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Infrared fixed point of the top Yukawa coupling in split supersymmetry

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Abstract

The severe constraints imposed on the parameter space of the minimal split supersymmetry model by the infrared fixed point solution of the top Yukawa coupling Y_t are studied in detail in terms of the value of the top quark mass measured at the Tevatron together with the lower bound on the lightest Higgs mass established by LEP. The dependence of the higgsino mass parameter μ, the gaugino coupling strengths $\tilde{g}_{u,d}$, $\tilde{g}'_{u,d}$ and of the Higgs quartic self coupling λ on the value of Y_t in the vicinity of the Landau pole is discussed. A few interesting features emerge, though the model is found to be disfavored within the infrared fixed point scenario because of the need to have several unnatural cancellations at work on account of the requirement of a low upper bound on $\tan\beta$.

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I. INTRODUCTION

The naturalness criterion has been one of the guiding principles in the formulation of the (Minimal) Supersymmetric Standard Model (M)SSM. Once this is accepted, a successful implementation of high scale gauge coupling unification obtains and the Lightest Supersymmetric Particle (LSP) emerges as a viable dark matter candidate. But, in view of the failure of the above criterion in dealing with the cosmological constant and in the light of the recently advanced landscape paradigm, an important question arises. Can one abandon the principle of naturalness, admit fine tuning and yet maintain the nice phenomenological aspects of the SSM at the same time? It has been emphasized [1, 2] that the successful unification of gauge couplings of the SSM can be retained even when all the scalars of the theory, except one finetuned light Higgs boson (akin to that in the Standard Model) lie far above the electroweak scale. Thus, despite the loss of the original motivation to cure the hierarchy problem, one can still have a supersymmetric theory with gauge coupling unification, which is free of many of the undesirable features of the SSM such as the flavor problem, fast proton decay via dimension five operators, generically large CP violation, a tightly constrained mass of the lightest Higgs etc. The gauginos and higgsinos of this theory are chosen to lie near the TeV scale to ensure gauge coupling unification at $\mathrm{M_{GUT}} \sim 10^{16}$ GeV as well as a stable LSP in the desirable mass range. This is the scenario of split supersymmetry, as named in Ref. [2].

Various theoretical and phenomenological aspects, characteristic of the above scenario, have been discussed in several recent works [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. One can identify a minimal split supersymmetry model described by six specific parameters : (1) a common mass $\widetilde{m}$ for the heavy scalars, (2) $\tan\beta$, where the angle β defines the combination of neutral $\mathrm{SU(2)_L}$-doublet Higgs fields which remains light, (3) the higgsino mass parameter $\mu(M_{GUT})$ at the GUT scale, (4) the gluino mass $m_{\tilde{g}}$, (5) the grand unification scale $\mathrm{M_{GUT}}$, and (6) the unified value of the gauge coupling strength α_G at $\mathrm{M_{GUT}}$. However, the last two are more or less fixed by the requirement of consistency with measurements of the three gauge coupling strengths at laboratory energies. It is thus convenient to discuss different phenomenological constraints in the space of the first four parameters. It has been already realized [2] that certain special constraints would ensue (on the parameter space of the minimal split SUSY

model, in particular) on account of the Landau pole [33, 34] in the top quark Yukawa coupling Y_t and the LEP lower bound on the mass of the Standard Model Higgs. However, a careful quantitative study of those, including the interrelation between the last mentioned two aspects, has been lacking and that is the aim of the present work.

We broadly embrace the philosophy of Refs.[2] and [5] in this paper. Our gluino and electroweak gaugino as well as higgsinos are envisioned to lie in the range of hundreds of GeV whereas $\widetilde{m}$ is taken to be much above 10 TeV and most likely around 10^9 GeV. Indeed we vary $\widetilde{m}$ all the way upto 10^{13} GeV beyond which scale one might encounter anomalously heavy isotopes [2]. We follow the RGE equations set up in Ref.[2] and numerically study the parameters of the minimal split SUSY model as $\widetilde{m}$ is varied with Y_t kept at its fixed point value or in its vicinity. Since the higgsino mass parameter $\mu(M_Z)$ and the gaugino couplings are sensitive to values of Y_t in this region, we study them as functions of the top mass m_t with $\widetilde{m}$ fixed. In Section II we first review the physics of the infrared fixed point of Y_t in MSSM and then extend the discussion to split supersymmetry. In Section III we consider the implications of this scenario for the Higss mass M_h, the higgsino mass parameter μ as well as the gaugino coupling strengths. Section IV contains our conclusion and the RGEs are relegated to the Appendix.

II. INFRARED FIXED POINT OF Y_t

Let us first review the fixed point behaviour [33] of the top Yukawa coupling in MSSM. In the low to moderate $\tan\beta$ region, the effects of the bottom and tau Yukawa coupling strengths can be ignored. With this approximation and, given gauge coupling unification at $\mathrm{M_{GUT}}$, one obtains a simple analytic relation [34, 35, 36, 37, 38, 39] at the one-loop level :

$$Y_t(t) = \frac{Y_t(0)E(t)}{1 + 6F(t)Y_t(0)}. \tag{1}$$

In Eqn. (1), $t = 2\ \ln(M_{GUT}/Q)$, $Y_t = \lambda_t^2/(4\pi)^2$, λ_t is the top Yukawa coupling strength in the Lagrangian, Q is the running scale variable, E and F are functions of the gauge couplings:

$$E(t) = (1+\beta_3 t)^{16/3b_3}(1+\beta_2 t)^{3/b_2}(1+\beta_1 t)^{13/9b_1}, \qquad F(t) = \int_0^t E(t')dt'. \tag{2}$$

The parameters $\beta_i\,(i = 1, 2, 3)$ in Eqn.(2) equal $b_i\alpha_G/(4\pi)$, where $(b_1, b_2, b_3) = (33/5,\ 1,\ -3)$ are the coefficients of the one-loop gauge β-function and $\alpha_G = \alpha_i(0)$ with the normalization

