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CONSTRAINTS ON BARYON-NONCONSERVING YUKAWA COUPLINGS IN A SUPERSYMMETRIC THEORY

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Abstract

The 1-loop evolution of couplings in the minimal supersymmetric standard model, extended to include baryon nonconserving (B) operators through explicit R -parity violation, is considered keeping only B superpotential terms involving the maximum possible number of third generation superfields. If all retained Yukawa couplings Y_i are required to remain in the perturbative domain ($Y_i < 1$) upto the scale of gauge group unification, upper bounds ensue on the magnitudes of the B coupling strengths at the supersymmetry breaking scale, independent of the model of unification. They turn out to be similar to the corresponding fixed point values reached from a wide range of Y_i (including all Y_i greater than unity) at the unification scale. The coupled evolution of the top and B Yukawa couplings results in a reduction of the fixed point value of the former.

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There is widespread interest today in baryon number violating (B) processes. Many studies have been made of B couplings in the context of baryogenesis, proton decay etc. Yet, one of the least investigated sources of baryon nonconservation is the set of B operators which occur in the superpotential of the minimal supersymmetric standard model (MSSM) extended to include explicit R -parity breaking terms [1].

In this paper we start with all such operators coming from the superpotential. We know from the Standard Model that Yukawa coupling strengths spectacularly reflect the generation hierarchy. The third generation ones are the strongest – followed by those of the second generation with the first generation couplings being the weakest. It would be reasonable to adopt a similar hypothesis for the B Yukawa couplings and retain only those which are the strongest – discarding the rest. Because of color antisymmetry, not all three superfields in the trilinear superpotential term can belong to the third generation. We deem it sufficient to retain only those terms in which the third generation superfields appear twice. We bound the magnitudes of the corresponding coupling strengths from above by utilizing constraints from Renormalization Group Evolution (RGE). The bounds result from the requirement that all Yukawa couplings in the theory remain in the perturbative domain upto the scale of the unification of gauge couplings [2] $M_U \sim 2 \times 10^{16}$ GeV.

R -parity $R_p \equiv (-)^{3B+L+2S}$ (with B, L, S as baryon number, lepton number and spin respectively) distinguishes between particles ($R_p = 1$) and sparticles ($R_p = -1$). The popular formulation [3] of the MSSM has R_p -

conservation built into it by fiat. Many recent studies suggest [4] nonetheless R_p couplings, allowed by renormalizability and supersymmetry, that admit both B and L violation. However, the simultaneous presence of both types of terms and the (naturalness-based) supposition that the supersymmetry breaking scale M_{SUSY} should be $\lesssim 0$ (TeV) would imply catastrophic proton decay unobserved in nature. This forces practitioners of explicit R_p models to consider *either* lepton *or* baryon nonconserving cases.

Thus two R_p scenarios are in vogue. There is one with purely lepton number violating interactions, respecting baryon number:

$$\text{Scenario 1 : } \mathcal{L}_\ell = \lambda_{ijk}(L_i L_j \bar{E}_k)_F + \lambda'_{ijk}(L_i Q_j \bar{D}_k)_F. \quad (1)$$

The second R_p scenario has purely baryon number violating terms with conserved lepton number:

$$\text{Scenario 2 : } \mathcal{L}_B = \lambda''_{ijk}[\bar{D}_i \bar{D}_j \bar{U}_k]_F. \quad (2)$$

Here λ, λ' and λ'' are Yukawa couplings with λ' and λ'' being antisymmetric in i, j . Moreover, $L, Q, \bar{E}, \bar{D}, \bar{U}$ stand for the doublet lepton, doublet quark, singlet antilepton, singlet d -antiquark, and singlet u -antiquark superfields respectively, with subscripts acting as generation indices.

