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ROLE OF THE SHORT DISTANCE EFFECT IN NON-LEPTONIC RADIATIVE DECAYS

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A B S T R A C T

Short distance contributions to non-leptonic radiative decays arise from the presence of the gradient of the antisymmetric tensor current octet in the Wilson expansion. They can cause large SU(3) violation, preliminary evidence of which already exists for the reaction $\Sigma^+ \to p\gamma$. Relations among various non-leptonic decay parameters are obtained taking proper account of such terms and some decisive tests are proposed for their existence.

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Non-leptonic radiative weak decays are unique in one respect. In the lowest orders of electricity and of weak forces *) they involve time-ordered products of three currents (two weak and one electromagnetic) between hadronic states. Consequently, the contributions to these amplitudes from weak structure effects in regions where the currents act at short distances in configuration space cannot be ignored. This was demonstrated recently [1] within Wilson's [2] scheme of broken scale invariance and operator product expansions at short distances. It was shown that the effective Lagrangian density for a transition of the type $H_1 \to H_2 \gamma$, with $H_{1,2}$ being hadronic states, can be written as

$$\mathcal{L}(0) = \frac{G}{\sqrt{2}} e \cos\theta \sin\theta \, m_H \mathcal{A}_\lambda \left[s_1 g^{\lambda\alpha} g^{\beta\gamma} + s_2 \varepsilon^{\lambda\gamma\alpha\beta} \right] \partial_\gamma T^{6+i7}_{\alpha\beta} \tag{1}$$

$$+ \text{ h.c. } + \text{ non-short part }^{**)}$$

In Eq. (1), G, ℓ, θ and $\mathcal{A}_\lambda$ have the usual meaning, m_H is a typical hadronic mass $\simeq 1$ GeV, $T^i_{\alpha\beta}$ is the antisymmetric tensor current which has the form $\bar{q}\sigma_{\alpha\beta}(\lambda^i/2)q$ in the free quark model and $s_{1,2}$ are unknown dimensionless constants ***) measuring the strength of the weak structure effects.

Diagrammatically, say in an intermediate vector boson picture, the short distance term arises from the high-momentum contribution to the loop integration in Fig. 1, while the non-short part originates from transitions of the type shown in Fig. 2. The important aspect of the former is that, as explained in Ref. 1), it provides a mechanism for significant SU(3) breaking effects proportional to the matrix elements of $T^i_{\alpha\beta}$ between specific hadronic states. The interesting feature of the latter is that the effective Lagrangian density $\mathcal{L}_W$ involved in the weak vertex is that of an ordinary non-leptonic transition which - within the scheme of Ref. 1) - is

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*) Throughout we stick to the current-current theory of weak interactions.

**) For recent discussions of non-leptonic radiative decays without the consideration of short distance terms, see Refs. 3) and 4).

***) $s_{1,2}$ are expected to be proportional to $\ell n\, \Lambda/m_H$ where Λ is a mass typifying the structure of weak interactions, see Ref. 1).

automatically octet-dominated *). In this letter we shall derive relations among different non-leptonic radiative decay parameters utilizing these properties. We shall use the behaviour under SU(3) symmetry of operators and states **) wherever possible. If λ is a SU(3) breaking parameter then in $< H_2|T^i_{\alpha\beta}|H_1 >$ and $< H_2|\mathcal{L}_w|H_1 >$ we shall ignore broken symmetric terms $O(\lambda)$ in comparison with SU(3) invariant terms $O(1)$; however - if terms $O(\lambda^{-1})$ are present - we shall be careful not to apply symmetry arguments. In this way we obtain two new predictions for two-body hyperon decay parameters. For kaon decays we do not have any definite prediction, but we can make qualitative statements which - if drastically violated - will have experimental consequences. A brief account of our arguments and results follows.

For two-body radiative hyperon decays $B_1 \to B_2\gamma$, B standing for a strongly stable baryon, the invariant amplitude can be written as [3),4)] :

$$\mathcal{M} = e\frac{G}{\sqrt{2}}\cos\theta\sin\theta\,\varepsilon^\mu k^\nu \bar{u}_2\,\sigma_{\mu\nu}(a+b\gamma_5)u_1\,, \tag{2}$$

where k and ε are the momentum and the polarization of the photon respectively. We shall denote the short-distance contributions to a, b by a^s, b^s and the non-short ones by a^{ns}, b^{ns} respectively. It will often be useful to split up the latter as $(a,b)^{ns} = (a,b)^{br.} + (a,b)^{str.}$. The superscript br. refers to the "bremsstrahlung" contribution corresponding to the case where the intermediate state in Fig. 2 is a single hyperon. The superscript str. applies to the "structure" contribution coming from all other intermediate states in the same. We shall use subscripts corresponding to the final hyperons produced in these decays.