$\alpha_1 = \frac{5}{3}\alpha_Y$ for the hypercharge coupling. Eqn.(1) implies that a large value (~ 3.5) at the GUT scale of the top Yukawa coupling λ_t in the Lagrangian corresponds to an infrared quasi-fixed point value of Y_t^f :

$$Y_t^f(t) = E(t)/6F(t). \tag{3}$$

The situation is somewhat different in split supersymmetry. Here all sfermions and the charged as well as the heavier CP even plus the CP odd Higgs bosons are very heavy and, as a first approximation, are taken to be degenerate[1]. Coupling strengths in the split theory at the scale $\widetilde{m}$ are obtained by matching its Lagrangian with that of the full MSSM valid at higher scales. In particular, the couplings of the light Higgs h in the split effective theory follow from matching conditions with the interaction terms of the Higgs doublet fields H_u and H_d in the full MSSM. Suppose we denote the top Yukawa coupling strength in the Lagrangian of the effective theory as h_t. If λ_t represents the coupling strength of the Yukawa interaction of the top with H_u in the full MSSM above $\widetilde{m}$, then we have [2]

$$h_t(\widetilde{m}) = \lambda_t^*(\widetilde{m}) \sin\beta, \tag{4}$$

The evolution of λ_t at scales greater than $\widetilde{m}$ is given at the one-loop level by Eqn. (1). However, below the scale $\widetilde{m}$, h_t evolves according to Eqn.(A.13) given in the Appendix with the matching condition of Eqn.(4). With this evolution also, an infrared fixed point is observed for $h_t = h_t^f$. Though an analytic expression for h_t^f becomes complicated, this striking behaviour can be seen numerically. The corresponding top quark pole mass is then given by [41]

$$M_t^{pole} = h_t^f(M_Z) v \left[1 + \frac{4\alpha_3(M_Z)}{3\pi} - 2Y_t'^f(M_Z)\right], \tag{5}$$

v being ≈ 246 GeV and $Y_t'^f = \left(h_t^f\right)^2/(4\pi)^2$. In our numerical calculations we have also taken into consideration the effects of bottom and tau Yukawa couplings.

In split supersymmetry $\tan\beta$ enters as an input parameter into the top mass via Eqn.(4). The experimental upper (lower) limit on the top mass then translates to an upper (lower) limit on $\tan\beta$. This feature is demonstrated in Fig. 1 for three values of $\widetilde{m}$, namely 10^4 GeV, 10^9 GeV and 10^{11} GeV. We have calculated the results numerically upto $\tan\beta = 40$

[1] Non-universal scalar masses in the split supersymmetry scenario have been considered [40].

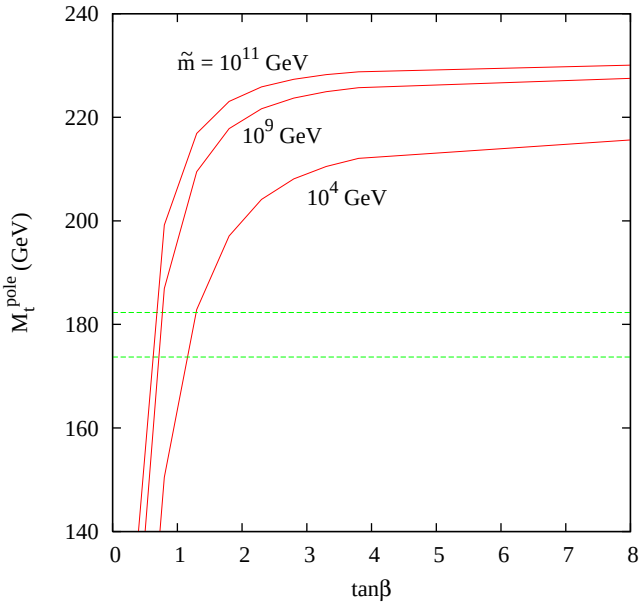

FIG. 1: Top pole mass at the infrared fixed point value as a function of $\tan\beta$ for three different values of $\widetilde{m}$ with the 1-σ band of the Tevatron measurement also shown.

but plotted them only in the small $\tan\beta$ region which is the most interesting part to look for in this context. The values of μ and $M_{1/2}$ at the GUT scale have been taken here to –600 GeV and 300 GeV, respectively. The 1σ error in M_t^{pole}, as currently quoted in the PDG listing [42]

$$M_t^{pole} = 178.0 \pm 4.3 \text{ GeV}, \tag{6}$$

in combination with its infrared fixed point value, puts bounds on $\tan\beta$ defined at the scale $\widetilde{m}$. An interesting new feature, different from what happens in the MSSM, is that $\tan\beta$ can now be lower than unity for large values of $\widetilde{m}$. However, the most important point is that the fixed point value of the top mass is now consistent with only a thin sliver of an allowed region in the $\tan\beta - \widetilde{m}$ plane, as shown in Fig.2. On the other hand, if we do not stick to the fixed point scenario, this severe restriction weakens considerably though a lower bound on $\tan\beta$ continues to exist and is correlated to the lower limit on M_t^{pole}.

The value of $\tan\beta$ in models of split supersymmetry depends upon [1, 30] what one assumes for the strength of the B-parameter, but it is generally difficult to keep $\tan\beta$ small. If $|B|$ is of the order of the EW symmetry breaking scale m_{EW} then $\tan\beta \sim \widetilde{m}^2/m_{EW}^2 > 100$ for $\widetilde{m}/m_{EW} > 10$, violating the upper bound $\lesssim 100$ on $\tan\beta$ coming from the need to keep the bottom Yukawa coupling strength perturbative, i.e. $\lesssim \mathcal{O}(1)$. On the other hand, in

