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DEEP INELASTIC ELECTRO- OR MUOPRODUCTION OF W BOSONS :
A TEST OF THE BILOCAL ALGEBRA

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ABSTRACT

We propose the study of the reactions $\ell N \rightarrow \ell W^\pm +$ anything (ℓ is a charged lepton, the W has a mass greater than 3 GeV and no strong interactions) at large leptonic momentum transfers as a test of the algebra of bilocal currents formulated recently by Fritzsch and Gell-Mann. The applicability of the algebra to these processes is first established by showing that in the Fourier transform of the squared matrix element all currents act on a lightlike ray. Using a parton analogy, explicit expressions are then given for the differential cross-sections in terms of the deep inelastic neutrino structure functions. The feasibility of this experiment at NAL is discussed briefly.

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1. INTRODUCTION

The properties of products of local operators on the light cone have evoked considerable current interest because of recent experimental results and new theoretical ideas. The regularities ¹⁾ observed in deep inelastic eN scattering at SLAC suggest ^{*)} that the commutator of two electromagnetic current densities, separated by lightlike distances, has the definite structure ²⁾ implied by simple models such as free quark field theory. This demonstrates the fruitfulness of the abstraction of general features of products of currents on the light cone from simple models without commitment to the specifics of such schemes. With this approach and motivated by free quark field theory, Fritzsch and Gell-Mann ³⁾ have extended the well-known equal time algebra of currents into a light cone algebra. Subsequently, it has been shown ⁴⁾ on the basis of canonical field theoretic manipulations that the leading light cone structure of this algebra is preserved even in the presence of vector, scalar or pseudoscalar gluons interacting with the quarks.

The new operators, introduced by Fritzsch and Gell-Mann to describe the extended algebra, are the $SU(3) \times SU(3)$ bilocal currents $J_\mu^i(x, y)$ and $J_\mu^5(x, y)$ which are defined near the light cone $(x-y)^2 = 0$. In the free quark model these operators can be written in terms of the quark field $q(x)$ as $\bar{q}(x) \gamma_\mu (\lambda^i/2) q(y)$ and $\bar{q}(x) \gamma_\mu \gamma_5 (\lambda^i/2) q(y)$, respectively ^{**)}. In a canonical quark vector gluon theory with an interaction $g \bar{q} \gamma_\mu q B_\mu$, B_μ being the gluon field, it is necessary to have for each current the additional multiplicative factor ⁴⁾ $\exp(-ig \int_y^x dz^\mu B_\mu(z))$ where dz^μ is lightlike. It will be convenient to introduce furthermore the symmetric and antisymmetric combinations of bilocal currents :

$$S_\mu^K(x, y) = \frac{1}{2} \left\{ J_\mu^K(x, y) + J_\mu^K(y, x) \right\}, \quad S_{\mu 5}^K(x, y) = \frac{1}{2} \left\{ J_{\mu 5}^K(x, y) + J_{\mu 5}^K(y, x) \right\} \quad (1)$$

$$A_\mu^K(x, y) = \frac{1}{2} \left\{ J_\mu^K(x, y) - J_\mu^K(y, x) \right\}, \quad A_{\mu 5}^K(x, y) = \frac{1}{2} \left\{ J_{\mu 5}^K(x, y) - J_{\mu 5}^K(y, x) \right\}$$

Now the connected part of the commutator of two currents on the light cone is postulated in Ref. 3) to be :

- *) Scaling implies that the leading operators in each term of the infinite expansion for the light cone commutator satisfy the rule $d = J+2$ (d being the dimension of the operator and its spin) which can be obtained by formal manipulations of specific field theories. The near-vanishing of the total longitudinal cross-section for highly virtual photons suggests that the fundamental fields carry spin $\frac{1}{2}$. See Ref. 2).
- **) Our metric and conventions are those of Bjorken and Drell.

$$\begin{aligned}
 [J_{\mu}^i(x), J_{\nu}^j(y)] &\equiv [J_{\mu 5}^i(x), J_{\nu 5}^j(y)] \equiv \partial_{\rho} D(z) \left\{ i f_{ijk} [S_{\mu\nu}^{\rho\sigma} S_{\sigma}^k(x,y) - i \epsilon_{\mu\nu}^{\rho\sigma} A_{\sigma 5}^k(x,y)] + \right. \\
 &\quad \left. + d_{ijk} [S_{\mu\nu}^{\rho\sigma} A_{\sigma}^k(x,y) - i \epsilon_{\mu\nu}^{\rho\sigma} S_{\sigma}^k(x,y)] \right\} \quad (2) \\
 [J_{\mu 5}^i(x), J_{\nu}^j(y)] &\equiv [J_{\mu}^i(x), J_{\nu 5}^j(y)] \equiv \partial_{\rho} D(z) \left\{ i f_{ijk} [S_{\mu\nu}^{\rho\sigma} S_{\sigma 5}^k(x,y) - i \epsilon_{\mu\nu}^{\rho\sigma} A_{\sigma}^k(x,y)] + \right. \\
 &\quad \left. + d_{ijk} [S_{\mu\nu}^{\rho\sigma} A_{\sigma 5}^k(x,y) - i \epsilon_{\mu\nu}^{\rho\sigma} S_{\sigma}^k(x,y)] \right\} \\
 S_{\mu\nu}^{\rho\sigma} &= \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} + \delta_{\mu}^{\sigma} \delta_{\nu}^{\rho} - g_{\mu\nu} g^{\rho\sigma}, \quad z = x-y, \quad D(z) = \frac{1}{2\pi} \epsilon(z) \delta(z^2).
 \end{aligned}$$