Our first remark is that in the first right-hand side term of Eq. (1) charge-conjugation invariance forces only $T^6_{\alpha\beta}$ to contribute to parity-conserving decays and only $T^7_{\alpha\beta}$ to parity-violating decays. Thus one can write

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*) The $\Delta I = \frac{3}{2}$ part is an exotic operator, carries an anomalously high dimension and is suppressed. See Ref. 1).

**) In the short distance contribution, the SU(3) breaking is contained in $s_{1,2}$ and we are free to use SU(3) symmetry on $< H_2|T^{6+i7}_{\alpha\beta}|H_1 >$.

$$(2\pi)^3\sqrt{\frac{E_j E_k}{m_j m_k}}\,\langle j\,|\,T^i_{\alpha\beta}\,|\,k\rangle = \bar{u}_j\,\sigma_{\alpha\beta}\,u_k\,(d_{ijk}D - i f_{ijk}F) \tag{3}$$

in SU(3) notation and obtain $a^s/s_1 = 2b^s/s_2$ for the various observable decays in terms of F and D. These are displayed in the Table. From this Table, we take the following relations which will be useful later

$$2a^s_n + a^s_\Lambda = \sqrt{3}\,a^s_{\Sigma^0} \tag{4}$$

and

$$\sqrt{2}\,b^s_{\Sigma^0} = -\,b^s_{\Sigma^-}\,, \tag{5a}$$

$$\sqrt{6}\,b^s_\Lambda = b^s_{\Sigma^-} - 2b^s_p\,, \tag{5b}$$

$$\sqrt{6}\,b^s_n = b^s_p - 2b^s_{\Sigma^-}\,, \tag{5c}$$

For charged hyperon decays we know [4] that

$$a^{br.}_{p,\Sigma^-} = \cdots \frac{\dfrac{\mu_{p,\Sigma^-}}{m_{p,\Sigma^-}} - \dfrac{\mu_{\Sigma^+,\Xi^-}}{m_{\Sigma^+,\Xi^-}}}{m_{\Sigma^+,\Xi^-} - m_{p,\Sigma^-}} = O(1)$$

and

$$b^{br.}_{p,\Sigma^-} = \cdots \frac{\dfrac{\mu_{p,\Sigma^-}}{m_{p,\Sigma^-}} - \dfrac{\mu_{\Sigma^+,\Xi^-}}{m_{\Sigma^+,\Xi^-}}}{m_{\Sigma^+,\Xi^-} + m_{p,\Sigma^-}} = O(\lambda).$$

In fact it is known on general grounds [3),5)] that

$$a^{ns}_{p,\Sigma^-} = O(1), \tag{6}$$

$$b^{ns}_{p,\Sigma^-} = O(\lambda). \tag{7}$$

Studying Eqs. (4)-(7), the only statement that we can make at this point is that if the preliminary experimental evidence [6] that $b_p = O(1)$ is confirmed, this would imply that b_p^s is $O(1)$, i.e., that s_2 is acting as an enhancement factor.

Coming now to neutral decays, we observe that for the "bremsstrahlung" contributions it was shown in Ref. 3) that

$$a^{br.}_{n,\Lambda,\Sigma^0} = O(\lambda^{-1}), \tag{8}$$

$$b^{br.}_{n,\Lambda,\Sigma^0} = O(\lambda). \tag{9}$$

Equation (8) is true because the denominator in the left-hand side is proportional to the difference of baryon masses whereas the numerator is $O(1)$ so long as there is no special cancellation between the f and d parameters *) of the weak mass. Equation (9) follows because the denominator now is proportional to the sum of the baryon masses whereas the numerator $\propto < B_2|\mathcal{L}_w^{p.v.}|B_1 > = O(\lambda)$. Equations (8) and (9) imply that we cannot (can) apply SU(3) arguments to the corresponding quantities $a^{ns}(b^{ns})$. Now it was further shown in Ref. 3) that the "structure" contributions to the parameter a for neutral decays are $O(1)$ and satisfy the relation :

$$\sqrt{3}\, a^{str.}_{\Sigma^0} = 2 a^{str.}_n + a^{str.}_\Lambda . \tag{10}$$

Combining Eqs. (4) and (10), we can write :

$$2(a - a^{br.})_n + (a - a^{br.})_\Lambda = \sqrt{3}\,(a - a^{br.})_{\Sigma^0}. \tag{11}$$

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*) These can be defined by $(2\pi)^3 \sqrt{(E_i E_j / m_i m_j)} < B_i|H_W^{p.c.}|B_j > = \bar{u}_i u_j (d_{6ij} d - i f_{6ij} f)$.