usual gravity-, gauge- or anomaly-mediated supersymmetry breaking, it is possible to have $|B|$ of the order of $\widetilde{m}$. In this case, one has $\tan\beta \sim \widetilde{m}/m_{EW}$ which allows somewhat larger splitting in the spectrum while keeping the value of $\tan\beta$ within the above-mentioned upper limit. However, it is still not sufficient to ensure that $\tan\beta$ remains within the allowed region of Fig. 2. We have just seen that in the infrared fixed point scenario in split supersymmetry the upper bound on $\tan\beta$ (as a function of $\widetilde{m}$) is very strong ($\tan\beta \lesssim 1$ for large values of $\widetilde{m}$). Thus, combining this observation with the above argument one can perhaps conclude that the infrared fixed point scenario is strongly disfavored in split supersymmetry in the context of gravity-, gauge- or anomaly-mediated supersymmetry breaking (with $|B| \sim \widetilde{m}$ or in the case when $|B| \sim m_{EW}$). In other words, if $\tan\beta$ is experimentally measured to be $\lesssim 1$ with a sparticle spectrum that contains physical charginos and neutralinos but with the scalars (except for one light Higgs) being out of the LHC energy reach, the infrared fixed point scenario can probably be retained but either at the cost of several unnatural cancellations having to work together [30] or having a direct mediation mechanism with D-term supersymmetry breaking ($|B| \gg \widetilde{m}$ and[2] $|\mu| \ll \widetilde{m}$) which introduces additional heavy matter fields or a new scale in the theory [5, 29].

III. IMPLICATIONS OF FIXED POINT FOR OTHER MASSES AND COUPLINGS

Let us now study how the light Higgs mass M_h changes with $\tan\beta$ when the top mass is at its fixed point value. As in the Standard Model, M_h in split supersymmetry can be written as

$$M_h = \sqrt{\lambda} v, \tag{7}$$

where λ is the strength of the quartic self-coupling of h, and v is as in Eqn.(5). The matching condition for the coupling λ at the scale $\widetilde{m}$ is

$$\lambda(\widetilde{m}) = \frac{[g^2(\widetilde{m}) + g'^2(\widetilde{m})]}{4}\cos^2 2\beta, \tag{8}$$

where g and g' are the respective $SU(2)_L$ and $U(1)_Y$ coupling strengths with $\alpha_1 = 5g'^2/(12\pi)$. The evolution of λ is governed by Eqn.(A.24) of the Appendix. The mass M_h also constrains

[2] Recall that the Higgs mass mixing term $B\mu$ needs to be of the same order as $\widetilde{m}^2$.

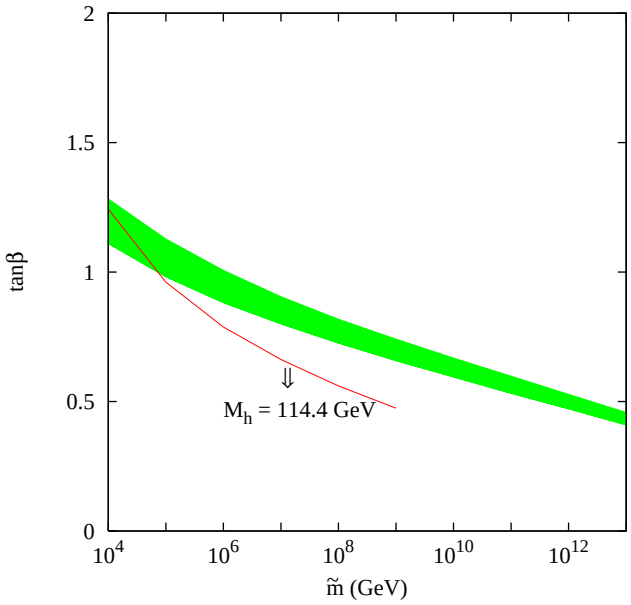

FIG. 2: Allowed region (coloured) in the $\widetilde{m}$ – $\tan\beta$ plane in the infrared fixed point scenario from the experimental limits on the top mass. The area below the solid line is disallowed by the LEP–2 lower limit [43] of $M_h > 114.4$ GeV. $M_{1/2}$ and μ at the GUT scale have been chosen at 300 GeV and –600 GeV respectively.

$\tan\beta$ as a function of $\widetilde{m}$, cf. Fig. 2.

It is also interesting to note how the quartic coupling $\lambda(M_Z)$ changes with the top mass near the fixed point value. In Fig. 3 we have shown this variation for a fixed $\tan\beta$ and $\widetilde{m}$ and for two values of the common gaugino mass $M_{1/2}$. In both the cases the fixed point value of the top mass is within the 1σ limit given in Eqn. 6. We can see from this figure that $\lambda(M_Z)$ shows some variation with the top mass near the fixed point. Accurate knowledge of chargino and neutralino masses (which will determine $M_{1/2}$) and of the top mass will enable one to obtain a precise value of $\lambda(M_Z)$ and then one can calculate the value of $\lambda(\widetilde{m})$ using the split susy RGE and verify the prediction given in Eqn.8. This figure is plotted for a fixed value of $\mu(M_{GUT})$ = -800 GeV but we have checked that the variation of $\lambda(M_Z)$ with M_t does not have any significant dependence on $\mu(M_{GUT})$ by varying the latter between -800 GeV and +800 GeV[3].

[3] In split supersymmetry, the neutralino and chargino masses (and hence $|\mu(M_Z)|$) cannot be much higher than $\mathcal{O}$(TeV). The latter requirement, together with the extremely small region of $\tan\beta$, i.e. $0.5 < \tan\beta < 1.3$ (cf. Fig.2), allowed in the infrared fixed point scenario, means that here a $|\mu(M_{\rm GUT})|$, much larger than $\mathcal{O}$(TeV), is disallowed since it will not be able to run down to an acceptable value of $|\mu(M_Z)|$.

