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Black hole thermodynamics from a possible model for internal structure

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Abstract. Treating a black hole as a relativistic gas of microblack holes (planckions) which have fermionic character, expressions for some thermodynamic quantities are obtained. These have the same structure as those obtained by Hawking by other considerations.

Keywords. Gravity; black hole thermodynamics; internal structure; relativistic gas.

1. Introduction

In two outstanding papers, one dealing with the thermodynamics of black holes and the other concerning the topological structures of space-time manifold at extremely short length scales, Hawking (1975, 1978) has laid the foundation for the interrelation between gravity, thermodynamics and quantum mechanics. The second paper addresses itself directly to the question of discrete nature of space-time. However, these papers do not provide a connection between the thermodynamic properties of a black hole and its possible internal structure. There is no classical way of treating the interior of the black hole as a system and deduce its parameters. Now that we have relations involving some thermodynamic parameters of a black hole *e.g.* entropy, temperature and energy, it should be possible to develop a model of the system which can yield meaningful results.

2. A possible model

It is well known that an alternative approach towards unification of fundamental forces is to take them as derived interactions from a more fundamental interaction involving matter (fundamental fermion-antifermion) fields (Sinha and Sudarshan 1978; Akama 1978; Terazawa and Akama 1980). In this approach the fundamental spinor fields generate the various other fields (*e.g.* bosons and gauge fields which give rise to various forces) and the space-time metric which becomes composite. In the context of black holes these pregeometric entities (fermionic in character) should be of fundamental significance.

Thus the central point of our model is to treat a black hole as a superdense object of elementary units which behave like fermions in the sense that no two elementary units can overlap. These may be pictured as a gas of microblack holes which may be akin to space-time foam (Hawking 1978). Thus in effect we shall be treating an assembly of microblack holes having fermionic character. In fact, we identify these microblack holes with the planck mass

$$M_P = (\hbar c/4G_N)^{1/2}, \tag{1}$$

which is obtained from

$$M_P c = \frac{\hbar}{2}\frac{c}{L_P},$$

$$L_P = (\hbar G_N/c^3)^{1/2}, \tag{2}$$

where G_N is the Newtonian gravitational constant and $\hbar$ and c are the usual universal constants. In the context of the present model these elementary units (planckions or unitons (Sivaram and Sinha 1979)) are to be treated as fundamental fermions. It is also assumed that the fundamental quantum principles such as the uncertainty and the exclusion principles hold inside a black hole.

The number N_P of planckions in a black hole of volume V_B is (Landau and Lifshitz 1969)

$$N_P = \frac{V_B g}{6\pi^2 \hbar^3} p_F^3 = \frac{V_B g}{6\pi^2} (c^3/\hbar G_N)^{3/2}, \tag{3}$$

where g is the degeneracy factor and we have taken the Fermi momentum

$$P_F = \hbar/L_P. \tag{4}$$

The number density of planckions is given by

$$n_P = \frac{N_P}{V_B} = \frac{g}{6\pi^2} (c^3/\hbar G_N)^{3/2}. \tag{5}$$

Thus the number density of such units is constant in the sense that it depends on the universal constants ($\hbar$, c and G_N).

The equilibirum size of the black hole can be derived by a very simple argument. The system is in equilibrium under two opposing forces namely, the gravitational attraction and the degeneracy pressure (P_0) of the constituent fermions (here the planckions). This gives

$$P_0 = \frac{G_N M_B}{R_0} M_P n_P, \tag{6}$$

where

$$P_0 = \frac{1}{4} (6\pi^2/g)^{1/3} \hbar c\, n_p^{4/3}. \tag{7}$$

From these relations (*cf* equations (1)–(7)) we get

$$R_0 = 2G_N M_B/c^2, \tag{8}$$

which is identical with the Schwarzschild radius of the black hole. In the above we have taken a low temperature situation $k_B T \ll c p_F$, k_B is the Boltzmann constant.

Now for an assembly of fermionic systems in equilibrium, we have

$$V_B T^3 = \text{constant}.$$

Thus we can write, using (8),

$$T = a\,(3/4\pi)^{1/3}\,\frac{1}{R_0}, \tag{9}$$

where a is a constant and from dimensional consideration we can put

$$a = \hbar c/k_B. \tag{10}$$

Thus

$$T = \frac{\hbar c}{k_B}\,(3/4\pi)^{1/3}\,\frac{1}{R_0}, \tag{11}$$

which on using (8) becomes

$$T = \frac{\hbar c^3}{2\,k_B\,G_N\,M_B}\,(3/4\pi)^{1/3}. \tag{12}$$

The expression (11) is similar to that obtained by Landsberg (1981) for an extremely relativistic fermion or boson gas for a closed universe in terms of the scale factor R. The relation (12) is close to the Hawking temperature for black holes apart from a small numerical factor.

The thermodynamic potential of an assembly of relativistic fermions is given by (Landau and Lifshitz 1969)

$$\Omega_B = \Omega_0 - \frac{(\mu\,k_B T)^2}{12\,(\hbar c)^3}\,g\,V_B, \tag{13}$$

where Ω_0 is constant. From this the volume entropy is easily calculated as

$$\begin{aligned} S_v &= -(\partial\Omega/\partial T)_V \\ &= \frac{2g}{9}\,(3/4\pi)^{1/3}\,\frac{4\pi\,k_B G_N M_B^2}{\hbar c}, \end{aligned} \tag{14}$$

where the Fermi wavevector is taken as the reciprocal of planck length giving

$$\mu = \hbar c k_F = \hbar c\,(c^3/\hbar G_N)^{1/2} = c^2\,(\hbar c/G_N)^{1/2}.$$

In order to calculate the surface entropy of the system of fermions, we consider the standard expression

$$S_s = -A\frac{\mathrm{d}\alpha}{\mathrm{d}T}, \tag{15}$$

where A is the total surface area and α the surface tension (negative for the present case) which is defined as the potential energy per unit area of the surface. A simple estimate of the surface tension may be given as

$$\alpha = -G_N\,M_B^2/R_0 4\pi R_0^2. \tag{16}$$

Inserting the value for R_0 obtained earlier this becomes

$$\alpha = -\frac{3}{g}\frac{1}{32\pi}-\frac{c^6}{G_N^2 M_B}, \tag{17}$$

which on making use of (12) becomes

$$\alpha = -(3\pi/16g^2)^{1/3}\frac{k_B T}{4\pi A_0}, \tag{18}$$

where

$$A_0 = L_P^2, \tag{19}$$

and represents the elementary surface area first introduced by Bekenstein (1972, 1973). Thus from (15) we get

$$S_s = (4\pi/3)^{1/3}\frac{k_B G_N M_B^2}{\hbar c}. \tag{20}$$

Again, apart from a small numerical factor, this is close to the Hawking result. Of course, in the present context, the total entropy

$$S = S_v + S_s.$$

From the volume entropy, we can get entropy per planckion as

$$s_p = \pi^2 (3/4\pi)^{1/3}\frac{M_p k_B}{M_B}. \tag{21}$$

It is interesting to note that like the temperature this also depends inversely on the mass of the black hole.

3. Concluding remarks

In the foregoing, we have attempted to describe black hole thermodynamics (*e.g.* temperature, entropy etc) from conventional points of view. For this, we have treated a black hole as an assembly of fermions having the size and mass of planckions. The expressions obtained are close to those of Hawking's apart from small numerical factors. These pregeometric objects with fermionic character may arise due to breakdown of metric structure of space-time under the extreme conditions of black holes.

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