Equation (11) was used in Ref. 3) to suggest a method of extracting the "bremsstrahlung" contributions from the data so that the f/d ratio for the weak mass could be determined. Since the form of Eq. (11) is unchanged by the presence of a short-distance term, this analysis still goes through. The "structure" contributions to the parity-violating parameters b are O(1) here. However, we can use not only SU(3) arguments but also the octet-dominance of $\mathcal{L}_W$ in the non-short contribution to obtain additional relations. Hence, following Lo [7], we have

$$b_n^{ns} = - b_\Lambda^{ns} = - \frac{b_{\Sigma^0}^{ns}}{\sqrt{3}} \equiv \beta. \tag{12}$$

Since $b = b^s + b^{ns}$, Eqs. (5) and (12) imply :

$$\begin{aligned} b_{\Sigma^0} &= - \frac{b_{\Sigma^-}}{\sqrt{2}} - \sqrt{3}\,\beta, \\ b_\Lambda &= \frac{1}{\sqrt{6}}\left(b_{\Sigma^-} - 2 b_p \right) - \beta, \\ b_n &= \frac{1}{\sqrt{6}}\left(b_p - 2 b_{\Sigma^-} \right) + \beta. \end{aligned} \tag{13}$$

Since β is an unknown parameter, Eqs. (13) contain two independent relations which can be experimentally tested. We shall write these relations as

$$b_\Lambda + b_n = - \frac{1}{\sqrt{6}}\left(b_{\Sigma^-} + b_p \right)$$

and

$$b_{\Sigma^0} - \sqrt{3}\, b_\Lambda = \sqrt{2}\left(b_p - b_{\Sigma^-} \right). \tag{14}$$

For a radiative non-leptonic K decay of the type $K_a \to \pi_b(p)\,\pi_c(q)\,\gamma(\varepsilon, k)$ the invariant amplitude can be written as

$$\mathcal{M} = \frac{G}{\sqrt{2}}\, e \cos\theta \sin\theta \left[B_{abc} + E_{abc}\, \overleftrightarrow{\varepsilon_\mu k_\nu}\, p^\mu q^\nu + i M_{abc}\, \varepsilon_{\mu\nu\alpha\beta}\, \overleftrightarrow{\varepsilon^\mu k^\nu}\, p^\alpha q^\beta \right] \tag{15}$$

In Eq. (15) the subscript a can stand for ±, L or S and the subscripts b, c for ± and 0 accordingly. E(M) stands for the parity-violating (parity-conserving) electric (magnetic) term, whereas B is the bremsstrahlung contribution given by (Q = charge) :

$$\mathcal{B}_{abc} = \left(\frac{p\cdot\varepsilon}{p\cdot k}Q_b + \frac{q\cdot\varepsilon}{q\cdot k}Q_c - \frac{(p+q)\cdot\varepsilon}{(p+q)\cdot k}Q_a\right)\mathcal{M}(K_a \to \pi_b \pi_c). \tag{16}$$

It is then clear that, because of CP invariance, we have :

$$\mathcal{B}_{L+-} = E_{L+-} = M_{S+-} = 0. \tag{17}$$

Moreover, we know that $K^{\pm} \to \pi^0 \pi^{\pm}$ are $\Delta I = \frac{3}{2}$ transitions whose amplitudes are down by a factor of 20 from a typical $\Delta I = \frac{1}{2}$ amplitude and that $\mathcal{M}(K_S \to \pi^+\pi^-)$ vanishes [8] in exact SU(3). Hence we have :

$$\mathcal{B}_{\pm 0 \pm} = O(\Delta I = 3/2) \simeq O(1/20),$$

$$\mathcal{B}_{S+-} = O(\lambda) \simeq O(1/10). \tag{18}$$

We now proceed to split $E_{\pm 0\pm}$, E_{S+-}, $M_{\pm 0 \pm}$, M_{L+-} (i.e., those non-bremsstrahlung terms which are non-vanishing) into short and non-short parts with superscripts s and ns respectively. For the short part we know once again from C invariance that only $T^7_{\alpha\beta}$ contributes to E and $T^6_{\alpha\beta}$ to M. Since the contributing tensor current carries $\Delta I = \frac{1}{2}$, we immediately have :

