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Four Zero Neutrino Yukawa Textures in the Minimal Seesaw Framework

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Abstract

We investigate, within the Type I seesaw framework, the physical implications of zero textures in the Yukawa couplings which generate the neutrino Dirac mass matrix m_D. It is shown that four is the maximal number of texture zeroes compatible with the observed leptonic mixing and the assumption that no neutrino mass vanishes. We classify all allowed four-zero textures of m_D into two categories with three classes each. We show that the different classes, in general, admit CP violation both at low and high energies. We further present the constraints obtained for low energy physics in each case. The rôle of these zero textures in establishing a connection between leptogenesis and low energy data is analysed in detail. It is shown that it is possible in all cases to completely specify the parameters relevant for leptogenesis in terms of light neutrino masses and leptonic mixing together with the unknown heavy neutrino masses.

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1 Introduction

Recent impressive experimental progress towards determining the masses and mixing angles of the three known light neutrinos has brought urgency to the task of unravelling the flavour structure of the neutrino mass matrix m_ν. It has been pointed out [1] that, in the case of Majorana neutrinos, it is not possible to completely determine the structure of m_ν from feasible experiments. This is one of the motivations for introducing some theoretical input aimed at reducing the number of free parameters. One interesting possibility is the imposition of zeroes in the elements of m_ν [1]. Another is to assume the vanishing of det m_ν [2]. Several papers have analysed in detail the consequences of imposing zeroes directly in the elements of m_ν, starting with at least two zero textures [1], [3]. Implications of single texture zeroes were studied in detail in [4]. In fact, the existence of vanishing mass matrix elements may reflect the presence of a family symmetry acting in the leptonic sector [5], [6]. It is then more natural to investigate and classify the appearance of zeroes in the fundamental mass matrix appearing in the Lagrangian rather than in m_ν which, at least within the seesaw framework, is a derived quantity. Therefore we focus our attention on the Yukawa couplings which lead to the neutrino Dirac mass matrix m_D, once spontaneous symmetry breaking occurs. One can then see how zeroes in m_D affect m_ν which we take to be related to m_D by the Type I seesaw relation. Throughout, by "texture" we shall refer to a configuration of m_D containing zeroes in some of its elements. In the Froggatt-Nielsen approach [7] texture zeroes correspond to **extremely suppressed** entries, which can be taken effectively as zeroes. The stability of zeroes in neutrino mass matrices under quantum corrections in type I seesaw models has been studied in Refs. [8], [9], [10]. One also needs to be aware that renormalisation group effects can be quite large in the case of quasi-degenerate (inverted hierarchical) light neutrino masses [11], [12].

In this paper we classify and analyse the physical implications of all neutrino Yukawa coupling matrices with four zero textures in the **Weak Basis** (WB) where the charged lepton and the righthanded Majorana neutrino mass matrices are diagonal and real. For simplicity, we work within the framework of the Type I seesaw, where three righthanded singlet neutrinos are added to the Standard Model (SM). The case of only two righthanded heavy neutrinos leads to one zero neutrino mass and in this case only one zero textures and some of the two zero textures are allowed experimentally [13], [14]. With three heavy righthanded neutrinos and the additional requirement that none of the physical neutrino masses vanishes, we show that four is the maximal number of zeroes in textures of m_D that are compatible with the available data on neutrino mixing. We organize all such four zero textures into classes and discuss the physical implications of each class. The imposition of texture zeroes in the Yukawa couplings has the advantage of allowing for the possibility of establishing a connection between low energy physics and physics at high energies, in particular leptogenesis [15].

It is by now established that new sources of CP violation beyond the Kobayashi-

Maskawa mechanism of the Standard Model (SM) are required in order to dynamically generate the observed Baryon Asymmetry of the Universe (BAU) [16], [17], [18], [19], [20], [21]. The scenario of baryogenesis through leptogenesis has been rendered especially appealing by the discovery of neutrino oscillations which provides evidence for nonvanishing neutrino masses. In general, there is no direct connection between CP violation at low energies and that entering in leptogenesis [22], [23]. It has been shown, however, that such a connection arises in models with texture zeroes in m_D [1], [13] [24], [25]. This is a question that we analyse in the present work for each one of the classes of allowed four zero textures and we conclude that it is possible in all cases to completely specify the parameters relevant for leptogenesis in terms of light neutrino masses and leptonic mixing together with the unknown heavy neutrino masses.

Texture zeroes are clearly not WB invariant. For definitness we analyse the allowed four zero textures in the WB in which both the charged lepton mass matrix and the heavy righthanded Majorana neutrino mass matrix are diagonal, as mentioned earlier. The question of how to recognise a flavour model corresponding to a given set of texture zeroes, when written in a different basis, was addressed in Ref [26]. It was shown there that some sets of texture zeroes imply the vanishing of certain CP-odd WB invariants. The relevance of CP-odd WB invariants in the analysis of texture zero ansätze is due to the fact that texture zeroes lead in general to a decrease in the number of CP violating phases.

This paper is organised as follows. In section 2 we set the notation and present our framework. We show in section 3, based on what is already known experimentally about leptonic mixing and on the condition that no neutrino mass vanishes, why textures with five or more zeroes in m_D are ruled out. In section 4 we enumerate the possible classes of four zero textures that are allowed and give for each one of them low energy relations leading to physical constraints. CP violation and related WB invariants for the surviving four zero textures are discussed in section 5. In section 6 we analyse the physical implications of four zero textures both for low energies and for leptogenesis. Our conclusions are summarised in section 7.

