x, \quad \text{V} = \text{const.}$

(*a*) *Chordwise Flow:*

Let

$$\frac{u}{\text{U}} = \sum_{s=1}^{6} c_s \eta_x^s$$

where

$$\eta_x = \frac{z}{\delta_x}. \tag{8.1}$$

From the boundary conditions (3.1) and (2.8) we have

$$\left.\begin{aligned} c_1 &= \frac{(12 - \text{M})\,\text{N} + 120}{\text{D}}; \quad c_2 = \frac{30\,(2\text{M} - \text{N})}{\text{D}}; \\ c_3 &= \frac{10\text{M}\,(2\text{M} - \text{N})}{\text{D}}; \\ c_4 &= \frac{5\,(-60 - 12\text{N} - 36\text{M} + 4\text{MN} - 9\text{M}^2)}{\text{D}}; \\ c_5 &= \frac{3\,(120 - 20\text{N} + 64\text{M} - 5\text{MN} + 12\text{M}^2)}{\text{D}}; \\ c_6 &= \frac{(-120 + 18\text{N} - 60\text{M} + 4\text{MN} - 10\text{M}^2)}{\text{D}}; \end{aligned}\right\} \tag{8.2}$$

where

$$D = M^2 + 12M + 60; \quad M = \frac{w_0 \delta_x}{\nu}; \quad N = \frac{U' \delta_x^2}{\nu}.$$

Here also we consider the two cases, one for suction corresponding to $M = -1$ and the other for injection corresponding to $M = +1$.

(i) *Suction with $M = -1$:*

The equation (2.11) with the help of (8.1) reduces to

$$\frac{dN}{dx} = -\frac{U'}{U} \frac{62048N^3 + 2373420N^2 - 42135696N + 250738488}{77560N^2 + 455742N - 9218160} \tag{8.3}$$

which on integration gives

$$\begin{aligned} 4.335175 - ln \frac{U}{U_0} &= 0 \cdot 812403 \, ln \, (N + 52 \cdot 617089) \\ &\quad + 0 \cdot 218798 \, ln \, (N^2 - 14 \cdot 365735N + 76 \cdot 800911) \\ &\quad - 0 \cdot 172510 \tan^{-1} \frac{N - 7 \cdot 182867}{5 \cdot 020690} \end{aligned} \tag{8.4}$$

taking $N = 0$ when $x = 0$.

The range of the applicability of the profile,

$$x_R = 0 \cdot 255. \tag{8.5}$$

(ii) *Injection with $M = +1$:*

The equation (2.11) with (8.1) reduces to

$$\frac{dN}{dx} = -\frac{U'}{U} \frac{101696N^3 + 3771444N^2 - 133273152N + 1015422408}{127120N^2 + 291906N - 21292560} \tag{8.6}$$

which on integration gives

$$\begin{aligned} 4 \cdot 465627 - ln \frac{U}{U_0} &= 0 \cdot 798063 \, ln \, (N + 61 \cdot 175531) \\ &\quad + 0 \cdot 225968 \, ln \, (N^2 - 24 \cdot 090061N + 163 \cdot 216998) \\ &\quad - 0 \cdot 025404 \tan^{-1} \frac{N - 12 \cdot 045035}{4 \cdot 258418} \end{aligned} \tag{8.7}$$

under the condition that $N = 0$ when $x = 0$.

The range of applicability of the profile,

$$x_R = 0{\cdot}1273. \tag{8.8}$$

(b) *Spanwise Flow*:

Let

$$\frac{v}{V} = \sum_{s=1}^{6} d_s \eta_y^s,$$

where

$$\eta_y = \frac{z}{\delta_y} \tag{8.9}$$

and

$$\left.\begin{aligned} d_1 &= \frac{120}{D}; \quad d_2 = \frac{60kM}{D}; \quad d_3 = \frac{20k^2M^2}{D}; \\ d_4 &= \frac{-45k^2M^2 - 180kM - 300}{D}; \\ d_5 &= \frac{36k^2M^2 + 192kM + 360}{D}; \\ d_6 &= \frac{-10k^2M^2 - 60kM - 120}{D} \end{aligned}\right\} \tag{8.10}$$

where

$$D = k^2M^2 + 12kM + 60; \quad k = \frac{\delta_y}{\delta_x}.$$

(i) *Suction with $M = -1$*:

The equation (2.12) with (8.9) reduces to

$$\begin{aligned} &(62048N^4 + 2373420N^3 - 42135696N^2 + 250738488N) \\ &\times [(8k^2 - 84k + 360)\{(18018N + 1297296)k^8 - (216216N \\ &+ 15567552)k^7 + \{1081080N + 77837760)k^6 - (556842N \\ &+ 69768270)k^5 + (23985N + 15618798)k^4 - (106704N \\ &+ 72953712)k^3 + (219093N + 162514800)k^2 - (118350N \\ &+ 119809440)k + (21420N + 30048480)\} - (k^3 - 12k^2 + 60k) \\ &\times \{(144144N + 10378368)k^7 - (1513512N + 108972864)k^6 \end{aligned}$$

$$+ (6486480N + 467026560)\, k^5 - (278410N + 348841350)\, k^4$$
$$+ (95940N + 62475192)\, k^3 - (320112N + 218861136)\, k^2$$
$$+ (438186N + 325029600)\, k - (118350N + 119809440)\}]$$
$$\times \frac{dk}{dN}$$

$$= -(46536N^3 - 730968N^2 + 11849688N - 125369244)$$
$$\times [(18018N + 1297296)\, k^8 - (216216N + 15567552)\, k^7$$
$$+ (1081080N + 77837760)\, k^6 - (556842N + 69768270)\, k^5$$
$$+ (23985N + 15618798)\, k^4 - (106704N + 72953712)\, k^3$$
$$+ (219093N + 162514800)\, k^2 - (118350N + 119809440)\, k$$
$$+ (21420N + 30048480)]\,(k^3 - 12k^2 + 60k) + (62048N^4$$
$$+ 2373420N^3 - 42135696N^2 + 250738488N)\,[18018k^8$$
$$- 216216k^7 + 1081080k^6 - 556842k^5 + 23985k^4 - 106704k^3$$
$$+ 219093k^2 - 118350k + 21420]\,(k^3 - 12k^2 + 60k)$$
$$- 1765764\,(77560N^2 + 455742N - 9218160)\,(k^3 - 12k^2$$
$$+ 60k - 120)\,(k^8 - 12k^7 + 60k^6). \qquad (8.11)$$

for $k < 1$ and the starting value is $k = 1$ when $N = 0$.

The solution of this equation is presented in Table VII for some chosen values of N from $N = 0$ to $N = 5$, taking into consideration the range of applicability of the chordwise profile.

TABLE VII

N=	0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0	5·0
k =	0·9988	0·9976	0·9964	0·9952	0·9927	0·9902	0·9876	0·9737	0·9575	0·9381	0·9141

(ii) *Injection with $M = +1$:*

The equation (2.12) with the help of (8.9) reduces to

$$(101696N^4 + 377144N^3 - 133273152N^2 + 1015422408N)$$
$$\times [(8k^2 + 84k + 360)\,\{(23166N + 1832688)\, k^8 + (277992N$$
$$+ 21992256)\, k^7 + (1389960N + 109961280)\, k^6 - (1078506N$$
$$+ 199086030)\, k^5 + (32175N + 23133006)\, k^4 + (114192N$$
$$+ 76982256)\, k^3 + (139539N + 75008304)\, k^2 - (130590N$$

$$+ 133175520)\,k + (28980\text{N} + 44644320)\} - (k^3 + 12k^2 + 60k)$$
$$\times \{(185328\text{N} + 14661504)\,k^7 + (1945944\text{N} + 153945792)\,k^6$$
$$+ (11119680\text{N} + 659767680)\,k^5 - (5392530\text{N} + 995430150)\,k^4$$
$$+ (128700\text{N} + 92532024)\,k^3 + (342576\text{N} + 230946768)\,k^2$$
$$+ (279078\text{N} + 150016608)\,k - (130590\text{N} + 133175520)\}]$$
$$\times \frac{dk}{d\text{N}}$$