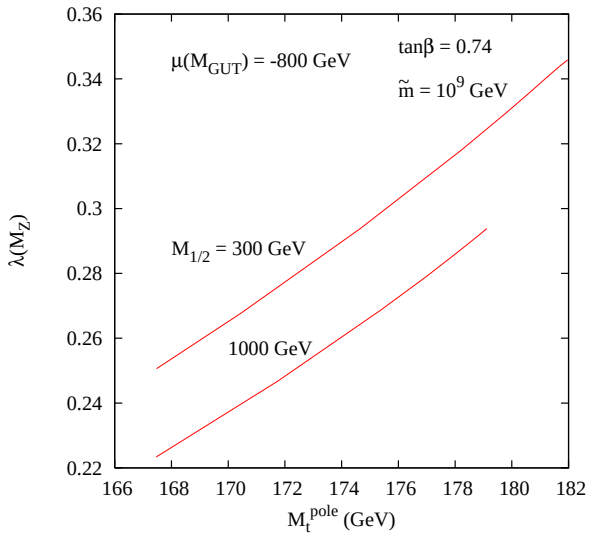

FIG. 3: Variation of the quartic coupling $\lambda(M_Z)$ near the top-quark fixed point for two values of $M_{1/2}$. Here, $\widetilde{m} = 10^9$ GeV and $\tan\beta$= 0.74. The value of $\mu(M_{GUT})$ is taken to be -800 GeV.

The fixed point behaviour of the top Yukawa coupling depends also on gauge coupling strengths. The unified coupling strength α_G and the grand unifying scale M_{GUT} are plotted in Fig.4 as functions of $\widetilde{m}$. In this figure α_G and M_{GUT} are shown to decrease with increasing $\widetilde{m}$. The effect of varying $\tan\beta$ in the allowed range of Fig.2 has been found to be negligible. The decrease is due to the fact that the effective particle content in split supersymmetry is smaller than in the MSSM; thus as $\widetilde{m}$ becomes larger, the running with split SUSY RG equations becomes longer and the coupling constants meet at a smaller scale with a smaller unified value. This feature has also been noticed in Ref.[2]. The values of α_2 and α_1 at the electroweak scale are ~ 0.0335 and 0.0168, respectively. An important point is that M_{GUT}, decreasing with $\widetilde{m}$, poses no threat to the longevity of the proton here since, as pointed out in Ref.[2], dimension five and six operators – relevant to proton decay – continue to remain suppressed. We have also considered the variation of the QCD coupling $\alpha_s(M_Z)$ with $\widetilde{m}$ with a result not very different from that of Ref.[2].

Consider now how other parameters, such as $\mu(M_Z)$ and gaugino coupling strengths vary with M_t in the neighborhood of the fixed point value. Fig. 5 shows precisely such a variation in $\mu(M_Z)$, plotted vs. M_t^{pole}, for various choices of $\mu(M_{\rm GUT})$ and $\widetilde{m}$= 10^9 GeV. The common gaugino mass at the GUT scale has been taken to be 300 GeV and $\tan\beta = 0.74$. Running with RGE's brings $\mu(M_{\rm GUT})$ to $\mu(M_Z)$. Evident from the figure is the fact that for this choice of $\tan\beta = 0.74$, the fixed point value of the top pole mass ($\sim$ 182 GeV) is within

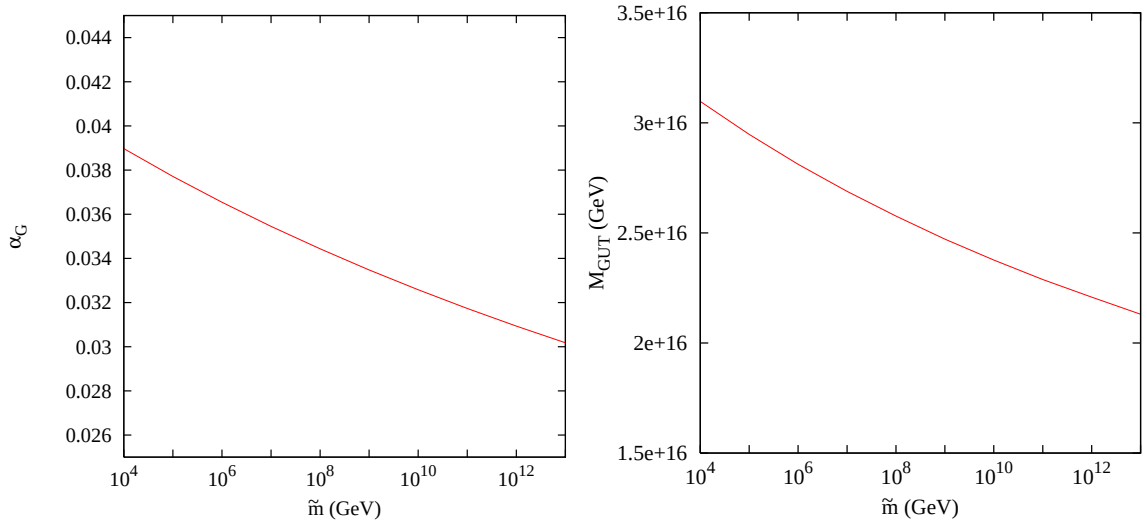

FIG. 4: Variations of α_G and $M_{\rm GUT}$ as functions of $\widetilde{m}$. Varying $\tan\beta$ in the allowed range of Fig.2 does not affect the curves.

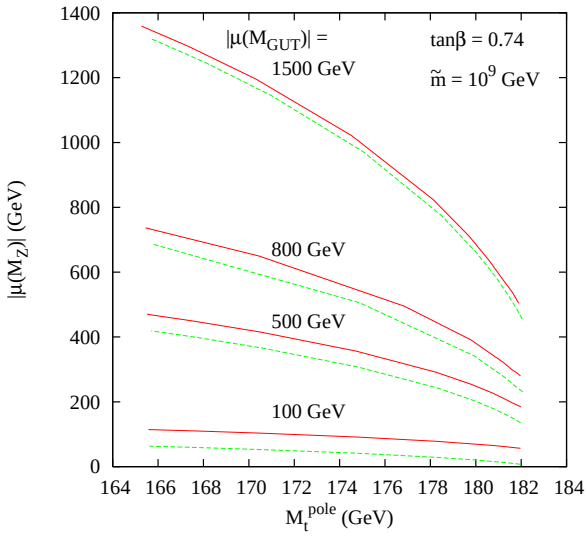

FIG. 5: Variation of $|\mu(M_Z)|$ near the top-quark fixed point for different values of $|\mu(M_{GUT})|$. Here, $\widetilde{m} = 10^9$ GeV and $\tan\beta$= 0.74 . The common gaugino mass at the GUT scale ($M_{1/2}$) is taken to be 300 GeV. Solid/Red lines correspond to negative μ and the dashed/green lines correspond to positive $\mu(M_{GUT})$.

