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# Colliding beam test of neutral isoscalar axial current

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**Abstract.** We discuss the condition for the presence of isoscalar parity violating neutral currents and notice that it is present in most gauge-theoretic models of weak interactions. We investigate the possibility of testing its presence in $e^+e^-$ colliding beams and find that parity violation in the reaction $e^+e^- \rightarrow \Lambda^\circ \bar{\Lambda}^\circ$ would provide an unambiguous test for such a current.

**Keywords.** Weak Interactions; gauge theories; neutral isoscalar axial current; hyperon pair production in $e^+e^-$ colliding beams.

## 1. Introduction

Recent experiments (Hasert *et al* 1973, Cundy 1974) have demonstrated the existence of strangeness-conserving hadronic weak neutral currents in neutrino induced reactions. It is clearly of extreme importance to establish the space-time and isospin properties of this current. In particular, one of the interesting questions to be answered is whether the hadronic weak neutral current contains an isoscalar axial vector part. The need to look for such an object has already been emphasized§ by Pais and Treiman (1974) on the grounds that it will be a "new" piece. There is, however, a further reason to investigate its presence or absence in the context of gauge theoretic models of weak interactions as we shall shortly explain. Pais and Treiman have suggested the search for a difference between the elastic scattering of neutrinos and antineutrinos off a deuterium target (an admittedly difficult experiment) which will expose the presence of an isoscalar axial vector weak neutral current. On the contrary, it should be experimentally much easier to detect such a current by studying polarization effects with appropriate hadronic isoscalar final states produced in electron-positron annihilation.

In this paper we shall first discuss the significance of the presence of an $I = 0$ axial vector neutral current vis-a-vis the models of weak interactions based on gauge theories. The importance of this question stems from the following fact. In the original version of the Salam-Weinberg (Salam 1968, Weinberg 1967, 1971) type gauge models based on the $SU(2) \times U(1)$ group (where strangeness-changing weak interactions are ignored) such a current is absent despite the presence of other types of weak neutral currents. On the contrary, when strangeness-

§ This was pointed out to us by K V L Sarma.

violating interactions are introduced through charged currents and yet strangeness-changing neutral currents are carefully excluded (to be in conformity with experiment), isoscalar axial vector neutral currents have to be included. Interestingly, in models based on the gauge-group SU(3) $\times$ U(1) (Schechter and Singer 1974, Gupta and Mani 1974) (where strangeness-changing weak interactions are included *ab initio*) the existence of $I = 0$ axial vector currents is assured right from the beginning. Thus, we see that it is of great theoretical interest to study the presence of such currents.

Our next objective is to devise a suitable experimental test to detect a weak neutral isoscalar axial current. We have already commented on the experimental difficulties of studying elastic neutrino and antineutrino scattering from isoscalar targets. On the other hand $(\nu, \bar{\nu})$ (p, n) elastic scattering can only provide bounds as the isovector part of the weak neutral current will also contribute. However, electron-positron annihilation in storage rings is a good process for the study of parity violation through polarization effects. Some authors (Love 1972, Cung *et al* 1972, Llwellyn-Smith and Nanopoulos 1974, Paschos 1974) have already noted that the presence or absence of parity violation in weak neutral currents can be checked by the observation of polarization effects in the reaction $e^+ + e^- \rightarrow \mu^+ + \mu^-$. We consider the reaction $e^+ + e^- \rightarrow \Lambda^0 + \bar{\Lambda}^0$. One may of course look for parity-violating effects with polarized $e^+$, $e^-$ beams. But even with unpolarized lepton beams, the longitudinal polarization of the $\Lambda^0$ should not be difficult to measure and should yield the desired information. A nonzero effect after the electron and positron momenta have been symmetrized as explained in section 3 is an unambiguous indication of the presence of a weak isoscalar neutral axial vector current coupling the electron-positron system to the $\Lambda^0\bar{\Lambda}^0$ channel. Experimental data on such effects should be forthcoming shortly because of presently operating high luminosity storage rings such as SPEAR from which the possibility of obtaining signals at the level of a few per cent will be shown.

In section 2 of the paper we bring out the connection between gauge models and the presence of a neutral isoscalar axial vector current. Section 3 contains the details of our suggestion to look for such a current in terms of the polarization of the $\Lambda^0$ in the reaction $e^+ + e^- \rightarrow \Lambda^0 + \overline{\Lambda^0}$.