$$E^s_{S+-} = E^s_{\pm 0 \pm} \tag{19a}$$

$$M^s_{L+-} = M^s_{\pm 0 \pm} \tag{19b}$$

Regarding the non-short part, we can use the octet-dominance of $\mathcal{L}_W$ (Fig. 2). The requirement of C invariance of the effective Lagrangian density imposes additional restrictions but it turns out that for parity-conserving decays there are still too many independent amplitudes and we cannot make any statement. However, for parity-violating decays, there is only one independent amplitude. If $F^{\mu\nu} = \overleftrightarrow{\partial^{\mu}} A^{\nu}$, P = the self-conjugate pseudoscalar meson octet of SU(3), Q = the charge matrix $(^{2/3}{-1/3}_{-1/3})$ and

$H = \lambda_6$ stands for the spurion corresponding to $\mathcal{L}_w$, we can take the effective Lagrangian density for the parity-violating non-short part to be as follows :

$$\mathcal{L}^{ns}_{pv}(0) = g F^{\mu\nu} \mathrm{Tr} \{ \partial_\mu P \partial_\nu P Q P H - \partial_\mu M \partial_\nu P H P Q \}$$

$$= g F^{\mu\nu} \{ -\tfrac{2}{3} \partial_\mu \pi^+ \partial_\nu K^0 \pi^- + \tfrac{\sqrt{2}}{3} \partial_\mu \pi^0 \partial_\nu K^+ \pi^- \quad (20)$$

$$+ \cdots + h.c. \} ,$$

where g is a suitable constant. Equation (20) immediately leads to the result

$$E^{ns}_{s+-} = -2 E^{ns}_{\pm 0 \pm} . \quad (21)$$

Considering Eqs. (19a) and (21), we see that since $E = E^s + E^{ns}$ for each mode, we do not have a definite prediction. However, the important point in this connection is that

$$\frac{E^s_{s+-}}{E^{ns}_{s+-}} = -\frac{1}{2} \frac{E^s_{\pm 0 \pm}}{E^{ns}_{\pm 0 \pm}} . \quad (22)$$

The factor $(-\frac{1}{2})$ in the right-hand side of Eq. (22) has the following implication. Although E, when non-vanishing, is in general expected to be $O(1)$, it may in fact be much smaller because of accidental near-cancellation between the short and non-short parts. However, because of Eq. (22), such a near-cancellation is not possible in both the $K_s \to \pi^+\pi^-\gamma$ and $K^\pm \to \pi^0\pi^\pm\gamma$ modes. Experimentally, on account of Eq. (18), the separation of the non-bremsstrahlung parts in these modes and the extraction of the electric term in $K^\pm \to \pi^0\pi^\pm\gamma$ may not turn out to be too difficult.

At present there are only crude indications *) of the magnitude of E_{+0+} and merely a very weak upper limit **) on E_{s+-}. However, we understand ***) that better results will be forthcoming in the near future.

Finally, it is hoped that the ideas presented here will induce experimenters to look harder at radiative non-leptonic decays of strange particles. In particular, if the SU(3) breaking parity-violating parameter b in the decay $\Sigma^+ \to p\gamma$ does turn out to be large, Eq. (14) can be used to test Wilson's [2] ideas of broken scale invariance and operator-product expansions at short distances assuming the current-current theory of non-leptonic weak interactions to be correct.

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*) If we take $\mathcal{M}(K^+ \to \pi^0 \pi^+) = (G/\sqrt{2})\cos\theta\ \sin\theta\ g_+\ m_K^4$, then three experimental groups [9-11] give the following results respectively : $(E_{+0+}/g_+) = (-103^{+69}_{-26})$, (86^{+52}_{-43}) and < 33. [Note that their parameter $\gamma = E/g_+(m_\pi/m_K)^4$.]