the 1σ experimental band and we should look into the variation of $\mu(M_Z)$ in this region of the parameter space. We can see that near the Landau pole the change in $|\mu(M_Z)|$ is sharp for larger values[3] of $|\mu(M_{GUT})|$, less so when the latter is closer to the EW scale. The value of $\mu(M_Z)$ can be determined (possibly along with $\tan\beta$) from the measurements of

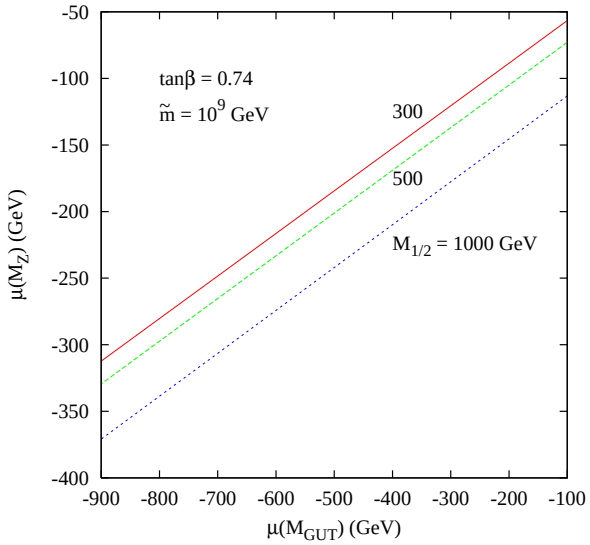

FIG. 6: Variation of $\mu(M_Z)$ near the top-quark fixed point for different values of common gaugino mass as a function of $\mu(M_{GUT})$. Here, $\widetilde{m} = 10^9$ GeV and $\tan\beta$= 0.74.

neutralino and chargino masses [44] at lepton colliders. Hence, with a precise measurement of the top mass and with the measured value of $|\mu(M_Z)|$ and $\tan\beta$, one can predict the value of $\mu(M_{\rm GUT})$ from the above plots for a given $\widetilde{m}$. Of course, it is true that this figure is drawn for a particular value of the common gaugino mass. In order to get some idea of the dependence of $\mu(M_Z)$ on the gaugino mass we have shown in Fig.6 the variation in $\mu(M_Z)$ as a function of $\mu(M_{GUT})$ at the fixed point for three different values of $M_{1/2}$ and for the same choice of $\tan\beta$ and $\widetilde{m}$ as in Fig.5. We have also checked that the gaugino mass parameters M_2 and M_1 show little variation as functions of top mass near the fixed point which we do not show here.

Another important split SUSY prediction is the inequality of the gauge and gaugino coupling strengths below the scale $\widetilde{m}$. This effect is large on account of the ultraheaviness of the sfermions and can be detected in collider experiments involving gaugino production. The part of the Lagrangian, containing the gaugino couplings, can be written in the notation of Ref. [2]

$$\mathcal{L}_{gaugino-int.} = \frac{h^\dagger}{\sqrt{2}}(\tilde{g}_u\sigma^a\tilde{W}^a + \tilde{g}'_u\tilde{B})\tilde{H}_u + \frac{h^T\epsilon}{\sqrt{2}}(-\tilde{g}_d\sigma^a\tilde{W}^a + \tilde{g}'_d\tilde{B})\tilde{H}_d + h.c. \qquad (9)$$

Here $\tilde{H}_{u,d}$ are the 'up,down type' higgsino fields, $\tilde{W}$ and $\tilde{B}$ are the Wino and the Bino respectively, h is the Higgs field and $\epsilon = i\sigma_2$. The boundary conditions of the gaugino

couplings at $\widetilde{m}$ are as follows :

$$\tilde{g}_u(\widetilde{m}) = g(\widetilde{m})\ \sin\beta, \quad \tilde{g}_d(\widetilde{m}) = g(\widetilde{m})\ \cos\beta \tag{10}$$

$$\tilde{g}'_u(\widetilde{m}) = g'(\widetilde{m})\ \sin\beta, \quad \tilde{g}'_d(\widetilde{m}) = g'(\widetilde{m})\ \cos\beta. \tag{11}$$

These couplings are then evolved to the electroweak scale using the renormalization group equations given in the Appendix. It is interesting to see the behaviour of these couplings near the infrared fixed point of the top mass. Following Ref.[7], one can define 'anomalous' gaugino couplings $\kappa_{u,d}$, $\kappa'_{u,d}$ by the following equations,

$$\kappa_u = 1 - \frac{\tilde{g}_u}{g\ \sin\beta}, \quad \kappa_d = 1 - \frac{\tilde{g}_d}{g\ \cos\beta}, \tag{12}$$

$$\kappa'_u = 1 - \frac{\tilde{g}'_u}{g'\ \sin\beta}, \quad \kappa'_d = 1 - \frac{\tilde{g}'_d}{g'\ \cos\beta}. \tag{13}$$

The behaviour of these anomalous gaugino couplings near the infrared fixed point top mass is shown in Fig.7. Measurements of gaugino couplings $\tilde{g}$ and gauge couplings g lead to the determination of $\widetilde{m}$, if $\tan\beta$ is known: according to Eqs. (10) and (11), the couplings $\kappa_{u,d}$ and $\kappa^{'}_{u,d}$vanish at the scale $\widetilde{m}$.