Furthermore, Fritzsch and Gell-Mann have postulated the following closed algebraic system for the bilocal currents :

$$\begin{aligned}
 [J_{\mu}^i(x,u), J_{\nu}^j(y,v)] &\equiv [J_{\mu 5}^i(x,u), J_{\nu 5}^j(y,v)] \equiv \partial_{\rho} D(x-v) (i f_{ijk} - d_{ijk}) (S_{\mu\nu}^{\rho\sigma} J_{\sigma}^k(y,u) + \\
 &\quad + i \epsilon_{\mu\nu}^{\rho\sigma} J_{\sigma 5}^k(y,u)) + \partial_{\rho} D(u-y) (i f_{ijk} + d_{ijk}) (S_{\mu\nu}^{\rho\sigma} J_{\sigma}^k(x,v) - i \epsilon_{\mu\nu}^{\rho\sigma} J_{\sigma 5}^k(x,v)) \quad (3) \\
 [J_{\mu 5}^i(x,u), J_{\nu}^j(y,v)] &\equiv [J_{\mu}^i(x,u), J_{\nu 5}^j(y,v)] \equiv \partial_{\rho} D(x-v) (i f_{ijk} - d_{ijk}) (S_{\mu\nu}^{\rho\sigma} J_{\sigma 5}^k(y,u) + \\
 &\quad + i \epsilon_{\mu\nu}^{\rho\sigma} J_{\sigma}^k(y,u)) + \partial_{\rho} D(u-y) (i f_{ijk} + d_{ijk}) (S_{\mu\nu}^{\rho\sigma} J_{\sigma 5}^k(x,v) - i \epsilon_{\mu\nu}^{\rho\sigma} J_{\sigma}^k(x,v)),
 \end{aligned}$$

where x, u, y, v are points on a lightlike ray. In a formal quark vector gluon theory⁴⁾ the last condition is necessary and sufficient for the validity of this bilocal algebra.

Most of the immediate experimental consequences of light cone current algebra involve the deep inelastic structure functions. Relations of this type appearing in Ref. 3) and additional ones discovered more recently⁵⁾ follow from Eq. (2). The experimental verification of these results will confirm the basic idea of a free field type of behaviour in light cone current commutators and the spin $\frac{1}{2}$ nature of the constituent fields. They will not, however, test the algebra of the bilocal operators themselves. Since Eqs. (2) do not constitute a closed algebraic system, the demonstration of their validity - while interesting in its own right - will not be a satisfactory experimental check on light cone current algebra. In order to test the completeness of the elegant hypothesis of Fritzsch and Gell-Mann, we thus need to have predictions based on Eqs. (3). To satisfy the condition of the relevant currents acting on a lightlike ray, these will necessarily involve reactions whose amplitudes contain at least two currents - each carrying a high virtual mass (real or imaginary).

It was pointed out in Ref. 3) that realistic predictions (i.e., those requiring experiments feasible in the near future) testing Eqs. (3) cannot be obtained by being confined strictly to commutators. One has to consider the generalization of the algebra to a set of relations for physical ordered *) products ⁶⁾ of currents. Now Fritzsch and Gell-Mann have suggested such a generalization for the connected part of the bilocal algebra based on the $i\epsilon$ prescription of free field theory. It may be worth while to mention here that the recently proposed ⁷⁾ tests of this generalized bilocal algebra in terms of experiments with colliding e^+e^- and e^-e^- beams involve the disconnected part of a commutator of two bilocal currents; for this part the above-mentioned generalization is surmised by the authors of Ref. 3) to be less reliable **) . On the other hand, the tests of the connected part proposed so far ⁸⁾ consider two-current amplitudes in which only one of the currents carries a high virtual mass; hence in the squared matrix element not all currents act on a lightlike ray. Fritzsch and Gell-Mann ³⁾ have recommended the study of the reaction $e^-p \rightarrow e^- \mu^+ \mu^- + \text{anything}$ at large electronic momentum transfers and with a high mass μ pair. The Compton terms in this reaction are - under the said conditions - theoretically ideal for such a test. Unfortunately, the Bethe-Heitler terms - which interfere coherently with the Compton terms - create a formidable technical obstacle against formulating the test in terms of experimentally measured quantities.

In view of the above discussion, it will be correct to say that so far there has not been any reliable, well-defined and experimentally feasible theoretical proposal to test the connected part of the generalized bilocal algebra. Such a proposal is precisely what the present paper contains. We consider the reaction $\ell N \rightarrow \ell W^\pm + \text{anything}$ at large negative values of the leptonic momentum transfer squared $-Q^2$. Here ℓ can be e or μ , W^\pm are the vector mediators of weak forces (with mass $M_W > 3$ GeV) and are assumed to be decoupled ^{***)} from strong interactions. Since both Q^2 and M_W^2 are large, each of the two virtual currents in the amplitude carries a high mass. We shall show (Section 3) that the applicability of the generalized

*) By this we mean the time ordered product together with necessary non-covariant "seagull" type terms that make up the full amplitude.

**) This is because the disconnected part, being more singular than the connected part, consists of products of distributions in configuration space for which the $i\epsilon$ prescription is ill-defined.

***) If the W^\pm have strong interactions, there will be a new fundamental length in the theory, namely M_W^{-1} , which will destroy asymptotic scale invariance.

bilocal algebra is now ensured in this case provided the momentum transfer squared t between the virtual photon and the W is also given a large negative value. Using the algebra in the said kinematic region one can then obtain an explicit expression for the differential cross-section in terms of the deep inelastic neutrino structure functions.

In Section 2 we consider the kinematics of the process and write the differential cross-section in terms of two structure functions W_1 and W_2 which in turn are related to four more basic structure functions V_1, V_2, V_3 and V_4 . Section 3 presents the light cone analysis according to which all the currents act on a lightlike ray provided the virtual photon is highly spacelike and the square of the momentum transfer between the γ and the W is large in magnitude. In Section 4 we explain the equivalence between the direct application of light cone algebra and the use of a parton analogy; we then use this analogy to obtain explicit expressions for V_1, V_2, V_3 and V_4 in terms of the deep inelastic neutrino structure functions. Section 5 contains a discussion of the feasibility of studying this reaction experimentally at the National Accelerator Laboratory.

