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Plasmon-magnon interaction in magnetic semiconductors

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Abstract. In appreciably doped semiconductors (e.g. EuO, $CdCr_2S_4$, etc.) plasmon and magnon energies are comparable. Therefore, there will be resonant interaction between these modes of excitations. On the basis of a new microscopic theory formulated for plasmon-magnon interaction, the effect of this interaction on the energies and lifetimes of plasmons and magnons has been calculated using the double-time Green's function. The energy shifts are very small and the lifetimes of plasmons, τ^p, and magnons, τ^m, are of the order of 10^{-2} and 10^{-3} sec respectively.

Keywords. Spin wave; plasmon; semiconductor.

1. Introduction

Interaction between plasmons and magnons in magnetic materials has not received much attention. The reason is that in ferromagnetic metals, the plasmon energy is extremely high ($\hbar\omega_p \sim 15$eV, where ω_p is the plasmon frequency) compared to the magnon energy. In fact plasmon modes in the absence of external radiations are not excited even at room temperature. The situation is rather different in doped semiconductors, where the plasmon energy may be comparable to magnon energies.

In recent years there has been considerable theoretical and experimental work in the field of ferromagnetic semiconductors (e.g., doped Eu chalcogenides, doped $CdCr_2S_4$). Theoretical work on these systems has been directed mainly towards explaining their interesting optical and electronic transport properties. Apart from some phenomenological discussions involving interaction of spin waves with the electrokinetic waves in the plasma of a magnetic semiconductor (Baryakhtar *et al* 1966) there does not appear to be any detailed microscopic theory of their interaction. Baryakhtar *et al* (1971) have discussed the Raman scattering of light near resonance between longitudinal plasma and spin wave modes in antiferromagnetic semiconductors.

In what follows, we consider a microscopic formulation of plasmon-magnon interaction in doped ferromagnetic semiconductors. After formulating the interaction Hamiltonian, its effect on the magnon and plasmon energies and lifetimes are considered. We found that there is no appreciable renormalization of magnon and plasmon energies. However, there is some effect on their lifetimes.

2. Interaction Hamiltonian

We consider a system which in the pure state is a ferromagnetic insulator (e.g., EuO). The magnetic moments in such a system are localized on the paramagnetic atoms (say in the d or f shells of the atoms). We shall not be concerned with the origin of exchange interaction involving the electrons of the paramagnetic ions and the electrons of the diamagnetic ions (Methfessel and Mattis 1968). We shall take this problem as solved. Thus, we start with an effective exchange Hamiltonian which describes the magnetic ordering in the solid. By introducing suitable impurities in such ferromagnetic solids (e.g., Gd in EuO) one can produce sufficient number of electrons in the otherwise empty conduction band. These electrons are delocalized and are responsible for the transport and other phenomena. The electronic band structure of these ferromagnetic semiconductors has not been solved to a reliable extent (Cho 1967, Kasuya and Yanase 1968). Nevertheless, there is evidence of the usual parabolic conduction band which is unsplit in the spin averaged state. However, it will be split up into spin up and spin down bands on taking into account the interactions with the localized spins.

The Hamiltonian of the system will comprise the electronic part, the exchange coupled spin system, and the electron-spin wave interaction part (Rys *et al* 1967, Woolsey and White 1968). In writing the Hamiltonian we shall introduce the magnon and plasmon variables straightaway. Thus, in the second quantized representation, the Hamiltonian of the system in the absence of an external magnetic field is given by

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{em}, \tag{2.1}$$

with

$$\mathcal{H}_0 = \sum_{k\sigma} \epsilon_{k\sigma}\, c^{\dagger}_{k\sigma}\, c_{k\sigma} + \sum_{\lambda} E_\lambda \left(b^{\dagger}_\lambda\, b_\lambda + \tfrac{1}{2}\right) + \sum \hbar\omega_p \left(a^{\dagger}_q\, a_q + \tfrac{1}{2}\right)$$

and

$$\mathcal{H}_{em} = -I\left(\frac{S}{2N}\right)^{\frac{1}{2}} \sum_{k\lambda} \left(c^{\dagger}_{k+\lambda\downarrow}\, c_{k\uparrow}\, b_\lambda + c^{\dagger}_{k+\lambda\uparrow}\, c_{k\downarrow}\, b^{\dagger}_\lambda\right) \tag{2.2}$$

$$+ \frac{IS}{2N} \sum_{k\lambda q\sigma} \sigma\, c^{\dagger}_{k\sigma}\, c_{k-q\sigma}\, b^{\dagger}_\lambda\, b_{\lambda+q} \tag{2.3}$$