Either (1) or (2) has been shown [4] to be consistent with some deeper fundamental theory and therefore can be seriously entertained. Moreover, cosmological bounds on λ, λ' and λ'' , once believed to be strong [5], have been shown [6] to be not generally valid. In particular, a GUT era leptogenesis makes λ''_{ijk} completely free [6] of cosmological constraints. Serious

phenomenological upper bounds do exist [7], of course, on the magnitudes of most of the λ and λ' components from the nonobservation of various leptonic rare decays, neutrino Majorana mass etc. Comparatively, the λ'' -components, apart from λ''_{211} and λ''_{311} [8] which have been strongly bounded from the lack of any observed $n\bar{n}$ oscillation, stand relatively unconstrained. It is therefore worthwhile to try to derive some upper bounds, however weak, on the magnitudes of the remaining λ'' components from the requirement of a perturbative behaviour. We also investigate the fixed point values of the top and (\bar{B}) Yukawa couplings. We discover that the former is somewhat reduced in strength from its value in the R_p conserving case while the latter are comparable to the above-mentioned bounds.

RGE is our basic tool in constraining the couplings of all superfields Φ^a . Here a is a generic index. Given a trilinear term $d_{abc}\Phi^a\Phi^b\Phi^c$ in the superpotential and the evolution scale μ , the RGE equation for d_{abc} is [9]

$$\mu \frac{\partial}{\partial \mu} d_{abc} = \gamma_a^i d_{ibc} + \gamma_b^j d_{ajc} + \gamma_c^k d_{abk}. \quad (3)$$

In (3) γ_a^i is the anomalous dimension matrix $Z^{-1/2}{}_a^k \mu \partial / \partial \mu Z^{1/2}{}_k^i$ where the renormalization constant Z relates the renormalized superfield Φ and the unrenormalized one Φ_0 by

$$\Phi_0^i = Z^{1/2}{}_a^i \Phi^a. \quad (4)$$

We would apply (3) to the Yukawa couplings of interest. The Yukawa part

of the Lagrangian density including $(\mathcal{B}, \mathcal{R}_p)$ terms is [1]:

$$\mathcal{L} = h_\tau [L_3 H_1 \bar{E}_3]_F + h_b [Q_3 H_1 \bar{D}_3]_F + h_t [Q_3 H_2 \bar{U}_3]_F + \lambda''_{233} [\bar{D}_2 \bar{D}_3 \bar{U}_3]_F + \lambda''_{133} [\bar{D}_1 \bar{D}_3 \bar{U}_3]_F. \quad (5)$$

As explained earlier, we include only the terms with the maximum possible number of third generation indices in the \mathcal{B} part of \mathcal{L} . Of course, we also have the three regular Yukawa terms coming from the third generation (characterized by the couplings h_t, h_b, h_τ) and ignore their lower generation counterparts. In (4) H_1 and H_2 are the Higgs superfields coupling to the up and down quarks respectively.

SUPERFIELDS Φ	REPRESENTATIONS $SU(3)_L \times SU(2)_L \times U(1)_Y$	ANOMALOUS DIMENSION $(4\pi)^2 \gamma_\Phi^\Phi$
L_3	$(1, 2, -1/2)$	$h_\tau^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_Y^2$
\bar{E}_3	$(1, 1, 1)$	$2h_\tau^2 - \frac{6}{5}g_Y^2$
\bar{D}_3	$(\bar{3}, 1, 1/3)$	$2h_b^2 + 6\lambda''_{233} + 6\lambda''_{133} - \frac{8}{3}g_c^2 - \frac{2}{15}g_Y^2$
\bar{D}_2	$(\bar{3}, 1, 1/3)$	$6\lambda''_{233} - \frac{8}{3}g_c^2 - \frac{2}{15}g_Y^2$
\bar{D}_1	$(\bar{3}, 1, 1/3)$	$6\lambda''_{133} - \frac{8}{3}g_c^2 - \frac{2}{15}g_Y^2$
\bar{U}_3	$(\bar{3}, 1, -2/3)$	$2h_t^2 + 6\lambda''_{133} + 6\lambda''_{233} - \frac{8}{3}g_c^2 - \frac{8}{15}g_Y^2$
Q_3	$(3, 2, 1/6)$	$h_b^2 + h_t^2 - \frac{8}{3}g_c^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_Y^2$
H_1	$(1, 2, -1/2)$	$h_\tau^2 + 3h_b^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_Y^2$
H_2	$(1, 2, 1/2)$	$3h_t^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_Y^2$

Table 1 : Properties of the relevent superfields.