2 Notation and framework

We work in the context of the minimal Type I seesaw with three generations of righthanded neutrinos which are singlets of SU(2). We do not extend the Higgs sector and therefore we do not include Majorana mass terms for lefthanded neutrinos. After spontaneous symmetry breaking, the leptonic mass terms are given by:

$$\begin{aligned} \mathcal{L}_m &= -[\overline{\nu_L^0} m_D \nu_R^0 + \frac{1}{2}\nu_R^{0T} C M_R \nu_R^0 + \overline{l_L^0} m_l l_R^0] + h.c. \\ &= -[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0] + h.c., \end{aligned} \quad (1)$$

where n_L is a column vector with $n_L^T = (\nu_L^0, (\nu_R^0)^c)$, while M_R, m_D, and m_l respectively denote the righthanded neutrino Majorana mass matrix, the neutrino Dirac mass matrix and the charged lepton mass matrix in family space. The superscript 0 signifies the fact that the corersponding fields are eigenstates of flavour. The matrix $\mathcal{M}$ is given by:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{2}$$

We assume that the scale of M_R is much higher than the electroweak scale $v \simeq 246$ GeV. Upon diagonalisation of the matrix $\mathcal{M}$, we are left with $\mathcal{D} = \mathrm{diag}(m_1, m_2, m_3, M_1, M_2, M_3)$, containing three light and three heavy Majorana neutrinos. The charged current interactions can then be written as

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \left(\overline{l_{iL}} \gamma_\mu K_{ij} \nu_{jL} + \overline{l_{iL}} \gamma_\mu G_{ij} N_{jL} \right) W^\mu + h.c., \tag{3}$$

where ν_j and N_j denote the light and the heavy neutrinos respectively and K and G are 3 x 3 blocks of a unitary 6 x 6 matrix that diagonalises the symmetric matrix $\mathcal{M}$. The light neutrino masses and mixing angles are obtained to an excellent approximation from:

$$U^\dagger m_{eff} U^\star = d, \tag{4}$$

where $m_{eff} = -m_D {M_R}^{-1} m_D^T \equiv m_\nu$ computed in the WB where m_l is diagonal, and $d = \mathrm{diag}\ (m_1, m_2, m_3)$. The unitary matrix U in Eq. (4) is the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) matrix [27] relating the mass eigenstate neutrinos ν_i $(i = 1, 2, 3)$ to the flavour eigenstate neutrinos ν_f $(f = e, \mu, \tau)$ by $\nu_{f_L} = U_{fi} \nu_{iL}$. It coincides with K in Eq. (3) up to corrections of order v^2/M^2, which we ignore. The Yukawa textures that we analyse are imposed in the weak basis where M_R and m_l are real and diagonal. In this WB, all CP violating phases are contained in m_D. From Eq. (4) and the definition of m_{eff}, one can write m_D in the Casas and Ibarra parametrisation [28]:

$$m_D = i U \sqrt{d} R \sqrt{D}, \tag{5}$$

where D stands for M_R in the WB where it is diagonal and R is a general complex orthogonal matrix. It is clear, by construction, that mixing and CP violation at low energies are blind to the matrix R. However this last matrix is relevant for leptogenesis.

3 Inadmissibility of more than four zero textures

We shall now demonstrate that, in the framework specified earlier, all five zero textures in m_D are ruled out. Let us start with the general form

$$m_D = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, \tag{6}$$

Since we are assuming that none of the neutrino masses vanishes we conclude from the definition of m_{eff} that the determinant of m_D cannot be zero. Therefore patterns of m_D with one full row of zeroes or one full column of zeroes are ruled out, as well as patterns with zeroes distributed in a quartet, i.e. in four elements (ij), (lk), (ik), (lj) where i,j,k,l can be 1, 2 or 3, with $l \neq$i, k$\neq$j. We are thus left with patterns where the zeroes are placed in such a way that invariably two rowns and two columns would have two zeroes simultaneously. Together with the requirement that there is no quartet of zeroes this leads to several different possibilities where, in each case, one nonzero entry of m_D is in a row and a column where all other entries are zeroes. Some examples are

$$\begin{bmatrix} a_1 & a_2 & 0 \\ 0 & 0 & b_3 \\ c_1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & 0 & b_3 \\ 0 & c_2 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & a_2 & 0 \\ 0 & b_2 & b_3 \\ c_1 & 0 & 0 \end{bmatrix},$$
$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & c_2 & c_3 \end{bmatrix}, \quad \begin{bmatrix} 0 & a_2 & a_3 \\ b_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & a_2 & a_3 \\ 0 & b_2 & 0 \\ c_1 & 0 & 0 \end{bmatrix}. \tag{7}$$

Since we work in a WB where M_R is diagonal the resulting matrix m_{eff}, for the five zero textures under discussion, is always block diagonal. Furthermore, the fact that we are in a WB where the charged lepton mass matrix is also diagonal, implies that these textures lead to two-family mixing only, which is already ruled out experimentally. Indeed, it is already known that there are two large mixing angles in the PMNS matrix and as a result all five zero textures are ruled out.