$$= - (76272\text{N}^3 - 1593816\text{N}^2 + 45344016\text{N} - 507711204)$$
$$\times (k^3 + 12k^2 + 60k)\,[(23166\text{N} + 1832688)\,k^8 + (277992\text{N}$$
$$+ 21992256)\,k^7 + (1389960\text{N} + 109961280)\,k^6 - (1078506\text{N}$$
$$+ 199086030)\,k^5 + (32175\text{N} + 23133006)\,k^4 + (114192\text{N}$$
$$+ 76982256)\,k^3 + (139539\text{N} + 75008304)\,k^2 - (130590\text{N}$$
$$+ 133175520)\,k + (28980\text{N} + 44644320)] + (101696\text{N}^4$$
$$+ 3771444\text{N}^3 - 133273152\text{N}^2 + 1015422408\text{N})\,(k^3 + 12k^2$$
$$+ 60k)\,[23166k^8 + 277992k^7 + 1389960k^6 - 1078506k^5$$
$$+ 32175k^4 + 114192k^3 + 139539k^2 - 130590k + 28980]$$
$$+ 2630628\,(127120\text{N}^2 + 291906\text{N} - 21292560)\,(k^3 + 12k^2$$
$$+ 60k + 120)\,(k^8 + 12k^7 + 60k^6) \qquad (8.12)$$

for $k < 1$, and the starting value is $k = 1$ when $\text{N} = 0$.

The solution of the equation is presented in Table VIII for some chosen values of N from $\text{N} = 0$ to $\text{N} = 4{\cdot}4$, taking into considerations the range of applicability of the chordwise profile.

TABLE VIII

N =	0·1	0·2	0·3	0·4	0·6	0·8	1·0	2·0	3·0	4·0	4·4
k =	0·9994	0·9989	0·9983	0·9978	0·9967	0·9956	0·9945	0·9892	0·9838	0·9784	0·9761

9. *Characteristics of Flow*:

We give below the characteristics of flow taking into account the effects of suction or injection.

(*a*) *Retarding Flow*

(i) *Suction with* $M = -1$:

Boundary l yer thickness:

$$\frac{\delta_x}{\sqrt{\nu}} = \sqrt{\frac{N}{U'}}\ ; \quad \delta_y = k\delta_x \tag{9.1}$$

Skin friction:

$$T_x = \frac{\tau_x}{\mu}\sqrt{\nu} = U\,\frac{2N+15}{7}\sqrt{\frac{U'}{N}}\ ; \ T_y = \frac{\tau_y}{\mu}\sqrt{\nu} = \frac{15}{7k}\sqrt{\frac{U'}{N}}\,. \tag{9.2}$$

Displacement thickness:

$$D_x = \frac{\delta_x{}^*}{\sqrt{\nu}} = \frac{17-N}{56}\sqrt{\frac{N}{U'}}\ ; \quad D_y = \frac{\delta_y{}^*}{\sqrt{\nu}} = \frac{17k}{56}\sqrt{\frac{N}{U'}}\,. \tag{9.3}$$

Momentum thickness:

$$\left.\begin{aligned} \theta_x &= \frac{\theta_x{}^*}{\sqrt{\nu}} = \frac{32760-819N-133N^2}{271656}\sqrt{\frac{N}{U'}} \\ \theta_y &= \frac{\theta_y{}^*}{\sqrt{\nu}} = \frac{32760}{271656}\,k\sqrt{\frac{N}{U'}} \end{aligned}\right\}. \tag{9.4}$$

ii) *Injection with* $M = +1$:

Boundary layer thickness:

$$\frac{\delta_x}{\sqrt{\nu}} = \sqrt{\frac{N}{U'}}\ ; \quad \delta_y = k\delta_x. \tag{9.5}$$