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ representations and the anomalous dimen-

sions γ_Φ^Φ of the superfields (diagonal elements of the anomalous dimension matrix), occurring in (3), are given in Table 1 (N.B. $Q = I_3 + Y$). The \bar{B} terms mix different generations at the 1-loop level. In particular, they lead to nonzero values of the following off-diagonal terms in the anomalous dimension matrices:

$$\begin{aligned}\gamma_{\bar{D}_1}^{\bar{D}_2} &= \frac{3}{8\pi^2} \lambda_{133}'' \lambda_{233}^{*''}, \\ \gamma_{\bar{D}_2}^{\bar{D}_1} &= \frac{3}{8\pi^2} \lambda_{133}^{*''} \lambda_{233}''.\end{aligned}\tag{5}$$

All other off-diagonal elements vanish.

We can now write down the evolution equations of the Yukawa couplings. (Soft supersymmetry breaking terms have dimensional strengths and they do not contribute to the evolution of dimensionless couplings.) The equations are

$$\begin{aligned}\mu \frac{\partial}{\partial \mu} h_\tau &= h_\tau \left(\gamma_{L_3}^{L_3} + \gamma_{H_1}^{H_1} + \gamma_{\bar{E}_3}^{\bar{E}_3} \right), \\ \mu \frac{\partial}{\partial \mu} h_b &= h_b \left(\gamma_{Q_3}^{Q_3} + \gamma_{H_1}^{H_1} + \gamma_{\bar{D}_3}^{\bar{D}_3} \right), \\ \mu \frac{\partial}{\partial \mu} h_t &= h_t \left(\gamma_{\bar{U}_3}^{\bar{U}_3} + \gamma_{H_2}^{H_2} + \gamma_{\bar{D}_3}^{\bar{D}_3} \right), \\ \mu \frac{\partial}{\partial \mu} \lambda_{133}'' &= \lambda_{133}'' \left(\gamma_{\bar{U}_3}^{\bar{U}_3} + \gamma_{\bar{D}_3}^{\bar{D}_3} + \gamma_{\bar{D}_1}^{\bar{D}_1} \right) + \lambda_{233}'' \gamma_{\bar{D}_1}^{\bar{D}_2}, \\ \mu \frac{\partial}{\partial \mu} \lambda_{233}'' &= \lambda_{233}'' \left(\gamma_{\bar{U}_3}^{\bar{U}_3} + \gamma_{\bar{D}_3}^{\bar{D}_3} + \gamma_{\bar{D}_2}^{\bar{D}_2} \right) + \lambda_{133}'' \gamma_{\bar{D}_2}^{\bar{D}_1}.\end{aligned}\tag{6}$$

Next, we define $t = 1/(2\pi)\ell n(\mu/\text{GeV})$ and, utilizing the entries in Table 1, write (6) in terms of the t -evolution of α_i 's and Y_i 's, where $\alpha_i = g_i^2/(4\pi)$ and

$Y_i = h_i^2/(4\pi)$, $Y_{ijk} = \lambda_{ijk}''^2/(4\pi)$. Thus

$$\begin{aligned}
\frac{d\alpha_Y}{dt} &= \left(2n_f + \frac{3}{5}\right) \alpha_Y^2, \\
\frac{d\alpha_2}{dt} &= (-6 + 2n_f + 1) \alpha_2^2, \\
\frac{d\alpha_3}{dt} &= (-9 + 2n_f) \alpha_3^2, \\
\frac{dY_\tau}{dt} &= \left(4Y_\tau + 3Y_b + 3\alpha_2 - \frac{9}{5}\alpha_Y\right) Y_\tau, \\
\frac{dY_b}{dt} &= \left(6Y_b + Y_t + Y_\tau + 6Y_B - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_Y\right) Y_b, \\
\frac{dY_t}{dt} &= \left(6Y_t + Y_b + 6Y_B - \frac{16}{3}\alpha_3 - \frac{13}{15}\alpha_Y\right) Y_t, \\
\frac{dY_B}{dt} &= \left(2Y_b + 2Y_t + 18Y_B - 8\alpha_3 - \frac{4}{5}\alpha_Y\right) Y_B.
\end{aligned} \tag{7}$$

In (7), n_f is the number of generations and we have defined Y_B as the sum $Y_{133} + Y_{233}$ for convenience.

