Pramāṇa, Vol. 11, No 6, December 1978, pp. 745–754, © printed in India

Explicit classical solutions to Euclidean Yang-Mills theory with spherical symmetry

AMITABHA MUKHERJEE and PROBIR ROY
Tata Institute of Fundamental Research, Bombay 400 005

MS received 22 May 1978; revised 6 October 1978

Abstract. All real classical solutions of the SU(2) Yang-Mills theory with spherical symmetry in 4-dimensional Euclidean space are constructed analytically and catalogued. The uniqueness of the solution of Belavin *et al* in possessing finite action is explicitly demonstrated.

Keywords. Euclidean Yang-Mills theory; Euclidean space; classical solutions; spherical symmetry.

1. Introduction

Classical solutions of the SU(2) Yang-Mills theory in Euclidean space have received considerable attention recently. (For a review see Jackiw *et al* 1977). The instanton or pseudoparticle solution of Belavin *et al* (1975) was the first to be explicitly constructed. This solution has finite total action, is spherically symmetric and is characterised by a topological quantum number (namely the Pontryagin index) $q = 1$ (-1 for the anti-instanton solution, obtained by a coordinate reflection). It has been shown to dominate the path-integral expression for the vacuum-to-vacuum transition amplitude in the quantum theory in Minkowski space-time. It thus plays an important role in understanding the structure of the gauge theory vacuum (t' Hooft 1976a, b; Jackiw and Rebbi 1976; Callan *et al* 1976). Finite action solutions where q is an arbitrary integer (' multi-instantons ') are also known (Witten 1976), but these are not spherically symmetric. The above solutions have the property that the field tensor $F_{\nu\mu}$ is self-dual or self-antidual, i.e.,

$$F_{\mu\nu} = \pm \tilde{F}_{\mu\nu}, \tag{1}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$.

In fact, this property was required *a priori* in obtaining the solutions. This is understandable since the condition of (1), which automatically satisfies the Yang-Mills equation of motion, is more tractable than the latter. However, (1) immediately implies a null value for the energy-momentum tensor $\theta_{\mu\nu}$ and hence can give information only on the vacuum rather than on states with non-zero energy. Some solutions with non-zero $\theta_{\mu\nu}$ are known (de Alfaro *et al* 1976; Cervero *et al* 1977;

Leznov and Saveliev 1978). In general they do not possess finite Euclidean action. Moreover, they do not have spherical symmetry but involve some external constant vector. However, one may argue that, in the description of a closed system, the absence of such external vectors is desirable on aesthetic grounds. Thus one is led to seek solutions with spherical symmetry and non-zero $\theta_{\mu\nu}$.

It is the purpose of this paper to demonstrate that the classical Euclidean SU(2) Yang-Mills theory with spherical symmetry is completely solvable. This we do by explicitly constructing all real solutions. The latter include the instanton and anti-instanton solutions, which are the only ones to satisfy (1). We explicitly verify, as earlier claimed by Calvo (1977), that all spherically symmetric solutions with non-zero energy have infinite total Euclidean action S_{Eucl}. Thus they cannot play any non-trivial role in quantum transitions which are proportional to exp $(-S_{\text{Eucl}}/\hbar)$. However, we hope that these explicit constructions will encourage efforts to find other (i.e., not spherically symmetric) solutions with non-vanishing $\theta_{\mu\nu}$, some of which may turn out to have finite action.

In §2 we discuss the reduction of the equation of motion under the assumption of spherical symmetry, and in §3 we solve the resulting equation and list the solutions. Some general remarks are made in the concluding § 4.

2. Consequences of spherical symmetry

Any vector field configuration, and hence in particular the gauge potential, can be written as the sum of longitudinal and transverse parts:

$$A^a_\mu = \partial_\mu f^a + \partial_\nu h^a_{\mu\nu}, \tag{2}$$

where f^a are scalars and $h^a_{\mu\nu}$ are antisymmetric tensors. In the absence of external vectors as required by spherical symmetry, all tensors must be constructed from the coordinate vector x_μ and the isotropic tensors $\delta_{\mu\nu}$ and $\epsilon_{\mu\nu\lambda\rho}$. (We always restrict our attention to a 4-dimensional Euclidean space). Clearly, no antisymmetric tensor of rank two under $O(4)$ can be so constructed. Thus all field configurations that are spherically symmetric in this strict sense can be written in the form $A^a_\mu = \partial_\mu f^a(r)$, where r is the radial co-ordinate: $r = (x_\mu x_\mu)^{1/2}$. Substitution in the definition of the field tensor,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a_{bc} A^b_\mu A^c_\nu, \tag{3}$$

leads to an identically vanishing field strength.*

Evidently, spherical symmetry in the above sense is too stringent a requirement, and would rule out, for example, the instanton solution. As is well known from the study of magnetic monopole theory, the resolution of this difficulty lies in extending the definition of spherical symmetry to allow for the mixing of tensor and internal symmetry degrees of freedom; instead of separate invariance under spatial and