2. NOTATION AND KINEMATICS

Figure 1 illustrates the two possible diagrams for the reaction $\bar{N} \rightarrow \bar{N} W^\pm + \text{anything (n)}$ in the lowest orders of the electromagnetic and semi-weak coupling constants e and $g = (GM_W^2)/\sqrt{2}$ respectively. The four momenta of the various particles are introduced in the figure. The black dot in Fig. 1a represents the interaction of the virtual photon via the hadronic electromagnetic current; the cross in either diagram stands for the interaction of the W via the hadronic weak (Cabibbo) current. Throughout this paper we shall ignore the Cabibbo angle for simplicity so that these two currents have the form

$$\begin{aligned} J_\mu^{\text{EM}} &= \bar{q} \gamma_\mu \frac{1}{2} (\lambda^3 + \frac{1}{\sqrt{3}} \lambda^8) q = \bar{q} \gamma_\mu \lambda_Q q, \\ J_\mu^{\pm W} &= \bar{q} \gamma_\mu (1 - \gamma_5) \frac{1}{2} (\lambda^1 \pm i \lambda^2) q = \bar{q} \gamma_\mu (1 - \gamma_5) \lambda_\pm q \end{aligned} \quad (4)$$

in the free quark model. The S matrix can be written in the lab. frame as

$$S_{fi} = \left(\frac{1}{2\pi}\right)^6 \frac{m_e \sqrt{M}}{\sqrt{VEP_e 2k_e}} \frac{8e^2}{q^2 + i\epsilon} i(2\pi)^4 \delta^{(4)}(p+q-k-p_n) \bar{U}(p_e') \gamma_\mu U(p_e) \epsilon^\mu M_{\mu\nu}^{n\pm} \quad (5)$$

In Eq. (5) E and E' are the lab energies of the initial and the final lepton, respectively, M is the mass of the proton, ϵ^{α} is the polarization of the W boson and $\mathcal{M}_{\mu\alpha}^{n\pm}$ is the invariant amplitude containing the normalization factors of the state n :

$$\begin{aligned} \left(\frac{1}{2\pi}\right)^3 \sqrt{\frac{M}{p_0}} \epsilon^{\alpha} \mathcal{M}_{\mu\alpha}^{n\pm} &= i \int d^4 x e^{-i \frac{q+k}{2} \cdot x} \epsilon^{\alpha} \langle n | T^* J_{\mu}^{EM}(\frac{x}{2}) J_{\alpha}^{\mp W}(-\frac{x}{2}) | p \rangle \pm \\ &\pm \frac{\epsilon^{\alpha}}{M_W^2 - t} \Gamma_{\mu\alpha}^{\beta} \langle n | J_{\beta}^{\mp W} | p \rangle, \\ \Gamma_{\mu\alpha}^{\beta} &= \left\{ g_{\mu\beta} (2k - q) + g_{\mu\gamma} q_{\gamma} - g_{\mu\alpha} k_{\gamma} + \chi (g_{\mu\gamma} q_{\alpha} - g_{\mu\alpha} q_{\gamma}) \right\} \left\{ g_{\beta\gamma} - \frac{(k - q)^{\beta} (k - q)^{\gamma}}{M_W^2} \right\} \end{aligned} \quad (6)$$

Here T^* stands for the covariant T product⁶⁾ and χ is the anomalous magnetic moment of the W . We now introduce two functions (to be specified later from dynamical considerations) $\mathcal{W}_{1,2}(Q^2, s, t, u)$ via the following definition:

$$\begin{aligned} \mathcal{W}_1^{\pm}(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2}) + \mathcal{W}_2^{\pm} \frac{1}{M^2} (p_{\mu} - \frac{P q}{q^2} q_{\mu})(p_{\nu} - \frac{P q}{q^2} q_{\nu}) + (\text{terms antisym. in } \mu, \nu) &\equiv \\ \equiv \sum_n \int d^4 k \Theta(k) \delta(k^2 - M_W^2) \delta(t - (k - q)^2) \delta(u - (k - p)^2) (2\pi)^3 \times & \\ \times \delta^{(4)}(p + q - k - p_n) \mathcal{M}_{\mu\beta}^{n\pm} \mathcal{M}_{\nu\alpha}^{n\pm} \left(-g_{\mu\beta}^{\alpha} + \frac{k^{\alpha} k^{\beta}}{M_W^2} \right). & \end{aligned} \quad (7)$$

Here $s = (p+q)^2$, $t = (k-q)^2$, $u = (k-p)^2$, $Q^2 = -q^2$ and M_W is the mass of the W boson. The initial proton is spin averaged. Now the differential cross-section can be written as:

$$\frac{d\sigma^{\pm}}{dQ^2 ds dt du} = \frac{\alpha^2 g^2}{4\pi^2 M Q^4} \cdot \frac{E'}{E} \left[\mathcal{W}_2^{\pm} \cos^2 \frac{\theta_L}{2} + 2 \mathcal{W}_1^{\pm} \sin^2 \frac{\theta_L}{2} \right], \quad (8)$$

where θ_L is the lepton scattering angle in the lab.

We also introduce four functions $V_{1,2,3,4}(Q^2, s, t, u)$ as follows:

$$\begin{aligned} \sum_n (2\pi)^3 \delta^{(4)}(p + q - k - p_n) \mathcal{M}_{\mu\beta}^{n\pm} \mathcal{M}_{\nu\alpha}^{n\pm} \left(-g_{\mu\beta}^{\alpha} + \frac{k^{\alpha} k^{\beta}}{M_W^2} \right) &= T_{\mu\nu}^{\pm} \equiv \\ \equiv V_1^{\pm} \left(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) + V_2^{\pm} (p_{\mu} - \frac{P q}{q^2} q_{\mu})(p_{\nu} - \frac{P q}{q^2} q_{\nu}) + & \\ + V_3^{\pm} (k_{\mu} - \frac{k q}{q^2} q_{\mu})(k_{\nu} - \frac{k q}{q^2} q_{\nu}) + V_4^{\pm} \left\{ (p_{\mu} - \frac{P q}{q^2} q_{\mu})(k_{\nu} - \frac{k q}{q^2} q_{\nu}) + \right. & \\ \left. + (k_{\mu} - \frac{k q}{q^2} q_{\mu})(p_{\nu} - \frac{P q}{q^2} q_{\nu}) \right\} + (\text{terms antisym. in } \mu, \nu). & \end{aligned} \quad (9)$$