The low energy constraints on Y_b, Y_t and Y_τ come from the following relations:

$$\begin{aligned}
\sqrt{4\pi Y_t(m_t)} &= m_t(m_t) \sqrt{1 + \tan^2 \beta} (174 \text{ GeV } \tan \beta)^{-1}, \\
\sqrt{4\pi Y_b(m_t)} &= m_b(m_t) \sqrt{1 + \tan^2 \beta} (174 \text{ GeV } \eta_b)^{-1}, \\
\sqrt{4\pi Y_\tau(m_t)} &= m_\tau(m_\tau) \sqrt{1 + \tan^2 \beta} (174 \text{ GeV } \eta_\tau)^{-1}.
\end{aligned} \tag{8}$$

In (8) $m_t(m_t)$, $m_b(m_b)$, $m_\tau(m_\tau)$ are the top, bottom and tau masses at $\mu = m_t$, $\mu = m_b$, $\mu = m_\tau$ respectively. We use $m_b(m_b) = 4.25 \pm .15$ GeV and $m_\tau(m_\tau) = 1.777$ GeV as input values. Moreover, with running effects from loops taken into account [9], one may take $\eta_b \equiv m_b(m_t)/m_b(m_b) = 1.54$ for $\alpha_s = .123 \pm .004$ and $\eta_\tau \equiv m_\tau(m_t)/m_\tau(m_\tau) \simeq 1$. In our notation, $\tan \beta$ is the ratio of vacuum expectation value of H_2 to that of H_1 . We know from

phenomenological analyses that [11] $\tan \beta$ lies between 1 and m_t/m_b . Once the values of Y_t , Y_b and Y_τ are fixed at the scale m_t , they can be evolved to the scale of SUSY breaking M_{SUSY} using non-SUSY RGE equations. In our calculations we have used M_{SUSY} to be 1 TeV since this is preferred by the data on gauge couplings for a unification scenario [2]. The top quark mass range is taken to be 125-185 GeV.

We have not assumed any boundary conditions on the Yukawa couplings at the unification scale. This makes our analysis independent of models of Yukawa coupling unification. We only require that all Yukawa coupling strengths remain perturbative ($Y_i < 1$). (Later, we will see the results in terms of the fixed points of these strengths). In our analysis this condition, when imposed, yields numerical upper bounds on the magnitude of $\lambda_{\vec{B}}$, where $\lambda_{\vec{B}}^2/(4\pi) = Y_{\vec{B}}$ (i.e. $|\lambda_{\vec{B}}| > |\lambda''_{133}|, |\lambda''_{233}|$) at the scale M_{SUSY} . The results have been collected in Table 2.

$m_t(\text{GeV}) \rightarrow$				
$\tan \beta$	125	145	165	185
1				Y_t
5	.63	.60	.56	diverges
10	.63	.60	.56	for any value
20	.62	.58	.56	of $Y_{133} + Y_{233}$
40	.62	.58	.56	at the scale
50	.59	.59	.53	M_{SUSY}

Table 2 : Upper bounds on $\lambda_{\vec{B}}$ as functions of $\tan \beta$ and m_t .

We can highlight the following points regarding the above results.

