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Gauge theories of weak and strong gravity

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Abstract. A review of some recent papers on gauge theories of weak and strong gravity is presented. For weak gravity, SL(2, C) gauge theory along with tetrad formulation is described which yields massless spin-2 gauge fields (quanta gravitons). Next a unified SL(2n,C) model is discussed along with Higgs fields. Its internal symmetry is SU(n). The free field solutions after symmetry breaking yield massless spin-1 (photons) and spin-2 (gravitons) gauge fields and also massive spin-1 and spin-2 bosons. The massive spin-2 gauge fields are responsible for short range superstrong gravity.

Higgs-fermion interaction can lead to baryon and lepton number non-conservation. The relationship of strong gravity with other forces is also briefly considered.

Keywords. Gauge theories; strong gravity; symmetry breaking; unification of forces.

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1. Introduction

It is well known that Einstein's general theory of relativity (GTR) (in a sense phenomenological theory of gravity) is based on a geometrical approach. However, there are other ways of looking at gravity. Through the work of Utiyama (1956), Kibble (1961) and others (*e.g.* Eguchi *et al* 1980; Ivanenko and Sardanashivly 1983), it has been realised that gravitational field can be regarded as a non-Abelian gauge field of Yang-Mills (1954) type. This field is self-interacting and the equations are nonlinear. While some authors (Salam 1977) consider Einstein's equations as gauge theory par excellence, others (*e.g.* Yang) contend that this is based on an unnatural interpretation of gauge fields. Nevertheless, the concept of gauge theory pervades GTR in the form of covariant derivatives.

In gauge theories, auxiliary fields appear when one considers invariance of the field equations under space-time dependent transformations. In GTR the role of these fields are played by Christoffel symbols for the group of coordinate transformations which are, in fact, space-time dependent. Over the last 15 years some important developments in the field of gravity have taken place. These are: (i) Superstrong short range gravity mediated by massive spin-two gauge bosons (Sivaram and Sinha 1979) in addition to weak gravity mediated by massless gravitons. (ii) Quantization of gravity: It had been hoped that like all other fields, gravity will also be quantized. Although there have been many interesting approaches to this greatest challenge of theoretical physics (for reviews see Isham *et al* 1981; Narlikar and Padmanabhan 1983), we do not have a solution to this problem as yet. (iii) Supergravity invokes supersymmetry between pairs of bosons and fermions, they being manifestations of the same super particle. Supersymmetry generalised to a local gauge invariance leads to a gravitational model as a mixture of spin-2 (graviton) and spin 3/2 (gravitino) particles. Gravitational interaction emerges within the framework of a unified gauge theory. There is intense activity in this field with the hope of unifying all fundamental forces in nature with extended supergravity models and Kaluza-Klein supergravity (van Nienwenhuizen 1981; Duff *et al* 1983). (iv) Another exciting development is the concept that gravity is not fundamental but is an induced effect arising from symmetry-breaking effect in quantum field theory. There is great promise but considerable work is needed before a clear picture emerges (Adler 1982; Zee 1982; Pagels 1983).

In the present article, we shall be mainly concerned with gauge theory of superstrong gravity and weak gravity with their possible connection with other interactions in nature, namely, weak, electromagnetic and strong interactions. Although the initial motivation for superstrong short range gravity came from the existence of massive spin-2 f-mesons (Isham *et al* 1971; Sivaram and Sinha 1973), it can be generalized to spin-2 gluons or gauge bosons. An appropriate gauge theory can permit the existence of both short range and infinite range gravity.

In what follows, we shall discuss a gauge theory formulation along with a concept of spontaneously broken symmetry (SBS) (Linde 1979) and Higgs (1966)-Kibble (1967) mechanism which will lead to the generation of massless and massive spin-1 and spin-2 gauge fields.

