Quark-lepton complementarity with quasidegenerate Majorana neutrinos

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A basis independent formulation of quark-lepton complementarity is implemented at a high scale for quasidegenerate Majorana neutrinos. It is shown that even with the renormalization group evolution in the minimal supersymmetric standard model, the scenario can be consistent with the data provided a nontrivial role is played by the Majorana phases. Correlated constraints are found on these phases and the neutrino mass scale using the current data. We also indicate how future accurate measurements of the mixing angles can serve as tests of this scenario and restrict the values of the Majorana phases.

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Neutrinos provide a fertile ground for novel and testable ideas due to the present availability and future prospects of more precise information [1] on their masses and mixing angles. We aim to combine three such ideas in this letter: (a) quark-lepton complementarity (QLC) [2, 3, 4, 5, 6, 7], (b) quasidegenerate neutrinos (QDN) [8], and (c) nontrivial Majorana phases. The possibility of such a combination was mentioned in [3], but we explicitly demonstrate its feasibility by use of the renormalization group (RG) evolution in a transparently analytic way.

QLC links the difference between the maximal (45°) and the measured ($33.8^{+2.4}_{-1.8}$ degrees [1]) value of the neutrino mixing angle θ_{12} to the Cabibbo angle $\theta_c = 12.6^\circ \pm 0.1^\circ$ [9]. This can be done by postulating the relation $\theta_{12} + \theta_c = 45^\circ$. However, the various implementations of this relation in the literature (e.g. in [3]) are fraught with basis ambiguities [10] and issues of scale [6]. In this letter, we follow a basis independent formulation of QLC from [3]:

$$U_{PMNS} = V^\dagger_{CKM} U_\nu^{\rm bimax} \, , \tag{1}$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata matrix unitarily transforming mass eigenstates of neutrinos to their flavor eigenstates, V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix [9] and $U_\nu^{\rm bimax}$ is the unitary matrix which diagonalizes the bimaximal form [11] of the neutrino Majorana mass matrix $\mathcal{M}_\nu^{\rm bimax}$. Eq. (1) yields

$$\theta_{12} + \theta_c/\sqrt{2} = 45^\circ + \mathcal{O}(\theta_c^2) \, . \tag{2}$$

The identification of eq. (1) as a statement of QLC becomes more transparent in the basis with $U_u = 1$, where U_f represents the unitary (mass $\to$ flavor) transformation of the left chiral components of the f $(= u, d, l)$ type of charged fermions which diagonalize the Yukawa coupling matrix combination $Y_f^\dagger Y_f$ [12]. Thus, in this basis, $Y_u^\dagger Y_u$ is diagonal in flavor space. It follows that V_{CKM} $(\equiv U_u^\dagger U_d)$ now equals U_d, so that a comparison between eq. (1) and the definition of U_{PMNS} $(\equiv U_l^\dagger U_\nu)$, with the assumption of U_ν being $U_\nu^{\rm bimax}$, now yields the quark-lepton symmetry relation $U_d = U_l$. Eq. (1), as it stands, is basis independent, however.

A quark-lepton symmetry relation such as (2) is expected to be valid at the GUT scale $\sim 10^{16}$ GeV. In our scenario, neutrino masses are generated by an effective dimension-5 operator $(l \cdot h)(l \cdot h)/\Lambda$ at the scale Λ and the mechanism that gives rise to this operator is immaterial. The mechanism may include right handed neutrinos, in which case we have to assume that the threshold effects [13] do not spoil the relation till $\Lambda \sim 10^{12}$ GeV, above which all the right handed neutrinos are expected to lie. All the other threshold effects are taken to be flavor blind. We thus postulate the relation (2) to hold at a scale $\Lambda \sim 10^{12}$ GeV. Our results are only logarithmically sensitive to the exact choice of scale.