4 Four zero textures

In this section, we classify all different possible four zero textures for m_D in a WB where M_R and m_l are real and diagonal with no vanishing neutrino mass. Among patterns of four zero textures in m_D, the nonvanishing det m_D condition rules out the occurrence of three of the zeroes in the same row or in the same column, as well as zeroes distributed in a quartet, as explained in the previous section. Block diagonal patterns such as

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & c_2 & c_3 \end{bmatrix}, \quad \begin{bmatrix} a_1 & 0 & a_3 \\ 0 & b_2 & 0 \\ c_1 & 0 & c_3 \end{bmatrix}, \quad \begin{bmatrix} a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \tag{8}$$

lead to two-family mixing only and are therefore ruled out.

The allowed remaining patterns can be split into two categories:

(i) those with two orthogonal rows;

(ii) those with two orthogonal columns and no pairs of orthogonal rows;

The first category can be divided into three classes corresponding to:

(i)(a) orthogonality of the first and second rows, leading to:

$$m_{eff\,12} = m_{eff\,21} = 0. \tag{9}$$

(i)(b) orthogonality of the first and third rows, leading to:

$$m_{eff\,13} = m_{eff\,31} = 0. \tag{10}$$

(i)(c) orthogonality of the second and third rows, leading to:

$$m_{eff\,23} = m_{eff\,32} = 0. \tag{11}$$

There are eighteen different cases in (i)(a). Six of them have two zeroes in the first row and two zeroes in the second row, as for example:

$$\begin{bmatrix} 0 & 0 & a_3 \\ 0 & b_2 & 0 \\ c_1 & c_2 & c_3 \end{bmatrix}, \begin{bmatrix} 0 & 0 & a_3 \\ b_1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{bmatrix}. \tag{12}$$

Another six different cases have two zeroes in the first row, one zero in the second row and one zero in the third row as in:

$$\begin{bmatrix} 0 & a_2 & 0 \\ b_1 & 0 & b_3 \\ 0 & c_2 & c_3 \end{bmatrix}, \begin{bmatrix} 0 & a_2 & 0 \\ b_1 & 0 & b_3 \\ c_1 & c_2 & 0 \end{bmatrix}. \tag{13}$$

Finally, six different cases are obtained with one zero in the first row, two zeroes in the second row and one zero in the third row as in:

$$\begin{bmatrix} 0 & a_2 & a_3 \\ b_1 & 0 & 0 \\ c_1 & 0 & c_3 \end{bmatrix}, \begin{bmatrix} 0 & a_2 & a_3 \\ b_1 & 0 & 0 \\ c_1 & c_2 & 0 \end{bmatrix}. \tag{14}$$

There are another eighteen different cases in (i)(b). These are obtained from those in (i)(a) exchanging the second with the third row. The cases in (i)(c) are also eighteen different ones obtained from those in (i)(a) by exchanging the first row with the third one. Each case in category (i) has one symmetric pair of nondiagonal zero entries in m_{eff}. Since m_{eff} is symmetric by construction, due to its Majorana character, off diagonal zeroes always come in pairs.

Textures in category (ii) are obtained by transposing those in category (i) and discarding those already considered in (i). There are eighteen cases altogether in category (ii). In all of these, two columns are orthogonal to each other, each having two zeroes, and there is one column without zeroes. This category can again be divided into three classes:

(ii)(a) six cases with two zeroes in the first row, these cases are given explicitly by:.

$$\begin{bmatrix} 0 & 0 & a_3 \\ 0 & b_2 & b_3 \\ c_1 & 0 & c_3 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & a_3 \\ b_1 & 0 & b_3 \\ 0 & c_2 & c_3 \end{bmatrix}, \quad \begin{bmatrix} 0 & a_2 & 0 \\ 0 & b_2 & b_3 \\ c_1 & c_2 & 0 \end{bmatrix},$$
$$\begin{bmatrix} 0 & a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & c_2 & c_3 \end{bmatrix}, \quad \begin{bmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ c_1 & 0 & c_3 \end{bmatrix}, \quad \begin{bmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_3 \\ c_1 & c_2 & 0 \end{bmatrix}. \tag{15}$$

These verify the conditions:

$$\left|m_{eff_{11}} m_{eff_{23}}\right| = \left|m_{eff_{12}} m_{eff_{13}}\right|, \qquad \arg(m_{11} m_{23} m_{12}^* m_{13}^*) = 0. \tag{16}$$

Note that $\arg(m_{ii} m_{jk} m_{ij}^* m_{ik}^*)$ is rephasing invariant. In Ref. [26] one example of this class was discussed.

(ii)(b) six cases with two zeroes in the second row, which are obtained from the patterns in (ii)(a) by interchanging the first with the second row. These verify the conditions:

$$\left|m_{eff_{22}} m_{eff_{13}}\right| = \left|m_{eff_{21}} m_{eff_{23}}\right|, \qquad \arg(m_{22} m_{13} m_{21}^* m_{23}^*) = 0. \tag{17}$$

(ii)(c) six cases with two zeroes in the third row, which are obtained from the patterns in (ii)(a) by interchanging the first with the third row. These verify the conditions:

$$\left|m_{eff_{33}} m_{eff_{12}}\right| = \left|m_{eff_{31}} m_{eff_{32}}\right|, \qquad \arg(m_{33} m_{12} m_{31}^* m_{32}^*) = 0. \tag{18}$$

Eqs. (16), (17) and (18) are of the form:

$$m_{eff_{ii}} m_{eff_{jk}} = m_{eff_{ij}} m_{eff_{ik}} \tag{19}$$

with i, j, k different from each other and no sum implied.

