Constraining four neutrino mass patterns from neutrinoless double beta decay

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Abstract

All existing data on neutrino oscillations (including those from the LSND experiment) imply a four neutrino scheme with six different allowed mass patterns. Some of the latter are shown to be disfavored by using a conservative upper bound on the $\beta\beta0\nu$ nuclear decay rate, if neutrinos are assumed to be Majorana particles. Comparisons are also made with restrictions from tritium β-decay and cosmology.

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Any observation of neutrinoless nuclear double beta decay would imply lepton nonconservation and a nonzero neutrino Majorana mass M_{ee}. The latter is defined as

$$M_{ee} = \sum_i m_i U_{ei}^2 \quad , \tag{1}$$

where m_i is the nonnegative ith physical Majorana mass and U_{ei} the matrix element which mixes the electrons neutrino ν_e with the mass eigenstate ν_i. There is now a considerable amount of flavor oscillation data from solar [1] and atmospheric [2] neutrinos, all of which can be accommodated within the standard picture of three neutrinos ν_e, ν_μ, ν_τ with tiny masses. The best fits yield two independent squared mass differences among the neutrinos: $\Delta_S \sim 4 \times 10^{-5}$ eV^2 for the solar case and $\Delta_A \sim 3 \times 10^{-3}$ eV^2 for the atmospheric one, the favored values of the corresponding mixing angles being $\sin^2 2\theta_S \sim 0.66$ and $\sin^2 2\theta_A \sim 1$. The following question then emerges: how is the overall mass scale of the neutrinos constrained? Specifically, how does one pin down the sum of the physical neutrino masses Σ_ν which controls the neutrino component of dark matter and hence neutrino effects on structure formation?

If we assume the neutrinos to be Majorana particles, there is a link between Σ_ν and M_{ee}. This link has been the subject of several recent investigations [3-8]. In particular, Barger et al [3] have given upper and lower bounds on Σ_ν in terms of M_{ee}, Δ_A and θ_S, neglecting Δ_S in comparison with Δ_A. When the small mixing angle relevant to unobserved neutrino oscillations at the CHOOZ reactor [9] is ignored, their inequalities become particularly simple, namely

$$2M_{ee} + \sqrt{M_{ee}^2 \pm \Delta_A} < \Sigma_\nu < \frac{2M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} \pm \Delta_A} \quad . \tag{2}$$

In eq.(2) the $+$ $(-)$ sign refers to the normal (inverted) three neutrino mass[3] hierarchy $m_1 \le m_2 < m_3$ $(m_1 < m_2 \le m_3)$. The inequality $M_{ee} > \sqrt{\Delta_A}$ is then automatically implied for the inverted hierarchy case. However, such considerations completely ignore another item of neutrino flavor oscillation information, namely the data [10] from the LSND experiment. These data can be explained by $\bar{\nu}_\mu \to \bar{\nu}_e$ (and $\nu_\mu \leftrightarrow \nu_e$) oscillations with a mass squared difference $\Delta_L = \mathcal{O}(1)$ eV^2 and a small mixing angle $\theta_L = \mathcal{O}(10^{-2})$. This note is addressed to a generalization of eq.(2) to include the LSND results.

A fourth light neutrino ν_s, which is not electroweak active and is hence called sterile, is needed along with ν_e, ν_μ and ν_τ to simultaneously explain the solar, atmospheric and LSND anomalies. Of course, it follows from the recent SNO [11] and Super-K [2] results that the final state to which the solar ν_e or the atmospheric ν_μ oscillates cannot be a purely sterile species. On the other hand, orthogonal linear combinations of ν_τ and ν_s are still allowable [12] final states in these oscillations.