Here,

$$\epsilon_{k\sigma} = \left(\frac{\hbar^2 k^2}{2m^*} - \frac{I\sigma S}{2}\right),\; E_\lambda = 2JS\,(1-\gamma_\lambda)$$

and

$$\hbar\omega_p = \hbar\,(4\pi n e^2/m^*\epsilon_0)^{1/2}$$

denote the electron, magnon and plasmon energies respectively and

$$\gamma_\lambda = \frac{1}{z} \sum_h e^{i\lambda\cdot R_h}$$

where m^* is the effective mass of the electron, N is the number of magnetic atoms, n is the carrier concentration, σ is the spin index and takes the values ± 1, J is the exchange integral between the nearest neighbour localized spins (S), I is the exchange integrals between the localized spin and the conduction electron, ϵ_0 is the static dielectric constant of the medium, z is the number of nearest neighbours for the magnetic atoms and $\mathbf{R}_h$ is the corresponding vector connecting the nearest neighbour $(c^{\dagger}_{k\sigma}, c_{k\sigma})$,

$(b_\lambda^\dagger, b_\lambda)$ and $(a_q^\dagger, a_q)$ are the creation and annihilation operators for electrons, magnons and plasmons respectively, k, λ, q signifying the corresponding wave-vectors.

The plasmon-magnon interaction arises from the last term in the electron-magnon interaction Hamiltonian (Eq. 2.3) as explained below. The electron operator in the last term of $\mathcal{H}_{em}$ is just the Fourier component of the electron density operator $\rho_{q\sigma}$, i.e.,

$$\sum_{k\sigma} \sigma c_{k-q\sigma}^\dagger c_{k\sigma} = \sum_{\sigma} \sigma \rho_{q\sigma} = \rho_{q\uparrow} - \rho_{q\downarrow} \tag{2.4}$$

At very low temperatures and concentrations ($n \sim 10^{18}$ cm^{-3}) and when the band splitting is larger than the Fermi energy of the electron, we can assume that

$$\rho_{q\downarrow} \approx 0$$

and therefore $\rho_{q\uparrow} \approx \rho_q$.

In the long wavelength region (i.e., for $q < q_c$) ρ_q can be approximately written in terms of the plasmon co-ordinates (Rickayzen 1959) by using the relation

$$\rho_q = -i \left(\frac{\Omega q^2}{4\pi e^2}\right)^{\frac{1}{2}} P_{-q}$$

$$\equiv \left(\frac{q^2 \Omega \, \hbar\omega_p}{8\pi e^2}\right)^{\frac{1}{2}} (a_q^\dagger + a_{-q}) \tag{2.5}$$

where Ω $(= \mathcal{N} a^3)$ is the volume of the system and a is the lattice constant. Therefore the third term of $\mathcal{H}_{em}$ becomes

$$\frac{I}{2\mathcal{N}} \left(\frac{\hbar\omega_p \Omega}{8\pi e^2}\right)^{\frac{1}{2}} \sum_{q<q_c} q \, b_\lambda^\dagger b_{\lambda+q} (a_q^\dagger + a_{-q}) + \frac{I}{2\mathcal{N}} \sum_{\substack{q>q_c \\ \sigma}} \sigma c_{k+q\sigma}^\dagger c_{k\sigma} b_\lambda^\dagger b_{\lambda+q} \tag{2.6}$$

3. Effects of plasmon-magnon interaction

Inasmuch as we are interested in the plasmon-magnon interaction only, we will not retain the first two terms of equation (2.3) and the second term of equation (2.6), because the first two terms of equation (2.3) do not involve density fluctuation operators. Also, the terms in $\mathcal{H}_{em}$ which give rise to the band splitting of electron energy has been taken in the electron Hamiltonian.