It can be checked that all allowed cases in category (i) as well as in category (ii) contain the same number of independent parameters in m_D and in all such cases one can rephase away three of the phases. The counting of independent parameters in m_{eff} is also the same in all cases, as will be seen in the next section. Moreover, we shall analyse in section 6 the implications of Eqs. (9), (10), (11), (16), (17), (18) corresponding to the two categories, each one with three different classes.

Notice that, although we are considering weak bases with the maximum number of zeroes allowed by experiment, together with the assumption that no neutrino mass vanishes, the resulting matrix m_{eff} contains at most one zero nondiagonal entry.

We are not considering here the possibility of fine-tuning between the parameters of m_D and those of M_R leading to additional zeroes due to special cancellations. This indicates that imposing texture zeros in the WB where m_l and M_R are diagonal does not allow to generate any of the two zero patterns considered in Ref. [1]. It is already known that not all of these patterns can realised through seesaw [29], [30]

5 CP Violation and Weak Basis Invariants

We start by recalling the general counting of the number of parameters contained in the lepton mass matrices and then consider the special case of textures with four zeros in m_D. In the WB where M_R and m_l are diagonal and real, leptonic mixing and CP violation are encoded in m_D, which is an arbitrary complex 3×3 matrix. The latter contains nine real moduli and nine phases. Of these, only six phases are physical, since three phases can be removed by simultaneous rephasing of ν_L, l_L. So m_D is left with nine real moduli plus six phases. Taking into account the three eigenvalues of M_R, we have in this WB a total of eighteen parameters including six phases. This equals the number of physical parameters to wit, three light neutrino masses, three heavy neutrino masses, plus six mixing angles and six CP violating phases in the first three rows of a 6×6 complex unitary matrix [31] which we have denoted as K and G in Eq. (3). It is interesting to notice that the number of independent physical phases in Eq. (5) is also six, three in the PMNS matrix and three required to parametrise the orthogonal complex matrix R.

Textures with four zeros in m_D lead to a strong reduction in the number of parameters, since there are only five real parameters and two phases after rephasing. This gives rise to interesting phenomenological implications which are analysed in detail in the next section. In particular, it will be shown that in all four zero textures classified by us, the matrix R, which plays an important rôle in leptogenesis, can be fully expressed in terms of low energy parameters entering in U and d. This establishes a direct connection between leptogenesis and low energy data. Moreover, this link exists both in the cases of unflavoured and flavoured leptogenesis.

In scenarios where flavour does not play an important rôle, the lepton number asymmetry resulting from the decay of the N_j heavy Majorana neutrino is given by [32]:

$$\begin{aligned} \varepsilon_{N_j} &= \frac{g^2}{{M_W}^2} \sum_{k \neq j} \left[\mathrm{Im} \left(({m_D}^\dagger m_D)_{jk} ({m_D}^\dagger m_D)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{({m_D}^\dagger m_D)_{jj}} \\ &\simeq \frac{g^2}{{M_W}^2} \sum_{k \neq j} \left[(M_k)^2 \mathrm{Im} \left((G^\dagger G)_{jk} (G^\dagger G)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{(G^\dagger G)_{jj}} . \end{aligned} \tag{20}$$

In Eq. (20) M_k are the heavy neutrino masses and we have neglected terms of order v^2/M_k^2. The variable x_k is defined as $x_k = M_k^2/M_j^2$ and the function $I(x_k)$ is given by

$I(x_k) = \sqrt{x_k}\,(1 + (1 + x_k)(\log x_k - \log(1 + x_k)))$. Eq. (20) has been obtained after summing over all charged leptons $l_i^{\pm}$ (i = e, μ , τ). From the Casas and Ibarra parametrisation we obtain:

$$m_D^\dagger m_D = \sqrt{D} R^\dagger d R \sqrt{D}. \tag{21}$$

In this framework, leptogenesis is insensitive to the low energy CP violating phases appearing in U and can occur even without CP violation at low energies [23]. Actually, leptogenesis depends on other parameters beyond ε_{N_j} and involves thermodynamic processes that have been analysed by several authors [33], [34] [35], [36]. It was pointed out recently that, under certain conditions, flavour matters in leptogenesis [37], [38], [39], [40], [41], [42]. In this case we must take into account the separate lepton i family asymmetry generated from the decay of the kth heavy Majorana neutrino which depends on the combinations [38] $\mathrm{Im}\Big((m_D^\dagger m_D)_{kk'}(m_D^*)_{ik}(m_D)_{ik'}\Big)$ and $\mathrm{Im}\Big((m_D^\dagger m_D)_{k'k}(m_D^*)_{ik}(m_D)_{ik'}\Big)$ Clearly, when one works with separate flavours, the matrix U does not cancel out and one is led to the interesting possibility of having viable leptogenesis even in the case of R being a real matrix [43], [44], [45], [46].