**) Bellotti et al. [12] write $\mathcal{M}(K_s \to \pi^+\pi^-\gamma)$ as $\mathcal{M}_{br.} + \mathcal{M}_E$ and claim (with $|\vec{k}| > 50$ MeV) that $|\mathcal{M}_E|^2/|\mathcal{M}_{br.}|^2 \leq 0.12$. For comparison, consider the ratio $\mathcal{M}(K_L \to \pi^+\pi^-\gamma)/\mathcal{M}(K_s \to \pi^+\pi^-\gamma,\ E_\gamma > 50\ \text{MeV}) \leq$ $\leq 3\times10^{-3}$ which measures the strength of the magnetic term $\mathcal{M}(K_L \to \pi^+\pi^-\gamma) = \mathcal{M}_M$, relative to the bremsstrahlung part $\mathcal{M}_{br.}$ in $K_s \to \pi^+\pi^-\gamma$. However, even this strong upper limit merely implies $M_{L+-}/g_{s+-} \leq 1.4$ where $\mathcal{M}(K_s \to \pi^+\pi^-) = (G/\sqrt{2})\cos\theta\ \sin\theta\ g_{s+-}\ m_K^4$.

***) G.L. Salmon - Private communication.

DECAY	$a^s/s_1 = 2b^s/s_2$
$\Sigma^+ \to p\gamma$	$\frac{D - F}{2}$
$\Xi^- \to \Sigma^-\gamma$	$\frac{D + F}{2}$
$\Xi^0 \to \Sigma^0\gamma$	$-\frac{D + F}{2\sqrt{2}}$
$\Xi^0 \to \Lambda\gamma$	$(1/2\sqrt{6})(3F - D)$
$\Lambda \to n\gamma$	$(-1/2\sqrt{6})(3F + D)$

Tabulation of short-distance parameters in hyperon decays.

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FIGURE CAPTIONS

Figure 1 Diagrams giving rise to short-distance effects.

Figure 2 Diagrams for the non-short part.

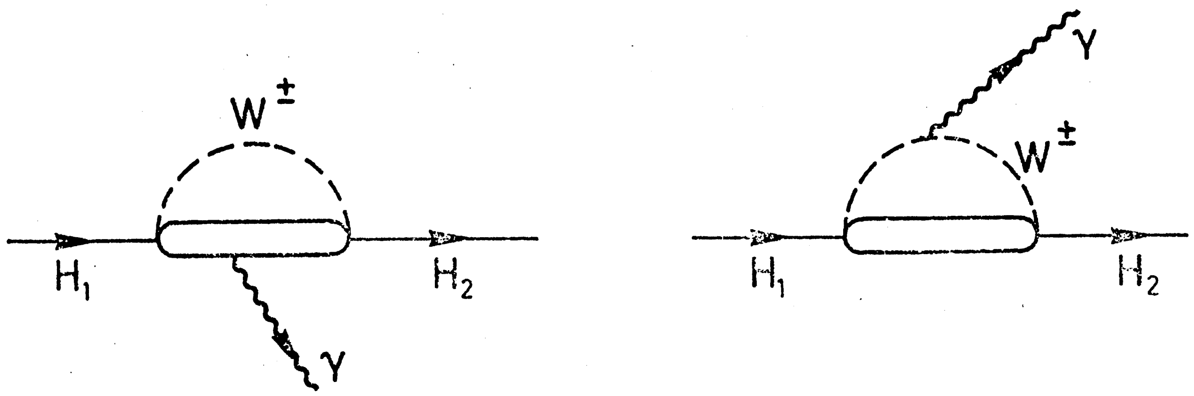

FIG.1

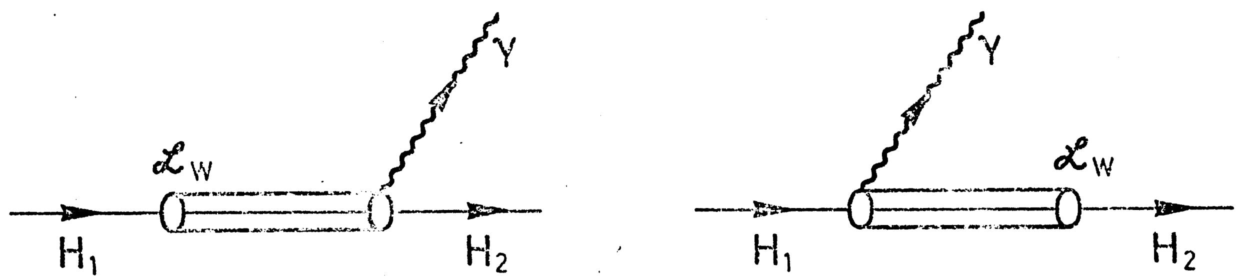

FIG.2