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Radiative Neutrino Mass Matrix for Three Active plus One Sterile Species

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Abstract

A simple unifying mass matrix is presented for the three active and one sterile neutrinos ν_e, ν_μ, ν_τ, and ν_s, using an extension of the radiative mechanism proposed some time ago by Zee. The total neutrino-oscillation data are explained by the scheme $\nu_e \leftrightarrow \nu_s$ (solar), $\nu_\mu \leftrightarrow \nu_\tau$ (atmospheric) and $\nu_e \leftrightarrow \nu_\mu$ (LSND). We obtain the interesting approximate relationship $(\Delta m^2)_{\rm atm} \simeq 2[(\Delta m^2)_{\rm solar}(\Delta m^2)_{\rm LSND}]^{1/2}$ which is well satisfied by the data.

Three neutrinos, each associated with a charged lepton (e, μ, τ), are now known. The invisible width of the Z boson, coming from the decay $Z \to \nu\bar{\nu}$, is also consistent[1] with exactly three such neutrinos. This means that if there is a fourth neutrino, either it has to be very heavy (with mass greater than $M_Z/2$) or it does not couple to Z. In particular, if it is light, then it must not have any electroweak gauge interactions. Such an object is often referred to as a "sterile" neutrino. The reason that this may be a necessary part of our understanding of particle physics is that there are at present three classes of neutrino experiments[2, 3, 4] which show evidence of neutrino oscillations with three very different Δm^2's, *i.e.* differences of mass-squares. If all three interpretations are correct, then we need four light neutrinos. (A possible but rather extreme three-neutrino scenario[5] is to have large anomalous ν_τ-quark interactions.) It is thus of theoretical interest to find a natural mechanism which explains the masses and mixings of these four neutrinos in the present experimental context.

A specific model for a 4×4 neutrino mass matrix was proposed[6] already some time ago.. The form of this matrix agrees with subsequent purely phenomenological analyses[7, 8] of all neutrino-oscillation data. Our present study concerns the possibility that all neutrino masses are zero at tree level, but are generated radiatively at one-loop to match the pattern in [6], using a mechanism first proposed by Zee[9]. We extend previous work[10, 11] on this topic to include a sterile neutrino[12] with the help of an extra U(1) gauge symmetry[13]. The resulting mass eigenvalues lead to the approximate relationship

$$(\Delta m^2)_{\mathrm{atm}} \simeq 2\sqrt{(\Delta m^2)_{\mathrm{solar}}(\Delta m^2)_{\mathrm{LSND}}} \qquad (1)$$

which is well satisfied by the data.

Our model extends the standard electroweak gauge model to include three singlet fermion fields ν_{sL}, N_R, and S_R, as well as 3 singlet scalar fields χ_1^+, χ_2^+, and χ_2^0. There are also two scalar doublets (ϕ_1^+, ϕ_1^0) and (ϕ_2^+, ϕ_2^0), where only one is needed in the minimal standard

fermions	L-parity	$SU(2)_L \times U(1)_Y$	$U(1)'$
$(\nu_i, l_i)_L$	$-$	$(2, -1/2)$	0
l_{iR}	$-$	$(1, -1)$	0
ν_{sL}	$-$	(1,0)	1
N_R	$+$	(1,0)	1
S_R	$+$	(1,0)	0
scalars	**L-parity**	$SU(2)_L \times U(1)_Y$	$U(1)'$
$(\phi^+_{1,2}, \phi^0_{1,2})$	$+$	(2,1/2)	0
χ^+_1	$+$	(1,1)	0
χ^+_2	$+$	(1,1)	1
χ^0_2	$+$	(1,0)	1

Table 1: List of fermion and scalar fields in our model.

model. To obtain radiative masses for the three doublet neutrinos, just $(\phi^+_{1,2}, \phi^0_{1,2})$ and χ^+_1 are enough[9, 11]. The more difficult task is to include the singlet neutrino ν_{sL} into a 4×4 radiative mass matrix of the same form. A natural way that this may come about is to have an extra gauge symmetry $U(1)'$ for the fields ν_{sL}, χ^+_2, and χ^0_2 which is broken at a higher ($\sim$ TeV) scale. The axial-vector anomaly, generated by ν_{sL}, is cancelled by N_R which transforms as ν_{sL} under $U(1)'$. We also add S_R which is trivial under $U(1)'$. A large mass for N_R is then ensured through the Yukawa interaction $\bar{S}_R {N_R}^C \chi^0_2$ since $\langle \chi^0_2 \rangle >\sim 1$ TeV. The particle content of the model is summarized in Table 1.