Next, we show explicitly that four zero textures lead in general to CP violation both at low and high energies. The strength of leptonic CP violation of Dirac-type, which can be observable through neutrino oscillations, is controlled by the WB CP-odd invariant [47]

$$I_1 \equiv \mathrm{tr}\,[h_{eff}, h_l]^3 = -6i\ \Delta\ I_{CP}, \qquad \text{with} \qquad I_{CP} \equiv \mathrm{Im}\ (h_{eff_{12}} h_{eff_{31}} h_{eff_{23}}), \tag{22}$$

where $h_{eff} = m_{eff} m_{eff}^\dagger$, $h_l = m_l m_l^\dagger$ and $\Delta = ({m_\mu}^2 - {m_e}^2)({m_\tau}^2 - {m_e}^2)({m_\tau}^2 - {m_\mu}^2)$. In order to show that this CP-odd invariant does not vanish, in spite of the four zeroes in m_D, we have to examine the structure of h_{eff}. For definiteness, let us consider the configuration

$$m_D = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & 0 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \tag{23}$$

belonging to case (c), category (i). Three phases can be rephased away so that one is left with only two phases which can be placed for instance at the entries (1,1) and (1,2). From Eq. (23) and the definition of m_{eff} one obtains the following structure for the latter:

$$m_{eff} \equiv \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & r_{13} \\ c_{12} & r_{22} & 0 \\ r_{13} & 0 & r_{33} \end{bmatrix}, \tag{24}$$

where entries labelled with a c are complex and those labelled with an r are real. With these choices there are two complex entries in m_{eff}. From this equation we obtain:

$$I_{CP} = \mathrm{Im}\ \Big[|m_{13}|^2 (m_{12}^2 m_{22}^* m_{11}^*) + |m_{12}|^2 (m_{11} m_{33} {m_{13}^*}^2) + {m_{12}}^2 {m_{13}^*}^2 m_{22}^* m_{33}\Big]. \tag{25}$$

One may note that each one of the three terms contributing to I_{CP} is rephasing invariant. It is clear that I_{CP} does not vanish for the m_{eff} of Eq. (24).

It is well known that at low energies there are three CP violating phases in U, one of the Dirac type and two of the Majorana type. The question of finding the CP-odd WB invariants that would be sufficient to control CP violation at low energies was first adressed in Ref. [47], and more recently in Ref. [48]. In particular it was pointed out that requiring the vanishing of the WB invariant of Eq. (22) together with the two WB invariants:

$$I_2 \equiv \text{Im tr}\left[h_l\, (m_{eff}\, m^*_{eff})\, (m_{eff}\, h^*_l\, m^*_{eff})\right], \tag{26}$$

$$I_3 \equiv \text{Tr}\left[\, (m^*_{eff}\, h_l\, m_{eff}\ ,\ h^*_l\right]^3 \tag{27}$$

provides in general, necessary and sufficient conditions for low energy CP invariance [48]. The invariant of Eq. (27), was first proposed in Ref. [49] where it was shown that it has the special feature of being sensitive to Majorana type CP violation even in the limit of three exactly degenerate Majorana neutrinos. Other relevant cases can be found in Ref. [50]. The fact that, for the four zero texture of Eq. (23), none of the three WB invariants vanishes in general, shows that this texture leads to both Dirac and Majorana-type CP violation at low energies. The same applies to the other four zero textures.

So far, we have only considered leptonic CP violation at low energies. Leptogenesis is a high energy phenomenon requiring CP violation. In the unflavoured case the relevant phases are those in $m_D^\dagger m_D$ as shown in Eq. (20). In this case one may also write a set of three independent WB invariants [22]

$$I_4 \equiv \text{ImTr}[h_D H M_R^* h_D^* M_R], \tag{28}$$

$$I_5 \equiv \text{ImTr}[h_D H^2 M_R^* h_D^* M_R], \tag{29}$$

$$I_6 \equiv \text{ImTr}[h_D H^2 M_R^* h_D^* M_R H], \tag{30}$$

where $h_D = {m_D}^\dagger m_D$ and $H = M_R^\dagger M_R$. These three would have to vanish if CP were to be conserved. The condition for the vanishing of I_4 was first given in Ref. [51]. The evaluation of these invariants, in the WB with diagonal M_R, shows that, in the case of a nondegenerate D and assuming no cancellations, they can all simultaneously vanish only if all imaginary parts of $(h_{Dij})^2$ are absent. Now it turns out that textures of category (ii) always have one zero off-diagonal entry in h_D due to the orthogonality of two columns of m_D, but the other two off-diagonal elements are in general nonzero. The same goes for those textures in category (i) that also have two orthogonal columns. The remaining textures in category (i) have, in general, three nonzero complex h_{Dij} but their phases are constrained to be cyclic, i.e., Im $(h_{D12} h_{D31} h_{D23}) = 0$. The fact that not all imaginary parts of $(h_{Dij})^2$ vanish simultaneously in any of the four zero textures, shows that they admit CP violation at high energies, relevant for leptogenesis.

6 Implications from models with four zero textures

6.1 Low energy physics

Let us start by summarising what is presently known about neutrino masses and leptonic mixing. We choose to parametrise the PMNS mixing matrix as [52]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P, \tag{31}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, with all θ_{ij} in the first quadrant, δ being a Dirac-type phase and $P = \text{diag}\,(1, e^{i\alpha}, e^{i\beta})$ with α and β denoting the phases associated with the Majorana character of neutrinos.