2. SL(2, C) gauge theory of weak gravity

Here we briefly outline an SL(2, C) gauge theory formulation of Einstein type theory of weak gravitation using the tetrad formalism and the concept of spontaneously broken symmetry (Sivaram and Sinha 1975; Dennis and Huang 1977). SL(2, C) group is homomorphic to the proper orthochronous Lorentz group. The tetrad formalism is ideally suited to display the gauge aspect of the theory. The special feature of the formalism is invariance under tetrad rotation. This corresponds to invariance under the gauge group SL(2, C) and is analogous to the invariance of the Yang-Mills (1954) fields under local isotopic spin rotation in SU(2).

In the tetrad formalism four reference vectors are constituted at each space-time point. This is in addition to the four-coordinate system. We shall denote the tetrad by

$$t^{\mu}_{a} \qquad (a = 0, 1, 2, 3 \text{ for tetrad components}$$

$$= 0, 1, 2, 3 \text{ for local coordinate indices}) \tag{1}$$

and their inverses t^{a}_{μ}, which satisfy

$$t^{\mu}_{a} t^{a}_{\nu} = \delta^{\mu}_{\nu}, t^{\mu}_{a} t^{b}_{\mu} = \delta^{b}_{a}. \tag{2}$$

The relationship between the Lorentz metric η_{ab} = diagonal (1, −1, −1, −1) and the metric $g_{\mu\nu}$ of general relativity is then given by

$$g_{\mu\nu} = t^{a}_{\mu} t^{b}_{\nu} \eta_{ab}. \tag{3}$$

Now the constant Dirac matrices (on flat space-time) satisfy the relation

$$\gamma_a \gamma_b + \gamma_b \gamma_a = 2\eta_{ab}. \tag{4}$$

One can define space-time dependent Dirac matrices

$$\gamma^\mu = t^\mu_a \gamma^a \tag{5}$$

which satisfy

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \tag{6}$$

The γ_a matrices of Dirac provide a representation. Using the expression

$$\gamma^a = L^a_b S(L)\gamma^b S^{-1}(L), \tag{7}$$

where S is the spacetime dependent spinor representation of the tetrad rotation L, the homomorphism

$$L^a_b \to S(L) \tag{8}$$

is defined. The transformation matrix S can be chosen as

$$S = \exp\left(\tfrac{1}{4} i g_w \theta_{ab}\sigma^{ab}\right) \tag{9}$$

where g_w represents the coupling constant,

$$\sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]. \tag{10}$$

The gauge potential and the gauge field for the present case are denoted by W_μ and

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + i g_w [W_\mu, W_\nu] \tag{11}$$

The transformation laws for $\gamma^\mu(x)$, W_μ and $W_{\mu\nu}$ under (9) (*i.e.* under SL(2, C)) are

$$\begin{aligned} \gamma^\mu &\to S\gamma^\mu S^{-1}, \\ W_\mu &\to SW_\mu S^{-1} + i(\partial_\mu S)S^{-1}, \\ W_{\mu\nu} &\to SW_{\mu\nu}S^{-1}. \end{aligned} \tag{12}$$

Further a spinor $\psi(x)$ obeys the transformation

$$\psi \to S\psi$$

and remains invariant under coordinate transformation. However,

$$\partial_\mu\psi \to S\partial_\mu\psi + (\partial_\mu S)\psi \tag{13}$$

i.e. the derivative is not a spinor. Hence we introduce the SL(2, C) covariant derivative

$$D_\mu\psi = \partial_\mu\psi + 2g_w W_\mu\psi. \tag{14}$$

The above relations are similar to those for Yang-Mills fields under isospin SU(2) transformation. In the tetrad formalism the Einstein-Hilbert type action becomes

$$A = \int d^4x \sqrt{-g}\,\{W_{\mu\nu} t^\mu_a t^\nu_b \eta^{ab} - 2\kappa \mathscr{L}\} \tag{15}$$

where $\sqrt{-g} \equiv \det(t^\mu_a)$

and $\mathscr{L}$ is the Lagrangian density of matter field (scalar, vector or spinor).