Skin friction:

$$\begin{aligned} T_x &= \frac{\tau_x}{\mu}\sqrt{\nu} = U\,\frac{5N+30}{23}\sqrt{\frac{U'}{N}}\ ; \\ T_y &= \frac{\tau_y}{\mu}\sqrt{\nu} = \frac{30}{23k}\sqrt{\frac{U'}{N}}\,. \end{aligned} \tag{9.6}$$

Displacement thickness:

$$D_x = \frac{\delta_x{}^*}{\sqrt{\nu}} = \frac{165-7N}{460}\sqrt{\frac{N}{U'}}\ ; \quad D_y = \frac{\delta_y{}^*}{\sqrt{\nu}} = \frac{33k}{92}\sqrt{\frac{N}{U'}}\,. \tag{9.7}$$

Momentum thickness:

$$\left.\begin{aligned} \theta_x &= \frac{\theta_x{}^*}{\sqrt{\nu}} = \frac{922350-2475N-2581N^2}{7331940}\sqrt{\frac{N}{U'}} \\ \theta_y &= \frac{\theta_y{}^*}{\sqrt{\nu}} = \frac{922350}{7331940}\,k\sqrt{\frac{N}{U'}} \end{aligned}\right\}. \tag{9.8}$$

The flow characteristics given above have been plotted in Figs. 1, 3, 5, 7.

(*b*) *Accelerating Flow*:

(i) *Suction with* $M = -1$:

Boundary layer thickness:

$$\frac{\delta_x}{\sqrt{\nu}} = \sqrt{\frac{N}{U'}} ; \quad \delta_y = k\delta_x \tag{9.9}$$

Skin friction:

$$T_x \equiv \frac{\tau_x}{\mu}\sqrt{\nu} = U\,\frac{11N + 120}{49}\sqrt{\frac{U'}{N}} ;$$

$$T_y \equiv \frac{\tau_y}{\mu}\sqrt{\nu} = \frac{120}{49k}\sqrt{\frac{U'}{N}} . \tag{9.10}$$

Displacement thickness:

$$D_x \equiv \frac{\delta_x^*}{\sqrt{\nu}} = \frac{26 - N}{98}\sqrt{\frac{N}{U'}} ; \quad D_y \equiv \frac{\delta_y^*}{\sqrt{\nu}} = \frac{13k}{49}\sqrt{\frac{N}{U'}} . \tag{9.11}$$

Momentum thickness:

$$\left.\begin{aligned} \theta_x &\equiv \frac{\theta_x^*}{\sqrt{\nu}} = \frac{9218160 - 151914N - 15512N^2}{86522436}\sqrt{\frac{N}{U'}} \\ \theta_y &\equiv \frac{\theta_y^*}{\sqrt{\nu}} = \frac{9218160}{86522436}\,k\sqrt{\frac{N}{U'}} \end{aligned}\right\} . \tag{9.12}$$

(ii) *Injection with* $M = +1$:

Boundary layer thickness:

$$\frac{\delta_x}{\sqrt{\nu}} = \sqrt{\frac{N}{U'}} ; \quad \delta_y = k\delta_x . \tag{9.13}$$

Skin friction:

$$T_x \equiv \frac{\tau_x}{\mu}\sqrt{\nu} = U\,\frac{13N + 120}{73}\sqrt{\frac{U'}{N}} ;$$

$$T_y \equiv \frac{\tau_y}{\mu}\sqrt{\nu} = \frac{120}{73k}\sqrt{\frac{U'}{N}} . \tag{9.14}$$

Displacement thickness:

$$D_x \equiv \frac{\delta_x^*}{\sqrt{\nu}} = \frac{310 - 9N}{1022}\sqrt{\frac{N}{U'}} ; \quad D_y \equiv \frac{\delta_y^*}{\sqrt{\nu}} = \frac{155k}{511}\sqrt{\frac{N}{U'}} . \tag{9.15}$$