## 2. Gauge theories and neutral isoscalar axial current

Consider first the Salam-Weinberg model, ignoring $\triangle S \neq 0$ charged currents. In this model as well as in others which are similar, a neutral isoscalar axial vector current does not exist. This is a consequence of the following argument.* In these theories the hadronic part of the generators $I_i^{\mathrm{W}}$ of the "weak isospin" $\mathrm{SU(2)_W}$ part of the gauge group $\mathrm{SU(2)_W} \times \mathrm{U(1)_W}$ is identified with the left-handed part of the hadronic isospin $I$. Thus, the two component spinor of left handed nonstrange quarks $\begin{pmatrix} \mathrm{p} \\ \mathrm{n} \end{pmatrix}_{\mathrm{L}}$ is an isodoublet under either. The hadronic electromagnetic charge is given by

* This general reasoning has been given to us by Tulsi Dass.

$$Q^{EM} = I_3^W + \frac{Y^w}{2}, \tag{1}$$

where $Y^w$ is the "weak hypercharge" associated with the $U(1)_w$ group. On the other hand, the Gell-Mann-Nishijima formula is

$$Q^{EM} = I_3 + \tfrac{1}{2}Y \tag{2}$$

In eq. (2) $I_3$ and $Y$ are the usual third component of isospin and hypercharge operators. Now $I_3$ can be broken up into $I_{3L} + I_{3R}$, corresponding to the left-handed and right-handed states. The identification of the two kinds of isospin alluded to earlier implies

$$I_3^W = I_{3L}. \tag{3}$$

Equations (1)–(3) lead to the result

$$\tfrac{1}{2}Y^w = I_{3R} + \tfrac{1}{2}Y. \tag{4}$$

The weak neutral currents couple to $I_3^W$ and $Y^w/2$. The isoscalar term is only contained in $Y/2 = (Y_L + Y_R)/2$, coming from eq. (4). This does not discriminate between left-handed and right-handed parts and couples only to the vector current. Hence, in these models there is no isoscalar axial vector weak neutral current.

It is well known that models of the type discussed in the previous paragraph can be modified to include strangeness-changing charged currents without leading to the presence of the unwanted $\triangle S = 1$ neutral currents. One well-known method of achieving this modification is to invoke the GIM (Glashow *et al* 1970) mechanism. This includes a fourth quark p′ and treats the two-component left-handed spinor $\begin{pmatrix} p' \\ \lambda_e \end{pmatrix}_L$ as an isodoublet. As a result, the identification of $I_3^W$ with $I_{3L}$ (*i.e.* eq. (3)) is immediately destroyed since p′ and $\lambda$ are individual isosinglets under hadronic isospin. Then isoscalar axial vector neutral currents can do occur through terms such as $\bar{p}'\gamma_\mu\gamma_5 p'$ and $\bar{\lambda}\gamma_\mu\gamma_5\lambda$. This is a general characteristic of almost all gauge models which possess $\triangle S = 1$ charged currents but only $\triangle S = 0$ neutral currents. These include models based on charm (Beg and Sirlin 1974) and in general they contain isoscalar axial vector currents that are neutral and strangeness-conserving. Instead of discussing each model separately, we can find the general condition which will lead to the presence of such a term. Let us first confine our attention to models based on the gauge group $SU(2)_W \times U(1)_W$. The coupling to the neutral gauge boson occurs through the covariant derivative.

$$D_\mu = \partial_\mu - igW_\mu^3 I_3^W - ig' W_\mu^0 \frac{Y^w}{2} \tag{5}$$

In eq. (5) $g$ and $g'$ are the two coupling constants and $W^3$, $W^0$ are the neutral vector bosons. These unphysical objects are related to the photon $A$ and the physical weak neutral boson $Z$ by means of the mixing relations ($\xi$ being the mixing angle):

$$W_\mu^3 = A_\mu \sin\xi + Z_\mu \cos\xi,$$

$$W_\mu^0 = A_\mu \cos\xi - Z_\mu \sin\xi. \tag{6}$$

It is clear from eqs (5) and (6) that the coupling between the photon and the neutral (desiredly electromagnetic) current is

$$g \sin \xi \, I_3^W + g' \cos \xi \, \frac{Y^W}{2}$$

which becomes the electromagnetic charge $Q$ only if

$$g \sin \xi = g' \cos \xi = e. \tag{7}$$

The coupling between the neutral current and the weak vector boson $Z$ is given by

$$g \cos \xi \, I_3^W - g' \sin \xi \, \frac{Y^W}{2} = e\left(\cot \xi \, I_3^W - \tan \xi \, \frac{Y^W}{2}\right). \tag{8}$$