In proceeding further, we have to use a modified invariance principle. The above action must be invariant under the group of general coordinate transformation. The action must also be invariant under Lorentz group of transformation at each space time point (tetrad rotation). As a consequence, the fields in question will be scalars or tensors under general coordinate transformation and scalars, tensors or spinors under Lorentz group. Further we must consider $W_{\mu\nu}$ to be a curvature tensor having second derivative of t^{μ}_{a}. On varying the action with respect to t^{μ}_{a}, then yields the equation

$$R^{b}_{\nu}-\tfrac{1}{2}Rt^{b}_{\nu}=\kappa T^{b}_{\nu}$$

$$R^{b}_{\nu}=R_{\mu\nu}t^{\mu}_{a}\eta^{ab} \tag{16}$$

$$T^{a}_{\mu}=\frac{1}{t}\left\{\frac{\partial(t\mathscr{L})}{\partial t^{\mu}_{a}}-\partial^{\alpha}\left(\frac{\partial(t\mathscr{L})}{\partial(\partial^{\alpha}t^{\mu}_{a})}\right)\right\} \tag{17}$$

which can also be expressed as

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=\kappa t^{-1}T_{\mu\nu}.$$

This has resemblance to Einstein's equation. However, the curvature tensor and $T_{\mu\nu}$ may not necessarily be symmetric.

3. Unified SL(2n, C) model of gauge fields

In the previous section we considered SL(2, C) gauge theory for weak gravity mediated by massless spin-2 gauge bosons.

We shall now discuss an extended model which can describe internal symmetries of elementary particles and also generate massive spin-2, spin-1 bosons besides the massless ones. For describing strongly interacting spin-2 mesons an SL(6, C) invariant form was chosen (Isham *et al* 1973; Dennis and Huang 1977). The relevant gauge group was SL(2, C) × SL(6, C). However, this gives a non-unified model with leptons providing the weak gravity and hadrons the strong gravity. This $f-g$ model has some non-renormalizable parts. The SL(8, C) gauge model of Huang and Dennis (1981a) includes leptons and quarks in a unified manner in a four-colour scheme using the subgroup SU(4). Through the Higgs mechanism of spontaneous symmetry-breaking (SSB) the spin-2 gluons (f) and fermions become massive leaving the gravitons (g) massless. Unlike the case of SL(2, C) × SL(6, C), the Higgs-fermion interaction is renormalizable. However the SL(8, C) model did not include spin-1 gauge fields and thus left out photons, W bosons and spin-1 gluons etc. Accordingly, we shall now discuss a unified SL($2n$, C) along with Higgs-Kibble mechanism which can generate both spin-1 and spin-2 gauge fields massless and massive (Huang and Dennis 1981b; Sinha 1983).

The transformation matrix of SL($2n$, C) group is

$$S=\exp\left(i\frac{g_c}{2}\left[\sum_{j=1}^{N}(\theta^{j}+i\theta^{j}_{5}\gamma_{5})\lambda_{j}+\sum_{J=0}^{N}\theta^{J}_{ab}\sigma^{ab}\lambda_{J}\right]\right) \tag{18}$$

where $N=n^2-1$; g_c = coupling strength

$$\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3,\ \lambda_0=1 \text{ and } \lambda_j\ (j=1,\ldots,N)$$

are the generators of SU(n) algebra. They obey the relation

$$[\lambda_i,\lambda_j]=2if_{ijk}\lambda_k,$$

$$\mathrm{Tr}(\lambda_j\lambda_k)=n\delta_{jk}. \tag{19}$$

The tetrad field for the present case is denoted by M^μ with components given by

$$M^\mu = \sum_{J=0}^{N} (M_J^{\mu a}\gamma_a + M_{5j}^{\mu a}\gamma_a\gamma_5)\lambda_J. \tag{20}$$