IV. CONCLUSION

In this paper we have studied the infra-red fixed point behaviour of the top Yukawa coupling and its associated phenomenology in split supersymmetry. In the fixed point scenario we find that only a thin band of the $\tan\beta - \widetilde{m}$ plane is allowed. This is a combined effect of the experimental limits in the measurement of the top mass and the position of the Landau pole. This observation makes the infrared fixed point scenario heavily disfavored in the context of split supersymmetry, since it requires additional unnatural cancellation of parameters (in usual gauge, gravity or anomaly mediated supersymmetry breaking) in order to keep $\tan\beta$ within the allowed limits. One should, however, note that such smaller values of $\tan\beta$ can possibly be obtained in the context of direct mediation of supersymmetry breaking with D-terms. Even if one does not assume the exact fixed point value for the top mass, there is still a lower limit on the parameter $\tan\beta$ as a function of $\widetilde{m}$, which can be less than unity for large values of $\widetilde{m}$. The LEP constraint that the Higgs must be heavier

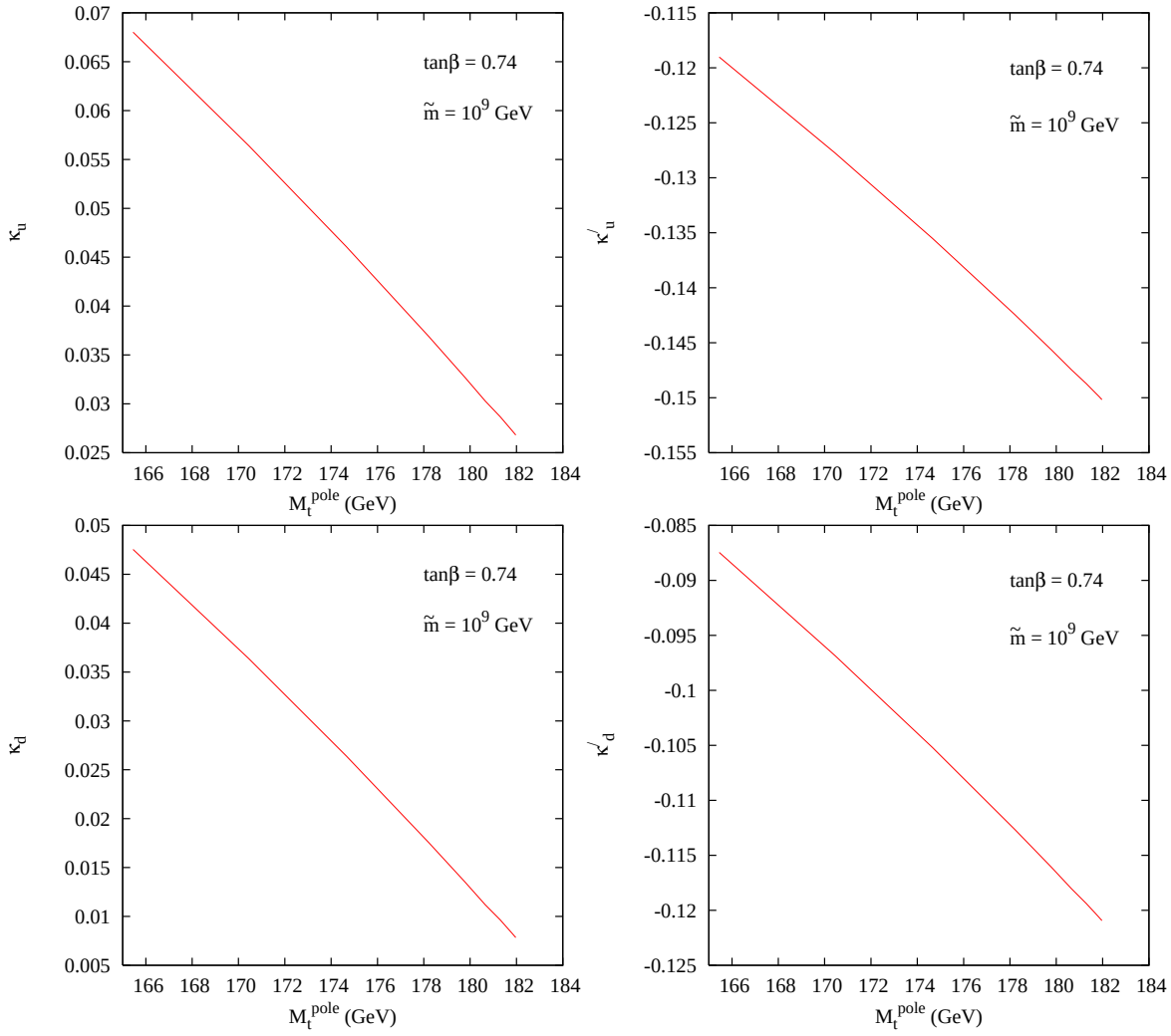

FIG. 7: Variation of the gaugino couplings (defined in Eqn. 12 and 13) near the top-quark fixed point for $\tan\beta = 0.74$ for which the fixed point value of $M_t^{pole} = 181.9$ GeV. Here, $\tilde{m} = 10^9$ GeV.

than 114.4 GeV puts additional restriction on minimal split SUSY parameters. We have studied various couplings as well as the value of the grand unifying scale in this scenario and, in particular, have drawn attention to the very interesting behaviour of the higgsino mass parameter $\mu(M_Z)$ near the the fixed point. We have also discussed the variations in the gaugino coupling strengths $\tilde{g}_{u,d}$, $\tilde{g}'_{u,d}$ and of the Higgs quartic self coupling λ, near the fixed point.

Note added in Proof: After this work was submitted, we saw a paper by Delgado and Giudice (hep-ph/0506217) which claims to have excluded the top-mass fixed point solution in split supersymmetry incorporated within an SU(5) GUT by assuming the corresponding boundary conditions for the soft scalar masses and by requiring the absence of charge and

color violating minima.

Acknowledgments

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APPENDIX

In this appendix we have written down the renormalization group equations for split supersymmetry which are taken from Ref. [2] but with the notations we have used in our numerical calculations.