Quasidegenerate neutrinos are very much allowed by present cosmological constraints [14] as well as neutrinoless double β-decay experiments [15]. From the model building point of view, the quasidegenerate spectrum can be obtained rather naturally through type II seesaw mechanism by invoking discrete symmetries like flavor $SO(3)$ [8]. It can also be produced in models with Abelian family symmetries [16], or with flavor symmetries like $L_\mu - L_\tau$ [17]. In the minimal supersymmetric standard model (MSSM), the neutrino masses and mixing angles may evolve significantly from Λ to the SUSY breaking scale λ via the RG equations [18, 19]. This evolution can potentially spoil the QLC signatures in the low energy data [3]. In this letter, we study the evolution of the QLC equation (1) analytically as well as numerically, including the effect of Majorana phases [19, 20], and show its consistency with the observed mixing angles in the QDN scenario.

We take the neutrino masses $m_{1,2,3}$ to be complex in general and parametrize their absolute values in terms of three real parameters m_0, ρ_A and ϵ_S as

$$\begin{aligned} |m_1| &= m_0(1-\rho_A)(1-\epsilon_S) \, , \\ |m_2| &= m_0(1-\rho_A)(1+\epsilon_S) \, , \\ |m_3| &= m_0(1+\rho_A), \end{aligned} \tag{3}$$

with m_0, the parameter setting the neutrino mass scale, and ϵ_S being required to be positive, while a positive (negative) ρ_A implies a normal (inverted) neutrino mass ordering. These parameters may be related to the solar and atmospheric mass squared differences ($\delta m_S^2 \sim 8 \times 10^{-5}$ eV^2 and $|\delta m_A^2| \sim 2\times 10^{-3}$ eV^2) and the the sum of the neutrino absolute masses through

$$\begin{aligned}
\delta m_S^2 &= |m_2|^2 - |m_1|^2 \approx 4m_0^2(1-\rho_A)^2\epsilon_S \ , \\
|\delta m_A^2| &= ||m_3|^2 - (|m_1|/2 + |m_2|/2)^2| = 4m_0^2|\rho_A| \ , \\
\Sigma_i |m_i| &= 3m_0(1-\rho_A/3) \ . \qquad (4)
\end{aligned}$$

In (4) we have made use of $|\epsilon_S| \ll 1$ (for instance, if $m_0 \simeq 0.2$ eV, one has $|\rho_A| \simeq 1.8\times 10^{-2}$ and $\epsilon_S \simeq 5\times 10^{-4}$) while neglecting terms which are $\mathcal{O}(\epsilon_S^2)$.

The RG evolution of the hierarchical charged fermion masses is known to be small [21], and we neglect it. The bimaximal neutrino mass matrix emerging at a high scale Λ gets modified at the low scale λ to yield [12, 22]

$$M_\lambda = \mathcal{I}_K \ \mathcal{I}_\kappa^T \ \mathcal{M}_\nu^{\rm bimax} \ \mathcal{I}_\kappa \ , \qquad (5)$$

where $\mathcal{I}_K \equiv \exp[-\int_{t(\Lambda)}^{t(\lambda)} K(t)dt]$ is the scalar factor that arises from the RG evolution due to the gauge couplings and the fermion-antifermion loops. In MSSM, we have $K(t) = -6g_2^2 - 2g_Y^2 + 6{\rm Tr}(Y_u^\dagger Y_u)$. Here $t(Q)$ is defined to be $t(Q) \equiv (16\pi^2)^{-1}\ln(Q/Q_0)$ with Q_0 an arbitrary scale.

The other factor $\mathcal{I}_\kappa$ in (5) is given by

$$\mathcal{I}_\kappa \equiv \exp[-\int_{t(\Lambda)}^{t(\lambda)} (Y_l^\dagger Y_l)(t)dt] \ . \qquad (6)$$

In the basis chosen for our QLC scenario,

$$Y_l^\dagger Y_l = V_{CKM} \ {\rm Diag}(y_e^2, y_\mu^2, y_\tau^2) \ V_{CKM}^\dagger \ . \qquad (7)$$