From Eqs. (7) and (9), in the asymptotic region (see next Section), one obtains :

$$\begin{aligned} W_1^{\pm} &= \frac{\pi}{s+Q^2} \left[\frac{V_1^{\pm}}{2} + \left\{ Q^2 \left(\frac{M_W^2 - u}{Q^2 + s} \right)^2 + \frac{(M_W^2 - u)(s+u)}{s+Q^2} - M_W^2 \right\} \frac{V_3^{\pm}}{4} \right], \\ W_2^{\pm} &= \frac{\pi M^2}{s+Q^2} \left[\frac{V_2^{\pm}}{2} + \frac{V_4^{\pm}}{(s+Q^2)^2} \left\{ (s+u)(s+Q^2) + 2(M_W^2 - u)Q^2 \right\} + \right. \\ &\quad \left. + \frac{V_3^{\pm}}{2(s+Q^2)^2} \left\{ (s+u)^2 + \frac{6(s+u)(M_W^2 - u)Q^2}{s+Q^2} + \frac{6Q^2(M_W^2 - u)^2}{(s+Q^2)^2} - \right. \right. \\ &\quad \left. \left. - 2M_W^2Q^2 \right\} \right]. \end{aligned} \quad (10)$$

Our theoretical considerations will be able to relate these V_i functions - in appropriate asymptotic regions - to the deep inelastic leptonic structure functions.

3. LIGHT CONE ANALYSIS

For the purpose of the coming discussion, it will be convenient to rewrite $T_{\mu\nu}^{\pm}$ by substituting Eq. (5) into Eq. (9). Introducing the four-vectors $r = q+k/2$, $s = k-q$, we then have :

$$\begin{aligned} T_{\mu\nu}^{\pm} &= \frac{1}{2\pi} \left(-g_{\mu\nu} + \frac{k_{\mu}k_{\nu}}{M_W^2} \right) (2\pi)^3 \frac{p_0}{M} \langle p | \int d^4x d^4y d^4z e^{-i\omega x} e^{i\omega y} e^{-i\omega z} \left[\bar{T} J_{\beta}^{\pm} \left(\frac{z-y}{2} \right) \times \right. \right. \\ &\quad \times V_{\nu} \left(\frac{y+z}{2} \right), \bar{T} V_{\mu} \left(\frac{x-z}{2} \right) J_{\alpha}^{\pm} \left(\frac{-x-z}{2} \right) \left. \right] \pm \frac{1}{16(M_W^2 - t)} \int d^4z e^{-i\omega z} \left\{ \bar{V}_{\mu}^{\alpha} \int d^4y \times \right. \\ &\quad \times e^{i\omega y} \left[\bar{T} J_{\beta}^{\pm} \left(\frac{z-y}{2} \right) V_{\nu} \left(\frac{y+z}{2} \right), J_{\alpha}^{\pm} \left(\frac{-x-z}{2} \right) \right] + \bar{V}_{\mu}^{\alpha} \int d^4x e^{-i\omega x} \left[J_{\alpha}^{\pm} \left(\frac{x}{2} \right), \bar{T} V_{\mu} \left(\frac{x-z}{2} \right) \times \right. \\ &\quad \times J_{\alpha}^{\pm} \left(\frac{-x+z}{2} \right) \left. \right] + \frac{\bar{V}_{\mu}^{\alpha} \bar{V}_{\nu}^{\beta}}{(M_W^2 - t)^2} \int d^4z e^{-i\omega z} \left[J_{\alpha}^{\pm} \left(\frac{z}{2} \right), J_{\beta}^{\pm} \left(\frac{-x-z}{2} \right) \right] | p \rangle. \end{aligned} \quad (11)$$

Here T^* is the physical ordered product, $J^{\pm W}$ (V^{EM}) refers to the W boson (electromagnetic) current and photon- W -photon vertex $\Gamma_{\mu\alpha\beta}$ is given by

$$\Gamma_{\mu\alpha\beta} = g_{\alpha\beta} (2k - q)_{\mu} - g_{\beta\mu} (k - q)_{\alpha} - g_{\alpha\mu} k_{\beta} + \lambda (g_{\beta\mu} q_{\alpha} - g_{\alpha\mu} q_{\beta}).$$

We record here two observations on Eq. (11) which will be useful later :

$$1) \quad T_{\mu\nu}^{\pm}(q, k) = -T_{\nu\mu}^{\pm}(-q, -k, J^+ \Rightarrow J^-);$$

2) since in going from Eq. (7) to Eq. (8) only terms symmetric in μ, ν survive, we need consider only the ($\mu = \nu$) symmetric part in the right-hand side of Eq. (11).

Let us now consider the asymptotic region where $Q^2, s \rightarrow \infty$ and $t \rightarrow -\infty$ in such a way that the ratios $Q^2/s, t/Q^2$ are fixed. Since M_W is very large, we shall assume that $k^2/Q^2, k^2/s$ and k^2/t are finite. In this region clearly $|r^2|, |s^2|$ and $|r \cdot s| \rightarrow \infty$ in such a way that the ratio between any two of them is finite. Then, because of the Riemann-Lebesgue lemma, the integrals in each right-hand side term of Eq. (11) are dominated²⁾ by the singularities in the regions $z^2 = 0(1/|s^2|) \rightarrow 0, y^2 = 0(1/|r^2|) \rightarrow 0$, and $x^2 = 0(1/|r^2|) \rightarrow 0$. Now, following Ref. 3), we write

$$\begin{aligned} r^{\mu} &= u e^{\mu} + a^{\mu}, \\ s^{\mu} &= v e^{\mu} + b^{\mu}, \end{aligned}$$

where e^{μ} is a lightlike vector and a^{μ}, b^{μ} are fixed timelike vectors. In our asymptotic region $|u|, |v| \rightarrow \infty$ in such a way that u/v is fixed. The requirement that the phase $sz = ve \cdot z + b \cdot z$ be bounded^{*)} in this limit requires that z be either proportional to or orthogonal to e . The choice $e = (1, 0, 0, 1)$ in a particular frame immediately shows the latter possibility to be consistent only with a spacelike z in contradiction with the above requirement that z^2 tend to zero. Hence, in this limit, z is proportional to e . Similarly, by considering the boundedness of the phases $r \cdot x$ and $r \cdot y$ it is easy to see that x and y also have to be proportional to e in this situation. Hence the points $0, x, y, z$ are effectively on a lightlike ray. Since in each term of the right-hand side of Eq. (11), the currents involved are acting on a lightlike ray relative to each other, the condition⁴⁾ for the applicability of the generalized bilocal algebra of Fritzsch and Gell-Mann is established.