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[3] The mass ordering $m_1 < m_2 < m_3$ with nonnegative m's has been chosen by definition.
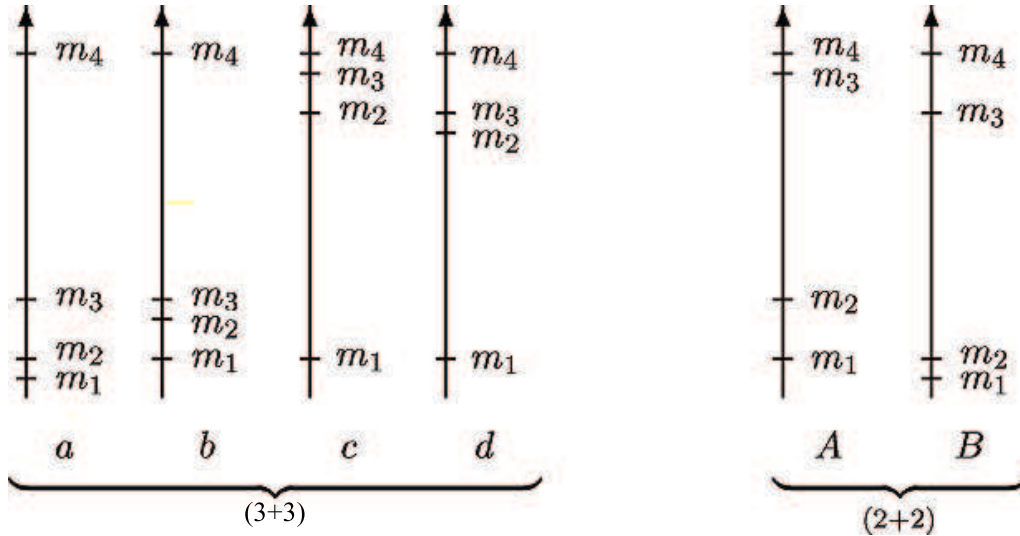
FIGURE 1. Six allowed of patterns of masses, grouped into two schemes, for the four neutrino scenario. Vertical separations symbolize mass squared differences pertinent to solar, atmospheric and LSND oscillations.

Comprehensive analyses [12-14] have recently been made of all current data on solar, atmospheric and LSND oscillations, together with constraints from other accelerator and reactor data, by considering the four neutrinos ν_e, ν_μ, ν_τ and ν_s. The conclusion is that the four neutrino picture is not excluded, though the required fits are not of particularly high quality.

Once one considers a four neutrino scenario, with the experimental input $\Delta_S <<$ $\Delta_A << \Delta_L$, the mass spectrum of the neutrinos becomes an issue of central importance. There are six possible four neutrino mass patterns, as shown in Fig. 1, that are a priori compatible [13–15] with the data[4] These group into two schemes called [16] **(3+1)** and **(2+2)**. The **(3+1)** scheme, consisting of four possibilities a, b, c, d (c.f. Fig.1) is characterized by three close-by neutrino masses separated from the fourth by a gap $\mathcal{O}\left(\sqrt{\Delta_L}\right)$. Here the sterile neutrino is only slightly mixed with the active ones. It is therefore a weak component in solar and atmospheric neutrino oscillations, but mainly provides a description of the LSND effect. In the **(2+2)** scheme, comprising two possibilities A and B (cf. Fig.1), there are two pairs of nearly degenerate states, separated by a gap $\mathcal{O}\left(\sqrt{\Delta_L}\right)$. In this pattern two orthogonal linear combinations of ν_s and ν_τ with comparable coefficients make up the final states to which the solar ν_e and the atmospheric ν_μ oscillate. Oscillation phenomenology alone cannot distinguish between different patterns within any of these schemes. However, a distinction does become possible when nuclear $\beta\beta0\nu$ decay is taken into account [17], assuming that the neutrinos are Majorana particles.

Turning towards mixing aspects, let us define the unitary transformation

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[4] Our ordering for the physical masses is always $m_1 < m_2 < m_3 < m_4$.

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$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} \quad . \tag{3}$$

The 4×4 matrix U can be written as a product of a 4×4 MNS type of a matrix V [18] times a Majorana phase matrix [19] diag. $(1 \; e^{i\alpha} \; e^{i\beta} \; e^{i\gamma})$. The Majorana phases α, β, γ make no contribution to neutrino oscillations but can affect $\beta\beta0\nu$ decay. The matrix V in general has [20] six angles and three phases. However, major simplifications occur when some experimental constraints are imposed. We demonstrate the way U is simplified in one case, namely for the pattern $(2+2)_B$. The form of U for other patterns can then be obtained by interchanging some columns.

In the $(2+2)_B$ pattern (c.f. Fig.1) ν_e resides largely in the state ν_1. Moreover, ν_1 and ν_2 are the oscillating pair for solar neutrinos and so, θ_{12}, the angle of rotation in the 1-2 plane, can be identified with the solar neutrino mixing angle θ_S. Any mixing between ν_e and the more massive states ν_3 and ν_4 is going to be strongly constrained by the Bugey experiment [21] which implies that

$$|V_{e3}|^2 + |V_{e4}|^2 < 10^{-2} \quad . \tag{4}$$

We shall interpret this result to mean that, for the mass pattern $(2+2)_{\mathbf{B}}$, it is a good approximation to let the elements V_{e3} and V_{e4} be zero and replace V_{e1} and V_{e2} by $\cos\theta_S$ and $\sin\theta_S$ respectively. Then the U matrix of eq.(3) becomes