Evolution between $M_{\rm GUT}$ and $\widetilde{m}$

The 2-loop renormalization group equations for the gauge couplings are given by

$$\frac{d\tilde{\alpha}_i}{dt} \;=\; -b_i\tilde{\alpha}_i^2 - \tilde{\alpha}_i^2[\sum_{j=1}^{3} B_{ij}\tilde{\alpha}_j - (d_i^t Y_t + d_i^b Y_b + d_i^\tau Y_\tau)], \tag{A.1}$$

where $t = 2\ln\frac{M_{\rm GUT}}{Q}$ and Q is the renormalization scale. $\tilde{\alpha}_i = \left(\frac{g_i}{4\pi}\right)^2$, $\mathrm{Y}_{t,b,\tau} = \left(\frac{\lambda_{t,b,\tau}}{4\pi}\right)^2$. We have used the GUT normalization condition $g_1^2 = (5/3)g'^2$. The β-function coefficients are given by

$$b \;=\; \left(\frac{33}{5}, 1, -3\right), \;\; B = \begin{pmatrix} \frac{199}{24} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} \tag{A.2}$$

$$d^t = \left(\frac{26}{5}, 6, 4\right), \;\; d^b = \left(\frac{14}{5}, 6, 4\right), \;\; d^\tau = \left(\frac{18}{5}, 2, 0\right) \tag{A.3}$$

The equations for the Yukawa couplings at the one loop level are given by

$$\frac{dY_t}{dt} \;=\; Y_t\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{15}\tilde{\alpha}_1\right) - 6Y_t^2 - Y_tY_b \tag{A.4}$$

$$\frac{dY_b}{dt} \;=\; Y_b\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{15}\tilde{\alpha}_1\right) - 6Y_b^2 - Y_tY_b - Y_bY_\tau \tag{A.5}$$

$$\frac{dY_\tau}{dt} \;=\; Y_\tau\left(3\tilde{\alpha}_2+\frac{9}{5}\tilde{\alpha}_1\right)-4Y_\tau^2-3Y_\tau Y_b \tag{A.6}$$

At the one loop level the equations for the gaugino masses and μ are given by

$$\frac{dM_i}{dt} \;=\; -b_i\tilde{\alpha}_i M_i \tag{A.7}$$

$$\frac{d\mu}{dt} \;=\; \left[\frac{3}{2}\tilde{\alpha}_2+\frac{3}{10}\tilde{\alpha}_1-\frac{3}{2}Y_t-\frac{3}{2}Y_b-\frac{1}{2}Y_\tau\right]\mu \tag{A.8}$$

Evolution between $\widetilde{m}$ and max (m_t, $M_{\tilde{\chi}_1^0}$)

Now,

$$\frac{d\tilde{\alpha}_i}{dt} \;=\; -b_i\tilde{\alpha}_i^2-\tilde{\alpha}_i^2[\sum_{j=1}^{3}B_{ij}\tilde{\alpha}_j-\{d_i^tY_t'+d_i^bY_b'+d_i^\tau Y_\tau'-d_i^W(\tilde{Y}_u+\tilde{Y}_d)-d_i^B(\tilde{Y}_u'+\tilde{Y}_d')\}], \tag{A.9}$$

where

$$d^W=\left(\frac{9}{20},\frac{11}{4},0\right),\;\; d^B=\left(\frac{3}{20},\frac{1}{4},0\right), \tag{A.10}$$

and

$$b=\left(\frac{9}{2},-\frac{7}{6},-5\right),\;\; B=\begin{pmatrix}\frac{104}{25} & \frac{18}{5} & \frac{44}{5}\\ \frac{6}{5} & \frac{106}{3} & 12\\ \frac{11}{10} & \frac{9}{2} & 22\end{pmatrix} \tag{A.11}$$

$$d^t=\left(\frac{17}{10},\frac{3}{2},2\right),\;\; d^b=\left(\frac{1}{2},\frac{3}{2},2\right),\;\; d^\tau=\left(\frac{3}{2},\frac{1}{2},0\right), \tag{A.12}$$

$\tilde{\mathrm{Y}}_u,\ \tilde{\mathrm{Y}}_d,\ \tilde{\mathrm{Y}}_u',\ \tilde{\mathrm{Y}}_d'$ are defined generically as $\tilde{\mathrm{Y}}=\frac{\tilde{g}^2}{(4\pi)^2}$ where the gaugino couplings ($\tilde{g}$'s) are defined in Eqn.(9) and $\mathrm{Y}'_{t,b,\tau}=\left(\frac{h_{t,b,\tau}}{4\pi}\right)^2$ with h_t and λ_t are related by Eqn.(4) and $h_{b,\tau}(\widetilde{m})=\lambda^*_{b,\tau}(\widetilde{m})cos\beta$.

Below the scale $\widetilde{m}$ the renormalization group equations of the Yukawa couplings at the one loop level are given by

$$\frac{dY_t'}{dt} \;=\; 3Y_t'\left(\frac{8}{3}\tilde{\alpha}_3+\frac{3}{4}\tilde{\alpha}_2+\frac{17}{60}\tilde{\alpha}_1\right)-\frac{1}{2}Y_t'(9Y_t'+3Y_b'+2Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d') \tag{A.13}$$

$$\frac{dY_b'}{dt} \;=\; 3Y_b'\left(\frac{8}{3}\tilde{\alpha}_3+\frac{3}{4}\tilde{\alpha}_2+\frac{1}{12}\tilde{\alpha}_1\right)-\frac{1}{2}Y_b'(3Y_t'+9Y_b'+2Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d') \tag{A.14}$$

$$\frac{dY_\tau'}{dt} \;=\; 3Y_\tau'\left(\frac{3}{4}\tilde{\alpha}_2+\frac{3}{4}\tilde{\alpha}_1\right)-\frac{1}{2}Y_\tau'(6Y_t'+6Y_b'+5Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d') \tag{A.15}$$

The gaugino mass equations are (including next-to-leading order corrections)

$$\frac{dM_3}{dt} \;=\; 9\tilde{\alpha}_3 M_3(1+c_{\tilde{g}}\tilde{\alpha}_3), \tag{A.16}$$

where $c_{\tilde{g}}=38/3$ in $\overline{MS}$ and $c_{\tilde{g}}=10$ in $\overline{DR}$.