Thus the truncated Hamiltonian is given by

$$\mathcal{H}_{tr} = \sum_{k\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{\lambda} \epsilon_\lambda (b_\lambda^\dagger b_\lambda + \tfrac{1}{2})$$

$$+ \sum_q \hbar\omega_p (a_q^\dagger a_q + \tfrac{1}{2})$$

$$+ \sum_{\lambda, q} A_q b_\lambda^\dagger b_{\lambda+q} (a_q^\dagger + a_{-q}) \tag{3.1}$$

where
$$A_q = \frac{Iq}{2\mathcal{N}} \left(\frac{\hbar\omega_p \Omega}{8\pi e^2}\right)^{\frac{1}{2}}. \tag{3.2}$$

For calculating the effects of the interaction terms in equation (3.1) on the renormalization of magnon and plasmon energies and on their lifetimes, we make use of the double-time Green's function technique. The energy Fourier transform of the relevant (magnon) Green's function is given by (with usual definition for Green's function (Zubarev, 1960)):

$$(E - \mathrm{E}_\lambda) \ll b_\lambda ; b_\lambda^\dagger \gg = \frac{\hbar}{2\pi} + \sum_q A_q \ll b_{\lambda+q} (a_q^\dagger + a_{-q}) ; b_\lambda^\dagger \gg . \tag{3.3}$$

On calculating the higher order Green's function $\ll b_{\lambda+q} (a_q^\dagger + a_{-q}) ; b_\lambda^\dagger \gg$ and decoupling the hierarchy at a suitable stage, the magnon Green's function becomes

$$\ll b_\lambda ; b_\lambda \gg_E = \frac{\hbar}{2\pi (E - E_\lambda - \Sigma^m (E, \lambda))} \tag{3.4}$$

where the self-energy $\sum^m (E, \lambda)$ is given by

$$\sum^m (E, \lambda) = \sum_q A_q^2 \left[\frac{(1 + n_q^p + n_{\lambda-q}^m)}{E - E_{\lambda-q} - \hbar\omega_p} + \frac{(n_p^p - n_{\lambda-q}^m)}{E - E_{\lambda-q} + \hbar\omega_p} \right], \tag{3.5}$$

and n_q^p and n_λ^m are the plasmon and magnon occupation numbers.

Similarly, we can calculate the plasmon Green's function. The equation of motion for plasmon Green's function is

$$(E - \hbar\omega_p) \ll a_q ; a_q^\dagger \gg_E = \frac{\hbar}{2\pi} + A_q \sum_\lambda \ll b_\lambda^\dagger b_{\lambda+q} ; a_q^\dagger \gg_E . \tag{3.6}$$

Calculating the higher order Green's function $\ll b_\lambda^\dagger \, b_{\lambda+q} ; a_q^\dagger \gg_E$ and using the decoupling approximation, we get the plasmon Green's function as

$$\ll a_q ; a_q^\dagger \gg_E = \frac{\hbar}{2\pi (E - \hbar\omega_p - \Sigma^p (E, q))} \tag{3.7}$$

where $\sum^p (E, q)$ is the plasmon self-energy and is given by

$$\sum^p (E, q) = A_q^2 \sum_\lambda \frac{n_{\lambda-q}^m - n_\lambda^m}{E - \mathrm{E}_\lambda - \mathrm{E}_{\lambda-q}} . \tag{3.8}$$

The self-energy can be written as the sum of real and imaginary parts using Dirac's identity.

$$\operatorname*{Lt}_{\eta \to 0} \frac{1}{x \pm i\eta} = P \frac{1}{x} \pm i\pi\delta(x) \tag{3.9}$$

As is well known the real and imaginary parts of the one particle Green's function gives the energy shift and lifetime of the corresponding quasiparticle. Hence, for magnons the energy shift is given by

$$\Delta E_\lambda^m = \mathrm{Re}\, \Sigma^m (E, \lambda).$$

At low temperatures (for $\hbar\omega_p \sim 0{\cdot}1 eV$) n^p is vanishingly small. Further in the long-wave length region $E_\lambda \approx 2JSa^2\lambda^2$. Thus, in this region we get

$$\Delta E_\lambda^m \approx \left(\frac{I^2 a^4 \hbar\omega_p}{256\pi^2 e^2 JS}\right)\left(\frac{\hbar\omega_p}{2JSa^2}\right)^{\frac{3}{2}} \tan^{-1}\left(q_m \sqrt{\frac{2JSa^2}{\hbar\omega_p}}\right) \tag{3.10}$$

An estimate with

$$m^* \approx m, \quad J \approx I \approx 6\times10^{-3}\text{eV}, \; a \sim 5\text{Å}, \; S = \tfrac{7}{2} \text{ and } \epsilon_0 \sim 10$$

gives

$$\Delta E_\lambda^m \sim 10^{-11}\, eV \text{ for } \lambda \sim 10^6 \text{ cm}^{-1}.$$

The shift is rather negligible.