4. PARTON ANALOGY AND CALCULATION IN MOMENTUM SPACE

In the previous section we have validated the application of the generalized bilocal algebra to the right-hand side of Eq. (11) in the asymptotic regions of present interest. In these limits it is legitimate to treat the currents there as free quark-currents and to carry out Wick contractions on their products till only bilocal currents remain between the spin averaged

*) This argument is similar to that given in Section III of the first paper quoted in Ref. 7).

forward nucleon states. This explicit substitution of Eqs. (3) and their generalized versions into Eq. (11) is possible but involves lengthy manipulations of Green's functions under Fourier integrals in configuration space. It is simpler *) to treat the problem in momentum space using an analogy with parton calculations. This is elaborated below.

Let us consider the deep inelastic scattering of a vector current J_μ^i off a spin averaged proton (Fig. 2a). We can define the structure functions $W_{1,2}^{ij}$ as :

$$W_1^{ij} \left(-q_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2^{ij}}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) = \\ = \frac{1}{\pi} \frac{p_0}{M} (2\pi)^3 \text{Abs.} i \int dx e^{iq \cdot x} \langle p | T J_\mu^i(x) J_\nu^j(0) | p \rangle.$$

In the Bj limit when $q^2 \rightarrow -\infty$, $p \cdot q \rightarrow \infty$ with $\omega = (2p \cdot q)/-q^2$ fixed, the quark parton calculation takes the Born diagram (Fig. 2b) for forward current scattering off massless **) partons with $\bar{p} = \omega^{-1} p$ as the parton momentum and substitutes the corresponding expression in the right-hand side of the above equation after smearing it with a distribution function $f(\omega)$. The result is ⁹⁾ :

$$F_2^{ij}(q, p) = \sum_{|q\rangle} \omega^{-1} f(\omega) \langle q | \frac{\lambda^i}{2} \frac{\lambda^j}{2} | q \rangle = \\ = \sum_{|q\rangle} \frac{\omega^{-1}}{2} [i f^S(\omega) f_{ijk}^{ij} + f^A(\omega) d_{ijk}^{ij}] \langle q | \frac{\lambda^k}{2} | q \rangle = (12) \\ = F_2^{ij}(-q, p)$$

where $|q\rangle$ is a one-parton state, F_2 is the Bjorken function

$$F_2 = \lim_{B_j} \frac{p \cdot q}{M} W_2, \quad f^S(\omega) = f(-\omega), \quad f^A(\omega) = -f^A(-\omega)$$

and $f(\omega) = f^S(\omega) + f^A(\omega)$. If we now define $A^k(\omega)$ and $S^k(\omega)$ by means of the relations :

$$A^k(\omega) p_\mu = \frac{p_0}{\pi} (2\pi)^3 \int d(z \cdot p) e^{-i\omega z \cdot p} \langle p | A_\mu^k(x, y) | p \rangle, \\ S^k(\omega) p_\mu = \frac{p_0}{\pi} (2\pi)^3 \int d(z \cdot p) e^{-i\omega z \cdot p} \langle p | S_\mu^k(x, y) | p \rangle, \quad (13)$$

$z = x - y$

we see that Eq. (12) is equivalent to the expression calculated by the direct use of light cone current algebra [Ref. 3), Eq. (A.12)], provided we make the identification :

*) This idea is due to H. Fritzsch, private communication.

**) Terms involving the parton mass correspond to non-leading singularities on the light cone ³⁾.

$$S^k(\omega) = \sum_{1q} f^S(\omega) \langle q | \frac{\lambda^k}{2} | q \rangle, \\ A^k(\omega) = \sum_{1q} f^A(\omega) \langle q | \frac{\lambda^k}{2} | q \rangle. \quad (14)$$

This equivalence being that of Born term calculations in configuration space and in momentum space is of general validity. It extends to amplitudes with two highly virtual currents (such as the one presently under consideration) whose cross-sections involve commutators of bilocal currents.

We here employ the identification of Eq. (14) in using a parton analogy to rewrite the right-hand side of Eq. (11) in terms of $S^k(\omega)$ and $A^k(\omega)$. This is strictly for simplicity of calculation and does not negate the fact that our final results really follow from Eqs. (2) and (3) generalized to include physical ordered products. The parton calculation of $T_{\mu\nu}^{\pm}$ in momentum space is related to the expression in the right-hand side of Eq. (11) in the same way as the Feynman rules for Born diagrams are related to the corresponding terms in the perturbation series for the U matrix written as integrals in configuration space. In the parton analogy we have to consider here the sum of the three Born diagrams illustrated in Fig. 3. In each case a quark parton of four-momentum $\bar{p} = \gamma^\mu p^\mu$ is scattered by virtual photon of momentum q into another quark and a W carrying the momentum k . The parton invariant amplitude can be written as :

$$\epsilon^* \bar{a}_{\mu\nu}^{\pm} = \bar{u}(\bar{p}) \left[\lambda_{\mp} \lambda_{\theta} \delta(1-\gamma_5) \frac{\bar{p} + \gamma}{q^2 + 2q \cdot \bar{p}} \gamma_{\mu} + \lambda_{\theta} \lambda_{\mp} \gamma_{\mu} \frac{\bar{p} - \gamma}{q^2 - 2q \cdot \bar{p}} \mp \right. \\ \left. \mp \lambda_{\mp} \frac{\epsilon^* \tilde{\Gamma}_{\mu\nu\rho}}{q^2 + 2q(\bar{p} - \bar{p})} \gamma^{\rho} (1 - \gamma_5) \right] U(\bar{p}). \quad (15)$$