The fermion Lagrangian density with local SL(2n, C) invariance

$$\mathscr{L}_F = i\,\mathrm{Tr}(\bar{\psi}M^\mu D_\mu\psi), \tag{21}$$

with $\quad \psi \to S\psi S^{-1}$,

and

$$D_\mu\psi = \partial_\mu\psi + ig_c G_\mu\psi. \tag{22}$$

The gauge potential G_μ behaves as

$$G_\mu \to SG_\mu S^{-1} - \frac{1}{ig_c}\partial_\mu SS^{-1}. \tag{23}$$

Similarly, the SL(2n, C) invariant Lagrangian density for the gauge field is

$$\mathscr{L}_{\dot{G}} = \frac{i\sqrt{-M}}{8ng_c}\mathrm{Tr}\{[M^\mu, M^\nu]G_{\mu\nu}\}, \tag{24}$$

where

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig_c[G_\mu, G_\nu], \tag{25}$$

$$M = \det M_{\mu\nu}, M_{\mu\nu} = \frac{1}{4n}\mathrm{Tr}\,(M_\mu M_\nu). \tag{26}$$

As for M^μ, the components of G_μ are given by

$$G_\mu = \sum_{j=1}^{N} (G_\mu^j + iG_\mu^{5j}\gamma_5)\frac{\lambda_j}{2} + \sum\frac{1}{4}G_{\mu[ab]}^J\sigma^{ab}\lambda_J. \tag{27}$$

At this stage we introduce the Higgs field ϕ satisfying the transformation

$$\phi \to S\phi S^{-1}$$

and have the components

$$\phi = \sum_{J=0}^{N} (\phi^J + i\phi_5^J\gamma_5 + \tfrac{1}{2}\phi_{ab}^J\sigma^{ab})\lambda_J \tag{28}$$

along with its conjugates $\bar{\phi} = \gamma_0\phi^\dagger\gamma_0$.

The Lagrangian densities for the Higgs scalar which are invariant locally under SL(2n, C) is given by

$$\begin{aligned}\mathscr{L}_\phi = \frac{\sqrt{-M}}{8n}&[a_1\,\mathrm{Tr}\,(D_\mu\bar{\phi}D_\nu\phi)M^{\mu\nu} + a_2\,\mathrm{Tr}\,(M^\mu D_\mu\bar{\phi}D_\nu\phi M^\nu)\\ &+ a_3\mathrm{Tr}(M^\nu D_\mu\bar{\phi}D_\nu\phi M^\mu) + a\mathrm{Tr}(D_\mu\bar{\phi}M^\mu D_\nu\phi M^\nu)\\ &+ a\mathrm{Tr}\,(D_\mu\bar{\phi}M^\nu D_\nu\phi M^\mu) + a_6\mathrm{Tr}(M^\mu D_\alpha\bar{\phi}D_\beta\phi M^\nu)M^{\alpha\beta}M_{\mu\nu}]\\ &- \sqrt{-M}\;V(\phi),\end{aligned} \tag{29}$$

where the coefficients satisfy the relation

$$a_6 = \tfrac{1}{2}a - \tfrac{1}{4}(a_1 + a_2 + a_3)$$

$$V(\phi) = \mu_0 + \frac{\mu_1}{2n}\mathrm{Tr}(\bar{\phi}\phi) + \frac{\mu_2}{2n}\mathrm{Tr}(\bar{\phi}\phi\bar{\phi}\phi) \tag{30}$$

$$+\frac{\mu_3}{2n}[\mathrm{Tr}(\bar{\phi}\phi)]^2 \tag{31}$$

and

$$D_\mu\phi = \partial_\mu\phi + ig_c[G_\mu, \phi]. \tag{32}$$

The form of $V(\phi)$ is such that for $\mu_1 < 0$ and $\mu_2, \mu_3 > 0$ we will have spontaneous symmetry breaking with the vacuum expectation value

$$\langle 0|\phi|0\rangle = \phi_0 = \pm(-\mu/(2\mu_2 + 8n\mu_2))^{1/2}. \tag{33}$$