* We assume that f^a and $h^a_{\mu\nu}$ do not have δ-function-like singularities.

internal gauge transformations, only a weaker invariance under appropriately combined transformations is required. This introduces other isotropic tensors into the solution, with explicit mixing of gauge and tensor indices. Now $h^a_{\mu\nu}$ is no longer constrained to vanish.

Although the above remarks apply to a large number of gauge groups, for the remainder of this paper we confine our discussion to SU(2) as the gauge symmetry. One can define a tensor $\eta_{a\mu\nu}$ (indeed, for SU(2) it is unique; 't Hooft 1976b) which is covariant under a combined rotation and gauge transformation. One representation is

$$\eta_{a\mu\nu} = \epsilon_{a\mu\nu4} + \delta_{a\mu}\,\delta_{\nu4} - \delta'_{a\nu}\,\delta_{\mu4},$$

where $a = 1, 2, 3$ and $\mu, \nu = 1, 2, 3, 4$. Spherically symmetric configurations with non-vanishing $F^a_{\mu\nu}$ follow from the substitution $h^a_{\mu\nu} = -\eta_{a\mu\nu}\, h(r)$ in (2) with $h(r)$ being an unknown function of r, i.e.,

$$\begin{aligned} A^a_\mu &= \partial_\mu f^a(r) - \eta_{a\mu\nu}\,\partial_\nu h(r) \\ &= \hat{x}_\mu f^{a\prime}(r) - \eta_{a\mu\nu}\,\hat{x}_\nu h'(r), \end{aligned} \tag{4}$$

where $F'(r) \equiv dF(r)/dr$ and $\hat{x}_\mu = x/|x|$. The Lagrangian density,

$$\mathscr{L} = \tfrac{1}{4} F^a_{\mu\nu}(x)\, F^a_{\mu\nu}(x), \tag{5}$$

then becomes a function of the 'coordinates' $f^{a\prime}(r)$ and $h'(r)$ and 'velocities' $f^{a\prime\prime}(r)$ and $h''(r)$. The substitution of (3) in (4) and (5) leads to

$$\mathscr{L} = \mathscr{L}_0 + g^2(h'(r))^2 f^{a\prime}(r) f^{a\prime}(r),$$

where $\mathscr{L}_0$ is independent of $f^{a\prime}$ and $f^{a\prime\prime}$ and involves only $h(r)$ containing the transverse degree of freedom. Thus the equation of motion for the longitudinal part of A^a_μ is

$$\frac{\partial\mathscr{L}}{\partial f^{a\prime}(r)} \equiv 2g^2\,(h'(r))^2 f^{a\prime}(r) = 0,$$

i.e., either $h'(r) = 0$ or $f^{a\prime}(r) = 0$. The first possibility, as shown earlier, corresponds to the trivial solution $F^a_{\mu\nu} = 0$. Hence we choose the second whereupon (4) becomes

$$\begin{aligned} A^a_\mu(x) &= -\eta_{a\mu\nu}\,\partial_\nu h(r) \\ &= -\eta_{a\mu\nu}\,\hat{x}_\nu h'(r). \end{aligned} \tag{6}$$

In more compact matrix notation (Sciuto 1977) take $A_\mu = gA^a_\mu\,\tau^a/2i$, $\sigma_{\mu\nu} = \eta_{a\mu\nu}\,\tau^a/2$ and $\ln \rho(r) = gh(r)$, so that

$$A_\mu = i\sigma_{\mu\nu}\,\partial_\nu(\ln \rho(r)). \tag{7}$$

The form of A_μ in (7) or the first line of (6) is precisely the one posited by Corrigan and Fairlie (1977) and Wilczek (1977). The only difference is that they allow $h(x)$ or $\rho(x)$ to have arbitrary spatial dependence whereas here we have functions of r alone. It is an interesting fact that a rather restrictive ansatz in the general situation turns out to be obligatory in the spherically symmetric case.