1. Though we have taken $M_{SUSY} \simeq 1$ TeV, the value preferred for gauge coupling unification [2], our bounds are insensitive to the magnitude of the supersymmetry breaking scale. The choice $M_{SUSY} \simeq m_t$ yields more or less the same bounds with some changes in the second decimal place.
2. An interesting point to note is that, towards higher values of the top mass, the top coupling Y_t itself diverges in the high t region and the upper bound on the baryon number violating couplings cannot be derived.
3. The constraint of perturbative unitarity at M_{SUSY} would directly imply that $Y_{\tilde{B}}(M_{SUSY}) < 1$, i.e. $\lambda_{\tilde{B}}(M_{SUSY}) < 3.54$. The dramatic effect of RGE and of the constraint $Y_{\tilde{B}}(\mu) < 1$ for $\mu < 2 \times 10^{16}$ GeV is to reduce this upper bound from 3.54 to 0.5 – 0.6. Thus our achievement here, after using RGE, is the reduction of the naive upper bound by a factor of six or seven.
4. One can try to introduce a further restriction by requiring [12] the unification of the three baryon number conserving Yukawa couplings. Here we note that the \tilde{B} Yukawa terms in the superpotential have a different matter parity from the B conserving ones. Therefore, even at the unification scale, there is no reason to expect the equality of the B -conserving and \tilde{B} couplings. However, the requirement of the

unification of Y_t, Y_b and Y_τ at that high scale mainly pushes the value of $\tan\beta$ towards the higher region. We see from Table 2 and Fig.1 that these bounds are contained in our result. In fact, the upper bounds do not vary much with $\tan\beta$ when the latter is large. However, Y_b and Y_τ start contributing substantially to the evolution of $Y_{\mathcal{B}}$ for higher values of $\tan\beta$. This reduces the upper bound somewhat.

5. Ours is only a 1-loop RGE analysis. Moreover, we have not included the threshold effects here since they are typically of the order of 2-loop terms [13] which have been ignored. Since ours is the initial investigation in this direction, we have tried to be approximate and simple, rather than very accurate and complicated.

There have been a number of studies [14] on the fixed point behaviour of the top quark Yukawa coupling Y_t in the MSSM. It has been found from the RGE that there is a fixed point value for $Y_t(m_t)$ starting from a large range of Y_t -values at the unification scale. With $\mathcal{B}, \mathcal{R}_p$ couplings present however, two questions arise automatically:

- What happens to the fixed point of Y_t now?
- Is there a fixed point in $Y_{\mathcal{B}}$?

We find that the evolutions of Y_t and $Y_{\mathcal{B}}$ are mutually dependent in a way that the fixed point in Y_t is reduced from the value it has in the MSSM. First,

we have calculated the approximate fixed point of the Yukawa couplings by taking arbitrarily large values of Y_t and $Y_{\vec{B}}$ at the unification scale. Such a scenario is possible in the SU(5) GUT where, at the unification scale, one has the relation $h_\tau = h_b \neq h_t$. It turns out that, in the presence of B -violating Yukawa couplings, the fixed point of h_t reduces from 1.06 in the MSSM to 0.88. We simultaneously get a fixed point value of the \vec{B} coupling $\lambda_{\vec{B}}$ around 0.59. On the other hand, we can make all Y 's including Y_τ and Y_b arbitrarily large at the unification scale. This scenario may appear in the SO(10) GUT in the presence of a $16 \times 16 \times 10$ coupling at the GUT scale. In this case the fixed point value of h_t reduces from 1.0 in the MSSM to nearly 0.86 in the presence of B -violation while the fixed point value of $\lambda_{\vec{B}}$ is about 0.65.

Let us summarize our conclusions regarding constraints on supersymmetric B -violating couplings. Upper bounds have been derived on the magnitudes of the generation-hierarchically largest \vec{B} Yukawa couplings, possible in the MSSM extended to include R_p - and B -violation, at the SUSY breaking scale. These are not very strong but are the first acceptable bounds on these hitherto unrestricted couplings. Moreover, the present work treats the evolution of \vec{R}_p, \vec{B} coupling strengths and their fixed point behavior in relation to that of the top Yukawa coupling. These bounds have emerged from the requirement of the validity of perturbative behavior at all energies below the unification scale. Similar considerations can also be applied to the other viable \vec{R}_P model with the violation of lepton (but not baryon) number. However, in that case the quadratic self-driving term would be smaller due

to the absence of any color factor in the 1-loop graphs containing leptons. So we expect less strong constraints there.

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Figure caption

Fig. 1 Three curves showing the variation on the upper bound on λ_B with respect to the free parameter $\tan\beta$ for three different values (in GeV) of the top mass.

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