The coefficients in (29) and (30) have been chosen to eliminate ghost states. Also the Lagrangian density $\mathscr{L}_\phi$ contains the interaction between Higgs field and the gauge field (*via* (31)) as a minimal coupling. This may suffice for the appearance of spin-1 gauge fields. To generate massive spin-2 gauge fields we need non-minimal coupling. This is simply achieved by noting that $[M_\mu, \phi]$ transforms under SL(2n, C) as $D_\mu\phi$. Accordingly, $\mathscr{L}_{\phi M}$ for non-minimal coupling is obtained by replacing $D_\mu\phi$ in $\mathscr{L}_\phi$ by $ig_w[M_\mu, \phi]$. The coefficients b_i of the various terms thus obtained are chosen such that the total free field Lagrangian is in the Pauli-Fierz (1939) form and satisfy reality conditions.

There will also be Higgs-fermion interaction term of the type

$$\mathscr{L}_{\phi F} = \sqrt{-M}[(c\bar{\psi}\phi\bar{\phi}\psi) + D(\bar{\psi}\phi\psi + \bar{\psi}\bar{\phi}\psi)]. \tag{34}$$

These forms on symmetry breaking *i.e.* $\langle\phi\rangle \neq 0$ will produce mass for the fermions. Further the second and third terms will permit non-conservation of baryon number and lepton number. The full Lagrangian density then turns out to be

$$\mathscr{L} = \mathscr{L}_F + \mathscr{L}_G + \mathscr{L}_\phi + \mathscr{L}_{\phi M} + \mathscr{L}_{\phi F}. \tag{35}$$

In order to include both spin-1 and spin-2 gauge bosons, a more general choice for ϕ_0 should be made. Thus

$$\phi_0 = C_0\mathbf{1} + \sum_r C_r\lambda_r \tag{36}$$

where λ_r are a subset of the SU(n) generators; C_0 and C_r are constant and $\mathbf{1}$ is the unit matrix.

In the following, we shall discuss the free field equations of the gauge fields on spontaneous symmetry breaking.

4. Free field equations for spin-1 and spin-2 gauge fields

This is achieved by expanding about the classical vacuum solution namely,

$$\phi = \phi_0, \quad M^\mu = \gamma^\mu \quad \text{and} \quad G_\mu = 0.$$

This leads to

$$M^\mu = \gamma^\mu + g_c \sum_{J=0}^{N} (Z_J^{\mu a}\gamma_a + Z_{5J}^{\mu a}\gamma_a\gamma_5)\lambda_J, \tag{37}$$

$$\phi = \phi_0 + \xi. \tag{38}$$

The resulting free field Lagrangian is not diagonal with respect to the internal group indices. This is diagonalised by using a suitable rotation (Huang and Dennis 1981b; Sinha 1983). It is convenient to introduce the notation

$$Z_0^{\mu a} = \tilde{g}^{\mu a}, \quad Z_j^{\mu a} = \tilde{f}_j^{\mu a} \tag{39}$$

The gauge field Lagrangian density then finally takes the form in the lowest orders of the couplings constant.

$$\mathscr{L}_{GF} = \mathscr{L}_g + \mathscr{L}_f + \mathscr{L}_R, \tag{40}$$

where $\mathscr{L}_R$ gives only the non-dynamical fields and leads only to constraint relations. Physically interesting field equations appear from the variations of $\mathscr{L}_g$ and $\mathscr{L}_f$. Thus the variation of $\mathscr{L}_g$ gives (Huang and Dennis 1981; Sinha 1983)

$$(\eta_{ac}\partial_\beta + \eta_{a\beta}\partial_c)\partial_\alpha \tilde{g}^{(\alpha a)} - \Box^2 \tilde{g}_{(\beta c)} + (\partial_\beta \partial_c + \eta_{\beta c}\Box^2)\tilde{g} = 0. \tag{41}$$