Since $y_e^2 \ll y_\mu^2 \ll y_\tau^2$, we can neglect y_e and y_μ. If, in addition, we neglect the elements of the CKM matrix that are of the order of θ_c^2 or smaller, only the {3–3} element of $Y_l^\dagger Y_l$ survives. Then we have

$$\mathcal{I}_\kappa \approx \ {\rm Diag}(1, 1, e^{-\Delta_\tau}) \ , \qquad (8)$$

where in the MSSM, one has

$$\Delta_\tau = m_\tau^2(\tan^2\beta + 1)(16\pi^2 v^2)^{-1}\ln(\Lambda/\lambda) \ . \qquad (9)$$

Here $v \equiv \sqrt{v_u^2 + v_d^2}$ where v_u and v_d are the vevs of the two neutral Higgs scalars, with $\tan\beta \equiv v_u/v_d$. For $\Lambda \sim 10^{12}$ GeV, $\lambda \sim 10^3$ GeV, $\tan\beta \sim 30$ and $v \sim 246$ GeV, we find that $\Delta_\tau \sim 6\times 10^{-3}$. Therefore, unless the coefficients of Δ_τ are $\mathcal{O}(10^2)$ or higher, we can neglect terms that involve two or more powers of Δ_τ. Henceforth, we keep only the terms linear in Δ_τ.

The mass matrix (5) in the flavor basis takes the following form at the low scale:

$$M_\lambda = \begin{pmatrix} A & B & -BX \\ B & C + A/2 & (C - A/2)X \\ -BX & (C-A/2)X & (C+A/2)Y \end{pmatrix} \mathcal{I}_K \qquad (10)$$

where we have used the notation $A \equiv (m_1 + m_2)/2$, $B \equiv (-m_1 + m_2)/(2\sqrt{2})$, $C \equiv m_3/2$, $X \equiv (1-\Delta_\tau)$, and $Y \equiv (1 - 2\Delta_\tau)$.

We parametrize the unitary matrix U_λ that diagonalizes M_λ by

$$\begin{aligned}
U_\lambda \ \equiv \ & {\rm Diag}(e^{i\phi_e\Delta_\tau}, e^{i\phi_\mu\Delta_\tau}, e^{i\phi_\tau\Delta_\tau}) R_{23}(\pi/4 + k_{23}\Delta_\tau) \times \\
& {\rm Diag}(1, 1, e^{i\delta}) R_{13}(k_{13}\Delta_\tau) {\rm Diag}(1, 1, e^{-i\delta}) \times \\
& R_{12}(\pi/4 + k_{12}\Delta_\tau) {\rm Diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i\alpha_3/2}) \ , \qquad (11)
\end{aligned}$$

where R_{ij} is the rotation matrix in the ij plane, α_i's are the Majorana phases, and the phases ϕ_e, ϕ_μ and ϕ_τ are required to diagonalize a general neutrino mass matrix [23]. Thus the new mixing angles are

$$\theta_{12} = \pi/4 + k_{12}\Delta_\tau \ , \theta_{23} = \pi/4 + k_{23}\Delta_\tau \ , \theta_{13} = k_{13}\Delta_\tau \ .$$

For $\Delta_\tau = 0$, we have $U_\lambda = U_\nu^{\rm bimax}$. We approximate the deviation of U_λ from $U_\nu^{\rm bimax}$ by keeping terms that are linear in Δ_τ. The current allowed ranges of the mixing angles are such that the deviations from QLC values without RG running are very small. Therefore, the approximation $|k_{ij}\Delta_\tau| \ll 1$ should be always valid so that we can neglect the higher order terms in $k_{ij}\Delta_\tau$. Furthermore, the Dirac phase δ, which is vanishing at the high scale and is generated only through the RG evolution, is retained only to the first order, and consequently plays no role in our $\mathcal{O}(\Delta_\tau)$ analysis.