Here the mass of the quark parton has been ignored *) in the leading contribution. Since the weak current is effectively conserved on the light cone, i.e.,

$$(k - q)^\beta J_\beta^{\pm W} \xrightarrow{\text{L.C.}} 0,$$

we have replaced the $\gamma^W \gamma$ vertex tensor $\Gamma_{\mu\nu\beta}$ by $\tilde{\Gamma}_{\mu\nu\beta}$ where

$$\tilde{\Gamma}_{\mu\nu\beta} = g_{\mu\beta} (2k - q)_\nu + (1 + \lambda) (q_\mu q_\nu - q_\nu q_\mu). \quad (16)$$

Now we can write $T_{\mu\nu}^{\pm}$ of Eq. (9) - for a proton target - as :

*) See footnote *) on p.8.

$$T_{\mu\nu}^{\pm} = \int_0^1 d\tau \frac{f(\tau)}{\tau} \frac{1}{4M} \delta[2\tau p \cdot (q-k) + t] \text{Tr} [\bar{P} B_{\mu\alpha}^{\mp} \bar{P} B_{\nu\beta}^{\pm}] \times (-g^{\alpha\beta} + \frac{k^\alpha k^\beta}{M_w^2}),$$

where

$$\begin{aligned} B_{\mu\alpha}^{\mp} &= \lambda_{\mp} \lambda_Q \gamma_{\mu} \frac{\bar{P} + q}{(\bar{P} + q)^2} \gamma_{\mu} (1 - \gamma_5) + \lambda_Q \lambda_{\mp} \gamma_{\mu} \frac{\bar{P} - q}{(\bar{P} - q)^2} \gamma_{\mu} (1 - \gamma_5) \mp \\ &\mp \lambda_{\mp} \frac{\tilde{\Gamma}_{\mu\alpha}^{\pm}}{M_w^2 - t} \gamma^{\mu} (1 - \gamma_5) \end{aligned} \quad (17)$$

and

$$\begin{aligned} B_{\nu\beta}^{\pm} &= \lambda_Q \lambda_{\pm} \gamma_{\nu} \frac{\bar{P} + q}{(\bar{P} + q)^2} \gamma_{\nu} (1 - \gamma_5) + \lambda_{\pm} \lambda_Q \gamma_{\nu} \frac{\bar{P} - q}{(\bar{P} - q)^2} \gamma_{\nu} (1 - \gamma_5) \mp \\ &\mp \lambda_{\pm} \frac{\tilde{\Gamma}_{\nu\beta}^{\pm}}{M_w^2 - t} \gamma^{\nu} (1 - \gamma_5). \end{aligned}$$

Finally, if we define

$$\eta \equiv -\frac{t}{2p \cdot (q-k)} = -\frac{t}{s+u+Q^2-M_w^2}, \quad (18)$$

we have :

$$T_{\mu\nu}^{\pm} = \frac{f(\eta)}{4M\eta|2p \cdot (q-k)|} \text{Tr} [\bar{P} B_{\mu\alpha}^{\mp} \bar{P} B_{\nu\beta}^{\pm}] (-g^{\alpha\beta} + \frac{k^\alpha k^\beta}{M_w^2}) \quad (19)$$

with $\bar{p} = \eta p$. In Eq. (19) $f(\eta) = f^S(\eta) + f^A(\eta)$ and f^S (f^A) multiplies that part of the trace that is symmetric (antisymmetric) under the transformation $q \rightleftharpoons -q$, $k \rightleftharpoons -k$ and $\lambda_+ \rightleftharpoons \lambda_-$.

By explicit calculation, we see - in comparison with Eq. (9) - that for a proton target

$$\begin{aligned} V_j^{\pm} &= \frac{1}{4M\eta|2p \cdot (q-k)|} \left[\langle q | \frac{1}{2} \left(\sqrt{\frac{2}{3}} \lambda^0 + \lambda^3 + \frac{1}{\sqrt{3}} \lambda^8 \right) | q \rangle \{ f^S(\eta) \pm f^A(\eta) \} V_{j1}^{\pm} + \right. \\ &\quad \left. + \langle q | \frac{1}{2} \left(\sqrt{\frac{2}{3}} \lambda^0 - \lambda^3 + \frac{1}{\sqrt{3}} \lambda^8 \right) | q \rangle \{ f^S(\eta) \mp f^A(\eta) \} V_{j2}^{\pm} \right], \quad (20) \end{aligned}$$

where j runs from 1 to 4 and V_{j1}^{\pm} and V_{j2}^{\pm} are explicit functions of $(\bar{p}+q)^2$, $(\bar{P}-q)^2$, t and Q^2 , i.e., of s, t, u, Q^2 and η which are given below. Substituting Eq. (14) into Eq. (20), and rewriting S^0, S^3, S^8 and A^0, A^3, A^8 in terms of the deep inelastic lepton structure functions following Ref. 3), i.e., the relations

$$\eta^{-1} [F_2^{vp}(\eta) + F_2^{\bar{v}p}(\eta)] = 4 \left[\sqrt{\frac{2}{3}} A^0(\eta) + \frac{1}{\sqrt{3}} A^3(\eta) \right],$$

$$F_3^{vp}(\eta) + F_3^{\bar{v}p}(\eta) = -4 \left[\sqrt{\frac{2}{3}} S^0(\eta) + \frac{1}{\sqrt{3}} S^3(\eta) \right],$$

$$\eta^{-1} [F_2^{vp}(\eta) - F_2^{\bar{v}p}(\eta)] = -4 S^3(\eta),$$

$$F_3^{vp}(\eta) - F_3^{\bar{v}p}(\eta) = 4 A^3(\eta),$$

we have :

$$V_j^+ = \frac{1}{8M\eta |2p.(q-k)|} \left[\left\{ \eta^{-1} F_2^{vp}(\eta) - F_3^{\bar{v}p}(\eta) \right\} V_{j1}^+ - \left\{ \eta^{-1} F_2^{\bar{v}p}(\eta) + F_3^{vp}(\eta) \right\} V_{j2}^+ \right],$$

$$V_j^- = \frac{1}{8M\eta |2p.(q-k)|} \left[- \left\{ \eta^{-1} F_2^{vp}(\eta) + F_3^{\bar{v}p}(\eta) \right\} V_{j1}^- + \left\{ \eta^{-1} F_2^{\bar{v}p}(\eta) - F_3^{vp}(\eta) \right\} V_{j2}^- \right]. \quad (21)$$