We have an unbroken discrete Z_2 symmetry, namely L-parity, to distinguish between two classes of fermions. The leptons now have odd L-parity, replacing the usual additive lepton number. This allows the four neutrinos to acquire Majorana masses. However, tree-level neutrino masses are forbidden by the assumed particle content of our model, even after the spontaneous breaking of the gauge symmetry. Note that ν_s does not get a Majorana mass because of $U(1)'$; it also does not get a Dirac mass by pairing up with N_R or S_R because of L-parity. More specifically, consider the following interaction Lagrangian density of the fields shown in Table 1.

$$\begin{aligned}\mathcal{L}_{int} &= \sum_{i,j} f_{ij}(\nu_{iL}l_{jL} - l_{iL}\nu_{jL})\chi_1^+ + \sum_i f_i' \bar{\nu}_{sL} l_{iR}\chi_2^+ + \sum_i h_i(\bar{\nu}_{iL}\phi_1^+ + \bar{l}_{iL}\phi_1^0)l_{iR} \\ &+\mu(\phi_1^+\phi_2^0 - \phi_1^0\phi_2^+)\chi_1^- + \mu'\chi_1^+\chi_2^-\chi_2^0 + h' N_R S_R \chi_2^{0*} + h.c., \qquad (2)\end{aligned}$$

where we have used the notation $\psi_i\zeta_j = \overline{\psi_i{}^C}\zeta_j$ for two fermion fields ψ and ζ. Evidently, f_{ij} is antisymmetric in its generation indices. We have assumed in (2) that (ϕ_2^+, ϕ_2^0) do not couple to leptons. This is easily achieved by a separate discrete Z_2 symmetry which is explicitly broken, but only by soft terms such as $\phi_1^-\phi_2^+ + \phi_1^{0*}\phi_2^0 + h.c.$ in the Higgs potential, as in the minimal supersymmetric standard model, for example. As shown below, the above interactions induce a radiative neutrino mass matrix for ν_e, ν_μ, ν_τ, and ν_s of the form

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & a & b & d \\ a & 0 & c & e \\ b & c & 0 & f \\ d & e & f & 0 \end{bmatrix}, \qquad (3)$$

which generalizes the 3×3 matrix of the Zee model [9] by including a fourth row and column.

In Fig. 1 we show the one-loop diagram linking ν_i and ν_j which contributes to the corresponding entry in $\mathcal{M}_\nu$. This is of course identical to that of Ref. [9] and [11]. Note that $i \neq j$ necessarily, hence only off-diagonal entries can be nonzero. Since $h_i = m_{l_i}/\langle\phi_1^0\rangle$, we obtain

$$a = f_{e\mu}(m_\mu^2 - m_e^2)\left(\frac{\mu v_2}{v_1}\right) F(m_{\chi_1}^2, m_{\phi_1}^2), \qquad (4)$$

$$b = f_{e\tau}(m_\tau^2 - m_e^2)\left(\frac{\mu v_2}{v_1}\right) F(m_{\chi_1}^2, m_{\phi_1}^2), \qquad (5)$$

$$c = f_{\mu\tau}(m_\tau^2 - m_\mu^2)\left(\frac{\mu v_2}{v_1}\right) F(m_{\chi_1}^2, m_{\phi_1}^2), \qquad (6)$$

where $v_{1,2} \equiv \langle \phi^0_{1,2} \rangle$, and the function F is given by

$$F(m_1^2, m_2^2) = \frac{1}{16\pi^2} \frac{1}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2}. \tag{7}$$