The values of k_{ij} are found to be

$$\begin{aligned}
k_{12} &= \frac{1}{4}\frac{|m_1 + m_2|^2}{(|m_2|^2 - |m_1|^2)} \ , \\
k_{23} &= \frac{1}{4}\left[\frac{|m_2 + m_3|^2}{(|m_3|^2 - |m_2|^2)} + \frac{|m_1 + m_3|^2}{(|m_3|^2 - |m_1|^2)}\right] \ , \\
k_{13} &= \frac{1}{4}\left[\frac{|m_2 + m_3|^2}{(|m_3|^2 - |m_2|^2)} - \frac{|m_1 + m_3|^2}{(|m_3|^2 - |m_1|^2)}\right] \ . \qquad (12)
\end{aligned}$$

The above expressions are valid as long as the values of m_i's and $\delta m_{S/A}^2$'s are described accurately by the $\mathcal{O}(\Delta_\tau)$ terms in their RG evolution. Though this condition always holds with the m_i's, for $\Delta_\tau \gtrsim \delta m_{S/A}^2/m_0^2$ the $\mathcal{O}(\Delta_\tau^2)$ terms dominate over the $\mathcal{O}(\Delta_\tau)$ terms in $\delta m_{S/A}^2$ [19] and eqs. (12) are no longer a good approximation. They are valid only for $\Delta_\tau \lesssim \delta m_S^2/m_0^2$, i.e. for $m_0\tan\beta \lesssim 3$ eV. Higher Δ_τ values also lead to $|k_{12}\Delta_\tau| \gg 1$, so that the evolution of θ_{12} is too large to be naturally accomodated with the current data.

Equations (12) are consistent with the running of angles computed in the general case [19], though they have been computed here in a much simpler way for the special case of bimaximal neutrino mixing. In terms of the parameters m_0, ρ_A, ϵ_S defined in eq. (3), the expressions (12) become

$$\begin{aligned}
k_{12} &= [(1 + \cos\alpha_2) + \epsilon_S^2(1 - \cos\alpha_2)]/(8\,\epsilon_S) \ , \\
k_{23} &= \frac{\Gamma}{8}[2 + \cos(\alpha_2 - \alpha_3) + \cos\alpha_3] + \frac{\rho_A}{2} + \mathcal{O}(\epsilon_S) \ , \\
k_{13} &= (\Gamma/8)[\cos(\alpha_2 - \alpha_3) - \cos\alpha_3] + \mathcal{O}(\epsilon_S) \ , \qquad (13)
\end{aligned}$$

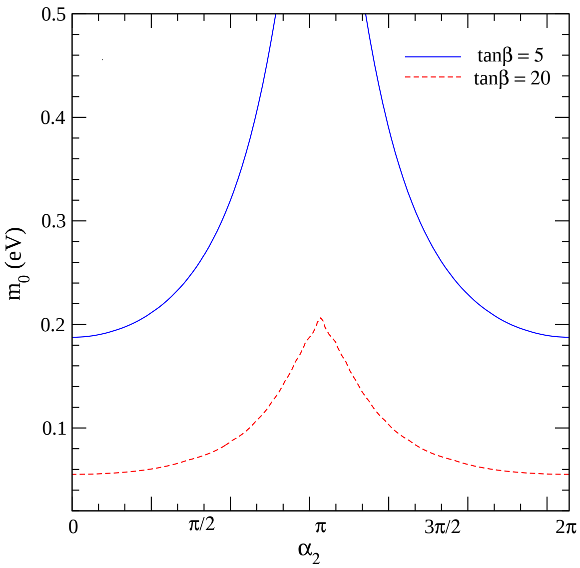

FIG. 1: Contours of θ_{12} =39.8° in the m_0 (eV)–α_2 (radians) plane shown for $\tan\beta = 5$ (20) by solid (dashed) lines. The regions above the contours are excluded by data for that particular $\tan\beta$ value.