Similarly, for a neutron target, we obtain (with S^3 and A^3 now changing sign) :

$$V_j^+ = \frac{1}{8M\eta |2p.(q-k)|} \left[\left\{ \eta^{-1} F_2^{vp}(\eta) - F_3^{\bar{v}p}(\eta) \right\} V_{j1}^+ - \left\{ \eta^{-1} F_2^{\bar{v}p}(\eta) + F_3^{vp}(\eta) \right\} V_{j2}^+ \right],$$

$$V_j^- = \frac{1}{8M\eta |2p.(q-k)|} \left[- \left\{ \eta^{-1} F_2^{\bar{v}p}(\eta) + F_3^{vp}(\eta) \right\} V_{j1}^- + \left\{ \eta^{-1} F_2^{vp}(\eta) - F_3^{\bar{v}p}(\eta) \right\} V_{j2}^- \right]. \quad (22)$$

In order to display the functions v_{jk}^\pm - with $j = 1, 2, 3, 4$ and $k = 1, 2$ - explicitly in terms of our primary variables s, t, u, Q^2 and η , it will be convenient to introduce some intermediate quantities. We define the secondary quantities ξ, a, b, σ, ρ in Table I and the tertiary quantities $A_k^\pm, B_k, X_k^\pm, Y_k^\pm, W_k^\pm, \Pi_k^\pm$ and \sum_k^\pm ($k = 1, 2$) in Table II. Then, by direct calculation of the trace in Eq. (19), we obtain :

$$\begin{aligned} V_{1k}^\pm &= \frac{1}{9\xi} (bX_k^\pm - aY_k^\pm)^2 + \frac{1}{2t} (b\sigma - a\rho)^2 - \frac{2}{9} (\xi + Q^2 t) A_k^\pm - \frac{2}{9} \xi B_k^2 + \sum_k^\pm - 2\Pi_k^\pm - 2W_k^\pm \\ V_{2k}^\pm &= \eta^2 \left\{ \left[\frac{(a-b)^2}{t\xi} + \frac{4Q^2}{\xi} \right] \left[\frac{2}{9} \xi B_k^2 - \frac{2}{9\xi} (bX_k^\pm - aY_k^\pm)^2 - \frac{1}{t} (b\sigma - a\rho)^2 + 2W_k^\pm + \right. \right. \\ &\quad \left. \left. + 4\Pi_k^\pm - \sum_k^\pm \right] - \frac{4Q^2}{9\xi} (X_k^\pm + Y_k^\pm)^2 + \frac{8Q^2}{9} A_k^\pm - \frac{2Q^2}{t} (\rho + \sigma)^2 \right\}, \quad (23) \end{aligned}$$

$$V_{3k}^{\pm} = \alpha^2 \left[\frac{2}{9} \xi B_k^2 - \frac{2}{9\xi} (bX_k^{\pm} - aY_k^{\pm})^2 - \frac{1}{t} (b\sigma - a\rho)^2 + 2W_k^{\pm} + 4\pi_k^{\pm} - \sum_k^{\pm} \right] - \frac{4Q^2 X_k^{\pm 2}}{9\xi} - \frac{2Q^2 \xi^2}{t},$$

$$V_{4k}^{\pm} = \gamma \left\{ -\left(\frac{\alpha^2}{t\xi} + \frac{Q^2 t + \xi}{\xi t} \right) \left[\frac{2}{9} B_k^2 - \frac{2}{9\xi} (bX_k^{\pm} - aY_k^{\pm})^2 - \frac{1}{t} (b\sigma - a\rho)^2 + 2W_k^{\pm} + 4\pi_k^{\pm} - \sum_k^{\pm} \right] + \frac{2Q^2}{9\xi} (2X_k^{\pm 2} + 9\pi_k^{\pm}) - \frac{2}{9} Q^2 (2A_k^{\pm} - 9\frac{\xi^2}{t}) \right\}.$$

Equations (21), (22) and (23) and Tables I and II, in conjunction with Eqs. (8) and (10) of Section 2, constitute our final result. We remark that, as mentioned in Section 3, only the part of $T_{\mu\nu}^{\pm}$ that is symmetric in μ, ν has been considered. Moreover, the property $T_{\mu\nu}^{\pm}(q, k) = -T_{\nu\mu}^{\pm}(-q, -k)$ is manifest if we remember that $S(\gamma)$ and $A(\gamma)$ are even and odd functions of γ respectively.

5. DISCUSSION OF EXPERIMENTAL FEASIBILITY

If the W has a mass in the range between 3 and 15 GeV, this experiment will be feasible at the National Accelerator Laboratory. There are now several NAL proposals to search for an intermediate vector boson in this mass range ¹⁰⁾. The reaction considered by us is not the best process for finding a W . However, once such an intermediate vector boson is found, it is an appropriate reaction to test the Fritzsch-Gell-Mann bilocal algebra in its generalized version, given the availability of sufficient knowledge about the deep inelastic neutrino structure functions. The relevant differential cross-sections can be expected to be measurably large on the basis of the following argument. The use of the bilocal algebra in the reaction under consideration is equivalent to a parton-⁹⁾ description of the process in the kinematic region specified above. An elaboration of this equivalence has been given in Section 4. Suffice it to focus here on the consequent implication that the cross-section for inelastic electro- or muoproduction of W^{\pm} in this region should be of the order of that for the corresponding "elastic" production $\ell N \rightarrow \ell W^{\pm} N'$ from bare pointlike nucleons. The latter was considered in the detailed investigation of the reaction $\ell N \rightarrow \ell W^{\pm} N'$ by Fearing, Pratap and Smith ¹¹⁾. They estimated, [cf., Fig. 6 of Ref. 11] $d\sigma/dQ^2 \sim 10^{-36} \text{ cm}^2/\text{GeV}^2$ for $Q^2 \sim 10(\text{GeV}/c)^2$, with an incident lepton beam - energy ~ 200 GeV and for a W of mass 5 GeV. Judging from the model dependent parton calculations by Mikaelian ¹²⁾ on the inelastic photoproduction of W bosons, we may justifiably expect the cross-section for our reaction to be in fact several times bigger than the numbers given in Ref. 11). The rather sharp forward W peak, obtained in the model calculations of Mikaelian may broaden considerably when $Q^2 \simeq 10(\text{GeV}/c)^2$.