In Fig. 2 we show the analogous one-loop diagram linking ν_i to ν_s. We find

$$d = (f_{e\tau} f'_\tau m_\tau + f_{e\mu} f'_\mu m_\mu) \left(\frac{\mu' u}{v_1} \right) F(m^2_{\chi_1}, m^2_{\chi_2}), \tag{8}$$

$$e = (f_{\mu\tau} f'_\tau m_\tau + f_{\mu e} f'_e m_e) \left(\frac{\mu' u}{v_1} \right) F(m^2_{\chi_1}, m^2_{\chi_2}), \tag{9}$$

$$f = (f_{\tau\mu} f'_\mu m_\mu + f_{\tau e} f'_e m_e) \left(\frac{\mu' u}{v_1} \right) F(m^2_{\chi_1}, m^2_{\chi_2}), \tag{10}$$

where $u \equiv \langle \chi^0_2 \rangle$. In the following, we will assume that $f'_e m_e$ is negligible. Moreover, while u is expected to be large compared to $v_{1,2}$, that can be compensated by m_{χ_2} being larger than m_{χ_1} or m_{ϕ_1}. Thus d, e, f need not be larger in magnitude than a, b, c. In any case, we have the important relationship

$$d = \frac{be}{c} \left(1 - \frac{m_\mu^2}{m_\tau^2} \right) + \frac{f f_{e\mu}}{f_{\tau\mu}}, \tag{11}$$

where m_e^2 in Eq. (5) has been neglected.

We make the same observation as in Refs. [9] and [11] that b and c are likely to be the dominant entries of $\mathcal{M}_\nu$ because they are proportional to m_τ^2. This means that ν_τ combines with a linear combination of ν_e and ν_μ to form a pseudo-Dirac particle. Let us also assume that $|f_{e\tau}| << |f_{\mu\tau}|$, so that $|b| << |c|$. Then the 2×2 submatrix spanning ν_e and ν_s is given by

$$\begin{aligned} \mathcal{M}_{\nu_e \nu_s} &= \begin{bmatrix} -2ab/c & d - be/c - af/c \\ d - be/c - af/c & -2ef/c \end{bmatrix} \\ &= \begin{bmatrix} -2ab/c & -f f_{e\mu}/f_{\mu\tau} - (be/c)(m_\mu^2/m_\tau^2) \\ -f f_{e\mu}/f_{\mu\tau} - (be/c)(m_\mu^2/m_\tau^2) & -2ef/c \end{bmatrix}, \end{aligned} \tag{12}$$

where we have used Eq. (11) and the fact that $|a/c| << |f_{e\mu}/f_{\mu\tau}|$. Hence

$$m_{\nu_e} \simeq -2\frac{ab}{c}, \quad m_{\nu_s} \simeq -2\frac{ef}{c}, \tag{13}$$

and for $m_{\nu_e} << m_{\nu_s}$, the $\nu_e - \nu_s$ mixing is $(cf_{e\mu}/ef_{\mu\tau} + bm_\mu^2/fm_\tau^2)/2$. This is assumed to be small, so as to satisfy the solar neutrino data. We now have

$$(\Delta m^2)_{\rm solar} \simeq 4\frac{e^2 f^2}{c^2}. \tag{14}$$

Since $\mathcal{M}_\nu$ has zero trace, it can easily be shown that the leading expressions for its eigenvalues are given by

$$-2\frac{ab}{c}, \quad c + \frac{ab}{c} + \frac{ef}{c}, \quad -c + \frac{ab}{c} + \frac{ef}{c}, \quad -2\frac{ef}{c}. \tag{15}$$

Hence the mass-squared difference between the two Majorana components of the pseudo-Dirac neutrino with mass c is

$$\Delta m^2 = 4(ab + ef) \simeq 4ef \simeq (\Delta m^2)_{\rm atm}. \tag{16}$$

Since this is for a $\nu_\mu - \nu_\tau$ mixing of 45°, we have taken it to explain the atmospheric neutrino data. Finally, the LSND data involve the mixing of ν_e and ν_μ, hence

$$(\Delta m^2)_{\rm LSND} = c^2, \tag{17}$$

with mixing given by b/c. Combining Eqs. (14), (16), and (17), we obtain Eq. (1), as claimed.