$$U_{(2+2)B} \simeq \begin{pmatrix} \cos\theta_S & \sin\theta_S & 0 & 0 \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} 1 & & & \\ & e^{i\alpha} & & \\ & & e^{i\beta} & \\ & & & e^{i\gamma} \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} \cos\theta_S & \sin\theta_S e^{i\alpha} & 0 & 0 \\ U_{\mu 1} & U_{\mu 2} e^{i\alpha} & U_{\mu 3} e^{i\beta} & U_{\mu 4} e^{\gamma} \\ U_{\tau 1} & U_{\tau 2} e^{i\alpha} & U_{\tau 3} e^{i\beta} & U_{\tau 4} e^{i\gamma} \\ U_{s1} & U_{s2} e^{i\alpha} & U_{s3} e^{i\beta} & U_{s4} e^{i\gamma} \end{pmatrix} \quad .$$

Thus the Majorana mass effective for $\beta\beta0\nu$ decay is given for this pattern by

$$M_{ee} = |m_1 \cos^2\theta_S + m_2 e^{2i\alpha} \sin^2\theta_S| \quad . \tag{6}$$

On the other hand, the nonnegative physical masses for the above pattern can be defined, ignoring Δ_S and Δ_A in comparison with $\Delta_L \equiv m_3^2 - m_1^2$, as

$$m_1 \simeq m_2 \equiv m \quad , \tag{7}$$

$$m_3 \simeq m_4 = \sqrt{m^2 + \Delta_L} \quad . \tag{8}$$

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Thus we can rewrite eq.(6) as

$$M_{ee} \simeq | \cos^2 \theta_S + \sin^2 \theta_S e^{2i\alpha} | m \qquad (9)$$

and note that in eq.(9) m gets minimized (maximized) when the two terms are in phase (out of phase) at the value M_{ee} $(M_{ee}/|\cos 2\theta_S|)$. Since the sum of the four neutrino Majorana masses

$$\Sigma_\nu^{(4)} \simeq 2m + 2\sqrt{m^2 + \Delta_L} \qquad (10)$$

is a monotonic function of m, we obtain the lower and upper bounds on $\Sigma_\nu^{(4)}$

$$2(M_{ee} + \sqrt{M_{ee}^2 + \Delta_L}) < \Sigma_\nu^{(4)} < 2\left(\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} + \Delta_L} \right) \qquad (11)$$

for the pattern $\mathbf{(2+2)}_B$.

Similar upper and lower bounds can be derived on $\Sigma_\nu^{(4)}$ as monotonic functions of M_{ee} for the other five allowed mass patterns. The derivation becomes very simple once it is realized that one can go from one pattern to another simply by interchanging a set of mass-eigenstate indices. The latter is tantamount to interchanging the corresponding columns in the matrix U. In each case $\Sigma_\nu^{(4)}$ is a monotonic function of m and the upper bounds are obtained by putting $m_{max} = M_{ee}/|\cos 2\theta_S|$. However, for the lower bounds, while $m_{min} = M_{ee}$ for the patterns $\mathbf{(3+1)}_a$, $\mathbf{(3+1)}_b$ and $\mathbf{(2+2)}_B$, it is Max.$(M_{ee}, \sqrt{\Delta_L})$ for $\mathbf{(3+1)}_c$, $\mathbf{(3+1)}_d$ and $\mathbf{(2+2)}_A$. We list all these results in Table 1, including eq.(12) for the pattern $\mathbf{(2+2)}_B$ along with statements on the necessary index interchanges. An inspection of the entries in this table tells us right away that the patterns $\mathbf{(2+2)}_A$, $\mathbf{(3+1)}_c$, and $\mathbf{(3+1)}_d$ are consistent only if the following inequality is satisfied:

$$M_{ee}^2 > \Delta_L \cos^2 2\theta_s \quad . \qquad (12)$$

Currently, the best fits [13] of all oscillation data in the four neutrino scenario, as given in Table 2 of Ref. [14], require Δ_L to be 1.74 eV^2 in the four patterns of the $\mathbf{(3+1)}$ scheme and 0.87 eV^2 in the two patterns of the $\mathbf{(2+2)}$ scheme. The present experimental upper bound[5] on M_{ee} can be given [22,25] as 0.35α eV, where α is the uncertainty in our knowledge of the nuclear matrix element involved in $\beta\beta 0\nu$ decay. It has been inferred [4] from a survey of all existing calculations that $\alpha < 2.8$. It would therefore be safe to regard 0.98 eV as a conservative upper bound on M_{ee}. On substituting these numbers, we find that the range of variation in the $\Sigma_\nu^{(4)}$ as a function of M_{ee} is sizeable for each of the pattern $\mathbf{(3+1)}_a$, $\mathbf{(3+1)}_b$ and $\mathbf{(2+2)}_B$. However, this range is found to be extremely restricted for each of the

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[5] A nonzero value of M_{ee} in the range 0.05 eV $< M_{ee} <$ 0.84 eV has recently been claimed [23], but there has been a strong criticism [24] of this alleged observation.