The axial part of the total weak neutral charge $Q^Z$ associated with the intermediate vector boson $Z$ may be written as

$$Q_5^Z = -\tfrac{1}{2}\left(Q^Z - \mathcal{P} Q^Z \mathcal{P}^{-1}\right), \tag{9}$$

where $\mathcal{P}$ is the parity operator. Since the electromagnetic charge operator $Q^{EM}$ commutes with $\mathcal{P}$, we have

$$[I_3^W, \mathcal{P}] = -\left[\frac{Y^W}{2}, \mathcal{P}\right]. \tag{10}$$

The use of eqs (8), (9) and (10) leads to the following expression for the weak neutral axial charge

$$Q_5^Z = \frac{e}{\sin 2\xi}\left[-I_3^W + \mathcal{P} I_3^W \mathcal{P}^{-1}\right]. \tag{11}$$

If the charge $Q_5^Z$ of eq. (11) is nonvanishing, then an axial neutral current is present. To check whether $Q_5^Z$ has an isoscalar part, it would be simplest to evaluate its matrix-element between two isoscalar states $|\, a\rangle$ and $|\, b\rangle$, say. If

$$\langle I = 0, \, b \,|\, Q_5^Z \,|\, I = 0, \, a\rangle \neq 0,$$

then clearly there must exist an isoscalar axial part of the weak neutral current. Now all models introduce the $\lambda$ quark in order to incorporate strangeness changing charged currents. Since the $\lambda$ quark is an isoscalar, one can test the presence of an $I = 0$ part in the charge $Q_5^Z$ by choosing $b = a = \lambda_L$. Using eq. (11) we see that if $\langle \lambda_L \,|\, I_3^W \,|\, \lambda_L\rangle$ and $\langle \lambda_R \,|\, I_3^W \,|\, \lambda_R\rangle$ are not the same, $Q_5^Z$ must contain a neutral isoscalar part. We now look at the various models in the literature. A survey of a number of these has been done in the work of Bjorken and Llewellyn-Smith (1973). From these it is clear that only the 2-2 model described in Appendix B of their paper treats $\lambda_L$ and $\lambda_R$ on equal footing. For every other model with weak neutral currents and strangeness-changing weak interactions,

$$\langle \lambda_L \,|\, I_3^W \,|\, \lambda_L\rangle \neq \langle \lambda_R \,|\, I_3^W \,|\, \lambda_R\rangle$$

and an isoscalar neutral axial vector current exists. On the other hand, in the 2-2 model the left-handed and right-handed parts of the quark $q$ are not treated equally in so far as weak isospin is concerned; $q_L$ belongs to an isodoublet while $q_R$ is a singlet. Hence, this model also contains a neutral isoscalar parity violating axial vector current.

The models based on the triplet charm of Beg and Zee (1973) also have isoscalar neutral axial currents. However, these are arranged in such a way that they do not contribute to matrix-elements associated with non-charmed particles.

Regarding models based on the gauge group SU(3) $\times$ U(1), one can carry out an analysis which is very similar to the one done with eqs (1) to (4). The analogue of eq. (1) here is

$$Q = I_3^W + \frac{Y^W}{2} + u \tag{12}$$

where $u$ is an extra U(1) quantum number. Equation (2) of course holds as before. However, now there is an identification not only between weak isospin and left-handed hadronic isospin (eq. 3) but also between weak hypercharge and the left-handed part of hadronic hypercharge

$$Y^W = Y_L. \tag{13}$$

Using eqs (2), (3), (12) and (13) one obtains

$$u = I_{3R} + \frac{Y^R}{2}. \tag{14}$$

In general there will be a weak neutral current coupling to $u$. Its isoscalar part contains only $Y_R$ and not $Y_L$ — a clear sign of the presence of parity-violation. Thus an isoscalar neutral axial vector current exists in these models too.