Momentum thickness:

$$\left.\begin{aligned}\theta_x &\equiv \frac{\theta_x^*}{\sqrt{\nu}} = \frac{21292560 - 97302N - 25424N^2}{192035844}\sqrt{\frac{N}{U'}}\\ \theta_y &\equiv \frac{\theta_y^*}{\sqrt{\nu}} = \frac{21292560}{192035844}\,k\sqrt{\frac{N}{U'}}\end{aligned}\right\}. \qquad (9.16)$$

The above flow characteristics have been plotted in Figs. 2, 4, 6, 8.

We shall now note some important conclusions:

(*a*) *Decelerating Flow*:

(i) Suction shifts the point of separation away from the leading edge and injection shifts it towards the leading edge, as is evident from the comparison table given below:

M	0	−1	+1
x_s	0.127	0.181	0.0914

(ii) Suction increases the ratio k of the two boundary layer thicknesses and the skin friction, whereas injection decreases them.

(iii) Suction decreases the displacement and momentum thicknesses, and injection increases them.

(*b*) *Accelerating Flow*:

(i) Suction increases the range of applicability, whereas injection decreases it as is shown by the following comparison table:

M	0	−1	+1
x_R	0.193	0.278	0.127

(ii) Suction decreases the ratio k and the displacement and momentum thicknesses, whereas injection increases them.

(iii) Suction increases the skin friction whereas injection decreases it.

10. Conclusion

In Part A we observe that a fifth degree profile predicts the point of separation closer than that predicted by Görtler's new series method for the chordwise retarding flow and a sixth degree profile gives a better range of applicability for the spanwise accelerating flow. We calculate the spanwise flow by taking a profile of the same degree for the corresponding case and by taking δ_x and δ_y different. We also note that for the decelerating flow the chordwise component of the skin friction is less than the spanwise component at all points of the plate and decreases more rapidly as we move downstream, as expected due to the separation in the chordwise flow. This conclusion is reversed for the accelerating flow. Since no exact solutions are available for the spanwise flow, it is hoped that the present approach gives good results as it works successfully for the chordwise flow.

In Part B we have studied the effect of suction or injection. We note that suction delays separation for the decelerating flow but increases the applicability of the profile for the accelerating flow. Besides, suction increases skin friction for both the accelerating and the decelerating flows, while the injection has the opposite effect.

11. SUMMARY

We have considered the boundary layer on a yawed semi-infinite plate for the main velocity distributions $U = 1 - x$, $V = \text{const.}$ and $U = 1 + x$, $V = \text{const.}$ We have followed the Pohlhausen technique for the study of both spanwise and chordwise flows. We have compared the flow characteristics for the fourth, fifth and sixth degree velocity profiles in the absence of suction or injection to study their applicability. We find that a fifth degree profile for the chordwise velocity predicts the point of separation closer to that predicted by Görtler's exact solution for the retarding flow, while a sixth degree profile gives a greater range of applicability of the solution for an accelerating flow than the other profiles. We have used these chordwise profiles in the study of the spanwise flow also assuming the same degree profile for the spanwise velocity. In Part B we have applied a normal suction or injection and its effect is represented by the parameter $M = w_0\delta_x/\nu$. We have studied only one case for injection $(M = 1)$ and one case for suction $(M = -1)$ using a fifth degree profile for the retarding flow and a sixth degree profile for the accelerating flow.

In particular, we mention the following:—

(i) We have taken the boundary layer thicknesses δ_x, δ_y different and solved the highly non-linear-ordinary differential equations for $k = \delta_y/\delta_x$,

(ii) We have taken two boundary conditions for both the chordwise and spanwise velocities on the plate following the suggestion of Bhatnagar and Jain.

Among the conclusions the following are worth mentioning:—

(*a*) Suction delays the separation in the decelerating flow and increases the range of applicability of the profile for the accelerating flow,

(*b*) Suction increases the skin friction for both the accelerating and decelerating flows considered here.

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