Current neutrino-oscillation data are consistent with $(\Delta m^2)_{\rm LSND} \sim 1$ eV^2 and $(\Delta m^2)_{\rm solar} \sim 6 \times 10^{-6}$ eV^2. In that case, $(\Delta m^2)_{\rm atm}$ is predicted by Eq. (1) to be about 5×10^{-3} eV^2, which is supported by the most recent data from Super-Kamiokande. In our model, ν_μ and ν_τ have the same mass $c \simeq 1$ eV and they mix maximally. Let $b \simeq 0.04$ eV, then the $\nu_\mu - \nu_e$ mixing parameter $(\sin^2 2\theta)_{\rm LSND}$ is $4b^2/c^2 \sim 6 \times 10^{-3}$, in good agreement with data. For $\nu_e - \nu_s$ oscillations, we let

$$\frac{f_{e\mu} c}{2 f_{\mu\tau} e} + \frac{b m_\mu^2}{2 f m_\tau^2} \simeq 0.04, \tag{18}$$

so that $(\sin^2 2\theta)_{\rm solar}$ is also about 6×10^{-3}, again in good agreement with data. More specifically, we can let $e \simeq 0.12$ eV and $f \simeq 0.01$ eV, then $m_{\nu_s} \sim 2ef/c \simeq 2.4 \times 10^{-3}$ eV.

Furthermore from Eq. (18), $f_{e\mu}/f_{\mu\tau}$ is now about 0.008 and from Eqs. (4) and (6), $a \sim 3\times10^{-5}$ eV, hence $m_{\nu_e} \sim 2ab/c \sim 2\times10^{-6}$ eV, justifying our assumption that $m_{\nu_e} << m_{\nu_s}$. We have thus a completely successful phenomenological picture of neutrino oscillations.

The model of Ref.[11] differs from ours in that ν_s is assumed there to acquire a tree-level mass which is just slightly bigger than the radiative mass of ν_e. [This is of course rather *ad hoc*, but it is necessary to satisfy solar data.] Let us compare its consequences with those of ours. In the former, the parameter a is forced to be large in magnitude because $4ab$ is identified there with $(\Delta m^2)_{\rm atm}$, resulting in $|f_{e\mu}| \sim |f_{\mu\tau}|$. This condition is subject to severe phenomenological constraints because $f_{e\mu}$ contributes to μ decay. In fact, in that scenario, $f^2_{e\mu} \sim f^2_{\mu\tau} < 7\times10^{-4}G_F.\ (\cos^2\phi M_1^{-2} + \sin^2\phi M_2^{-2})^{-1}$ where $M_{1,2}$ are the physical charged Higgs masses and ϕ is their mixing angle. In our model, because of Eq. (16), a can be and is very small, hence $|f_{e\mu}| << |f_{\mu\tau}|$, so that our $|f_{\mu\tau}|$ is not constrained to be small.

We note also that the form of Eq. (3) for the neutrino mass matrix with c as the dominant entry is not sufficient by itself to have the correct $\nu_e-\nu_s$ submatrix needed to explain the solar data. Without Eq. (11), which is an <u>automatic</u> consequence of our model, that submatrix would have dominant off-diagonal terms, *i.e.*

$$\mathcal{M}_{\nu_e\nu_s} \sim \begin{pmatrix} 0 & d \\ d & 0 \end{pmatrix}, \tag{19}$$

which would make ν_e and ν_s pseudo-Dirac partners with the requisite mixing of 45° in conflict with solar neutrino data.