## 3. Study of $e^+e^- \to \Lambda^0 \overline{\Lambda^0}$ to test neutral isoscalar axial current

We now investigate the possibility of experimentally testing the presence of such isoscalar axial neutral currents in $e^+e^-$ going into a specific hadronic channel $h$. Our aim is to detect appropriate parity violating effects. Let us consider the properties which the final state $h$ should have in order to lead to an observable effect. Such a parity violating effect comes from the interference of the two graphs (*a*) single photon exchange, and (*b*) single $Z^0$-exchange between the initial $e^+e^-$ and the final $h$. The matrix-element for the single photon exchange graph can be written as $j_\mu^{EM} J^{\mu,EM}$, whereas the diagram with the neutral current can be written as

$$v_\mu V^\mu + a_\mu A^\mu - a_\mu V^\mu - v_\mu A^\mu$$

in a self-evident notation where small letters stand for lepton currents and capitals for hadron currents.

Clearly the final state must have a component which is negative under charge-conjugation, otherwise the single photon exchange graph will be absent; in that case the total cross-section for the process will be very small. Secondly, the state must contain an $I = 0$ part with positive $C$ and abnormal spin-parity component in order to be able to couple to the neutral isoscalar weak current. Thus, the final hadronic state should not be an eigenstate of $C$. Finally, we do not want the $I = 1$ axial vector part of the weak neutral current to contribute, which means the final state should not have a component with $I = 1$ and positive $C$. This can be achieved by choosing the final state either as (*a*) positive under $G$-parity or as (*b*) isoscalar. The parity-violating effects come from the interference term

$$-J_\lambda^{EM} J^{\lambda,EM} [a_\mu V^\mu + v_\mu A^\mu].$$

We have to eliminate the effect of the parity-violating term $a^{\mu} V_{\mu} j_{\lambda}^{\mathrm{EM}} J^{\lambda,\mathrm{EM}}$ so that we are left only with the term of interest $v_{\mu} A^{\mu} j_{\lambda}^{\mathrm{EM}} J^{\lambda,\mathrm{EM}}$ where $A^{\mu}$ is the desired hadronic neutral weak isoscalar axial current. The former term involves the trace $\mathrm{Tr}\, \not{k}_{+}\gamma_{\alpha}\not{k}_{-}\gamma_{\beta}\gamma_{5} = 4\mathrm{i}\, \epsilon_{\mu\nu\alpha\beta} k_{+}^{\mu} k_{-}^{\nu}$ and can be eliminated by symmetrizing the respective e$^{+}$, e$^{-}$ four-momenta $k_{+}$ and $k_{-}$. Thus, the interference term involving the desired isoscalar weak axial neutral current $A^{\mu}$ can be isolated.

The lowest mass final states of the type (*a*) which show parity-non-conserving effects are 4 $\pi$ states.* To observe parity-violating effects in such a system with four particles might not be too easy. So we go to case (*b*) where an attractive possibility of $\Lambda^{0}\bar{\Lambda}^{0}$ exists. Here one measures the longitudinal polarization of the $\Lambda^{0}$ which is perhaps easier to determine. The final state $\Omega^{-}\bar{\Omega}^{-}$ will also fulfil the requirement; however, the production of such states may not be too large. Confining our discussion to $\Lambda^{0}\bar{\Lambda}^{0}$ final states and to initially unpolarized e$^{+}$, e$^{-}$ beams, it is straightforward to calculate the longitudinal polarization of the $\Lambda^{0}$. The relevant matrix-element can be defined after Budny (1973).