This equation has the invariance and the right symmetry to represent a massless spin-2 gauge field which can be identified with the graviton. This in turn implies that SL(2, C), which is a subgroup of SL(2n, C) has not been spontaneously broken. Similarly from $\mathscr{L}_f$ one finally gets

$$\Box^2 \tilde{f}_j^{[\beta d]} = \partial^d \partial_e \tilde{f}_j^{[\beta e]} - \partial^\beta \partial_e \tilde{f}_j^{[de]}, \tag{42}$$

which represents a massless spin-1 gauge field. There are still important remaining parts of $\mathscr{L}_f$. These can be separated into antisymmetric and symmetric parts. For the antisymmetric part one gets

$$\Box^2 \tilde{f}_{j[\beta c]} + \partial_c \partial^\alpha \tilde{f}_{j[\alpha\beta]} - \partial_\beta \partial^\alpha \tilde{f}_{j[\alpha c]} + m_{1j}^2 \tilde{f}_{j[\beta c]} = 0, \tag{43}$$

which because of the mass term m_{ij}^2 represents a massive spin-1 field. The symmetric part, on the other hand, leads to

$$2\eta_{a(\beta\partial_c)}\partial_\alpha \tilde{f}_j^{(\alpha a)} + \tfrac{1}{2}\eta_{\beta c}\Box^2 \tilde{f}_j - \Box^2 \tilde{f}_{j(\beta c)} - m_{2j}^2(d\tilde{f}_j \eta_{\beta c} + \tilde{f}_{j(\beta c)}) = 0, \tag{44}$$

which can be recast into the Pauli-Fierz form

$$(d+1)\,\Box^2 \tilde{f}_j - m_{2j}^2(2d + \tfrac{1}{2})\tilde{f}_j = 0, \tag{45}$$

d being a number. The above represent massive spin-2 gauge fields.

The foregoing shows how spin-1 and spin-2 gauge fields are both generated in an SL(2n, C) invariant model. This happens through the introduction of Higgs field ϕ having the transformation properties $\phi \to S\phi S^{-1}$. The manner in which spontaneous symmetry-breaking takes place is such that a massless spin-2 field (gravitons) are generated. The SL(2, C) group thus remains unbroken. Spin-1 gauge field arises from direct minimal coupling between ϕ and G_μ on spontaneous symmetry breaking. Massive spin-1 and spin-2 gauge fields arise from non-minimal coupling between ϕ and M^μ.

5. Nonconservation of baryon number

In § 3, a Lagrangian for Higgs-Fermion interaction was written with terms which lead to non-conservation of baryon and lepton numbers. We intend to discuss some explicit calculations (Huang and Dennis 1980).

For three different groups of Higgs fields we can write the Higgs-Fermion interaction as

$$\mathscr{L}_{\phi F} = -C_1 \bar{\psi}_\beta \phi_1 \bar{\Phi}_1 \psi_\beta - C_2 \bar{\psi}_2 \bar{\Phi}_2 \phi_2 \psi_L - C_3 (\bar{\psi}_\beta \phi_3 \psi_L + \bar{\psi}_L \bar{\Phi}_3 \psi_\beta). \tag{46}$$

The model preserves the fermion (F) number conservation where $F = 3B + L$, B and L being baryon and lepton numbers respectively. Quarks and nucleons can decay into leptons and antileptons and pions on the basis of the above mechanism. The model shows that positively-charged quarks are stable. Consequently, proton is much more stable. In fact the ratio of the life times of proton (t_p) to that of neutron (t_n) turns out to be

$$(t_p/t_n) \gg 1.$$

This differs radically from other theories where $(t_p/t_n) \approx 1$.