A third point concerns the fermion singlets N_R and S_R. They have even L-parity, which is unbroken in our model, hence they do not mix into the lepton sector. Both of them are massive, because the terms S_RS_R and $N_RS_R\chi_2^{0*} + h.c.$ are allowed in the Lagrangian density. The scale of $U(1)'$-breaking, i.e. $\langle\chi_2^0\rangle$ can be taken beyond 1 TeV, thereby pushing up these masses. It is to be noted that the off-diagonal terms in the $Z-Z'$ mass matrix are prohibited due to the absence of appropriate Higgs fields in the present model. We also assume that the kinetic mixing between the $U(1)_Y$ and $U(1)'$ gauge bosons is negligible. Hence our Z' couples at the tree level only to ν_{sL}, N_R, χ_2^+ and χ_2^0. Thus present experimental bounds [14] on a possible Z' with standard-model-like couplings do not apply. However, because ν_s mixes with ν_e radiatively, Z' develops a small coupling to ν_e. To avoid any possible conflict with nucleosynthesis or current electroweak phenomenology, we assume $M_Z \sim 1$ TeV or greater, which is of course natural since we already assumed $\langle \chi_2^0 \rangle \sim 1$ TeV or greater.

The charged scalar χ_1^+ contributes to the standard-model effective charged-current interaction due to the presence of the $f_{ij}(\nu_{iL} l_{jL} - l_{iL} \nu_{jL})\chi_1^+$ term in the Lagrangian density. The corresponding effects on processes, such as electron-neutrino scattering, are experimentally severely constrained. They give rise to the constraint $f_{e\mu}^2/M^2 < 0.036 G_F$ [11], where M is the mass of the charged scalar mediating the process. Since we have $|f_{e\mu}| << |f_{\mu\tau}|$, this is no problem for us. The proposed hierarchical relation $|f_{e\mu}| << |f_{\mu\tau}|$ is also consistent with the constraint from the branching ratio of the decay $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ being $(17.35 \pm 0.10)\%$ [14] since the latter only requires $f_{\mu\tau}^2/M^2 < 0.13\ G_F$.

In summary, we have demonstrated that the present results of solar, atmospheric as well as LSND experiments can be explained with three electroweak-active neutrinos and a sterile one with a minimal extension of the standard $SU(2)_L \times U(1)_Y$ electroweak gauge model. The extra $U(1)'$ gauge and Z_2 discrete symmetries are needed to avoid tree-level Majorana or Dirac mass terms. All neutrino masses are radiatively generated in one loop by an extension of the Zee model. Our proposal results in an interesting relationship $(\Delta m^2)_{\rm atm} \simeq 2[(\Delta m^2)_{\rm solar}(\Delta m^2)_{\rm LSND}]^{1/2}$ which is well satisfied by the present experimental data and will be critically tested with more accurate data forthcoming in the near future.

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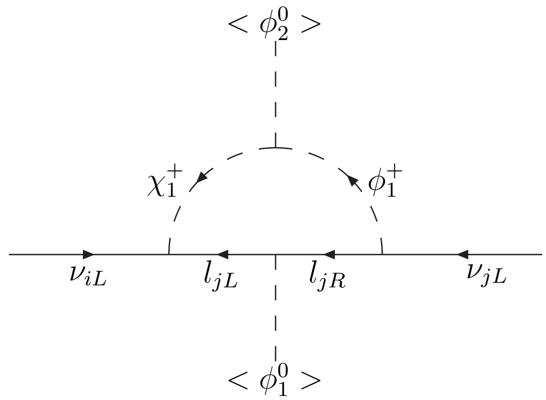

FIG. 1. One loop radiative $\nu_i - \nu_j$ $(i, j = e, \mu, \tau)$ mass due to charged Higgs exchange.

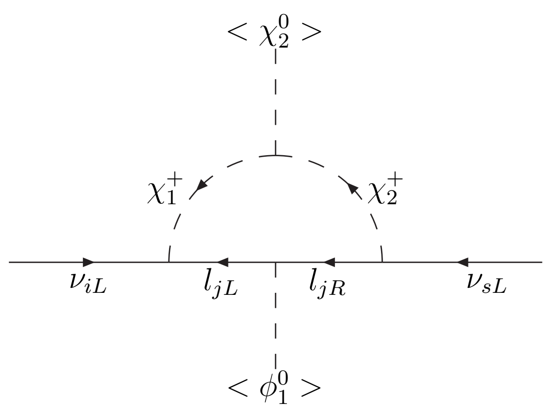

FIG. 2. One loop radiative $\nu_i - \nu_s$ $(i = e, \mu, \tau)$ mass due to charged Higgs exchange.