$$\frac{dM_2}{dt} \;=\; 6(\tilde{\alpha}_2-\frac{1}{2}\tilde{Y}_u-\frac{1}{2}\tilde{Y}_d)M_2-2\sqrt{\tilde{Y}_u\tilde{Y}_d}\mu \tag{A.17}$$

$$\frac{dM_1}{dt} \;=\; -\frac{1}{2}(\tilde{Y}_u'+\tilde{Y}_d')M_1-2\sqrt{\tilde{Y}_u'\tilde{Y}_d'}\mu \tag{A.18}$$

The renormalization group equation for the μ parameter below the scale $\widetilde{m}$ is given by

$$\frac{d\mu}{dt} \;=\; \left[\frac{9}{4}\left(\frac{\tilde{\alpha}_1}{5}+\tilde{\alpha}_2\right)-\frac{3}{8}(\tilde{Y}_u+\tilde{Y}_d)-\frac{1}{8}(\tilde{Y}_u'+\tilde{Y}_d')\right]\mu-\frac{3}{2}\sqrt{\tilde{Y}_u\tilde{Y}_d}M_2-\frac{1}{2}\sqrt{\tilde{Y}_u'\tilde{Y}_d'}M_1 \tag{A.19}$$

The equations for the gaugino couplings are given by

$$\begin{aligned}\frac{d\tilde{Y}_u}{dt} \;=\; & 3\tilde{Y}_u\left(\frac{11}{4}\tilde{\alpha}_2+\frac{3}{20}\tilde{\alpha}_1\right)-\frac{1}{4}\tilde{Y}_u(5\tilde{Y}_u-2\tilde{Y}_d+\tilde{Y}_u')-(\tilde{Y}_u\tilde{Y}_d\tilde{Y}_u'\tilde{Y}_d')^{1/2} \\ & -\frac{1}{2}\tilde{Y}_u(6Y_t'+6Y_b'+2Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d')\end{aligned} \tag{A.20}$$

$$\begin{aligned}\frac{d\tilde{Y}_u'}{dt} \;=\; & 3\tilde{Y}_u'\left(\frac{3}{4}\tilde{\alpha}_2+\frac{3}{20}\tilde{\alpha}_1\right)-\frac{3}{4}\tilde{Y}_u'(\tilde{Y}_u'+2\tilde{Y}_d'+\tilde{Y}_u)-3(\tilde{Y}_u\tilde{Y}_d\tilde{Y}_u'\tilde{Y}_d')^{1/2} \\ & -\frac{1}{2}\tilde{Y}_u'(6Y_t'+6Y_b'+2Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d')\end{aligned} \tag{A.21}$$

$$\begin{aligned}\frac{d\tilde{Y}_d}{dt} \;=\; & 3\tilde{Y}_d\left(\frac{11}{4}\tilde{\alpha}_2+\frac{3}{20}\tilde{\alpha}_1\right)-\frac{1}{4}\tilde{Y}_d(-2\tilde{Y}_u+5\tilde{Y}_d+\tilde{Y}_d')-(\tilde{Y}_u\tilde{Y}_d\tilde{Y}_u'\tilde{Y}_d')^{1/2} \\ & -\frac{1}{2}\tilde{Y}_d(6Y_t'+6Y_b'+2Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d')\end{aligned} \tag{A.22}$$

$$\begin{aligned}\frac{d\tilde{Y}_d'}{dt} \;=\; & 3\tilde{Y}_d'\left(\frac{3}{4}\tilde{\alpha}_2+\frac{3}{20}\tilde{\alpha}_1\right)-\frac{3}{4}\tilde{Y}_d'(\tilde{Y}_d'+2\tilde{Y}_u'+\tilde{Y}_d)-3(\tilde{Y}_u\tilde{Y}_d\tilde{Y}_u'\tilde{Y}_d')^{1/2} \\ & -\frac{1}{2}\tilde{Y}_d'(6Y_t'+6Y_b'+2Y_\tau'+3\tilde{Y}_u+3\tilde{Y}_d+\tilde{Y}_u'+\tilde{Y}_d')\end{aligned} \tag{A.23}$$

Now, the evolution equation for the Higgs quartic coupling λ is

$$\begin{aligned}\frac{d\tilde{\lambda}}{dt} &= -6\tilde{\lambda}^2 - \frac{1}{2}\tilde{\lambda}[-9\left(\frac{1}{5}\tilde{\alpha}_1 + \tilde{\alpha}_2\right) + 6(\tilde{Y}_u + \tilde{Y}_d) + 2(\tilde{Y}'_u + \tilde{Y}'_d) + 12Y'_t + 12Y'_b + 4Y'_\tau] \\ &\quad -\frac{9}{4}\left(\frac{1}{2}\tilde{\alpha}_2^2 + \frac{3}{50}\tilde{\alpha}_1^2 + \frac{1}{5}\tilde{\alpha}_1\tilde{\alpha}_2\right) + \frac{5}{2}(\tilde{Y}_u^2 + \tilde{Y}_d^2) + \tilde{Y}_u\tilde{Y}_d + \frac{1}{2}(\tilde{Y}'_u + \tilde{Y}'_d)^2 \\ &\quad +(\sqrt{\tilde{Y}_u\tilde{Y}'_u} + \sqrt{\tilde{Y}_d\tilde{Y}'_d})^2 + 6Y_t'^2 + 6Y_b'^2 + 2Y_\tau'^2, \end{aligned} \tag{A.24}$$

where $\tilde{\lambda} = \frac{\lambda}{(4\pi)^2}$.

Caution: If $M_{\tilde{\chi}_1^0} > m_t$, in the evolution from $M_{\tilde{\chi}_1^0}$ to m_t of the gauge couplings

$$b = \left(\frac{41}{10}, -\frac{19}{6}, -7\right), \quad B = \begin{pmatrix} \frac{109}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix} \tag{A.25}$$

$$d^t = \left(\frac{17}{10}, \frac{3}{2}, 2\right), \quad d^b = \left(\frac{1}{2}, \frac{3}{2}, 2\right), \quad d^\tau = \left(\frac{3}{2}, \frac{1}{2}, 0\right), \quad d^W = 0 = d^B. \tag{A.26}$$

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