Large additional enhancement factors for the cross-section may be secured by the use of targets of heavy nuclei if the scattering from the different nucleons can be shown to be incoherent. Thus we see that the differential cross-sections of present interest ought to be measurable at NAL if the existence of the W in the mass range mentioned earlier is established. At any rate, it would be very worth while for experimentalists to consider such a measurement in order to test the connected part of the generalized bilocal algebra of Fritzsch and Gell-Mann.

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TABLE I : SECONDARY QUANTITIES

$$\xi = \{ \eta s - (1-\eta) Q^2 \} \{ \eta u + (1-\eta) M_W^2 \} + Q^2 M_W^2$$

$$a = \eta (s + Q^2)$$

$$b = \eta (M_W^2 - u) - Q^2 - M_W^2$$

$$\sigma = \frac{t - x \eta (M_W^2 - u)}{M_W (M_W^2 - t)}$$

$$p = \frac{t + x [\eta (s + Q^2) - Q^2 - M_W^2]}{M_W (M_W^2 - t)}$$

TABLE II : TERTIARY QUANTITIES

$$A_1^{\pm} = \left\{ \frac{4}{\eta(s+Q^2)-Q^2} \mp \frac{3(1+x)}{M_w^2-t} \right\} \left\{ \frac{2}{M_w^2-\eta(M_w^2-u)} \mp \frac{3(1+x)}{M_w^2-t} \right\},$$

$$A_2^{\pm} = \left\{ \frac{2}{\eta(s+Q^2)-Q^2} \pm \frac{3(1+x)}{M_w^2-t} \right\} \left\{ \frac{4}{M_w^2-\eta(M_w^2-u)} \pm \frac{3(1+x)}{M_w^2-t} \right\},$$

$$B_1 = \frac{2}{M_w^2-\eta(M_w^2-u)} - \frac{4}{\eta(s+Q^2)-Q^2},$$

$$B_2 = \frac{4}{M_w^2-\eta(M_w^2-u)} - \frac{2}{\eta(s+Q^2)-Q^2}$$

$$X_1^{\pm} = -3 \frac{\eta(s+Q^2)+2t}{M_w^2-t} \pm \frac{2\eta(M_w^2-u)}{M_w^2-\eta(M_w^2-u)},$$

$$X_2^{\pm} = -3 \frac{\eta(s+Q^2)+2t}{M_w^2-t} \mp \frac{4\eta(M_w^2-u)}{M_w^2-\eta(M_w^2-u)},$$

$$Y_1^{\pm} = -3 \frac{M_w^2(1-\eta)+\eta u+2t+Q^2}{M_w^2-t} \mp 4 \frac{\eta(s+Q^2)-Q^2-M_w^2}{\eta(s+Q^2)-Q^2},$$

$$Y_2^{\pm} = -3 \frac{M_w^2(1-\eta)+\eta u+2t+Q^2}{M_w^2-t} \pm 2 \frac{\eta(s+Q^2)-Q^2-M_w^2}{\eta(s+Q^2)-Q^2},$$

$$W_1^{\pm} = 2t \left\{ \frac{-Q^2-2t+2M_w^2}{(M_w^2-t)^2} \mp \frac{2}{M_w^2-t} \right\} + \xi \left\{ \frac{-x^2(t+Q^2)-t+2xM_w^2}{M_w^2(M_w^2-t)^2} \mp \frac{2x}{M_w^2(M_w^2-t)} \right\},$$

$$W_2^{\pm} = 2t \left\{ \frac{-Q^2-2t+2M_w^2}{(M_w^2-t)^2} \pm \frac{2}{M_w^2-t} \right\} + \xi \left\{ \frac{-x^2(t+Q^2)-t+2xM_w^2}{M_w^2(M_w^2-t)^2} \pm \frac{2x}{M_w^2(M_w^2-t)} \right\},$$

$$\Pi_k^{\pm} = \frac{2}{9} X_k^{\pm} Y_k^{\pm} + \xi \frac{\{t+x\eta(s+Q^2)-x(Q^2+M_w^2)\} \{t-x\eta(M_w^2-u)\}}{M_w^2(M_w^2-t)^2},$$

$$\sum_1^{\pm} = \frac{1}{9} \left[16M_w^2 t \left(\frac{1}{M_w^2-\eta(M_w^2-u)} \mp \frac{3}{M_w^2-t} \right) \left(\frac{2}{\eta(s+Q^2)-Q^2} \mp \frac{3}{M_w^2-t} \right) - \frac{9t}{M_w^2} \left\{ 2 \mp \frac{(1+x)}{M_w^2} \frac{-t+M_w^2-Q^2}{M_w^2-t} \right\}^2 \right],$$

$$\sum_2^{\pm} = \frac{1}{9} \left[16M_w^2 t \left(\frac{2}{M_w^2-\eta(M_w^2-u)} \pm \frac{3}{M_w^2-t} \right) \times \left(\frac{1}{\eta(s+Q^2)-Q^2} \pm \frac{3}{M_w^2-t} \right) - \frac{9t}{M_w^2} \left\{ 2 \pm (1+x) \frac{-t+M_w^2-Q^2}{M_w^2-t} \right\}^2 \right].$$

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FIGURE CAPTIONS

Figure 1 : Diagrams for inelastic electro- or muoproduction of W bosons off a nucleon.

Figure 2 : Absorptive part of forward virtual Compton scattering :
a) from a proton,
b) from a parton.

Figure 3 : Parton diagrams for the reaction γ (virtual) + parton $\rightarrow W +$ parton.