So far, we have not used the full equation of motion

$$\partial_\mu F_{\mu\nu} + [A_\mu, F_{\mu\nu}] = 0,$$

where $F_{\mu\nu} = gF^a_{\mu\nu}\, \tau^a/2i$. With A_μ given by the Corrigan-Fairlie-Wilczek ansatz, this takes on a particularly simple form

$$\partial_\mu \left[\frac{\Box \rho}{\rho^3}\right] = 0,$$

i.e., $$\Box \rho = c\rho^3,$$

c being an integration constant (Sciuto 1977). In deriving this, use has been made of the fact that the matrices $\sigma_{\mu\nu}$ satisfy the $O(4)$ rotation algebra:

$$[\sigma_{\mu\nu}, \sigma_{\lambda\rho}] = i\,(g_{\mu\lambda}\, \sigma_{\nu\rho} + g_{\nu\rho}\, \sigma_{\mu\lambda} - g_{\mu\rho}\, \sigma_{\nu\lambda} - g_{\nu\lambda}\, \sigma_{\mu\rho}),$$

as follows from the definition $\sigma_{\mu\nu} = \eta_{a\mu\nu}\, \tau^a/2$. For spherically symmetric ρ, as in (7), we thus have

$$\rho'' + 3\rho'/r = c\rho^3. \tag{8}$$

Equation (8), although non-linear, is in fact exactly solvable. Before solving it, however, we first note below the form of the dependence of physically interesting quantities such as the action (Lagrangian) density $\mathscr{L}(x)$, the Pontryagin index density $q(x)$ and the energy-momentum tensor $\theta_{\mu\nu}(x)$ on $\rho(r)$.

Under the Corrigan-Fairlie-Wilczek ansatz, the field tensor $F_{\mu\nu}$ can be written, following Sciuto, in the form

$$F_{\mu\nu} = -i\,(\sigma_{\nu\lambda}\, T_{\mu\lambda} - \sigma_{\mu\lambda}\, T_{\nu\lambda}) - i\, E\, \sigma_{\mu\nu},$$

where $$T_{\mu\nu} = \partial_\mu (\ln \rho)\, \partial_\nu (\ln \rho) - \partial_\mu\, \partial_\nu (\ln \rho) - \tfrac{1}{4}\, \delta_{\mu\nu}\, [\partial_\alpha (\ln \rho)\, \partial_\alpha (\ln \rho) - \Box\, (\ln \rho)],$$

and $$E = \Box\rho/2\rho.$$

This yields

$$\mathrm{Tr}\,(F_{\mu\nu}\, F_{\mu\nu}) = -2\, T_{\mu\nu}\, T_{\mu\nu} - 6\, E^2, \tag{9}$$

$$\mathrm{Tr}\,(F_{\mu\nu}\, \tilde{F}_{\mu\nu}) = 2\, T_{\mu\nu}\, T_{\mu\nu} - 6\, E^2, \tag{10}$$

where $$\tilde{F}_{\mu\nu} = \tfrac{1}{2}\, \epsilon_{\mu\nu\alpha\beta}\, F_{\alpha\beta}.$$

Moreover, the self-dual and self-antidual parts of $F_{\mu\nu}$, defined as $F^{\pm}_{\mu\nu}$ by

$$F^{\pm}_{\mu\nu} = \tfrac{1}{2}(F_{\mu\nu} \pm \tilde{F}_{\mu\nu}),$$

satisfy $$\mathrm{Tr}\,[F'^{+}_{\mu\lambda} F'^{-}_{\nu\lambda} + (\mu \leftrightarrow \nu)] = ET_{\mu\nu}. \tag{11}$$

The left-hand sides of (9), (10) and (11) are proportional to $\mathscr{L}(x)$, $q(x)$ and $\theta_{\mu\nu}(x)$ respectively. These expressions are valid for an arbitrary function $\rho(x)$. For the spherically symmetric case $\rho \equiv \rho(r)$, we obtain

$$T_{\mu\nu} = T\left[\frac{1}{4}\delta_{\mu\nu} - \frac{x_\mu x_\nu}{r^2}\right], \tag{12a}$$

where $$T = \frac{\rho''}{\rho} - \frac{\rho'}{r\rho} - 2\left(\frac{\rho'}{\rho}\right)^2. \tag{12b}$$

Moreover, $$E = \frac{1}{2\rho}\left(\rho'' + \frac{3\rho'}{r}\right). \tag{13a}$$

In terms of E and T, the Lagrangian density is $3g^{-2}(T^2/4+E^2)$. If (8) is satisfied, (13a) reduces to

$$E = \tfrac{1}{2} c\rho^2. \tag{13b}$$

Hence, for any solution of the form given by (7), only T and E—as given by (12b) and (13b)—need be computed in order to know all physical properties of interest. It may be noted that for configurations that are either self-dual (T=0) or self-antidual (E=0), the action density is proportional to the Pontryagin index density and $\theta_{\mu\nu}$ vanishes identically.