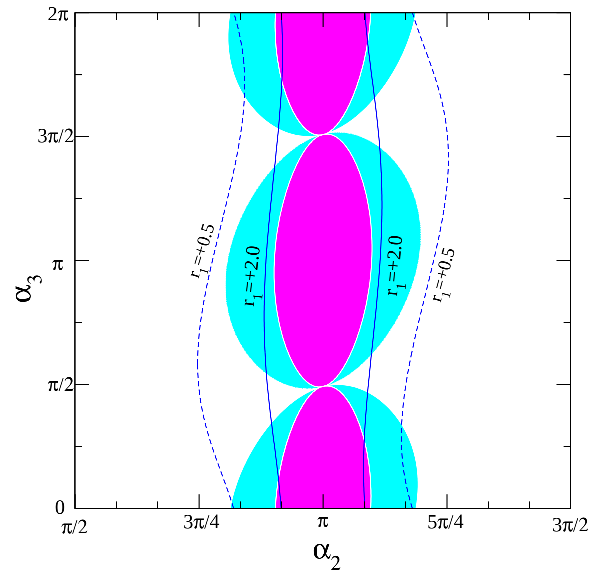

FIG. 2: The contours of the ratios $r_1 \equiv \Delta\theta_{23}/\Delta\theta_{12}$ and $r_2 \equiv \Delta\theta_{13}/\Delta\theta_{12}$ for normal mass ordering. The line contours are for $r_1 = 2.0$ (solid) and $r_1 = 0.5$ (dashed). The outer edges of the cyan (light) and magenta (dark) shaded regions at the centre correspond to $r_2 = 0.5$ (2.0) and those of the shaded regions at the top and bottom correspond to $r_2 = -0.5$ (-2.0).

where $\Gamma \equiv (1/\rho_A) - \rho_A$. One of the three Majorana phases $\alpha_{1,2,3}$ can be rotated away and we have chosen that to be α_1.

The following observations can now be made:

• Quasi-degeneracy of the neutrinos ($m_0^2 \gg \delta m^2_{S/A}$) is required for any significant enhancement of all k_{ij}'s: the magnitude of k_{12} is enhanced when $\epsilon_S = \delta m_S^2/[4m_0^2(1-\rho_A)^2] \ll 1$, whereas the magnitudes of k_{23} and k_{13} are enhanced for $\rho_A \delta m_A^2/(4m_0^2) \ll 1$.

• The values of the Majorana phases are crucial in deciding the values of k_{ij}'s: As $\alpha_2 \to 0$ we have $k_{12} \approx 1/(4\epsilon_S)$. When α_2 is nonzero, the value of k_{12} decreases rapidly. At $\alpha_2 = \pi$, we have k_{12} to be nearly as small as $\epsilon_S/4$. Both $|k_{23}|$ and $|k_{13}|$ are enhanced when $\alpha_3 = 0$ or $\alpha_3 = \alpha_2$. However, when $\alpha_2 = 0$, the magnitude of k_{13} is highly suppressed.

• k_{12} is independent of m_3 as well as α_3, and is always positive. k_{23} is positive (negative) for the normal (inverted) neutrino mass ordering. The sign of k_{13} depends on the ordering as well as the Majorana phases.

The net leptonic mixing matrix at the low scale is $V_{\rm PMNS} = V^\dagger_{\rm CKM} U_\lambda$ with the mixing angles given by $\theta_{ij} = \theta^0_{ij} + k_{ij}\Delta_\tau$, where $\theta^0_{12} \approx 35.4°, \theta^0_{23} \approx 42.5°, \theta^0_{13} \approx 8.9°$ are their QLC values at the high scale. Whereas θ^0_{12} and θ^0_{13} are known to an accuracy of $\approx \pm 0.1°$, the exact value of θ^0_{23} depends on the value of the CP violating phase δ in the CKM matrix [3], and is currently uncertain by nearly $\pm 1°$. The "deviations" $\Delta\theta_{ij} \equiv \theta_{ij} - \theta^0_{ij} \approx k_{ij}\Delta_\tau$ are observable quantities. From the earlier discussions $\Delta\theta_{12} > 0$, so that $\theta_{12} > 35.4°$ is a test for our scenario. Another test is the compulsion of normal (inverted) mass ordering for $\theta_{23} > \theta^0_{23}$ ($\theta_{23} < \theta^0_{23}$). Regarding θ_{13}, though the high scale value is $\theta^0_{13} = 8.9°$, allowed RG evolution in our scenario can make it anywhere between 0° and the extant upper bound of 13°.