Similarly the plasmon energy shift is given by

$$\Delta E_q^p = \text{Re}\, \Sigma^p(E, q) \approx \text{Re}\, A_q^2 \sum_\lambda \left(\frac{n_{\lambda-q}^m - n_\lambda^m}{E - E_\lambda - E_{\lambda-q}}\right) \tag{3.11}$$

The plasmon cut-off wave-vector (q_c) for a carrier concentration of $n \sim 10^{18}$ cm^{-3} is of the order of 10^6 cm^{-1}, i.e., it lies in the very long wavelength region. Under this condition we can assume $E \gg (E_{\lambda-q} - E_\lambda)$ and replacing E by $\hbar\omega_p$

$$\Delta E_q^p \approx \frac{I^2 a^5 q^4 (k_B T)^{\frac{3}{2}}\, \zeta(\frac{3}{2})}{256\pi^3 e^2 \hbar\omega_p}\left(\frac{\pi}{2JS}\right)^{\frac{1}{2}}, \tag{3.12}$$

where $\zeta\,(\frac{3}{2})$ is the Zeta function. For the choice of parameters given above

$$\Delta E_q^p \sim 10^{-12}\, eV \quad \text{for } q \sim 10^6 \text{ cm}^{-1}.$$

The lifetimes of the quasiparticles can be calculated from the imaginary part of Σ using the relation

$$\tau = \frac{\hbar}{\text{Im}\,\Sigma} \tag{3.13}$$

Therefore, for magnons

$$\frac{1}{\tau_\lambda^m} = \frac{\pi a^3 N}{\hbar(2\pi)^3}\Big[\int A_q^2\,(1 + n_q^p + n_{\lambda-q}^m)\,\delta\,(E - E_{\lambda-q} - \hbar\omega_p)\,d^3q$$

$$+ \int A_q^2\,(n_q^p - n_{\lambda-q}^m)\,\delta\,(E - E_{\lambda-q} + \hbar\omega_p)\,d^3q\Big] \tag{3.14}$$

The q integration can be performed inside a sphere of radius q_c. Under the above mentioned approximations

$$\frac{1}{\tau_\lambda^m} \approx \frac{I^2 q_m^4 a^4 \omega_p}{1024\pi^2 JS\lambda e^2} \quad \text{for } \lambda > \frac{q_m^2 + (\hbar\omega_p/2JSa^2)}{2q_m} \tag{3.15}$$

$$= 0 \text{ otherwise}$$

The estimated life-time

$$\tau_\lambda^m \sim 2\times10^{-3} \text{ sec} \quad \text{for } \lambda \sim 10^6 \text{ cm}^{-1}.$$

Similarly for plasmons, the life-time is given by

$$\frac{1}{\tau_q^p} \approx \frac{\hbar \omega_p^2 I^2 q a^2}{4096 J^2 S^2 e^2 \pi^2}\left[\coth\left\{\frac{(\hbar\omega_p + 2JSa^2q^2)^2}{8k_B T J S a^2 q^2}\right\} - 1\right] \tag{3.16}$$

and the estimated life-time

$$\tau_q^p \sim 10^{-2}\,\text{sec} \quad \text{for } q \sim 10^6\,\text{cm}^{-1}.$$

4. Discussion

In the foregoing sections, we have considered the plasmon-magnon interaction in ferromagnetic semiconductors. The interaction arises from the non spin-flip scattering part of the electron-magnon interaction Hamiltonian. The spin-flip part cannot give rise to plasmon-magnon interaction in the first order because no spin-flip is involved in the plasmon creation or annihilation process. However, the spin-flip parts can contribute to plasmon-magnon interaction in higher orders. The higher order interaction will involve operators such as $\rho_{q\uparrow}\rho_{q\downarrow} b_\lambda^\dagger b_{\lambda_1}$. Owing to the restriction that $\rho_{q\downarrow} \sim 0$ these processes will be extremely weak in the ferromagnetic semiconductors in question.

An alternative way of looking at the result is the following. The magnon self-energy $\Sigma^m(E, \lambda)$ is related to the density correlation when all the electrons have spin-up orientation. In the wave-vector region $q < q_c$ this corresponds to virtual exchange of plasmons.

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