3. Explicit solutions

We now proceed to give the solutions of (8). Observe first that (7), as well as T or E, is invariant under a constant rescaling of $\rho : \rho(r) \to \lambda\rho(r)$. Thus the value of the integration constant c is irrelevant to quantities of physical interest. However, c is a useful parameter in classifying the solutions of (8). The change of variables $\chi(r) = r\rho(r)$, $z = \ln r$ (cf. Marciano and Pagels 1976) takes (8) into the form

$$\frac{d^2\chi}{dz^2} - \chi - c\chi^3 = 0. \tag{14}$$

Equation (14) may be doubly integrated into

$$z - z_0 = \pm \int \frac{d\chi}{(\frac{1}{2}c\chi^4 + \chi^2 + \beta)^{1/2}}, \tag{15}$$

β and z_0 being integration constants. The inversion of (15) will lead to the solution $\chi(z)$ of (14). The integral can be evaluated in terms of inverse Jacobian elliptic functions (e.g., Abramowitz and Stegun 1970), the precise form depending on the values of c and the constant β. In tables 1, 2 and 3, we display the solutions in terms of ρ for the cases $c=0$, $c<0$ and $c>0$ respectively. Within each category, the solution is obtained in different forms, depending on the value of β. The constant z_0 just sets the scale of r, as is clear from (15). For each solution we tabulate the gauge-invariant quantities T and E defined in §2, together with their singularities.

It can be seen that the only solutions for which E and T are singularity-free everywhere are those corresponding to subcases (ii) and (iii) of table 1 and subcase (ii) of table 2. The second of these corresponds to the ' vacuum ' solution, and the third and the first to the instanton and anti-instanton solutions respectively. The family of solutions given by subcase (iii) of table 2 has the instanton as a limit ($m\to 1$), and is singular only at the point $r=0$. Thus it is not immediately clear that the total action is finite only for $m=1$. However, after some manipulations one can write for this subcase

$$\mathrm{Tr}\,(F_{\mu\nu}\,F_{\mu\nu}) = -\frac{6a^4}{r^4}\left[dn^4\,(u\,|\,m) + \frac{(1-m)^2}{dn^4\,(u\,|\,m)}\right], \tag{16}$$

Thus the total action is

$$\begin{aligned} S &= -\frac{1}{2g^2}\int d^4x\,\mathrm{Tr}\,(F_{\mu\nu}\,F_{\mu\nu}) \\ &= \frac{6\pi^2\,a^2}{g^2}\int_{-\infty}^{\infty} du\left[dn^4\,(u\,|\,m) + \frac{(1-m)^2}{dn^4\,(u\,|\,m)}\right] \end{aligned} \tag{17}$$

For $m\neq 1$, dn is periodic with period $2K(m)$, and the contribution from each period is positive; hence the integral diverges. For $m=1$, $dn\,u = \mathrm{sech}\,u$ and $S=8\pi^2/g^2$, which is the correct value for the instanton solution. The remaining solutions of tables 1 and 2 and all solutions of table 3 are singular at points other than the origin, and hence cannot have finite action. Hence no solution with spherical

Table 1. Solutions for $c=0$. All these solutions are self-antidual: $E=0$.

Subcase	ρ	T	Singularities	Remarks
(i) $\beta = a^2 > 0$	$\pm\, a\,\dfrac{(r^2-r_0^2)}{2r^2\,r_0}$	$\dfrac{-\,8r_0^2}{(r^2-r_0^2)^2}$	$r = r_0$	
(ii) $\beta = -\,a^2 < 0$	$a\,\dfrac{(r^2+r_0^2)}{2r^2\,r_0}$	$\dfrac{8r_0^2}{(r^2+r_0^2)^2}$	None	This is the singular form of the anti-instanton solution; see, e.g., Jackiw *et al* (1977)
(iii) $\beta = 0$	$\left.\begin{matrix}1/r_0\\ r_0/r^2\end{matrix}\right\}$	0	None	Trivial solution $F_{\mu\nu} = 0$

Table 2. Solutions for $c<0$. Here and in table 3, $B\equiv-2\beta/c$ and $u=\alpha(z-z_0)=\alpha\ln(r/r_0)$, where α depends on the subcase. Elliptic function notations are as in Abramowitz and Stegun (1970).