$$\langle 0 \,|\, j_{\alpha}^{\mathrm{EM}} \,|\, \mathrm{e^{+}e^{-}} \rangle = e\,\bar{v}(k_{+})\,\gamma_{\alpha}\, u(k_{-}) \tag{15}$$

$$\langle 0 \,|\, j_{\alpha}^{Z} \,|\, \mathrm{e^{+}e^{-}} \rangle = i\bar{v}(k_{+})\,\gamma_{\alpha}\,(g_{\mathrm{V}}^{0} + g_{\mathrm{A}}^{0}\gamma_{5})\, u(k_{-}) \tag{16}$$

$$\langle \Lambda^{0}\bar{\Lambda}^{0} \,|\, J^{\mathrm{EM}} \,|\, 0 \rangle = e\bar{u}(p)\,[F(Q^{2})\,(p-\bar{p})_{\alpha} + G(Q^{2})\,\gamma_{\alpha}]\, v(\bar{p}), Q = p + \bar{p} \tag{17}$$

$$\langle \Lambda^{0}\bar{\Lambda}^{0} \,|\, J_{\alpha}^{Z} \,|\, 0 \rangle = \bar{u}(p)\,[F_{\mathrm{V}}^{0}\,(p-\bar{p})_{\alpha} + G_{\mathrm{V}}^{0}\,\gamma_{\alpha} + H_{\mathrm{V}}^{0}\,(p+\bar{p})_{\alpha} + \\ + \{F_{\mathrm{A}}^{0}(p-\bar{p})_{\alpha} + G_{\mathrm{A}}^{0}\,\gamma_{\alpha} + H_{\mathrm{A}}^{0}\,(p+\bar{p})_{\alpha}\}\,\gamma_{5}]\, v(\bar{p}). \tag{18}$$

The expression for the longitudinal polarization of the $\Lambda^{0}$ can be obtained from the general differential cross section given by Budny (1973). It is easy to give expressions for parity-violating effects with polarized e$^{+}$, e$^{-}$ beams following the same author but we choose not to do so here. We also take the mass of the neutral intermediate vector boson $Z$ to be much larger than the momenta involved. One can show that if $P(\theta)$ is the longitudinal polarization of the $\Lambda^{0}$ coming at an angle $\theta$ with respect to the electron momentum $k_{-}$, then in the CM system

$$P \equiv \frac{P(\cos\theta) + P(-\cos\theta)}{2} \simeq \frac{A\,(1+\cos^{2}\theta) - BQ^{2}\sin^{2}\theta}{Q^{2}\,|\,F\,|^{2}\sin^{2}\theta + |\,G\,|^{2}(1+\cos^{2}\theta)} \tag{19}$$

where

$$A \simeq -2\,\mathrm{Re}\,(g_{\mathrm{V}}^{0}\, G^{*}\, G_{\mathrm{A}}^{0})\Big|_{Q^{2}} \frac{Q^{2}}{e^{2}\, M_{Z}^{2}} \tag{20}$$

and

$$B \simeq -2\,\mathrm{Re}\,(g_{\mathrm{V}}^{0}\, F^{*}\, F_{\mathrm{A}}^{0})\Big|_{Q^{2}} \frac{Q^{2}}{e^{2}\, M_{Z}^{2}}\,. \tag{21}$$

If the assumption $M_{Z}^{2} \gg Q^{2}$ is not made, a more general expression for $P$ can be

* See the Appendix of the paper by Pais and Treiman (1974).

easily derived following Budny's expressions. It should be noted that the form-factors $F(Q^2)$ and $G(Q^2)$ are related to the usual Sachs electromagnetic form-factors $G_E$, $G_M$ by

$$F(Q^2) = \frac{2M}{Q^2}\left[G_M(Q^2) - G_E(Q_2)\right] \tag{22}$$

and

$$G(Q^2) = G_M(Q^2) \tag{23}$$

Taking relatively large $Q^2$ and small $\theta$ (*i.e.*, large incident energies and a $\Lambda^0$ detected close to the beam line), eq. (19) simplifies to

$$\bar{P} \simeq \frac{A}{|G|^2} \simeq -\frac{2G_F Q^2}{e^2 |G_M|} \tag{24}$$

where

$$\left|\frac{g_V^0 G_A^0}{M_Z^2}\right|_{Q^2} \simeq G_F \simeq \frac{10^{-5}}{M_p^2} \tag{25}$$

Thus we can estimate

$$\bar{P} \simeq -1 \cdot 5 \times 10^{-4} \frac{Q^2}{|G_M|}, \tag{26}$$

where $Q^2$ is in GeV$^2$. Hence a signal at a few percentage level should be easily obtainable at SPEAR with $Q^2$ in the range of 70 GeV$^2$. The absence of such an effect will have very serious implications for most gauge-theories of weak interactions.

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