TABLE 1. Bounds on $\Sigma_\nu^{(4)}$ in six mass patterns of the four neutrino scenario, neglecting Δ_S, Δ_A in comparison with Δ_L.

Pattern	Interchange of mass eigenstate indices with respect to $(\mathbf{2+2})_B$	Lower bound on $\Sigma_\nu^{(4)}$	Upper bound on $\Sigma_\nu^{(4)}$
$(\mathbf{2+2})_B$	Not necessary	$2(M_{ee} + \sqrt{M_{ee}^2 + \Delta_L})$	$2\left(\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} + \Delta_L}\right)$
$(\mathbf{2+2})_A$	$1 \leftrightarrow 3$ $2 \leftrightarrow 4$	$2Max.\left(\sqrt{\Delta_L},\ M_{ee} + \sqrt{M_{ee}^2 - \Delta_L}\right)$	$2\left(\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} - \Delta_L}\right)$
$(\mathbf{3+1})_a$	Not necessary	$3M_{ee} + \sqrt{M_{ee}^2 + \Delta_L}$	$3\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} + \Delta_L}$
$(\mathbf{3+1})_b$	$1 \to 2 \to 3 \to 1$	$2M_{ee} + \sqrt{M_{ee}^2 - \Delta_A}$ $+ \sqrt{M_{ee}^2 + \Delta_L}$	$2\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} - \Delta_A}$ $+ \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} + \Delta_L}$
$(\mathbf{3+1})_c$	$1 \leftrightarrow 4$	$Max.(2\sqrt{\Delta_L} + \sqrt{\Delta_L - \Delta_A},\ 2M_{ee}$ $+ \sqrt{M_{ee}^2 - \Delta_A} + \sqrt{M_{ee}^2 - \Delta_L})$	$2\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} - \Delta_A}$ $+ \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} - \Delta_L}$
$(\mathbf{3+1})_d$	$1 \to 2 \to 3 \to 4 \to 1$	$Max.(2\sqrt{\Delta_L} + \sqrt{\Delta_L + \Delta_A},\ 2M_{ee}$ $+ \sqrt{M_{ee}^2 + \Delta_A} + \sqrt{M_{ee}^2 - \Delta_L})$	$2\frac{M_{ee}}{|\cos 2\theta_S|} + \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} + \Delta_A}$ $+ \sqrt{\frac{M_{ee}^2}{\cos^2 2\theta_S} - \Delta_L}$

remaining three patterns; therefore these three patterns, namely $(\mathbf{3+1})_c$, $(\mathbf{3+1})_d$ and $(\mathbf{2+2})_A$ are disfavored.

We can comment on the effective mass of the electron neutrino M_e which can be measured in tritium β-decay. The latter is given in our notation by [6]

$$M_e = \sqrt{\sum_i m_i^2 |U_{ei}|^2}\ . \tag{13}$$

There is an interesting inequality between M_{ee} and M_e which holds in all six cases

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[6] The validity of this expression for the effective mass extracted from endpoint measurements in tritium β-decay is discussed in Ref. [4].

as well as the three flavor case [26]. It can be expressed in two equivalent ways;