Subcase	ρ	E	T	Singularities	Remarks
(i) $B>0$	$\frac{1}{r}\left(\frac{2m}{\mid c\mid}\right)^{1/2}\alpha\, cn\,(u\mid m)$ $\alpha=(2m-1)^{1/2}$ $\frac{1}{2}<m<1$	$-\frac{1}{r^2}\frac{m}{(2m-1)}cn^2u$	$\frac{1}{r^2}[1-\alpha^2dn^2u$ $+\alpha^2msn^2u$ $-2\alpha^2sc^2udn^2u]$	$E: r=0$ $T: r=0$, $u=(2n+1)K(m)$	The limit $m\to1$ gives subcase (ii).
(ii) $B=0$	$\left(\frac{2}{\mid c\mid}\right)^{1/2}\frac{2r_0}{(r^2+r_0^2)}$	$-\frac{4r_0^2}{(r^2+r_0^2)^2}$	0	None	The instanton solution
(iii) $0>B>-\frac{1}{c^2}$	$\frac{1}{r}\left(\frac{2}{\mid c\mid}\right)^{1/2}\alpha\, dn\,(u\mid m)$ $\alpha=(2-m)^{1/2}$, $0<m<1$	$-\frac{1}{r^2}\cdot\frac{1}{(2-m)}dn^2u$	$\frac{1}{r^2}[1-\alpha^2mcn^2u$ $+\alpha^2msn^2u$ $-2\alpha^2m^2sd^2ucn^2u]$	$r=0$	(a) The limit $m\to1$ gives subcase (ii). (b) The limit $m\to0$ gives the singular solution $\rho=\frac{1}{\sqrt{\mid c\mid}}\frac{1}{r}$ studied by de Alfaro *et al* (1976).
(iv) $B<-\frac{1}{c^2}$	...	...	...	...	There are no real solutions in this range.

Table 3. Solutions for $c > 0$. (Notation as in table 2)

Subcase	ρ	E	T	Singularities	Remarks
(i) $B > 0$	$\pm\frac{1}{r}\left(\frac{2}{c}\right)^{1/2} \alpha\, ds\,(u \mid m)$ $\alpha = (2m-1)^{-1/2}$, $\frac{1}{2} < m < 1$	$\frac{\alpha^2}{r^2} ds^2\, u$	$\frac{1}{r^2}[1+\alpha^2-2m\alpha^2\, cd^2\, u]$	$E:\ r = 0$ $u = 2n\, K(m)$ $T:\ r = 0$	The limit $m \to 1$ gives subcase (ii).
(ii) $B = 0$	$\pm\left(\frac{2}{c}\right)^{1/2} \frac{2\, r_0}{r^2-r_0^2}$	$\frac{4\, r_0^2}{(r^2-r_0^2)^2}$	0	$r = r_0$	Self-dual but singular: instanton with imaginary scale
(iii) $0 > B > -\frac{1}{c^2}$	$\pm\frac{1}{r}\left(\frac{2}{c}\right)^{1/2} \alpha\, cs\,(u \mid m)$ $\alpha = (2-m)^{-1/2}$ $0 < m < 1$	$\frac{\alpha^2}{r^2} cs^2\, u$	$\frac{1}{r^2}[1+m\alpha^2-2\alpha^2\, dc^2\, u]$	$E:\ r = 0,\ u = 2n\, K(m)$ $T:\ r = 0,$ $u = (2n+1)\, K(m)$	The limits $m \to 1$ and $m \to 0$ correspond to subcases (ii) and (iv) respectively.
(iv) $B = -\frac{1}{c^2}$	$\frac{1}{r}\left(\frac{1}{c}\right)^{1/2} \cot u$ $\alpha = 2^{-1/2}$	$\frac{1}{2r^2}\cot^2 u$	$-\frac{1}{r^2}\tan^2 u$	$E:\ r = 0,\ u = n\pi$ $T:\ r = 0,$ $u = (2n+1)\,\pi/2$	
(v) $B < \frac{1}{c^2}$	$\pm\frac{1}{r}\left(\frac{2}{c}\right)^{1/2} \alpha\, \{cs\,(u \mid m)\ nd\,(u \mid m)\}$ $\alpha = 2^{-1/2}\,(1-2m)^{-1/2}$ $0 < m < \frac{1}{2}$	$\frac{\alpha^2}{r^2} cs^2\, u\, nd^2 u$	$\frac{1}{r^2}[1+2m\alpha^2+2m\alpha^2\, cn^2\, u$ $-2\alpha^2\, dc^2\, u]$	$E:\ r = 0,\ u = 2n\, K(m)$ $T:\ r = 0,$ $u = (2n+1)\, K(m)$	The limit $m \to 0$ gives subcase (iv).