6. Strong gravity and other forces

In this section, we briefly consider constants of strong gravity with those of other interactions, *e.g.*, strong and electromagnetic interactions. It has been shown earlier (Sivaram and Sinha 1973, 1979) that the field equations of strong gravity turn out to be Einstein type with a strong cosmological constant

$$\Lambda_f \sim 10^{28}\,\mathrm{cm}^{-2} \tag{47}$$

and the strong gravitational constant

$$G_f \sim 10^{38}\, G_N, \tag{48}$$

G_N being the Newtonian constant, which is of the order of strong interaction and is of short range (10^{-14} cm) and mediated by spin-2 gauge bosons of mass $m_f = 1{\cdot}5$ GeV.

As shown by Sivaram and Sinha (1979) the coupling constant of weak interaction (which has the same dimension as gravitational interaction) turns out to be

$$G_w = G_f\,(m_f/m_w)^2 \approx 10^{-5}, \tag{49}$$

where $m_w \sim 10^2$ GeV is the mass of the gauge boson mediating the weak interaction.

A solution of the strong gravity field equations coupled to the Yukawa field by Raut and Sinha (1981) gave the relation

$$\frac{g^2}{\hbar c} = \frac{\exp(2m_\pi r_p)c^3 r_p}{m_\pi G_f \hbar}\left[\frac{2\Lambda_f r_p^2}{3} - \frac{2G_f m_p}{c^2 r_p}\right]. \tag{50}$$

Here $(g^2/\hbar c)$ is the strong interaction coupling constant, m_p = proton mass, m_π $(= m\pi c/\hbar)$ is the inverse of the pion Compton length and r_p is the strong Schwarzschild radius or proton Compton length $\sim 2 \times 10^{14}$ cm. The numerical value of

$$g^2/\hbar c \approx 17 \tag{51}$$

which is very close to the observed value 14·5.

Indeed the recent work of Chela-Flores and Varela (1983) supports the above conclusion and the fact that the source of strong gravity lies in matter at hadronic density (Sivaram and Sinha 1979). Even the electric change of hadrons is related to strong gravity. Thus a solution of the field equations of strong gravity field coupled to the charge (Reissner-Nordstrom) field gives the relation (Raut and Sinha 1983)

$$\frac{e^2}{\hbar c} = \frac{\Lambda_f c^3 r_p^4}{\hbar G_f} \frac{1}{3 \times 308}, \tag{52}$$

which gives the value of the fine structure constant to be of the order of 1/141, pretty close to the observed value (1/137). This further renders a strong support to the relationship of strong gravity with other forces and the source of strong gravity. The basis of these calculations is that the strong gravitational force vanishes at the radius of an elementary particle (*e.g.* proton), *i.e.*

$$\left.\frac{\mathrm{d}V_s}{\mathrm{d}r}\right|_{r=r_p} = 0, \tag{53}$$

V_s being the strong gravitational potential. These calculations give strong support to the relevance of strong gravity in matter at extreme density particularly those occurring within hadrons.

7. Concluding remarks

In the foregoing sections, we have given a review of some recent developments in the field of gravity. The concept of superstrong short range gravity mediated by massive boson has emerged since 1971. In this article a discussion of gauge theories of both weak and strong gravity has been presented. SL(2, C) gauge theory describes weak gravity SL(2*n*, C), which has SU(*n*) subgroup for internal symmetry, coupled with Higgs-Kibble mechanism of spontaneous symmetry breaking gives a unified description. In this both spin-1 and spin-2 gauge fields appear. Further, the study of free fields give massless and massive spin-1 and spin-2 gauge field equations. The massless spin-2 gauge field is identified with gravitons showing that SL(2, C) symmetry remains unbroken. The constant form in the Higgs potential $V(\phi)$ i.e. μ_0 is related to the cosmological constant.

Taking into account Higgs-fermion interaction, one finds processes which lead to non-conservation of baryon and lepton numbers. In §6, the relationship of the coupling constants with the constants of strong gravity is given. The values presented for various interactions are in good agreement with those for the observed forces. Thus strong gravity field is related to all other forces. This is a feature of the gauge unification programme described here.

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