The current experimental bounds on neutrino masses and leptonic mixing are [52]:

$$\begin{aligned} \Delta m^2_{21} &= 8.0^{+0.4}_{-0.3} \times 10^{-5} \text{ eV}^2, & (32)\\ \sin^2(2\theta_{12}) &= 0.86^{+0.03}_{-0.04}, & (33)\\ |\Delta m^2_{32}| &= (1.9 \text{ to } 3.0) \times 10^{-3} \text{ eV}^2, & (34)\\ \sin^2(2\theta_{23}) &> 0.92, & (35)\\ \sin^2\theta_{13} &< 0.05, & (36) \end{aligned}$$

with $\Delta m^2_{ij} \equiv m^2_j - m^2_i$. The allowed ranges for the parameters listed above correspond to an impressive degree of precision. The angle θ_{23} may be maximal (i.e., $\pi/4$). In contrast, maximal mixing for θ_{12} is already ruled out experimentally. At the moment there is only an experimental upper bound on the angle θ_{13}. A value of θ_{13} close to the present bound would be good news for the prospects of detecting low energy leptonic CP violation, mediated through a Dirac-type phase. The strength of the latter is given by:

$$\mathcal{J}_{CP} \equiv \text{Im}\,[\,U_{11}U_{22}U^*_{12}U^*_{21}\,] = \frac{1}{8}\sin(2\,\theta_{12})\sin(2\,\theta_{13})\sin(2\,\theta_{23})\cos(\theta_{13})\sin\delta\,, \tag{37}$$

which would in this case be of order 10^{-2}, for $\sin\delta$ of order one. A similar quantity defined in terms of the elements of the Cabibbo Kobayashi Maskawa matrix is meaningful in the quark sector [53] [54], and the corresponding value is of the order of 10^{-5}. It is not yet known whether the ordering of the light neutrino masses is normal, i.e. $m_1 < m_2 < m_3$, or inverted, i.e. $m_3 < m_1 < m_2$. The scale of the neutrino masses is not yet established. The spectrum may vary from a large hierarchy between the two lightest neutrino masses to three quasi-degenerate masses. Examples of the possible extreme cases are:

(a) $m_1 \sim 0$ (or e.g. $\sim 10^{-6}$eV), $m_2 \simeq 0.009$eV, $m_3 \simeq 0.05$ eV corresponding to normal spectrum, hierarchical, or else:

$$\text{(b)} \qquad m_3 \sim 0 \ \ (\text{or e.g.} \sim 10^{-6}\text{eV}), \quad m_1 \simeq m_2 \simeq 0.05 \text{ eV}$$

corresponding to inverted spectrum, hierarchical, or else:

$$\text{(c)} \qquad m_1 \simeq 1\text{eV}, \quad m_2 \simeq 1\text{eV}, \quad m_3 \simeq 1\text{eV}$$

corresponding to almost degeneracy.

As explained below, the conditions obtained in section 4, are not all compatible with each of these scenarios. Finally, we note that it is not yet established whether or not neutrinos are Majorana particles and therefore at the moment there are no restrictions on the Majorana phases α, β

The low energy implications of patterns in category (i) which, as was already pointed out lead to one off diagonal set of zeroes in the symmetric matrix m_{eff} were studied in detail in Ref [4]. The main conclusions in this paper are that no off-diagonal entry in m_{eff} can vanish in the case of θ_{13} equal to zero. Implications of one zero in the first row of m_{eff} do not differ much in the two possible such cases due to the approximate $\mu - \tau$ exchange symmetry [55], [56], [57]. In this case all values of m_1 are allowed from extreme hierarchy to almost degeneracy; so are both possible orderings of neutrino masses. For $(m_{eff})_{23} = (m_{eff})_{32} = 0$ both normal hierarchy and inverted hierarchy are excluded.

Let us consider category (ii) which implies the constraints of Eq. (19). Taking into account Eq. (4) this equation can be written as:

$$\sum_{r<s} m_r m_s (U_{ir}U_{ks} - U_{is}U_{kr})(U_{ir}U_{js} - U_{is}U_{jr}) = 0 \qquad (38)$$

with i, j, k different from each other and no sum implied, and the indices r, s ranging from 1 to 3.

With the explicit parametrisation of Eq.(31) we obtain simple exact analytic relations for each of the classes in category (ii).

For class (ii) (a) the exact form of this constraint is:

$$\begin{aligned} -m_1 m_2 e^{2i\alpha} c_{23} s_{23} c_{13}^2 + \\ +m_1 m_3 e^{2i\beta}[c_{12}^2 c_{23} s_{23} + c_{12} s_{12}(c_{23}^2 - s_{23}^2) s_{13} e^{-i\delta} - s_{12}^2 c_{23} s_{23} s_{13}^2 e^{-2i\delta}] + \\ +m_2 m_3 e^{2i(\alpha+\beta)}[s_{12}^2 c_{23} s_{23} + c_{12} s_{12}(s_{23}^2 - c_{23}^2) s_{13} e^{-i\delta} - c_{12} c_{23} s_{23} s_{13}^2 e^{-2i\delta}] = 0 \end{aligned} \qquad (39)$$

An interesting feature of this expression is the fact that all terms sensitive to Dirac type CP violation are doubly suppressed since they are multiplied, either by s_{13}^2 or by the factor $(c_{23}^2 - s_{23}^2) s_{13}$, and it is already known experimentally that θ_{13} corresponds to small or no mixing and θ_{23} is maximal or close to maximal. Therefore, this expression can be very well approximated by:

$$- m_1 m_2 e^{2i\alpha} \ c_{23} s_{23} c_{13}^2 + m_1 m_3 e^{2i\beta} \ c_{12}^2 c_{23} s_{23} + m_2 m_3 e^{2i(\alpha+\beta)} \ s_{12}^2 c_{23} s_{23} = 0 \qquad (40)$$