The 3σ allowed range of θ_{12} is $\theta_{12} \in (29.3°, 39.8°)$ [1]. With $\Delta\theta_{12}\Delta_\tau$ necessarily positive, this implies $0 < k_{12}\Delta_\tau < 4.4°$. Strong constraints then ensue on m_0 and α_2, since $\Delta\theta_{12}$ is inversely proportional to the small quantity ϵ_S. In Fig. 1, we show the 3σ allowed values of m_0 and α_2 for two $\tan\beta$ values. The figure is obtained by solving the RG equations numerically with θ^0_{ij}'s as the initial conditions at the high scale, and marginalizing over α_3 and δ. The figure may be understood easily with our analytic expressions (13). At large $\tan\beta$, the value of α_2 has to be close to π in order to avoid an excessive k_{12} enhancement [3] (even this will not work if m_0 is too large). A nontrivial Majorana phase is thus essential. For smaller values of $\tan\beta$, however, no such tuning is required (see fig. 1).

The ratios $r_1 \equiv \Delta\theta_{23}/\Delta\theta_{12}$ and $r_2 \equiv \Delta\theta_{13}/\Delta\theta_{12}$ can be used to constrain the Majorana phases α_2 and α_3. In the QDN scenario, where $\rho_A \ll 1 \ll 1/\rho_A$, these constraints are independent of Δ_τ, and hence $\tan\beta$. We show in Fig. 2 the contours of constant r_1 and r_2 in the α_2–α_3 plane, for normal hierarchy. With inverted hierarchy, the signs of r_1 and r_2 are reversed. Note that $\alpha_2 = \alpha_3 = 0$ necessitates $r_1 \approx 2\delta m_S^2/\delta m_A^2 \approx 0.06$ and $r_2 = 0$, which implies $\theta_{23} \approx \theta^0_{23}$, $\theta_{13} = \theta^0_{13}$. Any deviation from this prediction will indicate non-zero Majorana phases. However, for these relations to be practically useful as a test, measurements of these angles accurate to within a couple of degrees are essential.

If $\alpha_2 \approx \pi$, which would be the case if θ_{12} is found to be very close to θ_{12}^0, the ratio $r_2/r_1 \approx -2\cos\alpha_3$ gives a direct measurement of α_3, with $|r_2/r_1| < 2$ serving as a weak test of this scenario.

Even without any knowledge of the Majorana phases, the measurements of the mixing angles can put a lower bound on Δ_τ, and hence on $\tan\beta$. With QDN we have the relations $|\Delta\theta_{12}| < \Delta_\tau/(4\epsilon_S)$, $|\Delta\theta_{23}| < \Delta_\tau/|2\rho_A|$, $|\Delta\theta_{13}| < \Delta_\tau/|4\rho_A|$ and additionally the combinations $|\Delta\theta_{23} \pm \Delta\theta_{13}| < \Delta_\tau/|2\rho_A|$. Once m_0 is known, the best of the above lower bounds on Δ_τ may be chosen to restrict $\tan\beta$ from below via eq. (9).

In conclusion, a basis independent formulation of QLC at a high scale can be consistent with the data even for the QDN scenario *provided a nontrivial role is played by the Majorana phases.* We have explicitly shown this numerically as well as through transparent analytic approximations for the RG evolutions of the mixing angles. Our new results are correlated constraints on the neutrino mass scale and the Majorana phases, as well as correlations among the neutrino mixing angles which can be tested by their precise measurements. Specifically, one of the major predictions of our scenario is $\theta_{12} > 35.4°$. Currently the data is consistent with this prediction to within 1σ. A further reduction of the error in θ_{12} [24] will clarify the situation.

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