$$M_{ee} < M_e < M_{ee}/|\cos 2\theta_S| \quad , \qquad\qquad (14a)$$

$$M_e|\cos 2\theta_S| < M_{ee} < M_e \quad . \qquad\qquad (14b)$$

The content of eqs.(14) is nontrivial since recent solar neutrino data suggest [1,13] that the concerned flavor mixing is not maximal i.e. $|\cos 2\theta_S| > 0$. The current experimental [27] upper bound on M_e is 2.2 eV. Which of the above inequalities becomes interesting will depend on whether a nonzero value of M_{ee} or M_e is discovered first.
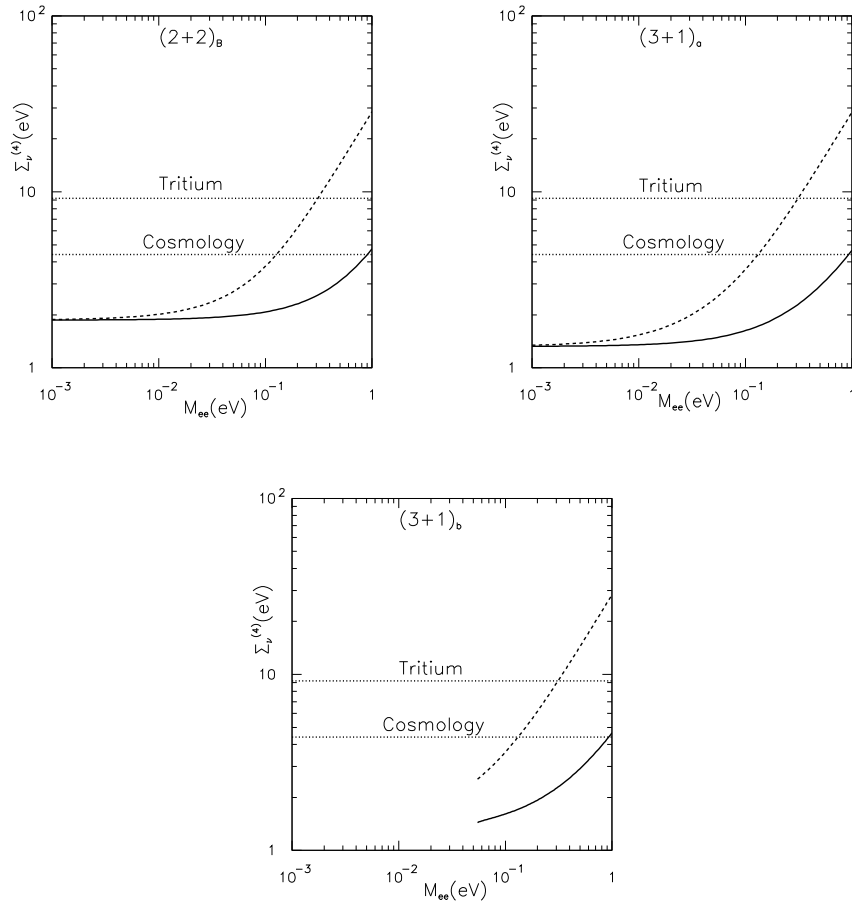

FIGURE 2. Plots of the upper (dashed) and lower (bold) bounds on $\Sigma_\nu^{(4)}$ as functions of M_{ee} for the mass patterns $(\mathbf{2+2})_B$, $(\mathbf{3+1})_a$ and $(\mathbf{3+1})_b$. Horizontal dotted lines show the upper bounds from tritium β−decay and cosmology.

A quantity of cosmological interest is the sum of neutrino masses contributing to the hot dark matter in the Universe. Galactic surveys and cosmic microwave background observations bound the latter from above by [28] by 4.2 eV. Big Bang

nucleosythesis considerations dictate that the density of a purely sterile neutrino species in the Universe is less[7] than that of an active one. But we are allowing substantial mixing between sterile and active neutrino types. As a result, the active density cannot significantly exceed the sterile density. Under the circumstances, it is not unreasonable to treat 4.2 eV as a cosmological upper bound on $\Sigma_\nu^{(4)}$.

We can make more precise estimates of the $\beta\beta 0\nu$ bounds on $\Sigma_\nu^{(4)}$. For the LMA solution of solar neutrino oscillations in the four neutrino scenario, we can take [13] $\sin^2 2\theta_S < 0.98$. Feeding in the earlier-mentioned best-fit values [13] of Δ_L, we plot the upper and lower bounds on $\Sigma_\nu^{(4)}$ as a function of M_{ee} in Fig. 2 for the mass patterns $(2+2)_B$, $(3+1)_a$, $(3+1)_c$. For comparison, the upper limits from cosmology and tritium β-decay are also shown. A reduction in the upper limit of the allowed range of values for $\sin^2 2\theta_S$ in the LMA solution will tighten the $\beta\beta 0\nu$ bounds on $\Sigma_\nu^{(4)}$, making them more competitive with the tritium and cosmology limits, while the next generation of $\beta\beta 0\nu$ experiments [25] are expected to lower the upper bound on M_{ee}. On the other hand, a significant improvement of the cosmological bound will enable a further discrimination among the surviving four neutrino mass patterns.

ACKNOWLEDGEMENT

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[7] This presupposes that the active-sterile mixing is small, specifically the effective $\Delta m^2 \sin^4\theta < 5\times 10^{-6}\ eV^2$, cf. Foot and Volkas [29] and references therein. Since this inequality is violated in the four neutrino oscillation fits [13–15], we can take the sterile density as comparable to active ones.

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