For class (ii) (b) we have the exact relation:

$$-m_1 m_2 e^{2i\alpha}\ c_{23} c_{13} s_{13} e^{i\delta} + m_1 m_3 e^{2i\beta}\ (c_{12} s_{12} s_{23} c_{13} + s_{12}^2 c_{23} c_{13} s_{13} e^{-i\delta}) + \\ m_2 m_3 e^{2i(\alpha+\beta)}\ (-c_{12} s_{12} s_{23} c_{13} + c_{12}^2 c_{23} c_{13} s_{13} e^{-i\delta}) = 0 \tag{41}$$

Class (ii) (c) exactly verifies:

$$-m_1 m_2 e^{2i\alpha}\ s_{23} c_{13} s_{13} e^{i\delta} + m_1 m_3 e^{2i\beta}\ (-c_{12} s_{12} c_{23} c_{13} + s_{12}^2 s_{23} c_{13} s_{13} e^{-i\delta}) + \\ m_2 m_3 e^{2i(\alpha+\beta)}\ (c_{12} s_{12} c_{23} c_{13} + c_{12}^2 s_{23} c_{13} s_{13} e^{-i\delta}) = 0 \tag{42}$$

This equation can be obtained from the previous one by interchanging s_{23} with c_{23} and by changing the sign of the terms that do not depend on the Dirac phase.

It is clear from these expressions that the main features of low energy physics coming out of these textures do not crucially depend on the possible existence of CP violation. In order to get a feeling of the main features of the implications of the constraints given by Eqs. (39), (41) and (42), let us take as a first approximation the Harrison, Perkins and Scott (HPS) mixing matrix [58]

$$U = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \tag{43}$$

which is consistent with present experimental data, and corresponds to θ_{23} maximal, θ_{13} zero and $c_{12} = 2/\sqrt{6}$ and no CP violation. Obviously, a detailed analysis would require the variation of θ_{13}, as well as of θ_{12} and θ_{23}, inside their allowed ranges and also to take into consideration the possibility of CP violation.

Eqs. (39) and (43) lead to

$$\frac{1}{2} m_1 m_2 - \frac{1}{3} m_1 m_3 - \frac{1}{6} m_2 m_3 = 0. \tag{44}$$

and, as already explained, Eq. (44) corresponds to ignoring terms with a double suppression. In the CP conserving limit, light neutrinos may have different CP parities [59], therefore there are several possible ways of obtaining the necessary cancellations. Normal ordering with strong hierarchy is ruled out since for $m_1 \ll m_2$ there would be only one dominant term, the one in $m_2 m_3$. Hierarchy in the masses implies that the magnitude of term in $m_2 m_3$ is close to $7 \times 10^{-5}\mathrm{eV}^2$. The strongest allowed hierarchy consistent with the above constraint favours the larger θ_{13} values; a numerical example obtained for maximal θ_{23} and the central value of θ_{12}, with cancellations already of order $10^{-10}\mathrm{eV}^2$ is:

$$m_1 = 0.00333071221\mathrm{eV}, \quad m_2 = -0.00954429898\mathrm{eV}, \quad \mathrm{m}_3 = 0.0501108132\mathrm{eV}, \\ s_{13} = 0.198669$$

Obviously the number of significant digits in the above numerical result is meaningless from the experimental point of view and is only given to be consistent with the degree of cancellation claimed above. Here m_2 is no more than a factor of three higher than m_1, which corresponds to a weak normal hierarchy. As m_1 decreases, cancellations cease to occur and the difference tends to the value of the dominant term which is the term in $m_2 m_3$. The situation would change for larger values of θ_{13} which are already ruled out. Likewise for instance, for a small solar angle, (θ_{12}), already excluded, as was pointed out in [25], where a particular example of a texture of this class was considered, since in this case the term in $m_2 m_3$ present in Eq. (40) would be suppressed by s_{12}^2. Strong inverse hierarchy is also ruled out since it would leave the dominant term in $m_1 m_2$ without the possibility of cancellation. Quasi-degeneracy can be accommodated within the present range of experimental values for the mixing angles.

Case (b) in category (ii) obeys the constraint of Eq. (41). The coefficient of $m_1 m_2$ is zero for the HPS matrix and in this case we are left with

$$-\frac{1}{3}m_1 m_3 + \frac{1}{3}m_2 m_3 = 0. \tag{45}$$

For nonzero θ_{13} the term in $m_1 m_2$ is suppressed but cannot be discarded for $m_3 \ll m_1, m_2$, i.e., inverse hierarchy. In the case of inverse hierarchy the necessary cancellation of the three terms may occur. Almost degeneracy can also be accommodated provided θ_{13} is different from zero. For $\theta_{13} = 0$ the coefficient of $m_1 m_3$ would be exactly equal to the coefficient of $m_2 m_3$ and this relation could not be verified, since it would imply $m_1 = m_2$.

Case (c) in category (ii) is very similar to case (b). The resulting equation for the HPS matrix coincides with Eq. (45) and the conclusions are the same as in case (b), category (ii).

All cases in category (ii) are thus incompatible with a strong hierarchy and normal ordering, i.e. for $m_1 \ll m_2$.