symmetry simultaneously satisfies the requirements of finite total action and non-vanishing $\theta_{\mu\nu}$.

4. Concluding remarks

We have constructed explicitly all real solutions to the classical Euclidean SU(2) Yang-Mills equation of motion under the assumption of spherical symmetry. Of these, apart from the vacuum solution, only the self-dual instanton solution of Belavin *et al* (1975) (and the anti-instanton, obtained by reflection of the x_4 coordinate) possesses finite action. Calvo (1977) had earlier claimed this result without explicitly solving the equation of motion. However, he had taken the spherically symmetric gauge potential to have the restricted form of an arbitrary radial function times the instanton solution. This amounts to making the Corrigan-Fairlie-Wilczek ansatz for the potential, and is not justified *a priori*. We have derived the CFW form as a consequence of spherical symmetry and thus removed this lacuna. Moreover, we have explicitly demonstrated the complete integrability of the classical Yang-Mills system with spherical symmetry by the actual analytical construction of all real solutions.

We have also looked for finite-action solutions possessing hyperbolic symmetry (i.e., depending only $\mathbf{x}^2 - x_0^2$) in Minkowski space-time, and failed to find any. When continued to Minkowski space, none of the solutions presented here (except the vacuum) has finite action. Hence it appears that any finite-action solution in Minkowski space must involve a fixed external space-time vector (e.g., de Alfaro *et al* 1976). A finite-action classical solution with hyperbolic symmetry has been constructed for the gauge group SO(3, 1) (Baaklini 1978). The non-compactness of the group plays an essential role in the construction, and it is our conjecture that such solutions do not exist for compact gauge groups. The total energy in Minkowski space is also infinite for the continued solutions, except for those that have $\theta_{\mu\nu} = 0$ identically.

Acknowledgements

We wish to thank Drs A K Raina and T R Ramadas for their helpful remarks.

References

Abramowitz M and Stegun I A 1970 *Handbook of Mathematical Functions* (New York: Dover Publications)

Baaklini N S 1978 *J. Phys.* **A11** L127

Belavin A A, Polyakov A M, Schwartz A S and Tyupkin Yu S 1975 *Phys. Lett.* **B59** 85

Callan C, Dashen R and Gross D 1976 *Phys. Lett.* **B63** 334

Calvo M 1977 *Phys. Rev.* **D16** 1061

Cervero J, Jacobs L and Nohl C R 1977 *Phys. Lett.* **B69** 351

Corrigan E and Fairlie D B 1977 *Phys. Lett.* **B67** 69

de Alfaro V, Fubini S and Furlan G 1976 *Phys. Lett.* **B65** 163

Jackiw R, Nohl C and Rebbi C 1977 Classical and semi-classical solutions of the Yang-Mills theory MIT preprint 675

Jackiw R and Rebbi C 1976 *Phys. Rev. Lett.* **37** 172

Leznov A N and Saveliev M V 1978 *Phys. Lett.* **B76** 108
Marciano W and Pagels H 1976 *Phys. Rev.* **D14** 531
Sciuto S 1977 Topics on Instantons, CERN lecture notes (to be published in *Rivista del Nuovo Cimento*)
't Hooft G 1976a *Phys. Rev. Lett.* **37** 8
't Hooft G 1976b *Phys. Rev.* **D14** 3432
Wilczek F 1977 in *Quark confinement and field theory* eds D Stump and D Weingarten (New York: Wiley)
Witten E 1976 *Phys. Rev. Lett.* **38** 121

Note :

After this work was completed, we received a paper by A Actor (Universität Dortmund preprint) which partly overlaps with ours. This paper contains an interesting method of generating new solutions from a known one. However, it is not concerned with the consequences of spherical symmetry as such.