6.2 Relating leptogenesis to low energy physics

Zero textures in m_D allow one to relate the matrix R, relevant to leptogenesis, to the light neutrino masses and low energy leptonic mixing. In fact, it is clear from Eq. (5) that each zero in m_D leads to an orthogonality condition between one column of the matrix R and one row of the matrix $U\sqrt{d}$ of the form:

$$(m_D)_{ij} = 0 \;\Rightarrow\; (U)_{ik}\sqrt{d}_{kk}R_{kj} = 0. \tag{46}$$

It was already pointed out [26] that the connection between leptogenesis and low energy physics could be easily established in a particular case that falls into category (ii), since in this case one can fully express the matrix R in terms of light neutrino masses and low energy leptonic mixing . The same is true for all other cases in category (ii) as well as for the cases that fall into category (i) as shown below. The example given in Ref. [26] can be generalised in the following way.

In category (i) there is always in m_D one column with two zeros and two columns with one zero each. Let l be the column with two zeros and a and b the columns with one zero only. In this case we can write

$$\left(\vec{R_l}\right)_i = \left(\varepsilon_{ijk}(U)_{pj}\sqrt{m_j}\ (U)_{qk}\sqrt{m_k}\right)\ \frac{1}{N_l}, \tag{47}$$

$$\left(\vec{R_a}\right)_i = \left(\varepsilon_{ijk}(U)_{rj}\sqrt{m_j}\ (R_l)_k\right)\ \frac{1}{N_a}, \tag{48}$$

$$\left(\vec{R_b}\right)_i = \left(\varepsilon_{ijk}(U)_{sj}\sqrt{m_j}\ (R_l)_k\right)\ \frac{1}{N_b}, \tag{49}$$

where p and q are the rows with zeros in the column l. Moreover, r and s are the rows where the zeros are in columns a and b respectively. The $\vec{R_i}$ are the columns of the matrix R and the N_i are complex normalization factors, with phases such that $\vec{R_i}^2 = 1$. It is easy to show that the columns $\vec{R_a}$ and $\vec{R_b}$ are indeed orthogonal to each other by using the constraint $m_{eff_{rs}} = 0$ valid for each case in category (i).

In Ref. [25] the relation between leptogenesis and CP violation at low energies in two cases falling into category (i), were analysed in detail in the case of hierarchical heavy Majorana neutrinos and also in the case of two fold quasi-degeneracy of the heavy neutrinos.

In category (ii) there is in m_D always one column without zeros and each of the other two columns has two zeros. Now let l be the column without zeros and a and b the columns with two zeros. In this case we can write:

$$\left(\vec{R_a}\right)_i = \left(\varepsilon_{ijk}(U)_{pj}\sqrt{m_j}\ (U)_{qk}\sqrt{m_k}\right)\ \frac{1}{N_a}, \tag{50}$$

$$\left(\vec{R_b}\right)_i = \left(\varepsilon_{ijk}(U)_{rj}\sqrt{m_j}\ (U)_{sk}\sqrt{m_k}\right)\ \frac{1}{N_b}, \tag{51}$$

$$\left(\vec{R_l}\right)_i = \varepsilon_{ijk}\left(\vec{R_a}\right)_j\ \left(\vec{R_b}\right)_k. \tag{52}$$

Here p and q are the rows with zeros in column a. Furthermore, r and s are the rows with zeros in column b, while N_i denote normalization factors. It is easy to show that $\vec{R_a}$ and $\vec{R_b}$ are indeed orthogonal to each other for each case in category (ii) by using Eq. (19).

All four zero textures analysed in this paper allow one to completely specify the matrix R, in terms of light neutrino masses and the elements of the PMNS matrix. It is clear that R can only be complex if there is CP violation at low energies.

7 Summary and Conclusions

We have made a systematic study of all allowed four zero textures in the neutrino Dirac mass matrix m_D, in the framework of Type I seesaw mechanism, without vanishing neutrino masses. In order for this study to be meaningful, one has to choose a specific weak basis (WB). Without loss of generality, we have chosen to work in the WB where the charged lepton and the righthanded neutrino mass matrices are both diagonal, real. Assuming that no neutrino mass vanishes and taking into account the experimental evidence that no leptonic family decouples from the other two, we have shown that four is the maximal number of zeros allowed in m_D. We have found the following remarkable result: the allowed four zero textures in the neutrino Yukawa coupling matrices automatically lead to one of two patterns in m_ν. Either the latter has only one pair of symmetic off-diagonal zeros or else it has no zero element but a vanishing subdeterminant condition. In the derivation of this result, we have implicitly assumed the absence of any fine-tuning between the parameters of m_D and those of M_R which would lead to special cancellations.

Our analysis also applies to scenarios where instead of zeroes, one has a set of extremely suppressed entries [60], as one often encounters in the Froggatt-Nielsen approach. Of course, renormalisation group effects, especially for quasi-degenerate and inverted hierarchical neutrinos [12] will change at least some of the zeroes in m_D into small entries.

We have also explored the phenomenological consequences of the above mentioned textures. In particular we have shown that they lead to a close connection between leptogenesis and low energy measurables such as neutrino masses and mixing angles. The establishment of such a connection in the leptonic sector between physics at low and very high energies is an important goal and provides an additional motivation for considering texture zeroes in the leptonic sector.

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